Question 1

1.1.

Let

*Exponential form*

*Alternate exponential form*

where ;

*principal of*

where ;

where ;

Therefore, we have an infinite number of values, all differing by integral multiples of 2πi.

It’s exponential form:

*also written as*

1.2.1.



The region is , which is the region above and including the line . The region is closed since it contains all its boundary points.

1.2.2.

Also,

Let where , then

The region is , which is the region below and not including the line . The region is closed since it contains all its boundary points.



Question 2

2.1.

Suppose that . The complex-valued function is differentiable at any point z in the complex plane:

The real part and the imaginary part are

And their partial derivatives are

All the partial derivatives are polynomial and thus continuous. Therefore, the function is differentiable wherever the Cauchy-Rieman equations are satisfied

If the Cauchy–Riemann equations are to hold at a point (x, y), it follows that and , or that .

We have that:

holds

Thus, holds:

or

Therefore, the Cauchy-Rieman equationsare satisfied at the lines and .

2.2.

In order to be analytic, a function needs the partial derivatives of the real part and the imaginary part should:

- satisfy Cauchy-Rieman equations and , and

- be continuous

There is no neighbourhood of any point throughout which is analytic as Cauchy-Rieman does not hold for an open set. Every neighbourhood of any point will have points which are not on the lines and . Therefore, is nowhere analytic.

Question 3

We can solve Laplace’s equation in any domain simply by taking the real part of any analytic function in that domain. Suppose that . The complex-valued function is differentiable at any point in the complex plane and the Cauchy-Rieman equations hold at any point ,

We have that:

Where the Cauchy-Rieman equations hold, and

And also

Question 4

Question 5

When , the image of the infinite strip is the upper half of the plane.

Let , where

Under the transformation ,

we have:

Under transformation where we have:

Under transformation where we have:

Which is a semi-circle in the upper half of the plane. The final sketch will not include points inside the circle.

So if and , then:

where

Question 6

6.1.

Example:



1

*is the singularity, inside of*

*Evaluate the function at*

6.2.

Example:



Singularities:

*is the singularity, inside of*

*is the singularity, outside of*

*Irrelevant: we only want to look at analytic/inside*

**Quotient rule**

*Evaluate the function at*