CS 289 Project S - Early Writeup

Abstract

Ancient astronomers measured movements of celestial bodies in the sky. Particularly, ancient Chinese astronomer Liu Hong kept a table of moon movements every day to make predictions. In this project, we figure out what he measured, collect these measurements using data in the most recent year and give basic descriptions. The output data contains the exact variables as in Liu Hong tables that can be useful for final projects. We also give instructions on unit transformation between Liu Hong's and modern units we used today to understand the difference in magnitude of data.

1 Introduction

The motion of the Moon around the Earth is very complicated due to the perturbation of the sun, the earth and other celestial bodies[1]. Liu Hong (A.D. 135-210), an ancient Chinese mathematician and astronomer, is the first Chinese astronomer who has provided us with the earliest detailed and complete theory of the Sun and Moon[2].In Liu Hong's book *Qian Xiang Li*, he affirmed the previous understanding of the non-uniformity of the Moon's motion. On the basis of the re-measurement, Liu Hong first determined that the distance between the two passes of the Moon was 27.5534 days, and created a new *Moon Departure Table*, a numerical table of the difference between the actual and average degrees of the moon every other day after the moon passes through the perigee, to correct the uneven motion of the Moon[3,4].

In this dataset, we used some modern datasets from NASA and processed them into the format used by Liu Hong. Our dataset is catered for further exploring the ideological source of Liu Hong's study, and constructing a mathematical prediction model with modern Machine Learning techniques.

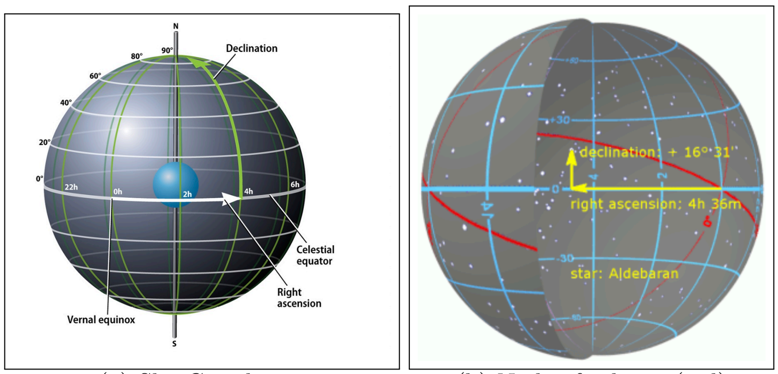
The remainder of this report is organized as follows. Section 2 introduces the methodology, which includes the description of data sources, important concepts and data processing methods. Section 3 describes the results from data processing and presents the preliminary statistic analysis. Section 4 discusses the contribution and limitations of the current study and potential future work.

2 Method

In this section, the raw dataset we adopted in this project is introduced and more details are discussed with comparison to Liu Hong's data. Necessary concepts and definitions are also provided. In the end, the data processing is presented.

Preprint. Work in progress.

Figure 1: Map of chosen obseriving sites



(a) Sky Coordinate (b) Node of ecliptic (red)

Figure 2: Equatorial coordinate system

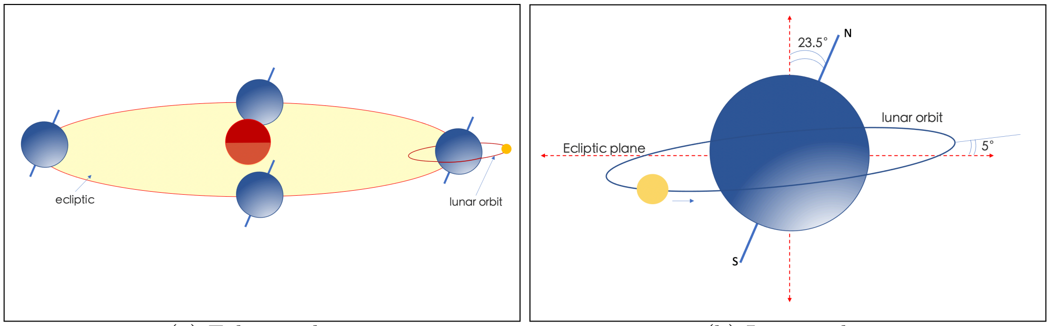
2.1 Raw data and comparision with Liu Hong's data

We first collected the moon ephemerides data from NASA HORIZONS System [5]. The main entries of concern include date, right ascension, and declination of the Moon with respect to the observing site in the International Celestial Reference Frame. The dates we used are from 2019-Oct-21 to 2020-Oct-20. In order to cover cities of different latitudes and longitudes, we choose as the observing sites Adelaide, Australia ($138°35'00.0''E$, $34°55'00.0''S$); Beijing, China ($116°22'59.9''E$, $39°54'59.8''N$); Addis Ababa, Ethiopia ($38°43'59.9''E$, $9°00'00.0''N$); London, England ($0°07'00.1''W$, $51°30'00.0''N$); Brasilia, Brazil ($47°55'00.1''W$, $15°52'00.1''S$); and Berkeley, CA ($122°16'15.6''W$, $37°52'09.8''N$), whose locations are illustrated in Figure 1.

The modern dataset adopted equatorial coordinate system including right ascension (RA) and declination (DEC) which correspond to longitude and latitude on Earth as shown in Figure 2. In Liu Hong's work, all motion of sun and moon are referred to the ecliptic as shown in Figure 3. So in his measurement, due to the difference between ecliptic plane and lunar orbit, the orbit of moon is always changing and the intersection between ecliptic plane and lunar orbit changes 0.054° every unit of time (day)[6]. In summary, Liu Hong collected four measurements as follows: 1) the daily average speed of moon movement, 2) the change between this speed in day t and the next day (t+1), 3) rate of lessening or increase (i.e. the difference between each day's speed and the mean speed in one cycle), and 4) the accumulated excess or deficit. These are variables we intend to calcualte and contain in our output dataset.

2.2 Relevant concepts and definitions

Units of Time The basic unit of time measurement throughout the system is the day, which begins at midnight and it is presumed that each day is of equal length.



(a) Ecliptic plane (b) Lunar orbit

Figure 3: Illustration on ancient coordinate system

	YYYY-Mon-Day	RA	DEC	dRA	dDEC	Parts	Rate	RLI	CRLI
0	2019-Oct-21	111.720292	22.360361	14.886875	−1.537056	14.966014	0.130855	1.231552	0.000000
1	2019-Oct-22	126.607167	20.823306	14.817708	−2.889806	15.096869	0.037379	1.362407	1.231552
2	2019-Oct-23	141.424875	17.933500	14.570583	−4.091889	15.134248	−0.006281	1.399785	2.593959
3	2019-Oct-24	155.995458	13.841611	14.264458	−5.037917	15.127967	−0.021997	1.393504	3.993744
4	2019-Oct-25	170.259917	8.803694	14.010167	−5.648500	15.105970	−0.034585	1.371507	5.387249

Figure 4: datahead

Units of Measurement In ancient China, astronomers used a set of units that have the same names (i.e. *du*, *fen*) but measure different values throughout history. For example, 1 *du* might equal 10, 12 or 32 *fen* depending on the dynasty. In Liu Hong's study, he used *du* and *fen* with relationship as 1 *du* = 19 *fen*. Although the conversion relation is different, it is certain throughout ancient Chinese history, that 1 circle is divided into 365.25 *du*. This is slightly different from modern Western units where 1 circle is divided into 360 degrees. Thus we can build the connection between Liu Hong's units and modern units in astronomy. For example, the first value of parts of lunar motion in Liu Hong's table is 276 *fen* = 14.53 *du* = 0.040 of a circle = 14.32 *arc degrees* = 859.07 *arc minutes* = 51544.48 *arc seconds*.

2.3 Data processing

The following description and formulas summarize the calculation procedure and mathematical justifications for the design of the output dataset:

1) Unify the unit of right ascension and declination to be *arc degrees*;

2) Calculate Parts of Lunar Daily Motion, which is in (1/19) *du* in Liu Hong's table and in *arc degrees* in our dataset. It is calculated as $f(t) = \sqrt{(RA_t - RA_{t-1})^2 + (DEC_t - DEC_{t-1})^2}$;

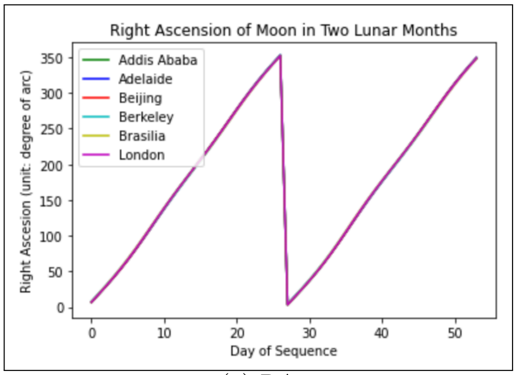
3) Calculate the Rate of Lessening or Increase (RLI), which gives the difference between Parts of Lunar Motion and the mean daily motion of 254/19 *du* in Liu Hong's table and in *arc degrees* in our dataset. It is calculated as $RLI_t = f(t) - \overline{f(t)}$;

4) Calculate the Accumulated Excess or Deficit (AED), which is the accumulated sum of the RLI. It is calculated as $AED_t = \sum_{i=2}^{n}[f(t-1) - \overline{f(t)}]$.

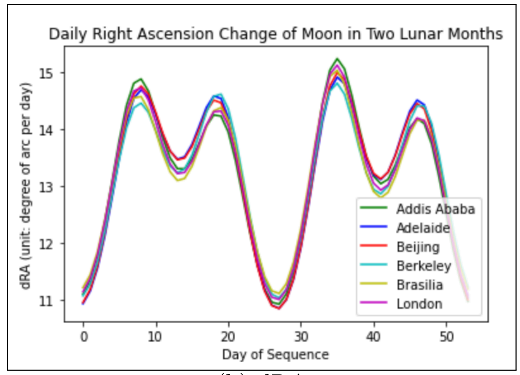
3 Results

3.1 Output dataset

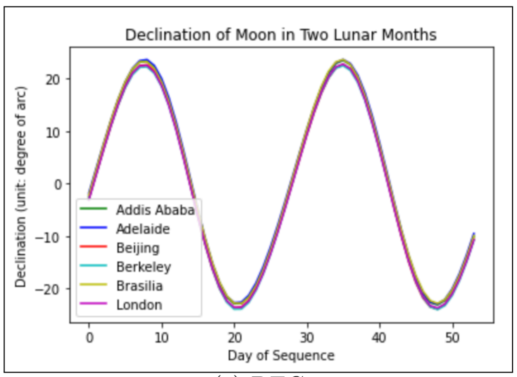
There are six output datasets in total, one for each observation site mentioned above. The output dataset has the following form as in Figure 4: Column 1 is the observation date. Column 2 and 3 are RA and DEC in *arc degrees*, which shows the location of moon using Equatorial coordinate system. Column 4 and 5 are the daily change of RA and DEC.



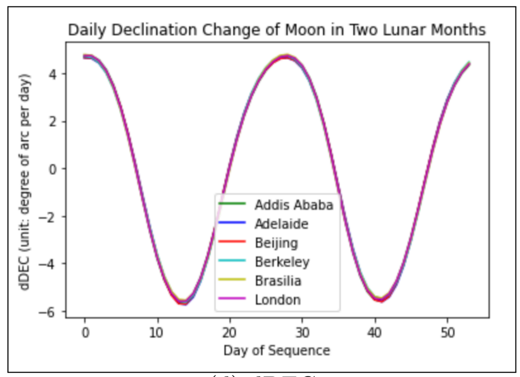
(a) RA



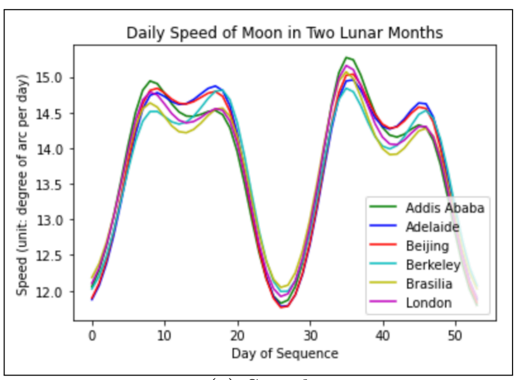
(b) dRA



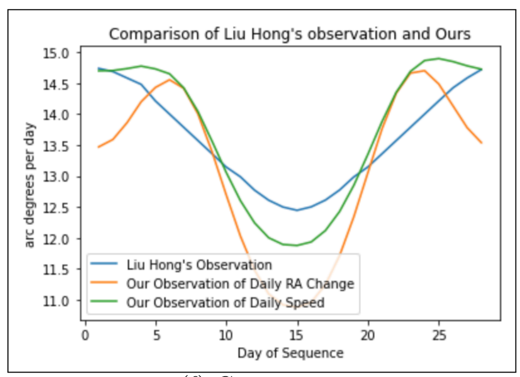
(c) DEC



(d) dDEC



(e) Speed



(f) Comparison

Figure 5: Preliminary analysis

Column 6 is the daily speed of lunar motion, with variable name 'Parts' in accordance with Liu Hong's naming. Column 7, 8 and 9 correspond to Rate, Rate of Lessening or Increase and Accumulated Excess or Deficit in Liu Hong's table. Column 8 is the difference between daily speed of lunar motion and the average value 13.7345.

3.2 Preliminary statistic analysis

As Figure 5 shows, we plotted the relationship between day of sequence and (a) RA; (b) daily RA change, denoted as dRA; (c) DEC; (d) daily DEC change, denoted as dDEC; and (e) speed, of lunar motion in two full lunar months, from 2019/11/9 to 2020/1/1.

From (a), it can be observed that the moon takes around 27 days to make a complete orbit around Earth, which corresponds to the fact that the sidereal period of Moon is 27.3

days[7]. Also, it seems that the Moon moves uniformly in RA since the trajectory looks like a straight line. However, (b) shows that it is not strict uniform motion in fact. The speed in the direction of RA varies from around 11 to 15 *arc degrees per day*, with an average of 13.167, which is pretty close to the constant that Liu Hong used as lunar mean daily motion, 254 *fen* = 13.176 *arc degrees*. The reason for the drop near the peak is still unclear and needs to be further studied.

In the DEC direction, lunar motion seems to follow sinusoidal function according to (c). We also checked the speed in this direction, which also seems to follow sinusoidal function as (d) indicates. This kind of change should be due to the previously introduced fact that there is a angle between the plane of the Moon's orbit and the equatorial plane of Earth; and that angle is changing slightly all the time. Therefore, the DEC of moon fluctuates around 0. Besides, the period of lunar motion in DEC seems to be a little longer than that of motion in RA.

Considering lunar motion in RA and DEC together, we get the total speed shown in (e). Since the movement in RA far outweighs that in DEC, the shape of total speed is more similar to that of dRA. However, we do not quite understand the difference between the observation of different sites and there doesn't seem to have a particular pattern between the difference and either longitude or latitude, which could be further studied in the final project.

In (f), we compared Liu Hong's observation of lunar speed and ours during a lunar month. Both data are aligned at the day with the lowest speed, Day 15. Although their tendency are similar, the differences are not negligible, which might be due to the limited measurement accuracy in the ancient time and the use of different reference frames (e.g. ecliptic or equatorial).

4 Conclusion

This study builds on the ancient work of Liu Hong. Liu Hong's methodology is analyzed and explored. Some modern datasets are utilized and processed into the format created by Liu Hong. This dataset can be used for further analysis and prediction of the positions of sun and moon. There are still some limitations in the preliminary analysis. From Figure 5(b) and (e), we can see the sudden drop of the dRA curve near the peak, the reasons of which haven't been figured out. Also, the observation results from different sites show different characteristics (RA, DEC). The reason of this phenomenon could be further studied.

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