# 002-定位-Improved Techniques for Grid Mapping With Rao-Blackwellized Particle Filters

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### 主要解决问题：

解决RBPF（Rao-Blackwellized particle filters）所存在的两个问题：①计算Importance Weights时需要的最优proposal distribution；②如何避免Resampling过程中粒子消耗的问题

### 解决思路：

- proposal distribution

  在本文中，在范围 $L^{(i)} = \left\{x \;\middle|\; p(z_t \mid m_{t-1}^{(i)}, x) > \epsilon\right\}.$ 内保留式（9）分子上的两个分量（而不是像之前一样将 $p(x_t \mid x_{t-1}^{(i)}, u_{t-1})$ 近似为常数**k**）

To efficiently draw the next generation of samples, we compute a Gaussian approximation $\mathcal{N}$ based on that data. The main differences to previous approaches is that we first use a scan-matcher to determine the meaningful area of the observation likelihood function. We then sample in that meaningful area and evaluate the sampled points based on the target distribution. For each particle $i$, the parameters $\mu_t^{(i)}$ and $\Sigma_t^{(i)}$ are determined individually for $K$ sampled points $\{x_j\}$ in the interval $L^{(i)}$. We furthermore take into account the odometry information when computing the mean $\mu^{(i)}$ and the variance $\Sigma^{(i)}$. We estimate the Gaussian parameters as

$$\begin{aligned}
\mu_t^{(i)} &= \frac{1}{\eta^{(i)}} \cdot \sum_{j=1}^{K} x_j \cdot p(z_t \mid m_{t-1}^{(i)}, x_j) \\
&\quad \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \qquad (15)\\
\Sigma_t^{(i)} &= \frac{1}{\eta^{(i)}} \cdot \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j) \\
&\quad \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \\
&\quad \cdot (x_j - \mu_t^{(i)})(x_j - \mu_t^{(i)})^T \qquad (16)
\end{aligned}$$

with the normalization factor

$$\eta^{(i)} = \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}). \qquad (17)$$

故，进一步可以得到importance weight为：

$$\begin{aligned}
w_t^{(i)} &= w_{t-1}^{(i)} \cdot p(z_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, u_{t-1}) \qquad (18)\\
&= w_{t-1}^{(i)} \cdot \int p(z_t \mid m_{t-1}^{(i)}, x') \cdot p(x' \mid x_{t-1}^{(i)}, u_{t-1})\, dx \\
&\simeq w_{t-1}^{(i)} \cdot \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \\
&= w_{t-1}^{(i)} \cdot \eta^{(i)}. \qquad (19)
\end{aligned}$$

Note that $\eta^{(i)}$ is the same normalization factor that is used in the computation of the Gaussian approximation of the proposal in Eq. (17).

- Resampling

$$N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} \left(\tilde{w}^{(i)}\right)^2}, \qquad ($$

where $\tilde{w}^{(i)}$ refers to the normalized weight of particle $i$.

$N_{eff}$可以衡量当前粒子集反映真实后验概率的能力

当$N_{eff} < N/2$时（N为粒子数）进行Resampling

**核心知识点:**

1. Mapping with Rao-Blackwellized Particle Filters

核心：估计地图m和机器人轨迹 $x_{1:t} = x_1, \ldots, x_t$ 的联合后验概率

$p(x_{1:t}, m \mid z_{1:t}, u_{1:t-1})$

其中，$z_{1:t} = z_1, \ldots, z_t$ 为观测数据，$u_{1:t-1} = u_1, \ldots, u_{t-1}$ 为里程计数据。

RBPF通常使用下述公式来高效的计算联合后验概率：

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t-1}) = p(m \mid x_{1:t}, z_{1:t}) \cdot p(x_{1:t} \mid z_{1:t}, u_{1:t-1}). \qquad (1)$$

（在给定$x_{1:t}$和$z_{1:t}$时，$p(m \mid x_{1:t}, z_{1:t})$ 可以比较容易计算得到，具体可参考论文*H.P.Moravec. Sensor fusion in certainty grids for mobile robots. AI Magazine, pages 61-74, Summer 1988*）

**故，问题转变为使用粒子滤波器来对后验概率** $p(x_{1:t} \mid z_{1:t}, u_{1:t-1})$ 进行估计。

=>使用Rao-Blackwellized SIR(Sampling Importance Resampling)进行粒子滤波：

（1）Sampling

下一个粒子$\{x_t^{(i)}\}$可以通过对proposal distribution **π**（本文解决的核心问题之一）采样从当前粒子$\{x_{t-1}^{(i)}\}$中获得。通常情况下，使用概率里程计运动模型作为proposal distribution。

（2）Importance Weighting

$$w_t^{(i)} = \frac{p(x_{1:t}^{(i)} \mid z_{1:t}, u_{1:t-1})}{\pi(x_{1:t}^{(i)} \mid z_{1:t}, u_{1:t-1})}.$$

（3）Resampling

3) *Resampling*: Particles are drawn with replacement proportional to their importance weight. This step is necessary since only a finite number of particles is used to approximate a continuous distribution. Furthermore, resampling allows us to apply a particle filter in situations in which the target distribution differs from the proposal. After resampling, all the particles have the same weight.

删去w值高的粒子=>导致粒子消耗（particle depletion）（本文解决的核心问题之二）

（4）Map Estimating

4) *Map Estimation*: For each particle, the corresponding map estimate $p(m^{(i)} \mid x_{1:t}^{(i)}, z_{1:t})$ is computed based on the trajectory $x_{1:t}^{(i)}$ of that sample and the history of observations $z_{1:t}$.

根据Doucet，我们使用一个递归算法来通过限制proposal π以满足下列条件来计算importance weights：

$$\pi(x_{1:t} \mid z_{1:t}, u_{1:t-1}) = \pi(x_t \mid x_{1:t-1}, z_{1:t}, u_{1:t-1}) \cdot\pi(x_{1:t-1} \mid z_{1:t-1}, u_{1:t-2}). \quad (3)$$

由此importance weights变为：

$$w_t^{(i)} = \frac{p(x_{1:t}^{(i)} \mid z_{1:t}, u_{1:t-1})}{\pi(x_{1:t}^{(i)} \mid z_{1:t}, u_{1:t-1})} \quad (4)$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{(i)}, z_{1:t-1}) p(x_t^{(i)} \mid x_{t-1}^{(i)}, u_{t-1})}{\pi(x_t^{(i)} \mid x_{1:t-1}^{(i)}, z_{1:t}, u_{1:t-1})} \cdot \underbrace{\frac{p(x_{1:t-1}^{(i)} \mid z_{1:t-1}, u_{1:t-2})}{\pi(x_{1:t-1}^{(i)} \mid z_{1:t-1}, u_{1:t-2})}}_{w_{t-1}^{(i)}} \quad (5)$$

$$\propto \frac{p(z_t \mid m_{t-1}^{(i)}, x_t^{(i)}) p(x_t^{(i)} \mid x_{t-1}^{(i)}, u_{t-1})}{\pi(x_t \mid x_{1:t-1}^{(i)}, z_{1:t}, u_{1:t-1})} \cdot w_{t-1}^{(i)}. (6)$$

Here $\eta = 1/p(z_t \mid z_{1:t-1}, u_{1:t-1})$ is a normalization factor resulting from Bayes' rule that is equal for all particles.

程序功能分块说明：

1. Proposal Distribution

A common approach – especially in localization – is to use a smoothed likelihood function, which avoids that particles close to the meaningful area get a too low importance weight. However, this approach discards useful information gathered by the sensor and, at least to our experience, often leads to less accurate maps in the SLAM context.

To overcome this problem, one can consider the most recent sensor observation $z_t$ when generating the next generation of samples. By integrating $z_t$ into the proposal one can focus the sampling on the meaningful regions of the observation likelihood. According to Doucet [5], the distribution

$$p(x_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, z_t, u_{t-1}) = \frac{p(z_t \mid m_{t-1}^{(i)}, x_t) p(x_t \mid x_{t-1}^{(i)}, u_{t-1})}{p(z_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, u_{t-1})} \tag{9}$$

is the optimal proposal distribution with respect to the variance of the particle weights. Using that proposal, the computation

故，importance weight变为：

$$\begin{aligned} w_t^{(i)} &= w_{t-1}^{(i)} \frac{\eta p(z_t \mid m_{t-1}^{(i)}, x_t^{(i)}) p(x_t^{(i)} \mid x_{t-1}^{(i)}, u_{t-1})}{p(x_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, z_t, u_{t-1})} && (10) \\ &\propto w_{t-1}^{(i)} \frac{p(z_t \mid m_{t-1}^{(i)}, x_t^{(i)}) p(x_t^{(i)} \mid x_{t-1}^{(i)}, u_{t-1})}{\frac{p(z_t \mid m_{t-1}^{(i)}, x_t) p(x_t \mid x_{t-1}^{(i)}, u_{t-1})}{p(z_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, u_{t-1})}} && (11) \\ &= w_{t-1}^{(i)} \cdot p(z_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, u_{t-1}) && (12) \\ &= w_{t-1}^{(i)} \cdot \int p(z_t \mid x') p(x' \mid x_{t-1}^{(i)}, u_{t-1}) \, dx'. && (13) \end{aligned}$$

When modeling a mobile robot equipped with an accurate sensor like, e.g., a laser range finder, it is convenient to use such an improved proposal since the accuracy of the laser range finder leads to extremely peaked likelihood functions. In the context of landmark-based SLAM, Montemerlo *et al*. [26] presented a Rao-Blackwellized particle filter that uses a Gaussian approximation of the improved proposal. This Gaussian is computed for each particle using a Kalman filter that estimates the pose of the robot. This approach can be used when the map is represented by a set of features and if the error affecting the feature detection is assumed to be Gaussian. In this work, we transfer the idea of computing an improved proposal to the situation in which dense grid maps are used instead of landmark-based representations.

$$L^{(i)} = \left\{ x \;\middle|\; p(z_t \mid m_{t-1}^{(i)}, x) > \epsilon \right\}.$$

在本文中，在范围 内保留式（9）分子上的

两个分量（而不是像之前一样将 $p(x_t \mid x_{t-1}^{(i)}, u_{t-1})$ 近似为常数**k**）

To efficiently draw the next generation of samples, we compute a Gaussian approximation $\mathcal{N}$ based on that data. The main differences to previous approaches is that we first use a scan-matcher to determine the meaningful area of the observation likelihood function. We then sample in that meaningful area and evaluate the sampled points based on the target distribution. For each particle $i$, the parameters $\mu_t^{(i)}$ and $\Sigma_t^{(i)}$ are determined individually for $K$ sampled points $\{x_j\}$ in the interval $L^{(i)}$. We furthermore take into account the odometry information when computing the mean $\mu^{(i)}$ and the variance $\Sigma^{(i)}$. We estimate the Gaussian parameters as

$$\begin{aligned}\mu_t^{(i)} &= \frac{1}{\eta^{(i)}} \cdot \sum_{j=1}^{K} x_j \cdot p(z_t \mid m_{t-1}^{(i)}, x_j) \\ &\quad \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \end{aligned} \tag{15}$$

$$\begin{aligned}\Sigma_t^{(i)} &= \frac{1}{\eta^{(i)}} \cdot \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j) \\ &\quad \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \\ &\quad \cdot (x_j - \mu_t^{(i)})(x_j - \mu_t^{(i)})^T \end{aligned} \tag{16}$$

with the normalization factor

$$\eta^{(i)} = \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}). \tag{17}$$

故，进一步可以得到importance weight为：

$$\begin{aligned} w_t^{(i)} &= w_{t-1}^{(i)} \cdot p(z_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, u_{t-1}) \qquad (18) \\ &= w_{t-1}^{(i)} \cdot \int p(z_t \mid m_{t-1}^{(i)}, x') \cdot p(x' \mid x_{t-1}^{(i)}, u_{t-1})\, dx \\ &\simeq w_{t-1}^{(i)} \cdot \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1}) \end{aligned}$$

$$= \quad w_{t-1}^{(i)} \cdot \eta^{(i)}. \tag{19}$$

Note that $\eta^{(i)}$ is the same normalization factor that is used in the computation of the Gaussian approximation of the proposal in Eq. (17).

2. Adaptive Resampling

   在resampling过程中，w值较低的粒子会被w值较高的所替代。

$$N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} \left(\tilde{w}^{(i)}\right)^2}, \quad ($$

where $\tilde{w}^{(i)}$ refers to the normalized weight of particle $i$.

   $N_{eff}$可以衡量当前粒子集反映真实后验概率的能力

   当$N_{eff} < N/2$时（N为粒子数）进行Resampling

3. 算法总结

**Algorithm 1** Improved RBPF for Map Learning

**Require:**

$\mathcal{S}_{t-1}$, the sample set of the previous time step

$z_t$, the most recent laser scan

$u_{t-1}$, the most recent odometry measurement

**Ensure:**

$\mathcal{S}_t$, the new sample set

$\mathcal{S}_t = \{\}$

**for all** $s_{t-1}^{(i)} \in \mathcal{S}_{t-1}$ **do**

$< x_{t-1}^{(i)}, w_{t-1}^{(i)}, m_{t-1}^{(i)} >= s_{t-1}^{(i)}$

// *scan-matching*

$x_t'^{(i)} = x_{t-1}^{(i)} \oplus u_{t-1}$

$\hat{x}_t^{(i)} = \operatorname{argmax}_x p(x \mid m_{t-1}^{(i)}, z_t, x_t'^{(i)})$

**if** $\hat{x}_t^{(i)} =$ **failure then**

$x_t^{(i)} \sim p(x_t \mid x_{t-1}^{(i)}, u_{t-1})$

$w_t^{(i)} = w_{t-1}^{(i)} \cdot p(z_t \mid m_{t-1}^{(i)}, x_t^{(i)})$

**else**

// *sample around the mode*

**for** $k = 1, \ldots, K$ **do**

$x_k \sim \{x_j \mid |x_j - \hat{x}^{(i)}| < \Delta\}$

**end for**

// *compute Gaussian proposal*

$\mu_t^{(i)} = (0, 0, 0)^T$

$\eta^{(i)} = 0$

**for all** $x_j \in \{x_1, \ldots, x_K\}$ **do**

$\mu_t^{(i)} = \mu_t^{(i)} + x_j \cdot p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_t \mid x_{t-1}^{(i)}, u_{t-1})$

$\eta^{(i)} = \eta^{(i)} + p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_t \mid x_{t-1}^{(i)}, u_{t-1})$

**end for**

$\mu_t^{(i)} = \mu_t^{(i)} / \eta^{(i)}$
$\Sigma_t^{(i)} = \mathbf{0}$
**for all** $x_j \in \{x_1, \ldots, x_K\}$ **do**
$\Sigma_t^{(i)} = \Sigma_t^{(i)} + (x_j - \mu^{(i)})(x_j - \mu^{(i)})^T \cdot$
$p(z_t \mid m_{t-1}^{(i)}, x_j) \cdot p(x_j \mid x_{t-1}^{(i)}, u_{t-1})$
**end for**
$\Sigma_t^{(i)} = \Sigma_t^{(i)} / \eta^{(i)}$
*// sample new pose*
$x_t^{(i)} \sim \mathcal{N}(\mu_t^{(i)}, \Sigma_t^{(i)})$

*// update importance weights*
$w_t^{(i)} = w_{t-1}^{(i)} \cdot \eta^{(i)}$
**end if**
*// update map*
$m_t^{(i)} = \text{integrateScan}(m_{t-1}^{(i)}, x_t^{(i)}, z_t)$
*// update sample set*
$\mathcal{S}_t = \mathcal{S}_t \cup \{< x_t^{(i)}, w_t^{(i)}, m_t^{(i)} >\}$
**end for**

$N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} (\tilde{w}^{(i)})^2}$

**if** $N_{\text{eff}} < T$ **then**
$\mathcal{S}_t = \text{resample}(\mathcal{S}_t)$
**end if**

1) An initial guess $x_t'^{(i)} = x_{t-1}^{(i)} \oplus u_{t-1}$ for the robot's pose represented by the particle $i$ is obtained from the previous pose $x_{t-1}^{(i)}$ of that particle and the odometry measurements $u_{t-1}$ collected since the last filter update. Here, the operator $\oplus$ corresponds to the standard pose compounding operator [24].
2) A scan-matching algorithm is executed based on the map $m_{t-1}^{(i)}$ starting from the initial guess $x_t'^{(i)}$. The search performed by the scan-matcher is bounded to a limited region around $x_t'^{(i)}$. If the scan-matching reports a failure, the pose and the weights are computed according to the motion model (and the steps 3 and 4 are ignored).
3) A set of sampling points is selected in an interval around the pose $\hat{x}_t^{(i)}$ reported scan-matcher. Based on this points, the mean and the covariance matrix of the proposal are computed by pointwise evaluating the

target distribution $p(z_t \mid m_{t-1}^{(i)}, x_j)p(x_j \mid x_{t-1}^{(i)}, u_{t-1})$ in the sampled positions $x_j$. During this phase, also the weighting factor $\eta^{(i)}$ is computed according to Eq. (17).

4) The new pose $x_t^{(i)}$ of the particle $i$ is drawn from the Gaussian approximation $\mathcal{N}(\mu_t^{(i)}, \Sigma_t^{(i)})$ of the improved proposal distribution.
5) Update of the importance weights.
6) The map $m^{(i)}$ of particle $i$ is updated according to the drawn pose $x_t^{(i)}$ and the observation $z_t$.

After computing the next generation of samples, a resampling step is carried out depending on the value of $N_{\text{eff}}$.

存在的问题:

改进的思路: