004-定位-LIPS：LiDAR-Inertial 3D Plane SLAM

序号：4
名称：LIPS：LiDAR-Inertial 3D Plane SLAM
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文献类型：IROS2018 会议论文
年份：2018
整理日期：2019年4约19日
论文链接：004-LIPS LiDAR-Inertial 3D Plane SLAM.pdf

主要解决问题

激光雷达传感器面临的主要问题之一是如何处理大量的无序3D点来提取有用的特征进行估计。传统的方式是使用ICP提取信息丰富的环境基元，如平面，进行位姿估计。而使用平面基元进行估计会在参数化过程中出现过度参数化的情况。

解决思路

利用最近点-面表示的奇点自由平面因子，并在基于图的优化框架中展示了它与惯性预积分测量的融合。得到的LiDAR惯性3D平面SLAM（LIPS）系统在定制的LiDAR模拟器和现实世界的实验中得到验证。
主要创新点

1. 提出最近点（CP）面表示，其奇点的分析，以及其在3D平面SLAM中作为平面表示和误差状态的用途。
2. 设计具有鲁棒性且具有相对平面锚定因子的新型LiDAR惯性三维平面SLAM（LIPS）系统，用于基于图的优化，有效地克服了CP表示的奇异性问题。
3. 开发用于评估LiDAR辅助定位算法的通用LiDAR模拟器。
4. 通过蒙特卡罗模拟和实际实验验证所提出的LIPS系统。

核心知识点

1. 蒙特卡罗模拟
   通过构造符合一定规则的随机数来解决数学上的各种问题。对于那些由于计算过于复杂而难以得到解析解或者根本没有解析解的问题，蒙特卡罗模拟是一种有效求出数值解的方法。
   2、问题的构建
   A.基于图的优化

   $$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \sum_i ||\mathbf{r}_i(\mathbf{x})||^2_{\mathbf{P}_i}$$

   最大概率优化问题

   其中r_i为观测z_i有关的零均值测量残差（residual),P_i为观测方差，$||\mathbf{v}||^2_{\mathbf{P}} = \mathbf{v}^\top \mathbf{P}^{-1}\mathbf{v}$

   $$\widetilde{\mathbf{x}}^* = \underset{\widetilde{\mathbf{x}}}{\operatorname{argmin}} \sum_i ||\mathbf{r}_i(\hat{\mathbf{x}}) + \mathbf{J}_i \widetilde{\mathbf{x}}||^2_{\mathbf{P}_i}$$

   使用error state $\widetilde{\mathbf{x}}$:定义新的优化问题

where $\mathbf{J}_i = \frac{\partial \mathbf{r}_i(\hat{\mathbf{x}} \boxplus \tilde{\mathbf{x}})}{\partial \tilde{\mathbf{x}}}$ is the Jacobian of i-th residual with respect to the error state. We define the generalized update operation, $\boxplus$, which maps the error state to the full state. After solving the linearized system, the current state estimate is updated as $\mathbf{x} = \hat{\mathbf{x}} \boxplus \tilde{\mathbf{x}}$.

B.LIPS系统

定义每个时刻机器人状态为一个16*1的向量

$$\mathbf{x}_I = \begin{bmatrix} {}^I_G\bar{q}^\top & \mathbf{b}_g^\top & {}^G\mathbf{v}_I^\top & \mathbf{b}_a^\top & {}^G\mathbf{p}_I^\top \end{bmatrix}^\top$$

where the quaternion ${}^I_G\bar{q}$ represents the rotation, ${}^I_G\mathbf{R}$, from global frame $\{G\}$ to the IMU frame $\{I\}$, the velocity ${}^G\mathbf{v}_I$ is of the IMU seen from the global frame, position ${}^G\mathbf{p}_I$ is the IMU position seen in the global frame, and $\mathbf{b}_g$ and $\mathbf{b}_a$ are the gyroscope and accelerometer biases respectively. We

定义最小表示误差状态

$$\tilde{\mathbf{x}}_I = \begin{bmatrix} {}^I_G\boldsymbol{\delta\theta}^\top & \tilde{\mathbf{b}}_g^\top & {}^G\tilde{\mathbf{v}}_I^\top & \tilde{\mathbf{b}}_a^\top & {}^G\tilde{\mathbf{p}}_I^\top \end{bmatrix}^\top$$

$\tilde{x}_I = [{}^I_G\delta\theta^T \ \tilde{b}_g^T \ {}^G\tilde{v}_I^T \ \tilde{b}_a^T \ {}^G\tilde{p}_I^T]^T$

其中 $\delta\bar{q} = \bar{q} \otimes \hat{\bar{q}}^{-1} \simeq [\frac{1}{2}\delta\theta^T \ 1]^T$

四元数乘法操作.

可以用传统加法误差模型更新，例 ${}^G\tilde{p}_I = {}^Gp_I - {}^G\hat{p}_I$

误差 实际 估计

待估计的全部状态X包括m个IMU历史状态和k个平面基元: $X = [X_{I(1)} \dots X_{I(m)} \ \Pi_1 \dots \Pi_k]$

估计方法：使用因子图进行优化，连续**IMU**预积分因子+**3D**平面因子

LIPS的最终cost函数为：

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left[\sum_i ||\mathbf{r}_{Ii}(\mathbf{x})||^2_{\mathbf{R}_{Ii}} + \sum_j ||\mathbf{r}_{\Pi j}(\mathbf{x})||^2_{\mathbf{R}_{\Pi j}} \right]$$

where $\mathbf{r}_{Ii}(\mathbf{x})$ and $\mathbf{r}_{\Pi j}(\mathbf{x})$ are the zero mean residuals associated with the continuous preintegration and anchored CP planes measurements, respectively. While, $\mathbf{R}_{Ii}$ and $\mathbf{R}_{\Pi j}$ are the covariances of the continuous preintegration and anchor CP plane measurements, respectively.

程序功能分块说明

1. 连续IMU预积分

highly informative constraint. Continuous preintegration is based on *closed-form* expressions of the IMU measurement dynamics rather than the discrete approximations used in previous works [32]. We model the linear acceleration and angular velocity inertial measurements as:

$$\boldsymbol{\omega}_m = \boldsymbol{\omega} + \mathbf{b}_w + \mathbf{n}_w \tag{8}$$

$$\mathbf{a}_m = \mathbf{a} + \mathbf{b}_a + \mathbf{n}_a + {}^I_G\mathbf{R}\,{}^G\mathbf{g} \tag{9}$$

where ${}^G\mathbf{g}$ is the gravity in the global frame, $\boldsymbol{\omega}$ is the angular velocity, $\mathbf{a}$ is the linear acceleration, and $\mathbf{n}_w$, $\mathbf{n}_a$ are the continuous measurement noises. The underlying standard IMU dynamics are given by [33]:

$${}^I_G\dot{\bar{q}} = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega}_m - \mathbf{b}_w - \mathbf{n}_w)\,{}^I_G\bar{q} \tag{10}$$

$$\dot{\mathbf{b}}_w = \mathbf{n}_{wb} \tag{11}$$

$${}^G\dot{\mathbf{v}}_k = {}^G_k\mathbf{R}\left(\mathbf{a}_m - \mathbf{b}_a - \mathbf{n}_a\right) - {}^G\mathbf{g} \tag{12}$$

$$\dot{\mathbf{b}}_a = \mathbf{n}_{ab} \tag{13}$$

$${}^G\dot{\mathbf{p}}_k = {}^G\mathbf{v}_k \tag{14}$$

随机游动噪声

where $\mathbf{n}_{wb}$, $\mathbf{n}_{ab}$ are the random walk noises and $\boldsymbol{\Omega}(\cdot)$ is:

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -\lfloor\boldsymbol{\omega}\times\rfloor & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^\top & 0 \end{bmatrix} \tag{15}$$

预积分的核心是因式分解上述五个方程的积分：

$${}^G\mathbf{p}_{k+1} = {}^G\mathbf{p}_k + {}^G\mathbf{v}_k\Delta T - \frac{1}{2}{}^G\mathbf{g}\Delta T^2 + {}^G_k\mathbf{R}\,{}^k\boldsymbol{\alpha}_{k+1} \tag{16}$$

$${}^G\mathbf{v}_{k+1} = {}^G\mathbf{v}_k - {}^G\mathbf{g}\Delta T + {}^G_k\mathbf{R}\,{}^k\boldsymbol{\beta}_{k+1} \tag{17}$$

$${}^{k+1}_G\bar{q} = {}^{k+1}_k\bar{q} \otimes {}^k_G\bar{q} \tag{18}$$

where ΔT is the difference between the bounding LiDAR pose timestamps (t_k, t_{k+1}) and ${}^k\boldsymbol{\alpha}_{k+1}$, ${}^k\boldsymbol{\beta}_{k+1}$ are defined by the following integrations of the IMU measurements:

$${}^k\boldsymbol{\alpha}_{k+1} = \int_{t_k}^{t_{k+1}}\int_{t_k}^{s} {}^k_u\mathbf{R}\left(\mathbf{a}_m - \mathbf{b}_a - \mathbf{n}_a\right) du\,ds \tag{19}$$

$${}^k\boldsymbol{\beta}_{k+1} = \int_{t_k}^{t_{k+1}} {}^k_u\mathbf{R}\left(\mathbf{a}_m - \mathbf{b}_a - \mathbf{n}_a\right) du \tag{20}$$

泰勒展开：

$$
{}^{k}\boldsymbol{\alpha}_{k+1} \simeq {}^{k}\breve{\boldsymbol{\alpha}}_{k+1} + \frac{\partial \boldsymbol{\alpha}}{\partial \mathbf{b}_a}\Big|_{\bar{\mathbf{b}}_a} \Delta \mathbf{b}_a + \frac{\partial \boldsymbol{\alpha}}{\partial \mathbf{b}_w}\Big|_{\bar{\mathbf{b}}_w} \Delta \mathbf{b}_w \tag{21}
$$

$$
{}^{k}\boldsymbol{\beta}_{k+1} \simeq {}^{k}\breve{\boldsymbol{\beta}}_{k+1} + \frac{\partial \boldsymbol{\beta}}{\partial \mathbf{b}_a}\Big|_{\bar{\mathbf{b}}_a} \Delta \mathbf{b}_a + \frac{\partial \boldsymbol{\beta}}{\partial \mathbf{b}_w}\Big|_{\bar{\mathbf{b}}_w} \Delta \mathbf{b}_w \tag{22}
$$

$$
{}^{k+1}_{k}\bar{q} \simeq \bar{q}(\Delta \mathbf{b}_w)^{-1} \otimes {}^{k+1}_{k}\breve{\bar{q}} \tag{23}
$$

where ${}^{k}\breve{\boldsymbol{\alpha}}_{k+1}$, ${}^{k}\breve{\boldsymbol{\beta}}_{k+1}$, ${}^{k+1}_{k}\breve{\bar{q}}$ are the preintegrated measurements evaluated at the current bias estimates. In particular, ${}^{k+1}_{k}\breve{\bar{q}}$ can be found using the zeroth order quaternion integrator [29]. We define the quaternion which models multiplicative orientation corrections due to changes in the linearized bias as:

$$
\bar{q}(\Delta \mathbf{b}_w) = \begin{bmatrix} \frac{\boldsymbol{\theta}}{||\boldsymbol{\theta}||} \sin \frac{||\boldsymbol{\theta}||}{2} \\ \cos \frac{||\boldsymbol{\theta}||}{2} \end{bmatrix}, \ \boldsymbol{\theta} = \frac{\partial \bar{q}}{\partial \mathbf{b}_w}\Big|_{\bar{\mathbf{b}}_w} \left(\mathbf{b}_{w(k)} - \bar{\mathbf{b}}_w\right)
$$

where $\Delta \mathbf{b}_w := \mathbf{b}_{w(k)} - \bar{\mathbf{b}}_w$ and $\Delta \mathbf{b}_a := \mathbf{b}_{a(k)} - \bar{\mathbf{b}}_a$ are the differences between the true biases and the bias estimate used as the linearization point. The new preintegration mea-

最终测量残差为

最终测量残差

$$
r_I(\mathbf{x}) = \begin{bmatrix}
2\mathrm{vec}\left({}^{k+1}_{G}\bar{q} \otimes {}^{k}_{G}\bar{q}^{-1} \otimes {}^{k+1}_{k}\breve{\bar{q}}^{-1} \otimes \bar{q}(\Delta \mathbf{b}_w)\right) \\
\mathbf{b}_{w(k+1)} - \mathbf{b}_{w(k)} \\
\begin{pmatrix} {}^{k}_{G}\mathbf{R}\left({}^{G}\mathbf{v}_{k+1} - {}^{G}\mathbf{v}_{k} + {}^{G}\mathbf{g}\Delta T\right) \\ -{}^{k}\breve{\boldsymbol{\beta}}_{k+1} - \frac{\partial \boldsymbol{\beta}}{\partial \mathbf{b}_a}\Big|_{\bar{\mathbf{b}}_a} \Delta \mathbf{b}_a - \frac{\partial \boldsymbol{\beta}}{\partial \mathbf{b}_w}\Big|_{\bar{\mathbf{b}}_w} \Delta \mathbf{b}_w \end{pmatrix} \\
\mathbf{b}_{a(k+1)} - \mathbf{b}_{a(k)} \\
\begin{pmatrix} {}^{k}_{G}\mathbf{R}\left({}^{G}\mathbf{p}_{k+1} - {}^{G}\mathbf{p}_{k} - {}^{G}\mathbf{v}_{k}\Delta T + \frac{1}{2}{}^{G}\mathbf{g}\Delta T^2\right) \\ -{}^{k}\breve{\boldsymbol{\alpha}}_{k+1} - \frac{\partial \boldsymbol{\alpha}}{\partial \mathbf{b}_a}\Big|_{\bar{\mathbf{b}}_a} \Delta \mathbf{b}_a - \frac{\partial \boldsymbol{\alpha}}{\partial \mathbf{b}_w}\Big|_{\bar{\mathbf{b}}_w} \Delta \mathbf{b}_w \end{pmatrix}
\end{bmatrix}
$$

where $\mathrm{vec}(\cdot)$ returns the vector portion of the quaternion (i.e., the top three elements) and the bias errors are the difference between biases in the bounding states.

2. 图优化中平面因子的确定

A.最近点平面描述

“最近点（CP）”表示可以被描述为驻留在平面上并且最接近当前帧的原点的3D点

during optimization. This CP representation can be described using the Hesse normal vector ${}^{G}\mathbf{n}$ and distance scalar ${}^{G}d$:

$$ {}^{G}\mathbf{\Pi} = {}^{G}\mathbf{n}\ {}^{G}d \tag{24} $$

$$ \begin{bmatrix} {}^{G}\mathbf{n} \\ {}^{G}d \end{bmatrix} = \begin{bmatrix} {}^{G}\mathbf{\Pi}/\|{}^{G}\mathbf{\Pi}\| \\ \|{}^{G}\mathbf{\Pi}\| \end{bmatrix} \tag{25} $$

It is important to point out that without special care, this representation still has a singularity when the value of ${}^{G}d$ approaches zero. Any plane ${}^{G}\mathbf{\Pi}$ that intersects our frame they are extracted from. The singularity in practice would only arise if we transform a local CP plane, ${}^{L}\mathbf{\Pi}$, into a frame where the plane intersects its origin (see Figure 2).

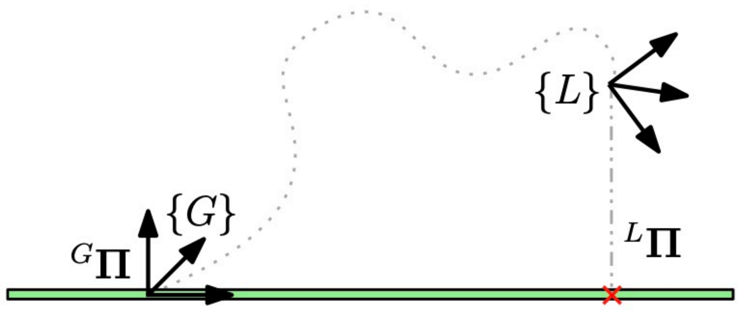

Fig. 2: A visual representation of the closest point on the plane. Also shown is an example of a local plane parameter ${}^{L}\mathbf{\Pi}$ that is well defined, while the global plane representation ${}^{G}\mathbf{\Pi}$ is ill-defined.

B.锚面因子

解决奇点问题:

To overcome the aforementioned singularity issue of the CP representation, we parameterize the plane in the first observation frame, guaranteeing that the distance to the plane will be non-zero (from here forward this will be denoted the "anchor" frame/state). As seen in Figure 3, the transform of

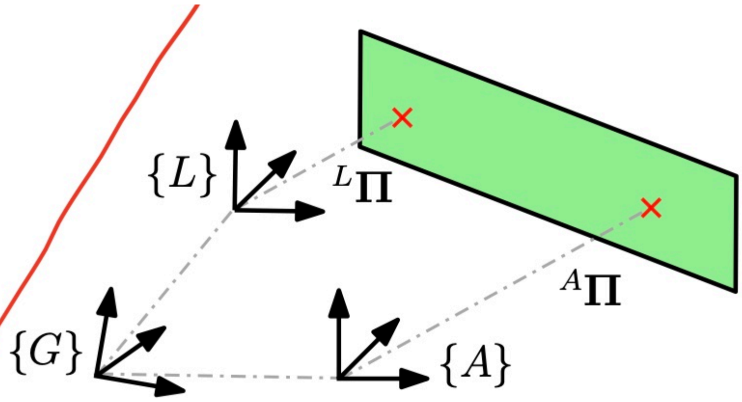

Fig. 3: Pictorial view of a closest point plane representation seen in the local $\{L\}$ frame which can be transformed into its anchor frame $\{A\}$ and vice versa.

从一帧到另一帧的平面表示变换不是直接的3D点变换，而是需要再新帧中计算CP】

使用Hesse平面表示法，我们可以将锚坐标系{A}中表示的平面映射到局部坐标系{L}

$$\begin{bmatrix} {}^L\mathbf{n} \\ {}^L d \end{bmatrix} = \begin{bmatrix} {}^L_A\mathbf{R} & 0 \\ -{}^A\mathbf{p}_L^\top & 1 \end{bmatrix} \begin{bmatrix} {}^A\mathbf{n} \\ {}^A d \end{bmatrix} \tag{26}$$

where ${}^L_A\mathbf{R}$ is the relative rotation between the local and anchor LiDAR frames, ${}^A\mathbf{p}_L$ is the position of the local LiDAR frame seen from the anchor LiDAR frame.

利用 ${}^G\mathbf{\Pi} = {}^G\mathbf{n}\ {}^G d$ 及上述{An,Ad}的关系，我们可以得到

$${}^L\mathbf{\Pi}(\mathbf{x}) = \left({}^L_A\mathbf{R}\,{}^A\mathbf{n}\right)\left({}^A d - {}^A\mathbf{p}_L^\top\,{}^A\mathbf{n}\right)$$

For a given plane measurement, ${}^L\widehat{\mathbf{\Pi}}$, we have the following residual for use in graph optimization (see (7)):

$$r_\Pi(\mathbf{x}) = {}^L\mathbf{\Pi}(\mathbf{x}) - {}^L\widehat{\mathbf{\Pi}} \tag{28}$$

C.点到面压缩

为了得到对应于平面的无序点云的子集，可以使用RANSAC或其他平面分割方法

be used. We model each point measurement ${}^L\mathbf{p}_{mi}$ as a true measurement ${}^L\mathbf{p}_i$ being corrupted by some zero mean Gaussian noise:

$${}^L\mathbf{p}_{mi} = {}^L\mathbf{p}_i + \mathbf{n}_p, \quad \mathbf{n}_p \sim \mathcal{N}(0, \mathbf{R}_d) \tag{29}$$

首先将提取的点子集压缩为本地CP并匹配可用于优化的协方差。我们可以从制定加权最小二乘优化问题开始，我们寻求最小化提取点与本地CP测量Ln之间的点到面的距离

$${}^L\mathbf{\Pi}^* = \underset{{}^L\mathbf{\Pi}}{\operatorname{argmin}} \sum_i \left\| \frac{{}^L\mathbf{\Pi}^\top}{\|{}^L\mathbf{\Pi}\|}\,{}^L\mathbf{p}_{mi} - \|{}^L\mathbf{\Pi}\| \right\|^2_{W_i^{-1}} \tag{30}$$

where W_i is the inverse variance of the noise that corrupts the ith measurement. In practice, we also introduce a robust Huber loss to minimize the effect of outliers during optimization (see [36]). We minimize the above cost function using the Gauss-Newton method of iterative linearization of the residual about the current best estimate. Formally, we solve for the correction, ${}^L\widetilde{\mathbf{\Pi}}$, to our linearization point ${}^L\widehat{\mathbf{\Pi}}$:

$${}^L\widetilde{\mathbf{\Pi}} = -\left(\sum_i \mathbf{J}_i^\top\, W_i\, \mathbf{J}_i\right)^{-1}\left(\sum_i \mathbf{J}_i^\top W_i\, r_i({}^L\widehat{\mathbf{\Pi}})\right)$$

where we have the following:

$$\mathbf{J}_i = \frac{{}^L\mathbf{p}_{mi}^\top}{\|{}^L\widehat{\mathbf{\Pi}}\|} - \left({}^L\mathbf{p}_{mi}^\top\,{}^L\widehat{\mathbf{\Pi}}\right)\frac{{}^L\widehat{\mathbf{\Pi}}^\top}{\|{}^L\widehat{\mathbf{\Pi}}\|^3} - \frac{{}^L\widehat{\mathbf{\Pi}}^\top}{\|{}^L\widehat{\mathbf{\Pi}}\|} \tag{31}$$

$$W_i = \left(\frac{{}^L\widehat{\mathbf{\Pi}}^\top}{||{}^L\widehat{\mathbf{\Pi}}||}\,\mathbf{R}_d\,\frac{{}^L\widehat{\mathbf{\Pi}}}{||{}^L\widehat{\mathbf{\Pi}}||}\right)^{-1} \tag{32}$$

最终局部最近点的协方差矩阵为:

$$\mathbf{P}_\Pi = \left(\sum_i \mathbf{J}_i^\top\, W_i\, \mathbf{J}_i\right)^{-1}$$

存在的问题

改进的思路