001-定位-Improving Grid-based SLAM with Rao-Blackwellized Particle Filters by Adaptive Proposals and Selective Resampling

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主要解决问题：

解决RBPF（Rao-Blackwellized particle filters）所存在的两个问题：①计算Importance Weights时需要的最优proposal distribution；②如何避免Resampling过程中粒子消耗的问题

解决思路：

- proposal distribution

$$p(x_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, z_t, u_t) \simeq \mathcal{N}(\mu_t^{(i)}, \Sigma_t^{(i)})$$

$$\mu_t^{(i)} = \frac{1}{\eta}\sum_{j=1}^{K} x_j p(z_t \mid m_{t-1}^{(i)}, x_j) \quad (7)$$

$$\Sigma_t^{(i)} = \frac{1}{\eta}\sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j)(x_j - \mu_t^{(i)})(x_j - \mu_t^{(i)})^T . (8)$$

Here $\eta = \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j)$ is a normalizer. Observe that the computation of $\mu_t^{(i)}$ and $\Sigma_t^{(i)}$ as well as the scan-matching process are carried out for each particle. The $\{x_j\}$ are chosen to cover an area dependent on the last odometry reading uncertainty $x_j \in \{x_t \mid p(x_t \mid x_{t-1}, u_t) > \chi\}$, and with a density depending on the grid map resolution. In our current system we apply a scan-matching routine similar to that of Hähnel *et al.* [10].

- Resampling

$$N_{eff} = \frac{1}{\sum_{i=1}^{N}\left(w^{(i)}\right)^2}.$$

N_{eff}可以衡量当前粒子集反映真实后验概率的能力

当$N_{eff} < N/2$时（N为粒子数）进行Resampling

核心知识点:

1. Rao-Blackwell Theorem
   统计学里，该定理以一个任意的原始估计为起点，寻找最小方差无偏估计量（MVUE）。
2. Rao-Blackwellized Mapping
   核心：估计后验概率$p(x_{1:t} \mid z_{1:t}, u_{0:t})$，其中$x_{1:t}$为机器人可能的轨迹，$z_{1:t}$为观测数据，$u_{0:t}$为里程计数据。
   而后估计轨迹和地图的联合后验概率
   $$p(x_{1:t}, m \mid z_{1:t}, u_{0:t}) = p(m \mid x_{1:t}, z_{1:t})p(x_{1:t} \mid z_{1:t}, u_{0:t}).$$
   （在给定$x_{1:t}$和$z_{1:t}$时，$p(m \mid x_{1:t}, z_{1:t})$可以比较容易计算得到，具体可参考论文*H.P.Moravec. Sensor fusion in certainty grids for mobile robots. AI Magazine, pages 61-74, Summer 1988*）
   =>使用Rao-Blackwellized SIR(Sampling Importance Resampling)：
   （1）Sampling
   下一个粒子$\{x_t^{(i)}\}$可以通过对proposal distribution $\pi(x_t \mid z_{1:t}, u_{0:t})$.（本文解决的核心问题之一）采样从当前粒子$\{x_{t-1}^{(i)}\}$中获得。
   （2）Importance Weighting

$$w^{(i)} = \frac{p(x_t^{(i)} \mid z_{1:t}, u_{0:t})}{\pi(x_t^{(i)} \mid z_{1:t}, u_{0:t})}. \quad (2)$$

—实际状态

—额外计算

   （3）Resampling

w值较低的粒子会被w值较高的粒子所取代=>减少所需要的粒子数量

并删去w值高的粒子=>导致粒子消耗（particle depletion）（本文解决的核心问题之二）

3) *Resampling*: Particles with a low importance weight w are typically replaced by samples with a high weight. This step is necessary since only a finite number of particles are used to approximate a continuous distribution. Furthermore, resampling allows to apply a particle filter in situations in which the true distribution differs from the proposal.

降采样

删除w值高的粒子(即不好的粒子)，故导致粒子消耗

4) *Map Estimating*: for each pose sample $x_t^{(i)}$, the corresponding map estimate $m_t^{(i)}$ is computed based on the

（4）Map Estimating

计算一定条件下的$m_t^{(i)}$ $p(m_t^{(i)} \mid x_{1:t}^{(i)}, z_{1:t})$.

程序功能分块说明：

1. Proposal Distribution

   现有的方案：

   - a. 考虑到粒子权重的方差&&马尔科夫假设，Doucet提出最优proposal distribution为：

$$p(x_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, z_t, u_t) = \frac{p(z_t \mid m_{t-1}^{(i)}, x_t) p(x_t \mid x_{t-1}^{(i)}, u_t)}{\int p(z_t \mid m_{t-1}^{(i)}, x') p(x' \mid x_{t-1}^{(i)}, u_t) dx'}.$$

   - b. $p(x_t \mid x_{t-1}, u_t)$

做两个近似：

- 实验证明，在间隔$L^{(i)}$内，$p(z_t \mid m_{t-1}^{(i)}, x_t)$ 在分子上起主要作用，故我们假设在

$L^{(i)} = \{x \mid p(z_t \mid m_{t-1}^{(i)}, x) > \epsilon\}.$ 内，$p(x_t \mid x_{t-1}, u_t)$ 近似为常数**k**。

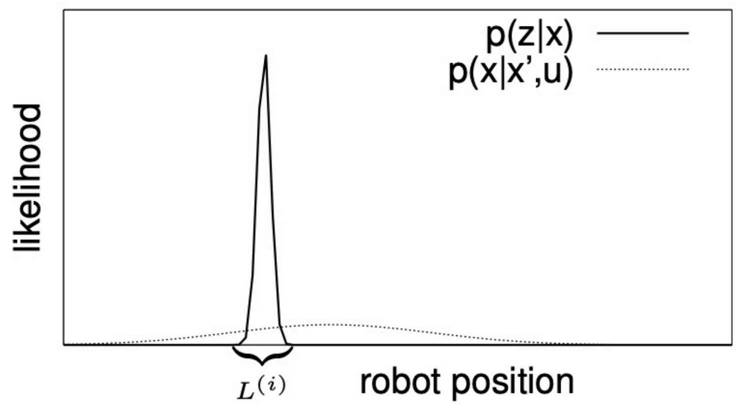

Fig. 1. The two components of the motion model. Within the interval $L^{(i)}$ the product of both functions is dominated by the observation likelihood. Accordingly the model of the odometry error can safely be approximated by a constant value.

故，a中的式子可以近似为

$$p(x_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, z_t, u_t) \simeq \frac{p(z_t \mid m_{t-1}^{(i)}, x_t)}{\int_{x' \in L^{(i)}} p(z_t \mid m_{t-1}^{(i)}, x') dx'}$$

- 局部高斯分布近似 $p(x_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, z_t, u_t) \simeq \mathcal{N}(\mu_t^{(i)}, \Sigma_t^{(i)})$

$$\mu_t^{(i)} = \frac{1}{\eta} \sum_{j=1}^{K} x_j p(z_t \mid m_{t-1}^{(i)}, x_j) \tag{7}$$

$$\Sigma_t^{(i)} = \frac{1}{\eta} \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j)(x_j - \mu_t^{(i)})(x_j - \mu_t^{(i)})^T. \tag{8}$$

Here $\eta = \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j)$ is a normalizer. Observe that the computation of $\mu_t^{(i)}$ and $\Sigma_t^{(i)}$ as well as the scan-matching process are carried out for each particle. The $\{x_j\}$ are chosen to cover an area dependent on the last odometry reading uncertainty $x_j \in \{x_t \mid p(x_t \mid x_{t-1}, u_t) > \chi\}$, and with a density depending on the grid map resolution. In our current system we apply a scan-matching routine similar to that of Hähnel *et al.* [10].

故，可以进一步推出importance weight $w^{(i)}$为：

$$\begin{aligned}
w_t^{(i)} &= w_{t-1}^{(i)} p(z_t \mid m_{t-1}^{(i)}, x_{t-1}^{(i)}, u_t) \\
&= w_{t-1}^{(i)} \int p(z_t \mid m_{t-1}^{(i)}, x') p(x' \mid x_{t-1}^{(i)}, u_t) dx' \quad \approx k \\
&\simeq w_{t-1}^{(i)} k \int_{x' \in L^{(i)}} p(z_t \mid m_{t-1}^{(i)}, x') dx' \\
&\simeq w_{t-1}^{(i)} k \sum_{j=1}^{K} p(z_t \mid m_{t-1}^{(i)}, x_j) \\
&= w_{t-1}^{(i)} k \eta
\end{aligned} \tag{9}$$

效果如下：

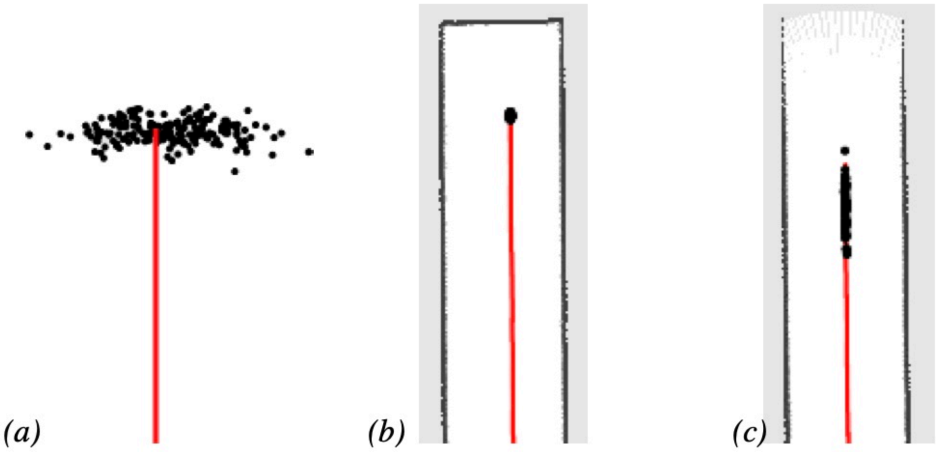

Fig. 2. Proposal distributions typically observed during mapping. In a featureless open space the proposal distribution is the raw odometry motion model *(a)*. In a dead end corridor the particles the uncertainty is small in all of the directions *(b)*. In an open corridor the particles distributes along the corridor *(c)*.

2. Selective Resampling

$$N_{eff} = \frac{1}{\sum_{i=1}^{N} \left(w^{(i)}\right)^2}.$$

N_{eff}可以衡量当前粒子集反映真实后验概率的能力

当$N_{eff} < N/2$时（N为粒子数）进行Resampling

存在的问题:

改进的思路: