Leading Cruise Control in Mixed Traffic Flow

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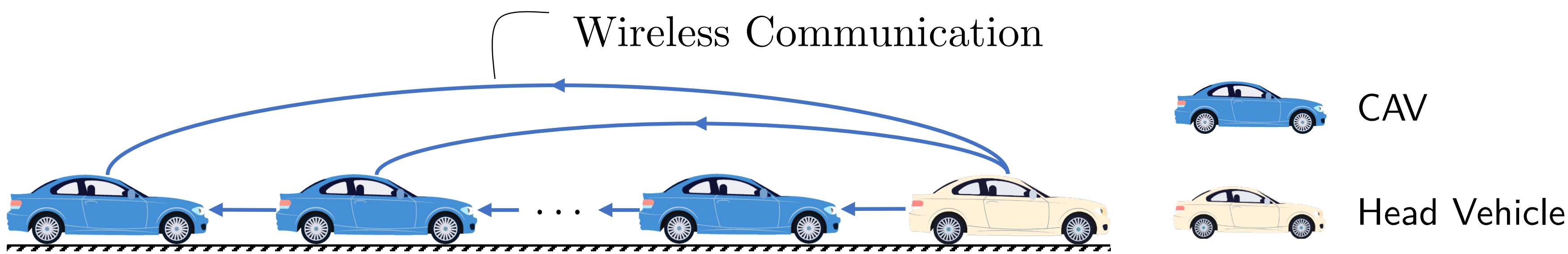
1.1 Existing CAV Control Frameworks

Background: Vehicle-to-vehicle and vehicle-to-infrastructure communications have provided new opportunities for better traffic mobility and smoother traffic flow, contributing to the emergence of Connected and Autonomous Vehicles (CAVs).

Typical extension from Adaptive Cruise Control (ACC):

Cooperative Adaptive Cruise Control (CACC)



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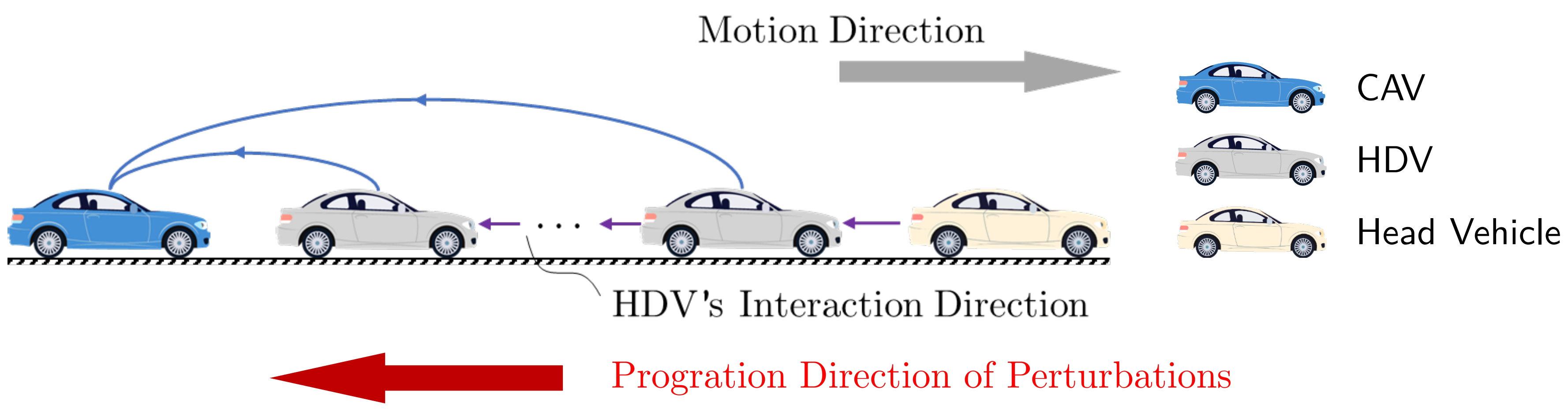
1.1 Existing CAV Control Frameworks

Long-term transition phase: **mixed traffic flow**, i.e., coexistence of human-driven vehicles (HDVs) and CAVs

Connected Cruise Control [G. Orosz, 2016]



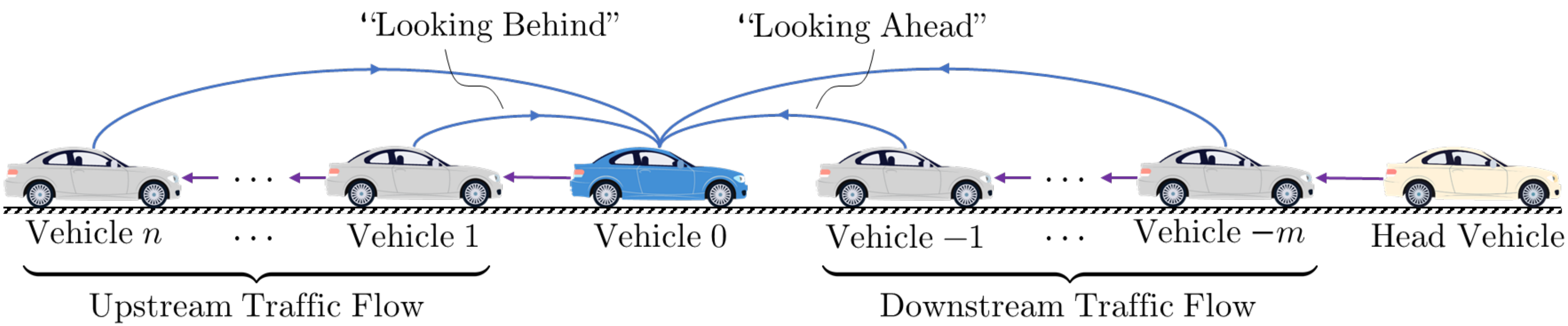
- Explicitly consider the car-following dynamics of multiple HDVs ahead
- Aim at attenuating perturbations ahead and achieving a better car-following behavior

Note: Due to the front-to-rear reaction dynamics of HDVs, the behavior of one individual vehicle will simultaneously have a significant impact on the upstream traffic flow containing the vehicles behind.

Notion of Leading Cruise Control

Motivation: How to incorporate the subsequent influence of the perturbations on the upstream traffic flow behind the CAV into CAV control has not been clearly addressed.

Leading Cruise Control (LCC)

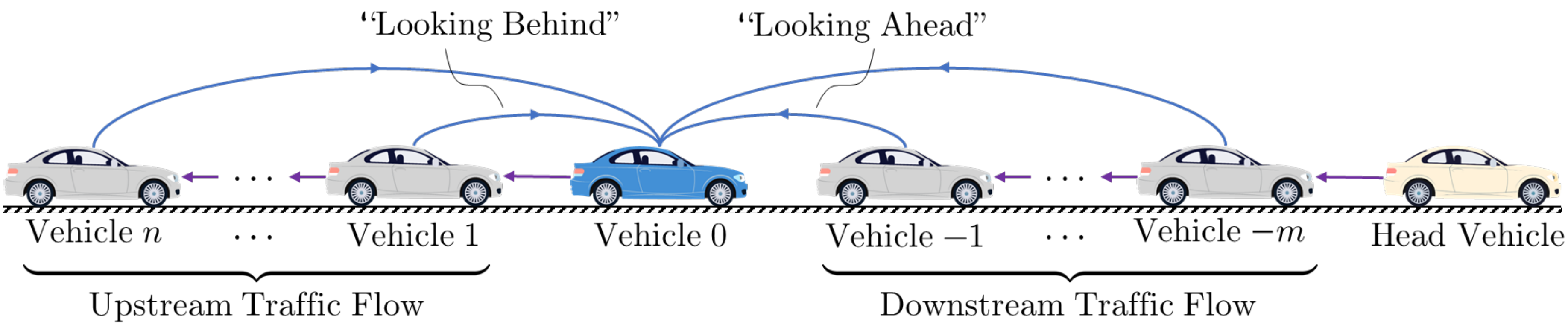


Follow the vehicles ahead: adapt to the motion condition of downstream traffic flow

Lead the vehicles behind: improve the performance of upstream traffic flow

Feature: 1) Explicit consideration of an individual vehicle as both a leader and a follower in traffic flow. 2) Adequate Employment of V2V connectivity: both "looking ahead" and "looking behind".

Leading Cruise Control (LCC)



Question

- How to depict the **dynamics** of LCC?
- Why is it possible to lead the motion of the HDVs **behind**?
- What is the improvement of LCC in dampening the perturbations **ahead**?

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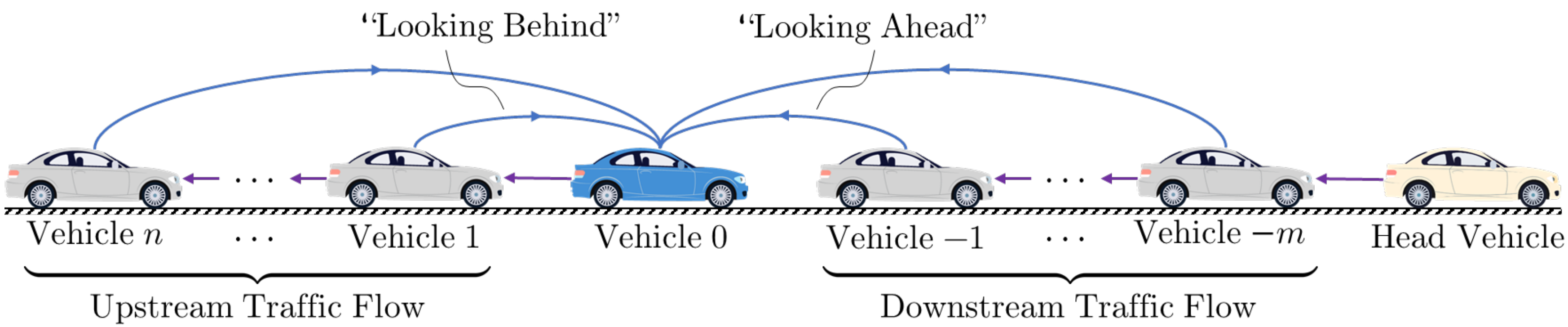
2.1 Scenario Setup

Single-lane straight road



$\Downarrow$

Set of following HDVs behind

$$\mathcal{F} = \{1, 2, \ldots, n\}$$

$\Downarrow$

Set of preceding HDVs ahead

$$\mathcal{P} = \{-1, -2, \ldots, -m\}$$

- The position, velocity and acceleration of vehicle $i \ (i \in \{0\} \cup \mathcal{F} \cup \mathcal{P})$ is denoted as p_i , v_i and a_i , respectively.
- There exists a vehicle in the front of this series of vehicles, whose information is not received by the CAV via V2V, and its velocity is represented as v_{h} .

2.2 Longitudinal Dynamics of Individual Vehicles

Car-following dynamics of HDVs

General formulation

$$\dot{v}_i(t) = F(s_i(t), \dot{s}_i(t), v_i(t)), i = 2, \ldots, n$$

Equilibrium equation

$$F(s_i^*, 0, v_i^*) = 0$$

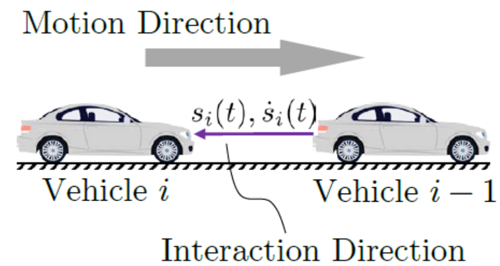
Deviation from the equilibrium state

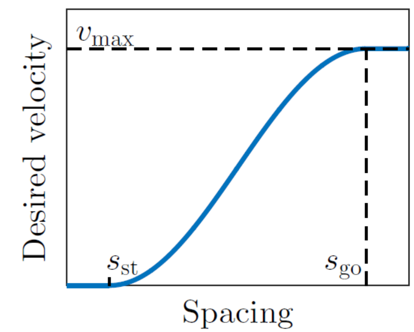
$$\tilde{s}_i(t) = s_i(t) - s^*, \tilde{v}_i(t) = v_i(t) - v^*$$

Linearized car-following model

$$\begin{cases} \dot{\tilde{s}}_i(t) = \tilde{v}_{i-1}(t) - \tilde{v}_i(t), \\ \dot{\tilde{v}}_i(t) = \alpha_1 \tilde{s}_i(t) - \alpha_2 \tilde{v}_i(t) + \alpha_3 \tilde{v}_{i-1}(t). \end{cases}$$

$$\alpha_1 = \frac{\partial F}{\partial s_i}, \alpha_2 = \frac{\partial F}{\partial \dot{s}_i} - \frac{\partial F}{\partial v_i}, \alpha_3 = \frac{\partial F}{\partial \dot{s}_i}$$





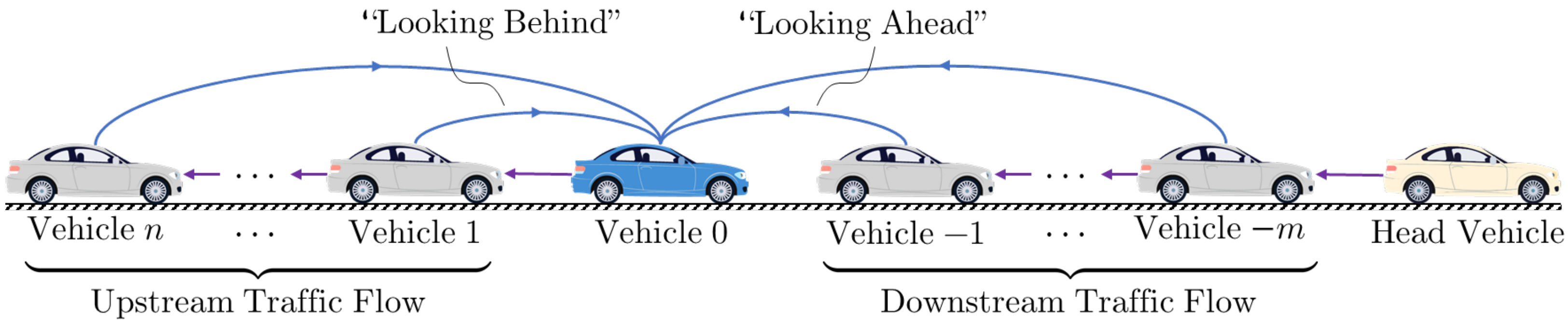
Longitudinal dynamics of CAV

$$\begin{cases} \dot{\tilde{s}}_0(t) = \tilde{v}_{-1}(t) - \tilde{v}_0(t), \\ \dot{\tilde{v}}_0(t) = u(t). \end{cases}$$

Control input $u(t)$

→ acceleration signal of CAV

Modeling General LCC Systems



System states $x(t)=\left[\tilde{s}_{-m}(t),\tilde{v}_{-m}(t),\ldots,\tilde{s}_0(t),\tilde{v}_0(t),\ldots,\tilde{s}_n(t),\tilde{v}_n(t)\right]^T$

Dynamics model for LCC: $\dot{x}(t)=Ax(t)+Bu(t)+H\tilde{v}_{\mathrm{h}}(t)$

$$A=\begin{bmatrix} P_1 & & & & & & & & \\ P_2 & P_1 & & & & & & & \\ & \ddots & \ddots & & & & & & \\ & & P_2 & P_1 & & & & & \\ & & & S_2 & S_1 & & & & \\ & & & & P_2 & P_1 & & & \\ & & & & & \ddots & \ddots & & \\ & & & & & & P_2 & P_1 \end{bmatrix},$$

$$B=\left[b_{-m}^T,\ldots,b_{-1}^T,b_0^T,b_1^T,\ldots,b_n^T\right]^T,$$

$$H=\left[h_{-m}^T,\ldots,h_{-1}^T,h_0^T,h_1^T,\ldots,h_n^T\right]^T.$$

$$P_1=\begin{bmatrix}0 & -1\\ \alpha_1 & -\alpha_2\end{bmatrix},P_2=\begin{bmatrix}0 & 1\\ 0 & \alpha_3\end{bmatrix};$$

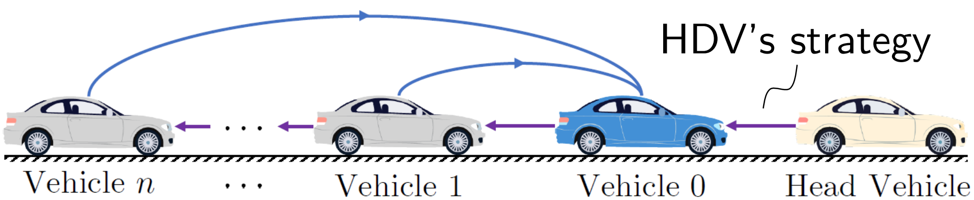
$$S_1=\begin{bmatrix}0 & -1\\ 0 & 0\end{bmatrix},S_2=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix};$$

$$b_0=\begin{bmatrix}0\\ 1\end{bmatrix},b_i=\begin{bmatrix}0\\ 0\end{bmatrix};$$

$$h_{-m}=\begin{bmatrix}1\\ \alpha_3\end{bmatrix},h_j=\begin{bmatrix}0\\ 0\end{bmatrix}.$$

Most current frameworks for CAVs in mixed traffic flow have been limited to focusing on the state of the HDVs ahead. One straightforward distinction of LCC lies in the explicit consideration of those vehicles behind.

Special Case 1: Car-following LCC (CF-LCC)

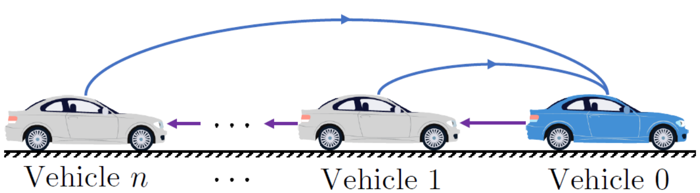


$$x_c(t) = [\tilde{s}_0(t), \tilde{v}_0(t), \tilde{s}_1(t), \tilde{v}_1(t), \ldots, \tilde{s}_n(t), \tilde{v}_n(t)]^T$$

$$\dot{x}_c(t) = A_c x_c(t) + B_1 \hat{u}(t) + H_1 \tilde{v}_h(t)$$

$$A_c = \begin{bmatrix} P_1 & & & \\ P_2 & P_1 & & \\ & \ddots & \ddots & \\ & & P_2 & P_1 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \; H_1 = \begin{bmatrix} 1 \\ \alpha_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Special Case 2: Free-Driving LCC (FD-LCC)



$$x_f(t) = \left[-p_0(t), \tilde{v}_0(t), \tilde{s}_1(t), \tilde{v}_1(t) \ldots, \tilde{s}_n(t), \tilde{v}_n(t)\right]^T$$

$$\dot{x}_f(t) = A_f x_f(t) + B_1 u(t)$$

$$A_f = \begin{bmatrix} S_1 & & & \\ P_2 & P_1 & & \\ & \ddots & \ddots & \\ & & P_2 & P_1 \end{bmatrix}.$$

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3.1 Controllability Results

Controllability of two special LCC and general LCC

Theorem 1: The FD-LCC system with no vehicle ahead and n HDVs behind is completely controllable, if the following condition holds

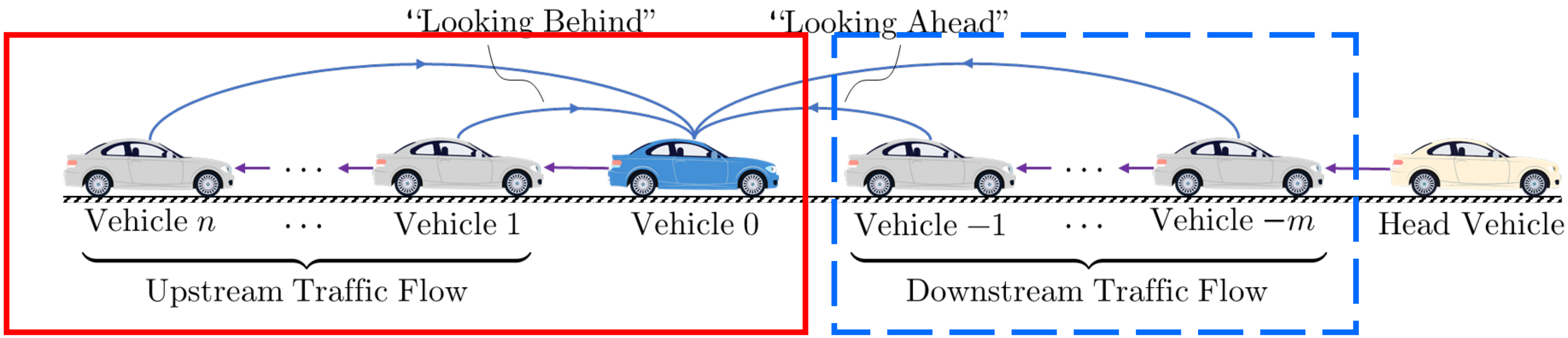
$$\alpha_1 - \alpha_2\alpha_3 + \alpha_3^2 \neq 0 \tag{1}$$

Proof: Popov-Belevitch-Hautus (PBH) controllability test

Corollary 1: The CF-LCC system where the CAV adopts the HDVs' strategy to follow one vehicle ahead and considers n HDVs behind is completely controllable, if the condition (1) holds.

Theorem 2: Consider the general LCC system with m vehicles ahead and n HDVs behind. The subsystem consisting of the states of the vehicles ahead is uncontrollable, while the subsystem consisting of the states of the CAV and the vehicles behind is controllable, if the condition (1) holds.

3.2 Physical Interpretation



$\Downarrow$

The CAV has no influence on the HDVs ahead.

$\Downarrow$

The CAV control input has complete control of the motion of the HDVs behind.

$\Downarrow$ $\Downarrow$

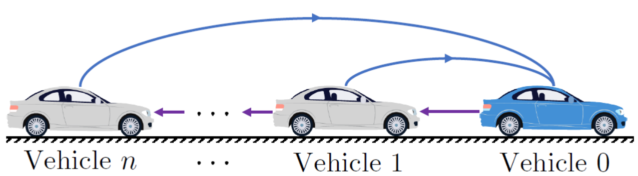
- The CAV has a potential to act as a sophisticated leader with global consideration, e.g., aiming to improve the performance of the entire upstream traffic flow.
- This result generalizes the previous stabilizability results in a closed ring road, where it has been shown that one single CAV can stabilize the entire traffic flow [Cui et al., 2019; Zheng et al., 2020].

3.3 Traffic Simulation

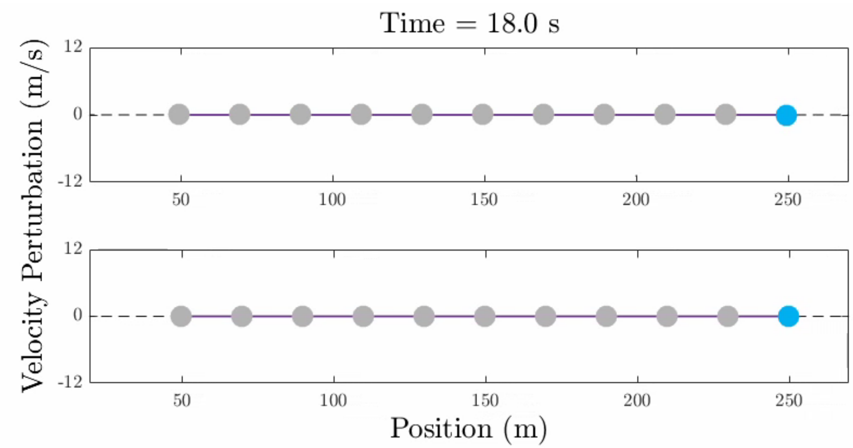
Simulation Setup (the perturbation happens in an HDV behind)

- $n = 10$; at $t = 20\,\mathrm{s}$, vehicle 1 brakes at $-5\,\mathrm{m/s^2}$ for one second.
- A static state feedback controller for CAV with consideration of two following HDVs.

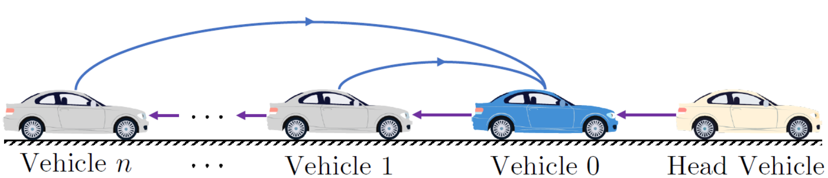
FD-LCC



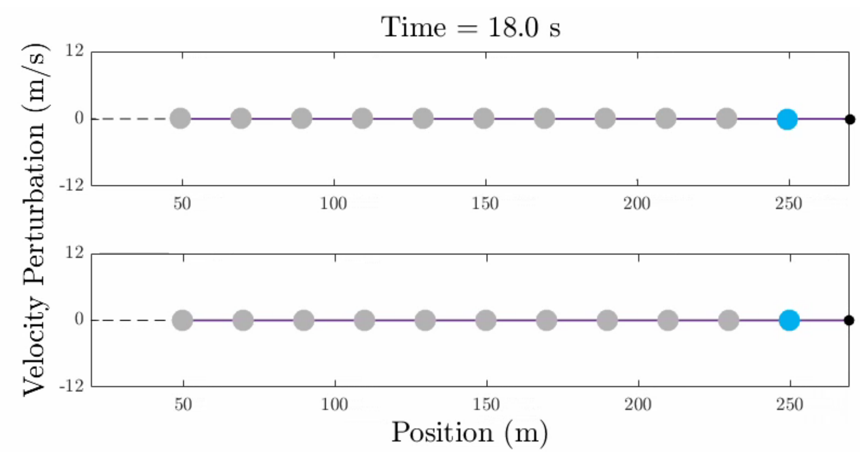
Reduce velocity perturbations by 42.98%.



CF-LCC



Reduce velocity perturbations by 25.72%.



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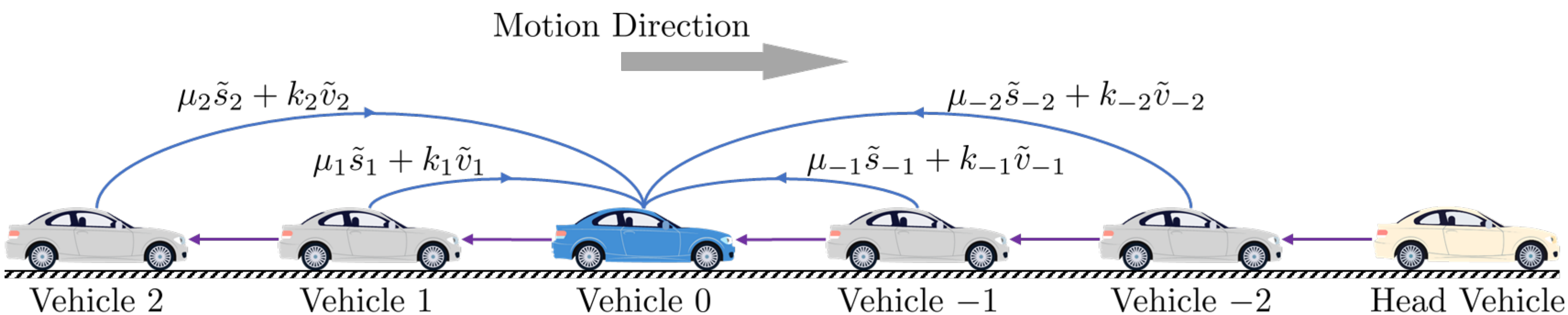
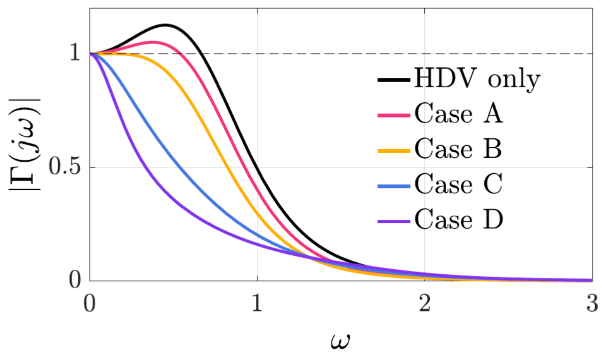
Head-to-Tail String Stability of LCC Systems

We have derived the head-to-tail transfer function of general LCC systems with m vehicles ahead and n vehicles behind.



	μ_{-2}	k_{-2}	μ_{-1}	k_{-1}	μ_1	k_1	μ_2	k_2
Case A	1	-1	0	0	0	0	0	0
Case B	1	-1	1	-1	0	0	0	0
Case C	1	-1	1	-1	-1	-1	0	0
Case D	1	-1	1	-1	-1	-1	-1	-1

Parameter setup in feedback gains



Frequency-domain response

"Looking behind" could further improve the CAV's capability in dampening traffic waves.

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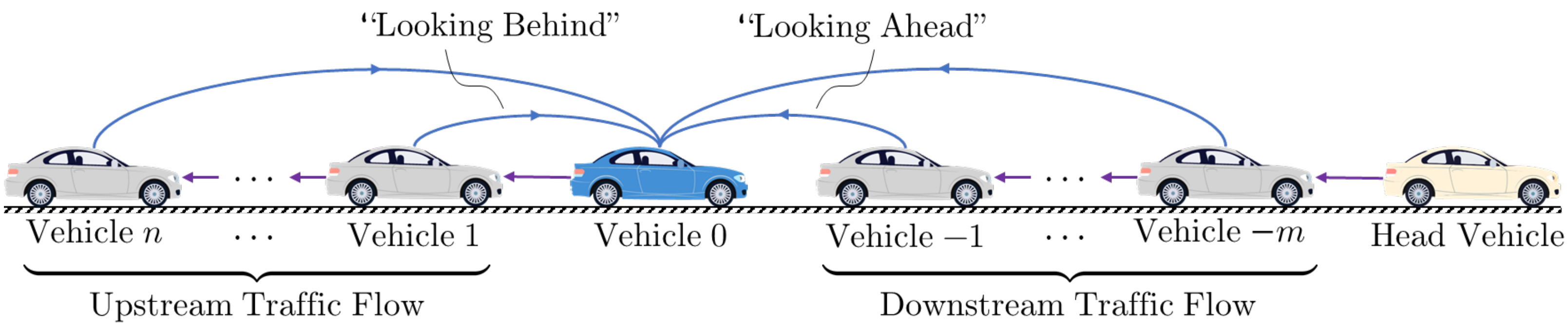
Take-Home Messages



Message 1: Notion of Leading Cruise Control (LCC)

- Retain the car-following operation + consider the influence on vehicles behind.
- Adequate employment of vehicle-to-vehicle connectivity.

Message 2: Controllability results

- The CAV has a potential to act as a leader to actively lead the motion of its following vehicles and improve the performance of the entire upstream traffic flow.

Message 3: Head-to-tail string stability results

- LCC could strengthen the capability of CAVs in reducing traffic instabilities and smoothing traffic flow.

Future work: design more sophisticated controllers for LCC.

Thank you for your attention!

- Wang, J., Zheng, Y., Chen, C., Xu, Q., & Li, K. (2020). Leading Cruise Control in Mixed Traffic Flow. arXiv preprint arXiv:2007.11753.

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Backup

Head-to-Tail String Stability of LCC Systems

A.1 Definition

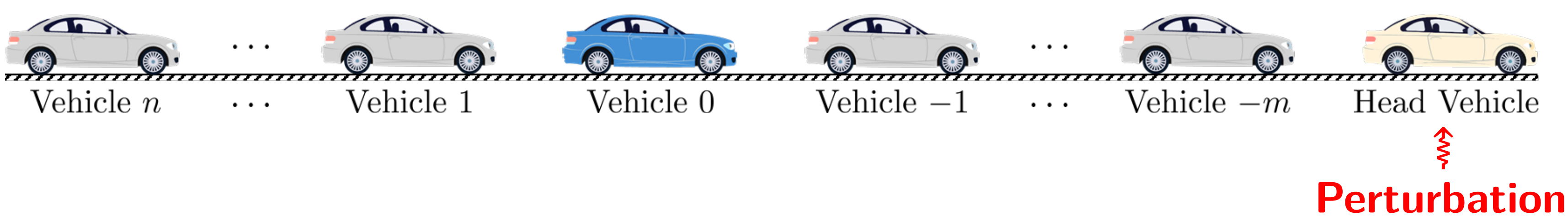
Head-to-tail string stability

Given a series of consecutive vehicles, denote the velocity deviation of the vehicle at the head and the one at the tai $\tilde{v}_{\mathrm{h}}(t)$ and $\tilde{v}_{\mathrm{t}}(t)$, respectively. The head-to-tail transfer function is defined as

$$\Gamma(s) = \frac{\widetilde{V}_{\mathrm{t}}(s)}{\widetilde{V}_{\mathrm{h}}(s)},$$

where $\widetilde{V}_{\mathrm{h}}(s), \widetilde{V}_{\mathrm{t}}(s)$ denote the Laplace transform of $\tilde{v}_{\mathrm{h}}(t)$ and $\tilde{v}_{\mathrm{t}}(t)$, respectively. Then the system is called head-to-tail string stable if

$$|\Gamma(j\omega)|^2 < 1, \forall \omega > 0$$

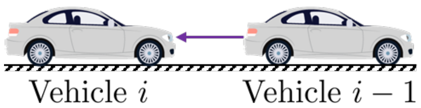


Motivation: Previous research mostly focuses on the transfer function from the head vehicle to the CAV itself, instead of a certain vehicle behind the CAV. It is worth noting that the perturbations will continue to propagate upstream after reaching the CAV.

A.2 Head-to-Tail Transfer Function of LCC

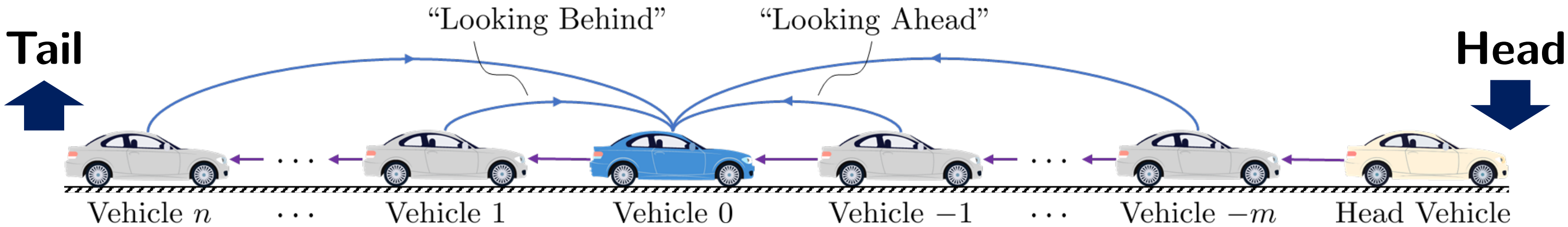
Local transfer function of one HDV

$$\frac{\widetilde{V}_i(s)}{\widetilde{V}_{i-1}(s)} = \frac{\alpha_3 s + \alpha_1}{s^2 + \alpha_2 s + \alpha_1} = \frac{\varphi(s)}{\gamma(s)}$$



Control input of the CAV

$$u(t) = \alpha_1 \tilde{s}_0 - \alpha_2 \tilde{v}_0 + \alpha_3 \tilde{v}_{-1} + \sum_{i \in \mathcal{F} \cup \mathcal{P}} \left(\mu_i \tilde{s}_i(t) + k_i \tilde{v}_i(t)\right)$$



Head-to-tail transfer function of LCC:

$$\Gamma(s) = \frac{\widetilde{V}_n(s)}{\widetilde{V}_\mathrm{h}(s)} = \frac{\varphi(s) + \sum_{i \in \mathcal{P}} H_i(s) \left(\frac{\varphi(s)}{\gamma(s)}\right)^{i+1}}{\gamma(s) - \sum_{i \in \mathcal{F}} H_i(s) \left(\frac{\varphi(s)}{\gamma(s)}\right)^{i}} \cdot \left(\frac{\varphi(s)}{\gamma(s)}\right)^{n+m},$$

where $H_i(s) = \mu_i \left(\gamma(s)/\varphi(s) - 1\right) + k_i s,\ i \in \mathcal{F} \cup \mathcal{P}$.

A.2 Head-to-Tail Transfer Function of LCC

Special cases of head-to-tail transfer function of LCC

- Case 1: $\mu_i = k_i = 0,\ i \in \mathcal{F} \cup \mathcal{P}$ $\Leftrightarrow$ A platoon of $n+m+1$ HDVs

$$\Gamma_1(s) = \left(\frac{\varphi(s)}{\gamma(s)}\right)^{n+m+1}$$

- Case 2: $\mu_i = k_i = 0,\ i \in \mathcal{F}$ $\Leftrightarrow$ "Looking ahead" only

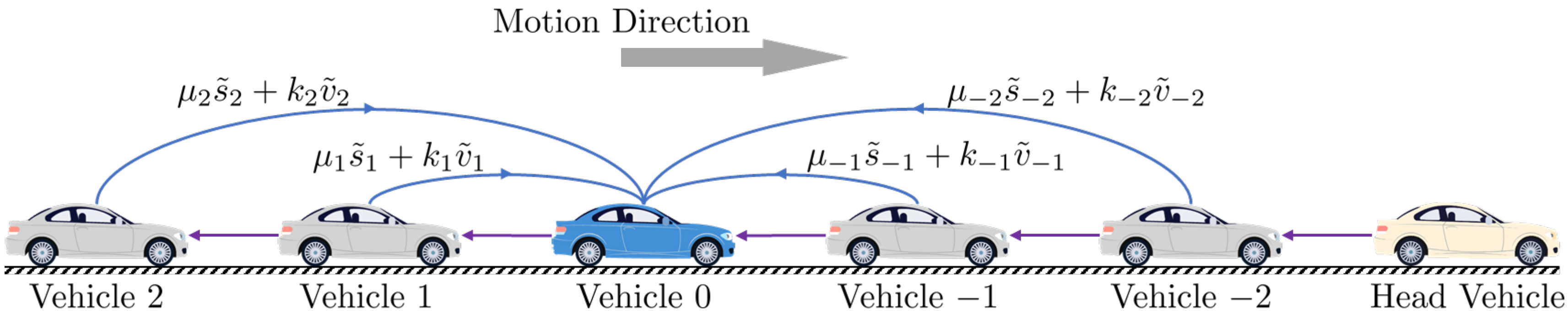
$$\Gamma_2(s) = \frac{\varphi(s) + \sum_{i \in \mathcal{P}} H_i(s) \left(\frac{\varphi(s)}{\gamma(s)}\right)^{i+1}}{\gamma(s)} \cdot \left(\frac{\varphi(s)}{\gamma(s)}\right)^{n+m}$$

- Case 3: $\mu_i = k_i = 0,\ i \in \mathcal{P}$ $\Leftrightarrow$ "Looking behind" only

$$\Gamma_3(s) = \frac{\varphi(s)}{\gamma(s) - \sum_{i \in \mathcal{F}} H_i(s) \left(\frac{\varphi(s)}{\gamma(s)}\right)^{i}} \cdot \left(\frac{\varphi(s)}{\gamma(s)}\right)^{n+m}$$

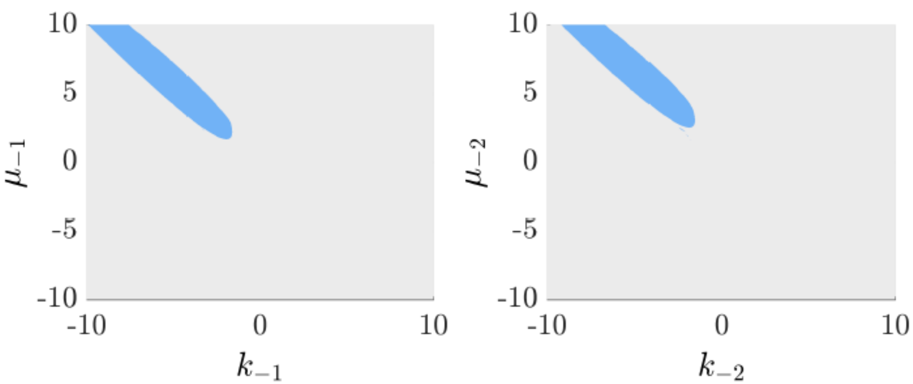
The incorporation of either the state of the preceding vehicles or the following vehicles brings a significant change to the head-to-tail transfer characteristic of mixed traffic flow, but the changes of the two types work in different ways.

Scenario setup: $m = 2, n = 2$

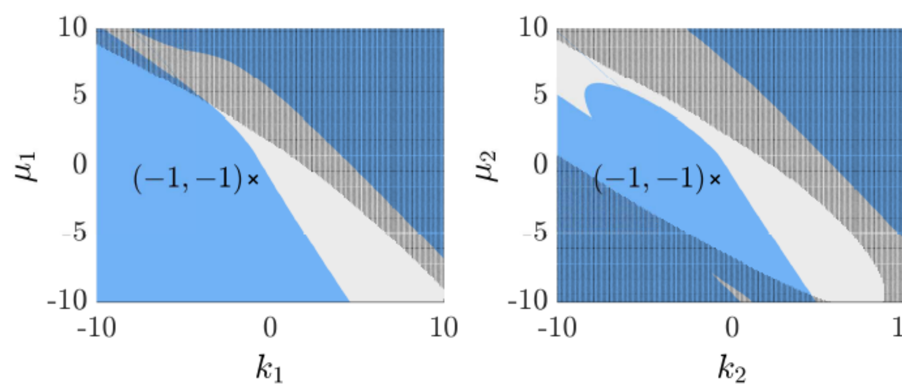


Numerical Study A: string stable region when monitoring one HDV

"looking ahead" string stable region

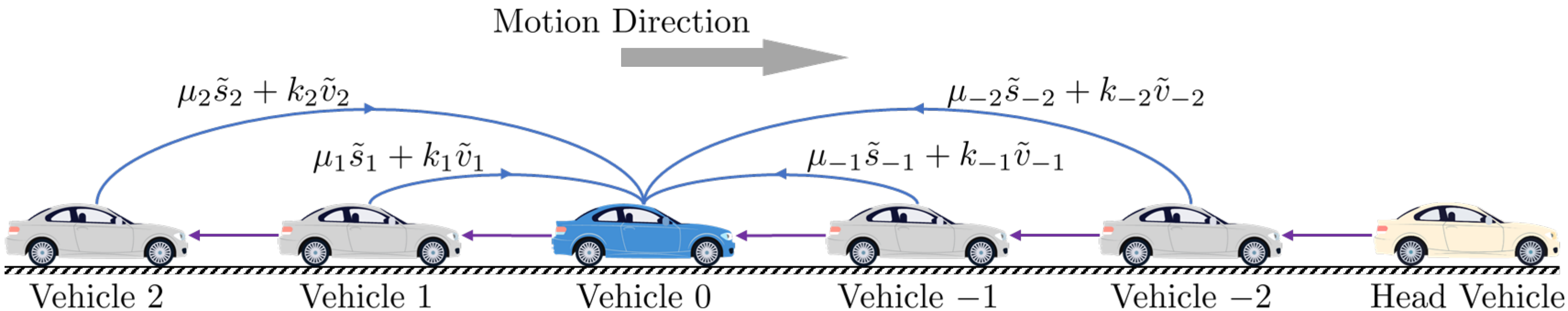


"looking behind" string stable region



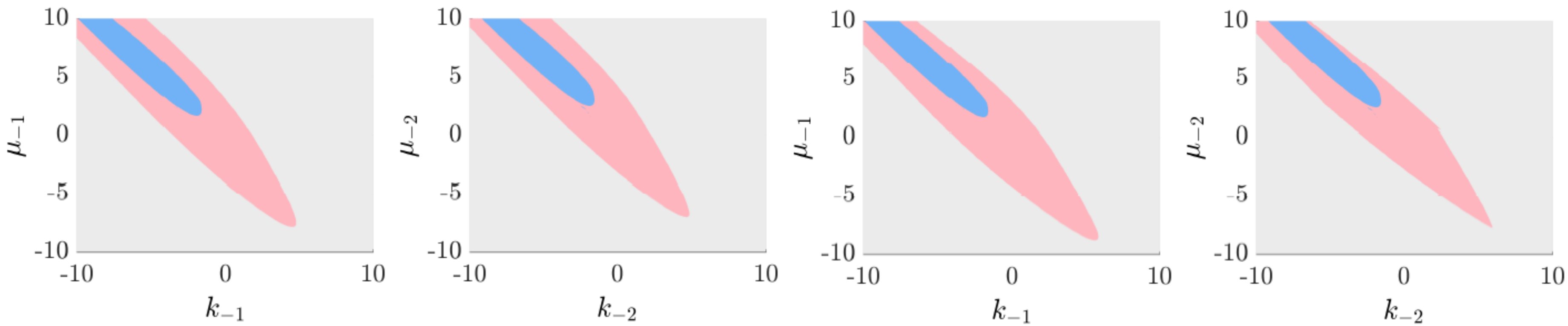
"Looking behind" only has more string stable choices.

Scenario setup: $m = 2, n = 2$



Numerical Study B: influence of "looking behind" on the original "looking ahead" string stable region (blue domain → original region; red domain → expanded region)

$\mu_1 = -1, k_1 = -1$ $\quad\quad$ $\mu_2 = -1, k_2 = -1$



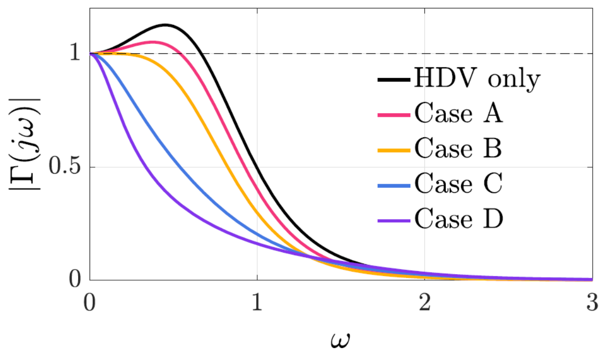
"Looking behind" simultaneously allows for more "looking ahead" string stable choices.

A.3 Numerical Results

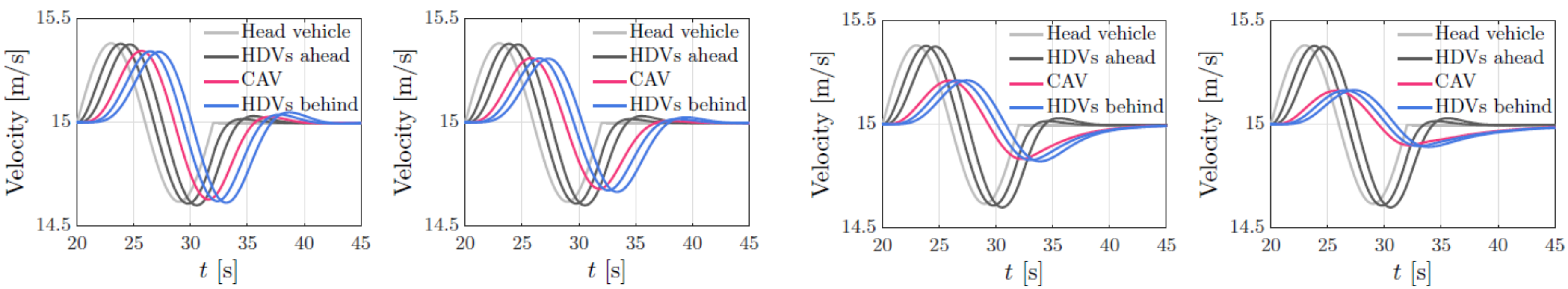
Numerical Study C: case study in mitigating traffic perturbations

	μ_{-2}	k_{-2}	μ_{-1}	k_{-1}	μ_1	k_1	μ_2	k_2
Case A	1	-1	0	0	0	0	0	0
Case B	1	-1	1	-1	0	0	0	0
Case C	1	-1	1	-1	-1	-1	0	0
Case D	1	-1	1	-1	-1	-1	-1	-1

Parameter setup in feedback gains



Frequency-domain response



(a) Case A (b) Case B (c) Case C (d) Case D

Time-domain response

"Looking behind" could further improve the CAV's capability in dampening traffic waves.