Optimal Formation of Autonomous Vehicles in Mixed Traffic Flow

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1.1 Cooperative Formation of Autonomous Vehicles

Background: The emergence of autonomous vehicles (AVs) is expected to improve traffic flow. In particular, **cooperative formation and control of multiple AVs** promises to revolutionize road transportation systems.

Typical example: vehicle platooning

PATH

SARTRE

Energy ITS

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- **Major objective:** regulate adjacent vehicles to maintain a same desired velocity while keeping a pre-specified inter-vehicle distance
- **Performance:** achieve higher traffic efficiency, better driving safety and lower fuel consumption

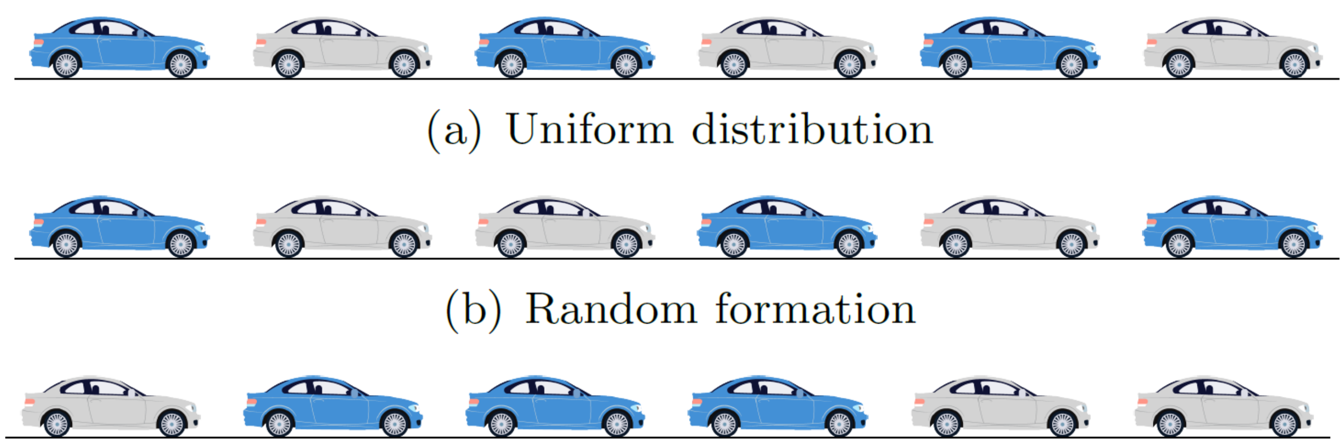
1.2 Practical challenge for implementation of platooning

Long-term transition phase: **mixed traffic flow**, i.e., coexistence of human-driven vehicles (HDVs) and AVs

Maneuvers for forming a platoon, e.g., joining, leaving, merging, and splitting, are wave triggers, and **might even cause undesired congestions** [Jesús Mena-Oreja et al., 2018].

Motivation: Existing results suggest that forming a platoon for AVs might not be necessary in mixed traffic flow. Is there any other possibility?

Examples for other possibilities



(a) Uniform distribution

(b) Random formation

(c) Platoon formation

Question

- Which formation of AVs could achieve the optimal system-wide performance for the entire traffic flow?
- In particular, does the prevailing platoon formation perform better than other formations in mixed traffic flow?

⇓ **Approach:** set function optimization formulation ⇓

Answer

- There exist two predominant optimal formations: uniform distribution and platoon formation, depending on traffic parameters.
- The prevailing platoon formation is not always the optimal choice, and it might have the least potential when HDVs have a poor string stability behavior.

⇓ ⇓

Suggest more opportunities for the formation of AVs, beyond platooning, in mixed traffic flow.

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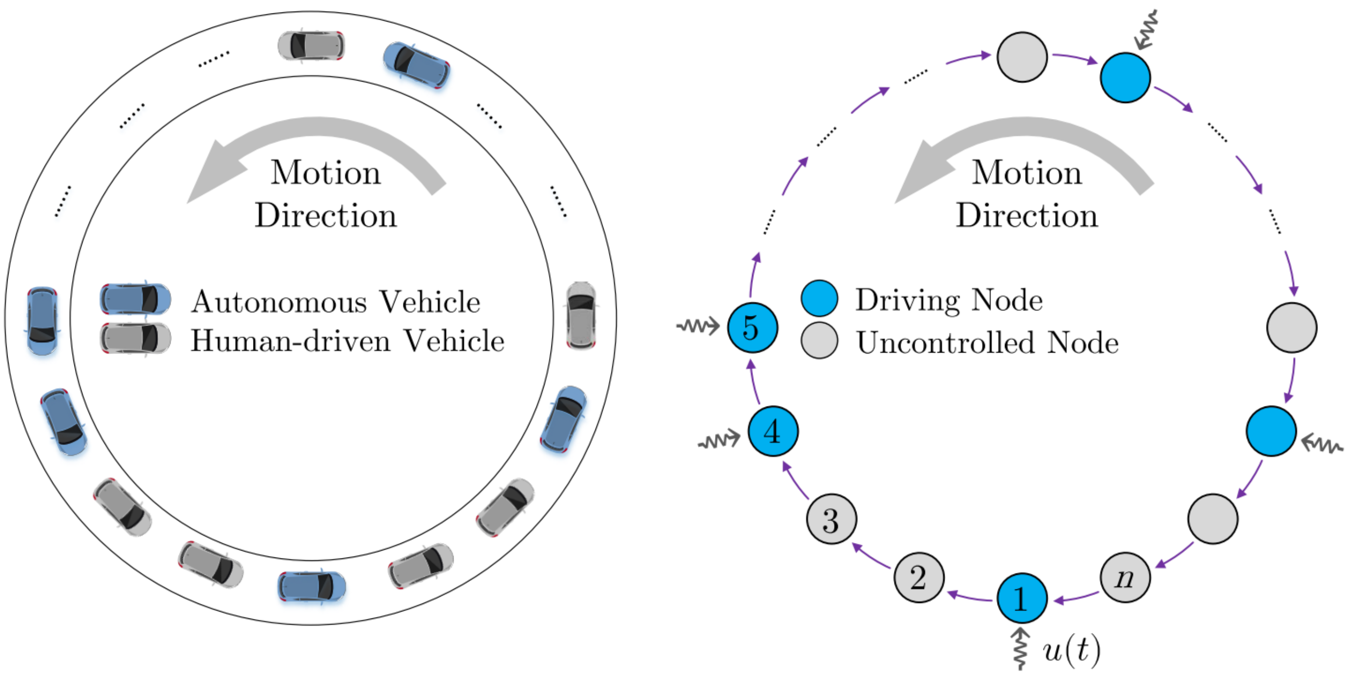
Scenario: single-lane ring road

A widely used setup in the literature

- Represent a simplified closed traffic system with no boundary conditions
- Correspond to a straight road of infinite length and periodic traffic dynamics
- Validation of existing experimental results [Yuki Sugiyama et al., 2008; Raphael E. Stern et al., 2018]



Set-variable representation: characterize the formation of the AVs by their spatial locations in mixed traffic flow: $S = \{i_1, \ldots, i_k\} \subseteq \Omega = \{1, 2, \ldots, n\}$.

Dynamics model of the mixed traffic system

HDV's longitudinal dynamics $(i \notin S)$

- $\dot{v}_i(t) = F(s_i(t), \dot{s}_i(t), v_i(t))$
- Deviation from the equilibrium state

  $\tilde{s}_i(t) = s_i(t) - s^*, \tilde{v}_i(t) = v_i(t) - v^*$
- $\begin{cases} \dot{\tilde{s}}_i(t) = \tilde{v}_{i-1}(t) - \tilde{v}_i(t), \\ \dot{\tilde{v}}_i(t) = \alpha_1 \tilde{s}_i(t) - \alpha_2 \tilde{v}_i(t) + \alpha_3 \tilde{v}_{i-1}(t). \end{cases}$

AV's longitudinal dynamics $(i \in S)$

- Control input

  $u_i(t)$ → acceleration signal
- Longitudinal dynamics

  $\begin{cases} \dot{\tilde{s}}_i(t) = \tilde{v}_{i-1}(t) - \tilde{v}_i(t), \\ \dot{\tilde{v}}_i(t) = u_i(t). \end{cases}$

Lumped states for the traffic system

$$x(t) = [\tilde{s}_1(t), \ldots, \tilde{s}_n(t), \tilde{v}_1(t), \ldots, \tilde{v}_n(t)]^T,\ u(t) = [u_{i_1}(t), \ldots, u_{i_k}(t)]^T,\ \omega(t) = [\omega_1(t)\ , \ldots, \omega_n(t)]^T.$$

State-space model of mixed traffic system: $\dot{x}(t) = A_S x(t) + B_S u(t) + H\omega(t).$

$$A_S = \begin{bmatrix} 0 & M_1 \\ \alpha_1 (I_n - D_S) & P_S \end{bmatrix} \in \mathbb{R}^{2n \times 2n};$$

$$B_S = [\mathbb{e}_{i_1}, \mathbb{e}_{i_2}, \ldots, \mathbb{e}_{i_k}] \in \mathbb{R}^{2n \times k};$$

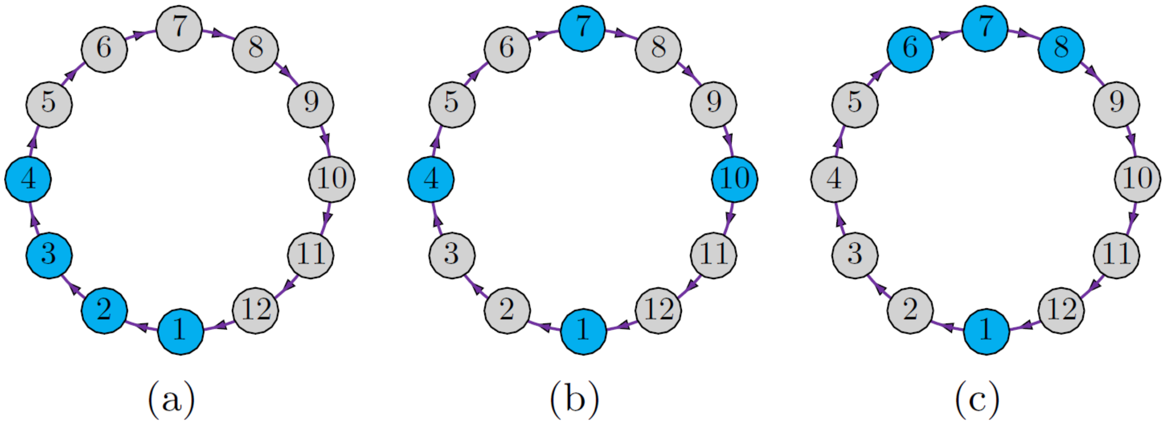
$$H = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \in \mathbb{R}^{2n \times n}.$$

$$M_1 = \begin{bmatrix} -1 & & \cdots & 1 \\ 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & -1 \end{bmatrix}; \quad P_S = \begin{bmatrix} -\alpha_2 \bar{\delta}_1 & & \cdots & \alpha_3 \bar{\delta}_1 \\ \alpha_3 \bar{\delta}_2 & -\alpha_2 \bar{\delta}_2 & & \\ & \ddots & \ddots & \\ & & \alpha_3 \bar{\delta}_n & -\alpha_2 \bar{\delta}_n \end{bmatrix}.$$

$$D_S = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_n);\ \ \delta_i = 0, \text{ if } i \notin S;\ \delta_i = 1, \text{ if } i \in S.\ \ \bar{\delta}_i = 1 - \delta_i.$$

2.3 Problem statement of optimal formation

Illustrations of possible formations in a ring-road system ($n = 12, k = 4$)



(a) Platoon formation, *i.e.*, $S = \{1, 2, 3, 4\}$.
(b) Uniform distribution, *i.e.*, $S = \{1, 4, 7, 10\}$.
(c) Abnormal formation, *i.e.*, $S = \{1, 6, 7, 8\}$.

Problem statement: Assume there are k AVs in the ring-road mixed traffic system. Find an optimal formation, *i.e.*, $S = \{i_1, \ldots, i_k\} \subseteq \Omega$, for the AVs, which achieves the optimal system-wide performance for the entire traffic flow.

General formulation: $$\max_{S} \; J(S), \quad \text{s.t. } S \subseteq \Omega, |S| = k.$$

Remained factors: 1) AVs' control strategy $u(t)$; 2) performance value function $J(S)$.

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3.1 Redesign the optimal controller

Redesigned strategy: Given a formation S of the AVs, consider a static feedback controller $u = -K_S x$, $K_S \in \mathbb{R}^{2n\times k}$. The control objective is to achieve an optimal performance for the global traffic system via controlling the AVs under this formation.

Performance output

$$z(t) = \begin{bmatrix} Q^{\frac{1}{2}} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ R^{\frac{1}{2}} \end{bmatrix} u(t),$$
$$Q^{\frac{1}{2}} = \mathrm{diag}\left(\gamma_s, \ldots, \gamma_s, \gamma_v, \ldots, \gamma_v\right),\ R^{\frac{1}{2}} = \mathrm{diag}(\gamma_u, \ldots, \gamma_u)$$

Closed-loop system

$$\dot{x}(t) = (A_S - B_S K_S)x(t) + Hw(t),$$
$$z(t) = \begin{bmatrix} Q^{\frac{1}{2}} \\ -R^{\frac{1}{2}} K_S \end{bmatrix} x(t).$$

$\mathcal{H}_2$ optimal control

- Denote $G(S)$ as the transfer function from disturbance w to output z

$$\min_{K_S} \|G(S)\|_2^2$$

Reformulation of optimal formation

Convex reformulation of optimal control [Yang Zheng, et al., 2020]

$$\min_{K_S} \|G(S)\|_2^2 \quad \Leftrightarrow \quad \begin{aligned} \min_{X,Y,Z} \ & \mathrm{Tr}(QX) + \mathrm{Tr}(RY) \\ \text{s.t.} \ & (A_S X - B_S Z) + (A_S X - B_S Z)^T + HH^T \preceq 0, \\ & \begin{bmatrix} Y & Z \\ Z^T & X \end{bmatrix} \succeq 0, \ X \succ 0, \end{aligned}$$

Interpretation: The optimal value of $\min_{K_S} \|G_S\|_2^2$ indicates the minimum influence of perturbations on the entire traffic flow when the AVs are optimally controlled, thereby revealing the maximum potential of a given formation S of the AVs.

Reformulation of optimal formation problem

$$\begin{aligned} \max_{S} \quad & J(S) = -\min_{K_S} \|G_S\|_2^2 \\ \text{s.t.} \quad & S \subseteq \Omega, |S| = k \end{aligned}$$

Feasibility: It is proved that the ring-road mixed traffic system with one or more AVs is stabilizable, leading to the existence of stabilizing feedback gains. [Yang Zheng, et al., 2020]

Submodularity analysis

Definition of submodularity

A set function $f : 2^{\Omega} \to \mathbb{R}$ is called submodular if for all $A \subseteq B \subseteq \Omega$ and all elements $e \in \Omega$, it holds that

$$f(A \cup \{e\}) - f(A) \geq f(B \cup \{e\}) - f(B).$$

- For a submodular and monotone increasing set function, greedy algorithm can return a near-optimal solution with performance guarantees [Tyler H. Summers, et al., 2016].

Counterexample for submodularity of $J(S)$:

$$\text{Set } \alpha_1 = 0.5, \alpha_2 = 2.5, \alpha_3 = 0.5,\ \gamma_s = 0.01, \gamma_v = 0.05, \gamma_u = 0.1$$

$$\text{Let } S_1 = \{4, 9, 10\} \text{ and } S_2 = \{2, 3, 4, 9, 10\}, \text{ which implies } S_1 \subseteq S_2.$$

$$\begin{array}{l} J_2\left(S_1 \cup \{1\}\right) = -0.5982, J_2\left(S_1\right) = -0.5003; \\ J_2\left(S_2 \cup \{1\}\right) = -0.7860, J_2\left(S_2\right) = -0.6910. \end{array} \Rightarrow \begin{array}{l} J_2\left(S_1 \cup \{1\}\right) - J_2\left(S_1\right) = -0.098 \\ \leq J_2\left(S_2 \cup \{1\}\right) - J_2\left(S_2\right) = -0.095. \end{array}$$

To obtain the true optimal formation solution, the greedy algorithm cannot provide any guarantees, while the brute force method is one straightforward approach.

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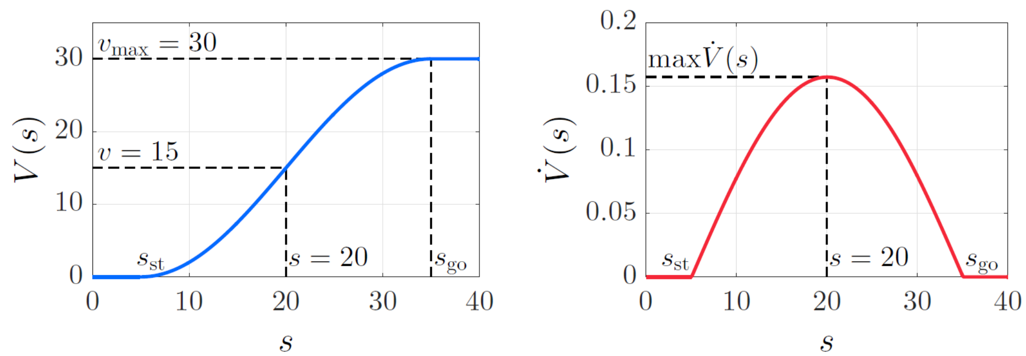
4.1 Numerical setup

Specific model for HDVs: optimal velocity model (OVM) [Jin I. Ge, et al., 2017]

$$F(\cdot) = \alpha\left(V\left(s_i(t)\right) - v_i(t)\right) + \beta \dot{s}_i(t)$$



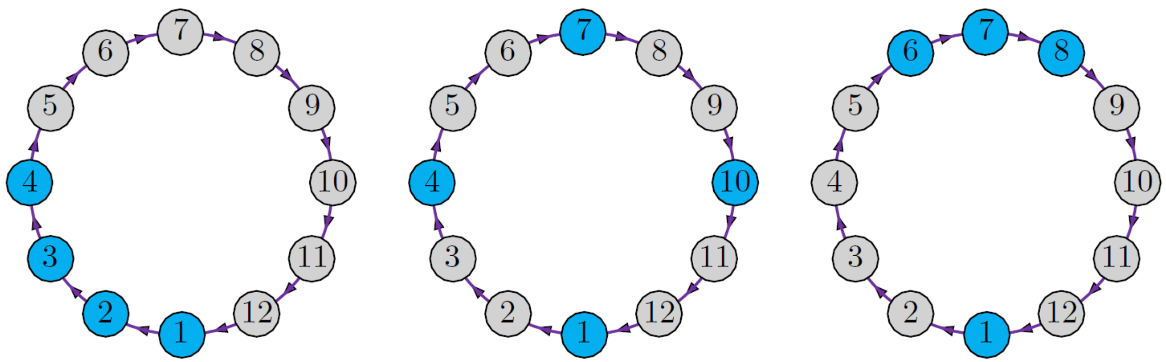
String stability

- A string of multiple vehicles is called string unstable if the amplitude of certain oscillations are amplified along the propagation upstream the traffic flow.
- Strict string stability condition for OVM after linearization [Gábor Orosz, et al., 2010]

$$\alpha + 2\beta \geq 2\dot{V}(s^*)$$

Define a string stability index: $\xi := \alpha + 2\beta - 2\dot{V}\left(s^*\right)$. A larger value of α, β or $\left|s^* - 20\right|$ leads to a larger value of ξ, i.e., a better string stability behavior of HDVs.

Fix $n = 12, v_{\max} = 30, s_{\mathrm{st}} = 5, s_{\mathrm{go}} = 35, \gamma_s = 0.01, \gamma_v = 0.05, \gamma_u = 0.1$



(a) (b) (c)

Table 1: Optimal Formation ($n = 12$, $k = 4$)

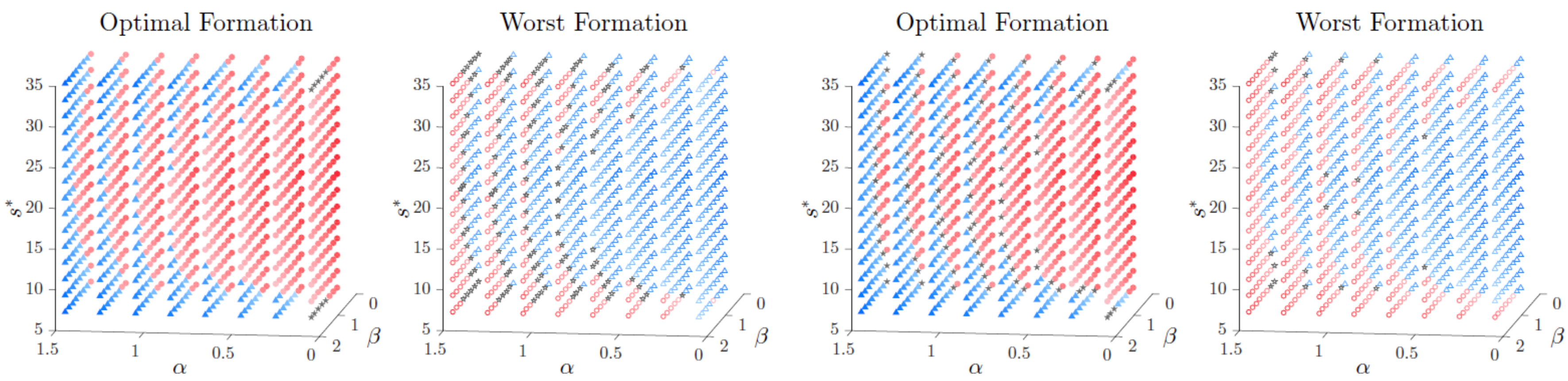
α	β	s^*	numerical solution
1.4	1.8	10	platoon formation (a)
0.6	0.9	20	uniform distribution (b)
0.9	1.3	16	abnormal formation (c)

- The abnormal formation is essentially a transition pattern between platoon formation and uniform distribution.

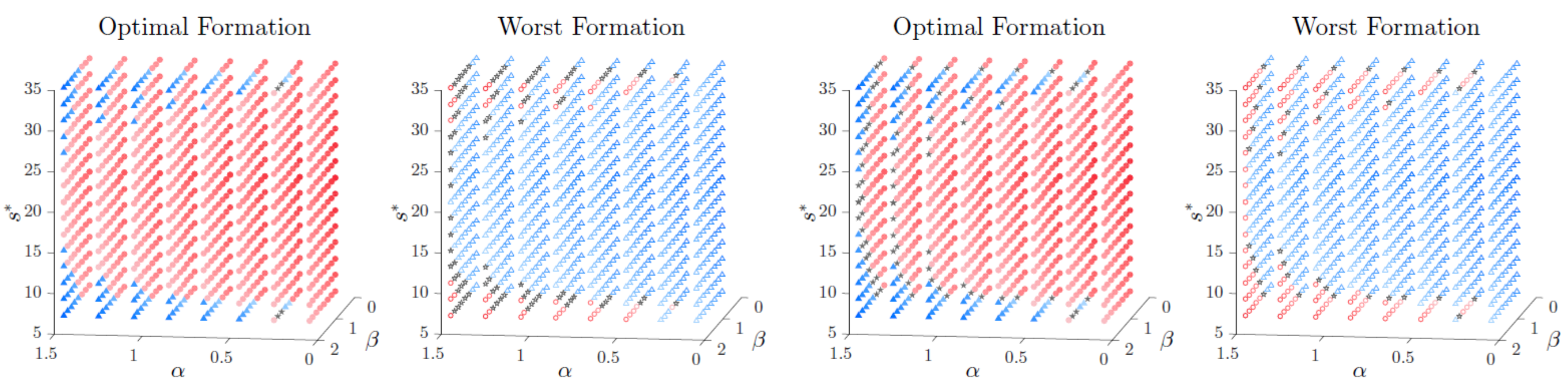
4.3 Two predominant optimal formations

Consider various parameter setups (various HDVs' behaviors)

- The worst formation can be also obtained by the optimal formation problem.

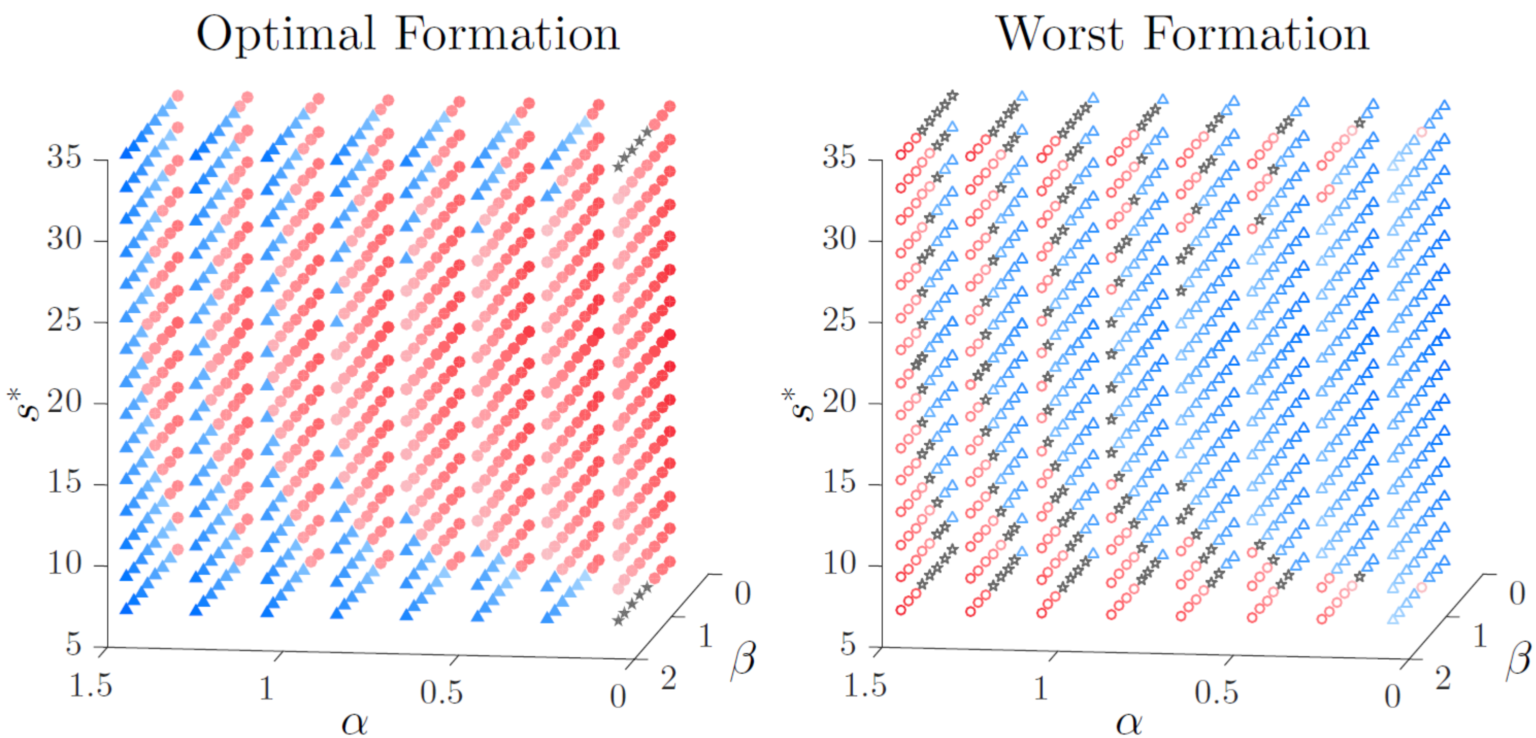


(a) $n = 12, k = 2$ (b) $n = 12, k = 4$



(c) $n = 12, k = 2$ (d) $n = 12, k = 4$

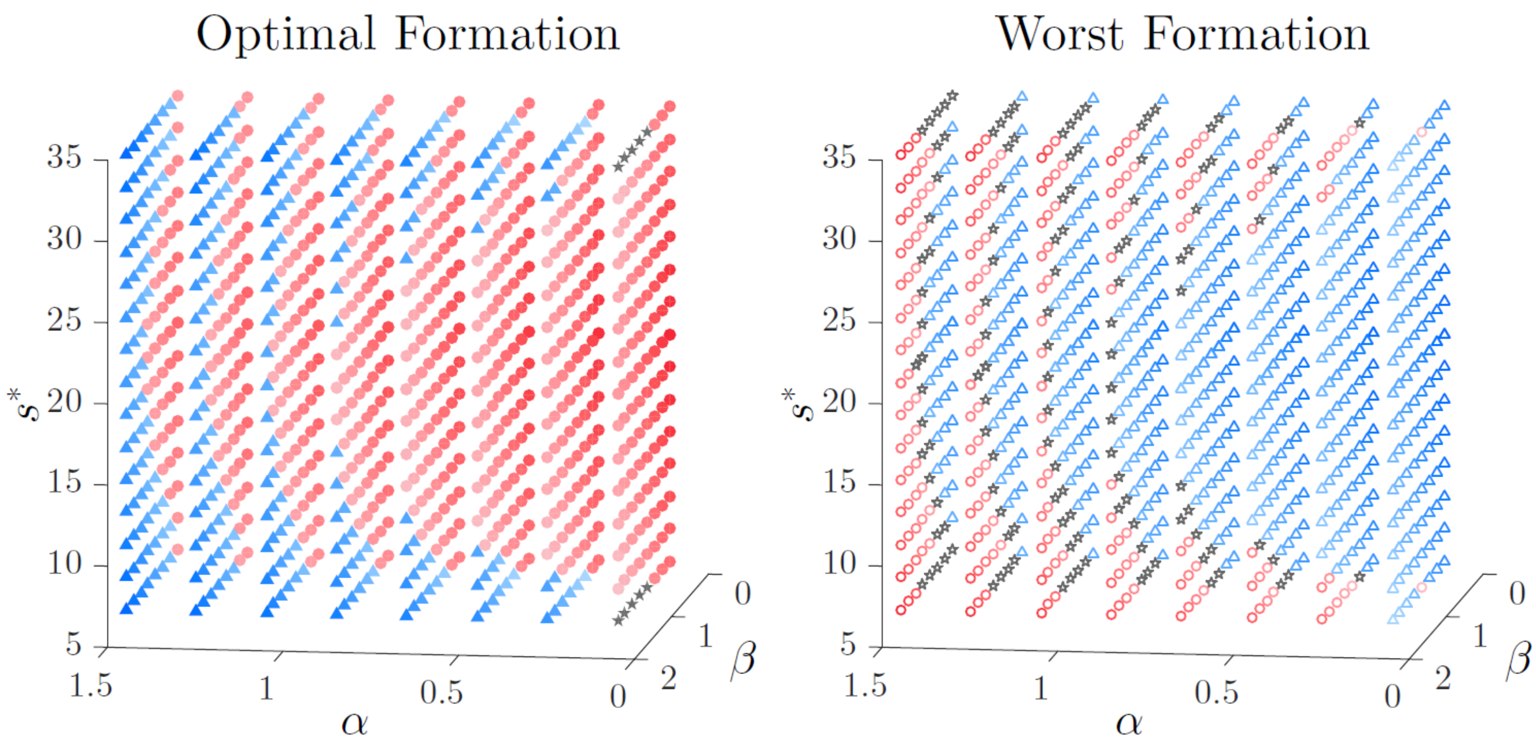
4.3 Two predominant optimal formations



red circle	blue triangle	gray stars
uniform distribution	platoon formation	abnormal formation

Observation 1: There exist two predominant patterns for optimal or worst formations: platoon formation and uniform distribution.

4.3 Two predominant optimal formations



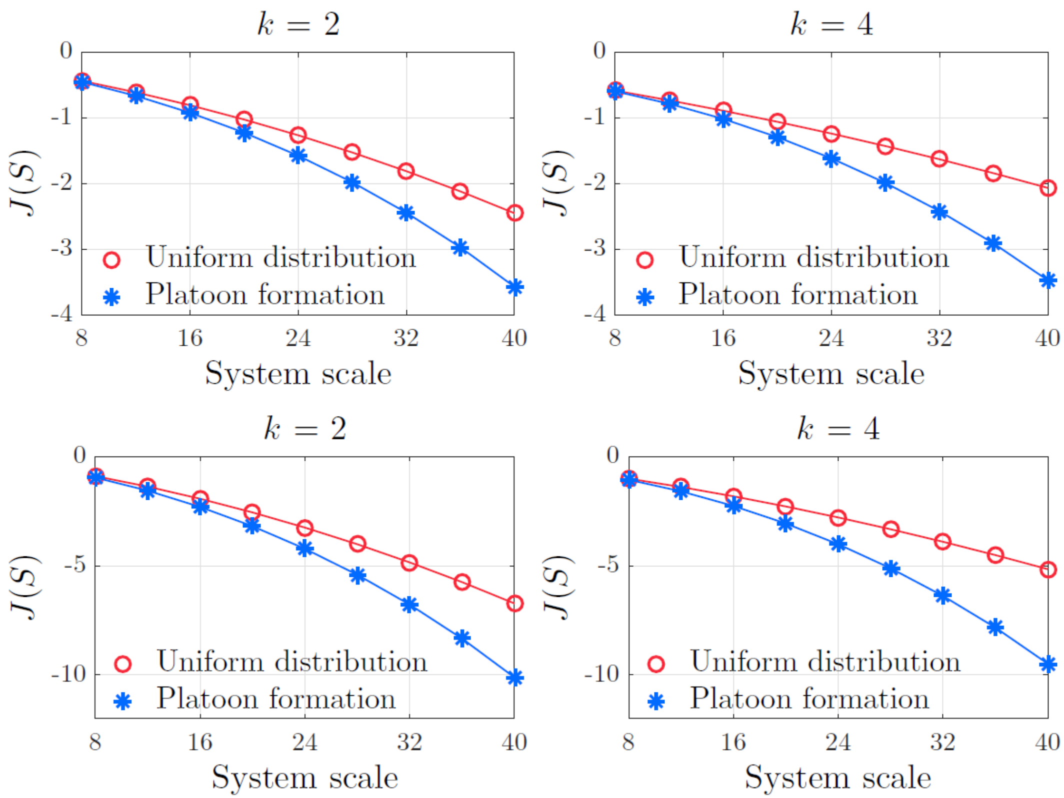
- Color darkness denotes the changing direction of HDV's string stability
  dark blue → light blue → light red → dark red: $\xi = \alpha + 2\beta - 2\dot{V}\left(s^*\right) \downarrow$

Observation 2: The specific optimal formation depends on the HDVs' string stability. Platooning becomes the worst choice when HDVs have a poor string stability behavior.

Vary the system scales

Change system scales under a fixed number of autonomous vehicles

- a typical setup for HDVs: $\alpha = 0.6, \beta = 0.9, s^* = 20, v_{\max} = 30, s_{\text{st}} = 5, s_{\text{go}} = 35$



Observation 3: The performance gap between the two formations is rapidly enlarged as the system scale grows up.

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5 Conclusions

Question

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$\Downarrow$ **Approach:** set function optimization formulation $\Downarrow$

Answer

- There exist **two predominant optimal formations**: uniform distribution and platoon formation, depending on traffic parameters.
- The prevailing platoon formation is **NOT** always the optimal choice, and it might have **the least potential** when HDVs have a poor string stability behavior.

$\Downarrow$ $\Downarrow$

Suggest more opportunities for the formation of AVs, beyond platooning, in mixed traffic flow.

Thank you for your attention!

- Keqiang Li, Jiawei Wang, Yang Zheng. (2020). Optimal Formation of Autonomous Vehicles in Mixed Traffic Flow. arXiv:2004.00397.

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