

A Novel Road Traffic Risk Modeling Approach Based on the Traffic Safety Field Concept

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ABSTRACT

Previous research on driving safety field theory has shown that driving risk can be perceived as the danger approaches. However, the influence factors of the driver, vehicle, and road are difficult to determine. This study proposes a novel road traffic risk modeling approach based on the dynamic analysis of vehicles in driving scenarios. By analyzing the generation process of traffic risks between vehicles and the road environment, the equivalent forces of vehicle-vehicle and vehicle-road are presented and theoretically calculated. Then the prediction steering angle and trajectory are considered in the determination of traffic risk influence area. Next, the weights of the equivalent force in different prediction trajectories are determined by real free driving experimental data. Finally, the road traffic risk was described as a field of equivalent force. Every road user generates their own field of equivalent force in the road traffic risk map, which was time-varying.

INTRODUCTION

In 2015, nearly 190,000 crashes, causing more than 58,000 fatalities and 200,000 injuries, were reported to police in China, (TMBPSM, 2016). Traffic accidents are a major public safety problem in developing countries such as China, as they cause enormous economic losses and can ruin families' lives.

Fortunately, intelligent driving technologies such as ADAS and autonomous vehicles have been developed to avoid vehicle crashes and minimize the impact of accidents (Ji 2017). Traffic risk identification is one of the key technologies in ADAS products and autonomous driving. The collision distance and collision time logic algorithm are mainly used in longitudinal risk identification. The typical collision distance-based algorithms include the Mazda model, Honda model, JHU model, Jaguar model, the fixed car-following distance model and the critical safety distance model (Van Winsum 1999; Lee and Peng 2005; Pei et al. 2012). The typical collision time-based algorithms usually take the time to collision into account such as TTC (time to collision), TTCi (inverse time to collision), THW (time headway) and so on (Ward et al. 2015; Balas et al. 2006; Sharifi et al. 2016). For lateral safety, the car's current position (CCP), time to lane cross (TLC), and variable rumble strip (VRBS) are mainly used in driver assistance products (Risack et al. 2000; Pilutti et al. 2003; Mammara et al. 2004). However, the traffic risk cannot be described as a continuous variable by using these methods, which are artificially divided into longitudinal and lateral direction.

Besides, some researchers have studied the traffic risk from the angle of statistics constantly, such as the road accident rate analysis (Kuliczowska 2016) and the process analysis of traffic conflicts and crashes (Davis et al. 2016). But these types of methods cannot assess the traffic risk

in time. Another way to research traffic risk is surrogate safety measures modeling (Pirdavani et al. 2010 and 2011; Wu and Jovanis 2012a, b). However, many of these measures have not been used in models, because of the structure of the model or difficulties in measuring them in existing models (Young et al. 2014).

Our previous research (Wang et al. 2014, 2015, and 2016) presented a driving safety field theory based on the artificial potential field concept. This method is able to represent risks of driving caused by drivers, vehicles, roads, and other traffic factors. Nevertheless, the driving safety field model contains a number of undetermined constants. Moreover, the calibration task of these undetermined constants is difficult by using existing technologies.

In this paper, we presented a novel road traffic risk modeling approach according to the “field” concept. The traffic risk caused by a vehicle was quantitatively described according to the kinetic energy of the vehicle by considering the distance between the vehicle and other road users or environment. Its influence range was expressed by considering the vehicle dynamical state and traffic environment conditions. Hence, the traffic risk can be described by a relatively accurate method.

The rest of this paper is arranged as follows. In the second section, the road traffic risk is described by considering the process of free driving, car-following, cut-in and arbitrary two-vehicle scenarios. The third section presents a traffic risk range model according to the steer angle and turning-probability. The fourth section describes the concept of traffic safety field. In the fifth section, the model applications are introduced and a simple application for autonomous driving in driver’s perspective is described. The sixth section presents the discussions of this study. The last section presents the conclusions of this study.

ROAD TRAFFIC RISK

The rapid development of the social economy and the progress of human civilization make us more dependent on transportation tools. However, while we enjoy the convenience of traffic, we suffer the harmfulness of traffic accidents. Humans, vehicles, and the road environment constitute a closed loop traffic system, and traffic risks always exist in this system. Actually, the traffic risks are equivalent to collision events. Moreover, the potential traffic risks exist in near-crash scenarios. In order to reduce traffic risks, we need to analyze each influence factor of traffic risks in humans, vehicles, and the road environment. In addition, statistics show 80% of road traffic accidents occurred in the straight road scenario (TMBPSM 2016). Therefore, this scenario became the main research object in this paper.

The road traffic risk caused by a single moving object

The essence of the collision process is converting kinetic energy into frictional heat energy, elastic, and plastic deformation. In the traffic system, each moving objects contains kinetic energy, including the vehicles, pedestrians, and cyclists. Therefore, the degree of traffic risk can be described according to the value of kinetic energy of moving objects. For a single moving object i , its kinetic energy can be written in the following form.

$$E_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i v_i \cdot v_i = \frac{1}{2} m_i v_i \cdot \frac{(v_i - 0)}{\Delta x_i} \cdot \Delta x_i \quad (1)$$

where, E_i , m_i , and v_i are the kinetic energy, mass, and velocity of moving object i respectively, and the Δx_i means the distance between one point in front of i with i itself.

Set $F_i = 1/2 m_i v_i \cdot (v_i - 0) / \Delta x_i$, therefore,

$$E_i = F_i \cdot \Delta x_i \quad (2)$$

where, F_i denotes the equivalent force in the traffic environment exerted by the moving object i . It is measured in Newton.

When an obstacle j appears in front of the moving object i , a relationship between the moving object i and the obstacle j emerges. Here we use E_{ij} to describe this relationship.

$$E_{ij} = \frac{1}{2} m_i v_i \cdot (v_i - v_j) \cdot \frac{|x_i - x_j|}{|x_i - x_j|} = \frac{1}{2} m_i v_i \cdot \frac{v_i - v_j}{|x_i - x_j|} \cdot |x_i - x_j| \quad (3)$$

where, $v_j=0$, it is the velocity of the obstacle j . x_j is the longitudinal position of the obstacle j . According to Eq. (3), the $(v_i - v_j)/|x_i - x_j|$ means the quotient of relative velocity and the relative distance between the moving object i and the obstacle j . This physical quantity in automotive engineering represents the inverse of time to collision (TTC_i). Therefore, the Eq. (3) can be written as follows:

$$E_{ij} = \frac{1}{2} m_i v_i \cdot TTC_i \cdot |x_i - x_j| \quad (4)$$

similarly, we set $F_{ij} = 1/2 m_i v_i \cdot TTC_i$, which indicates the internal equivalent force between the moving object i and the obstacle j . Its units are Newton.

The road traffic risk in car-following scenario

The car-following scenario is a typical scenario in the traffic environment. With the rapid increase in auto production, vehicles usually travel as platoons or clusters on the city road and highway. The car-following scenario is shown in Figure 1. s_{ij}^* denotes the occupied space of the vehicle i and the vehicle j in the traffic environment, s_{ij} the space headway of this two vehicles, L_j the length of vehicle j , and the x_i , x_j denotes the longitudinal position of the vehicle i and vehicle j respectively.

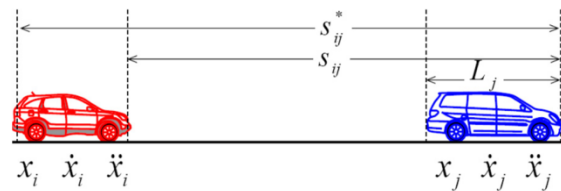


Figure 1. Car-following scenario.

The traffic risks caused by vehicle i and vehicle j are the same as the single moving object form. Therefore the traffic risks are defined as follows:

$$E_i = \frac{1}{2} m_i v_i \cdot \frac{(v_i - 0)}{\Delta x_i} \cdot \Delta x_i \quad (5)$$

$$E_j = \frac{1}{2} m_j v_j \cdot \frac{(v_j - 0)}{\Delta x_j} \cdot \Delta x_j \quad (6)$$

Next, the collision event only can exist between the front of the following vehicle i and the end of the leading vehicle j in this car-following scenario. In other words, if we set Event A to denote the scenario that vehicle i crashes into vehicle j , and Event B to denote that vehicle j crashes into vehicle i , the probability of Event A must be greater than zero, and the probability of Event B equal to zero absolutely. Therefore we define the following vehicle as an active-

collision participant (ACP) and the leading vehicle as a passive-collision participant (PCP). The traffic risk between the ACP and PCP are defined as follows:

$$E_{ij} = \frac{1}{2} m_i v_i \cdot \frac{(v_i - v_j)}{|x_i - x_j|} \cdot |x_i - x_j| = \frac{1}{2} m_i v_i \cdot TTCi \cdot |x_i - x_j| \quad (7)$$

similarly, we set $F_{ij} = 1/2 m_i v_i \cdot TTCi$, which indicates the internal equivalent force between the vehicle i and the vehicle j . Its units are Newton.

Hence, the traffic risk of the road environment in the car-following scenario can be defined as follows:

$$E = E_i + E_j + E_{ij} \quad (8)$$

The road traffic risk in arbitrary scenario

The above section described the relationship between two vehicles in the car-following scenario. However, the car-following scenario often accompanies the cut-in scenario. The cut-in scenario is shown in Figure 2. (x_i, y_i) (x_j, y_j) v_i and v_j denote the positions and velocities of vehicle i and vehicle j respectively. v_{ij} and d_{ij} denote the relative velocity and distance between vehicle i and vehicle j , respectively, d_{ij}^* the minimum relative distance, θ_{ij} the interior angle of v_{ij} and d_{ij} , θ_{ij}^* from v_i to, and the counterclockwise direction is positive. Therefore, the maximum force on vehicle j which loaded by vehicle i is calculated as follows:

$$F_{ij, \max} = \frac{1}{2} m_i \vec{v}_i \cdot \frac{\vec{v}_{ij}}{\sqrt{d_{ij}^2 - d_{ij}^{*2}}} = \frac{1}{2} m_i v_i \cdot \frac{v_{ij} \cos(\theta_{ij} + \theta_{ij}^*)}{d_{ij} \cos \theta_{ij}} = \frac{1}{2} m_i v_i \cdot \frac{v_{ij} (\cos \theta_{ij}^* - \tan \theta_{ij} \sin \theta_{ij}^*)}{d_{ij}} \quad (9)$$

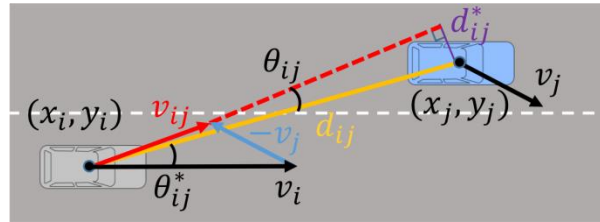


Figure 2. Cut-in scenario.

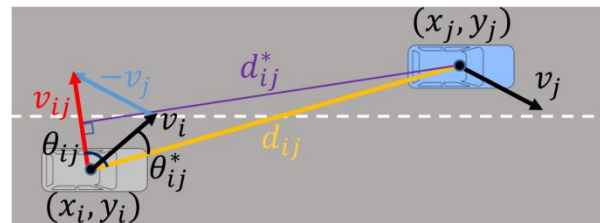


Figure 3. Arbitrary two-vehicle scenario.

In addition, Figure 2 shows an instantaneous scenario; the cut-in action of vehicle j is a continuous process, all variables in Figure are time-varying, $F_{ij, \max}$ has the same properties as well. The arbitrary two-vehicle scenario (Figure 3) can be analyzed by using the same method. Hence, the traffic risk between vehicle i and vehicle j is derived as follows:

$$E_{ij} = F_{ij, \max} \cdot d_{ij} = \frac{1}{2} m_i v_i \cdot \frac{v_{ij} (\cos \theta_{ij}^* - \tan \theta_{ij} \sin \theta_{ij}^*)}{d_{ij}} \cdot d_{ij} = \frac{1}{2} m_i v_i \cdot v_{ij} (\cos \theta_{ij}^* - \tan \theta_{ij} \sin \theta_{ij}^*) \quad (10)$$

Similarly, the traffic risk of the road environment in the arbitrary two-vehicle scenario can also be described as Eq. (8).

THE RANGE OF ROAD TRAFFIC RISK

The road traffic risk is always produced by the influence of road users and the road traffic environment. It is related to the motion states of road users and the road environment conditions. We considered road traffic accidents to occur because the road users did not recognize the traffic risk caused by others or did not sufficiently control the traffic risk they just caused. Hence the road traffic accident took place, which makes sense. In response to this situation, the range of road traffic risk is proposed in this paper. Meanwhile, the mathematic model of this risk range is established.

Road users must follow traffic rules and laws in the road traffic environment. In general, reverse driving is forbidden. Vehicles turn around and change lanes should obey the traffic signs. We assume the driver always follows the traffic rules and laws in this paper. The vehicle can only drive forward, change lanes, and adjust speed in a one-way lane. Therefore, when a car is in a free driving state on a straight road, we assume its velocity and steer angle are continuous constant values. The position of this vehicle in the road environment at the next moment can be predicted. The motion trajectory can be predicted according to each predictive position. The details are illustrated in Figure 4.

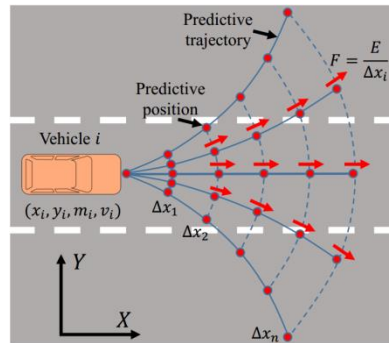


Figure 4. Predictive trajectories and positions diagram.

As shown in Figure 4, the red points indicate the prediction positions, the blue curves are the prediction trajectories based on the prediction positions, the F indicates the influence of each position by vehicle i , and the turning radius R can be calculated using the equivalent linear two-wheel vehicle model as follows:

$$R(t) = \left[1 + K v_i^2(t) \right] \frac{L}{\delta(t)} \quad (11)$$

where K indicates the stability factor, L is the wheelbase of vehicle i , and δ denotes the steering angle. v_i is the velocity of vehicle i .

When vehicle i drives at a constant velocity with negligible side slip angle, the predicted positions (x_{ip}, y_{ip}) at a time horizon t_p with a command steer angle δ can be calculated as follows:

$$\begin{bmatrix} x_{ip} \\ y_{ip} \end{bmatrix} = \begin{bmatrix} x_{t_0} + \int_{t_0}^{t_p} v_i(t) \cdot \cos \frac{v_i(t)}{R(t)} dt \\ y_{t_0} + \int_{t_0}^{t_p} v_i(t) \cdot \sin \frac{v_i(t)}{R(t)} dt \end{bmatrix} \quad (12)$$

Assume the vehicle i always under control and driving stability. The maximum value of velocity and turning radius should be subject to the road conditions. The motion states of vehicle i are subject to the following formulas:

$$F_x^2 + F_y^2 = \phi F_z \quad (13)$$

$$F_x = m_i g f + \frac{C_D A v_i^2(t)}{21.15} \quad (14)$$

$$F_y \geq m_i \frac{v_i^2(t)}{R(t)} \quad (15)$$

where F_x and F_y denote the longitudinal and lateral force of vehicle i respectively, F_z the ground reaction forces, ϕ the adhesion coefficient, f the rolling resistance coefficient; C_D the air resistance coefficient, A the windward area of vehicle i .

Based on Eq.(11), (13), (14), and (15), the relationship between steer angle δ and velocity v_i can be derivated as follows:

$$|\delta(t)| \leq \left[\frac{K}{M} + \frac{1}{M \cdot v_i^2(t)} \right] \sqrt{N - 2F_f W \cdot v_i^2(t) - W^2 \cdot v_i^4(t)} \quad (16)$$

where

$$W = \frac{C_D A}{21.15} \quad (17)$$

$$N = \phi^2 F_z^2 - m_i^2 g^2 f^2 \quad (18)$$

$$M = m_i / L \quad (19)$$

$$F_f = m_i g f \quad (20)$$

With the increase of driving velocity of vehicle i , the allowable steer angle δ decreased according to Eq. (16). Meanwhile, the steer angle δ_i subject to the mechanical structure of vehicle i . The maximum value is equal to the steer angle limit δ_{\max} . Generally, the $\delta_{\max} \in [-\pi/4, \pi/4]$ for a passenger car.

$$|\delta(t)| \leq \delta_{\max} \quad (21)$$

Next, the probable motion trajectory of vehicle i should have a certain boundary according to the steer angle range, and the motion states of vehicle i are stabilized within this boundary absolutely. As shown in Figure 5, the black dotted curves denote the left and right limit of the prediction trajectory. When vehicle i is driving straight on the road, the driver may make the following actions: drive straight, turn to the left lane, and turn to the right lane. We assume the driver will turn the steering wheel. Then the steering angle and the turning-probability are δ_k and p_k , respectively. Therefore, the turning-probability p_k can be defined as follows:

$$\sum_{k=-n}^n p_k = 1 \quad (22)$$

$$\delta_k = k * \Delta\delta, k \in [-n, n] \quad (23)$$

where, $k, n \in \mathbb{Z}$. $\Delta\delta$ indicates the increment of the steering angle. In addition, δ_0 denotes straight driving, δ_k indicates turning left if k is a positive integer, otherwise, the δ_k denotes turning right.

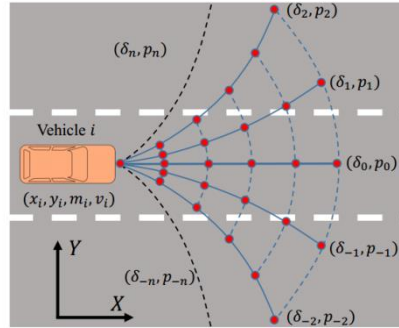


Figure 5. Trajectories based on the steer angle and turning-probability.

However, it is difficult to predict steer angle of the driver and assign it a corresponding value for the turning-probability. To solve this problem, we use real free driving experimental data. The details of the experimental route are shown in Figure 6. This free driving database contains a significant amount of original experiment data of 33 actual experienced drivers, including GPS data and vehicle data. In addition, this database contains about 32.5 hours and more than 1,160,000 measuring points of highway experiment data. Therefore, we count all the highway experiment statistics data to analyze the steering angle of the drivers. The probability of steering angles in highway section basically presents the Gauss normal distribution. The details of the result are shown in Figure 7. The Gauss normal distribution is defined as follows:

$$p_k(\delta_k) = f(\delta_k | \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\delta_k - \mu)^2}{2\sigma^2}} = \frac{1}{3.7704 * \sqrt{2\pi}} e^{-\frac{\delta_k^2}{28.4318}} \quad (24)$$

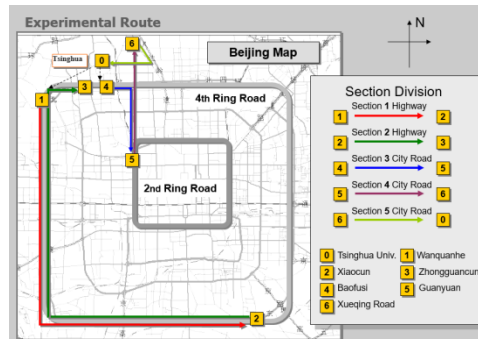


Figure 6. The Experimental route.

THE TRAFFIC SAFETY FIELD CONCEPT

This subsection describes a new concept of the traffic safety field. The traffic risk and its distribution region are described in the above subsections. We use a series of equivalent force to describe the potential impact of the traffic environment which is caused by a road user. Guided by this principle, the road environment will be covered with this kind of force when road users are moving on the road, including vehicles, pedestrians, and cyclists. Furthermore, as previously mentioned, the traffic risk is caused by the active-collision participant (ACP) and the passive-

collision participant (PCP). Active or passive is a relative concept. In the real traffic environment, each road user can display as an ACP or PCP in different times and spaces. The safety rate of the road environment can be quantized by analyzing the range and the distribution of the equivalent force. Therefore, we named this force range as the traffic safety field. The value of equivalent force decreases with the distance between the predicted point and the road user increases. Similarly, the value of equivalent force decreasing laterally to both sides, and the weight of the equivalent force is defined as w_k in the pictorial diagram of Figure 8(a).

Meanwhile, the weight w_k are defined as follows:

$$w_k = \frac{p_k(\delta_k)}{p_0(\delta_0)} \quad (25)$$

where, $k \in [-n, n]$ and $k, n \in \mathbb{Z}$. $p_0(\delta_0)$ denotes the probability of vehicle stayed at angle δ_0 by the driver in the next moment, $p_k(\delta_k)$ the probability of vehicle steered to angle δ_k by the driver in the next moment.

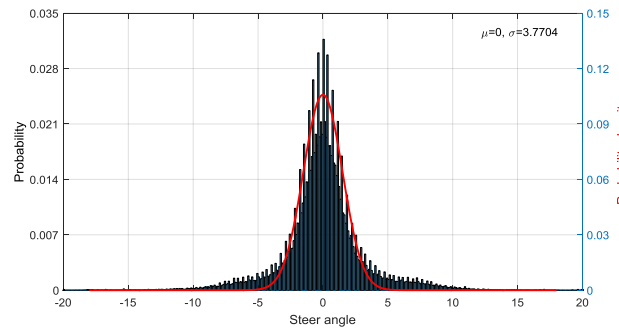


Figure 7. The distribution of turning-probability in highway section.

Based on Eq.(25), the equivalent force in each predictive position can be calculated as follows:

$$F_{ki} = \frac{E}{\Delta x_i} \cdot w_k = \frac{1}{2} \frac{w_k m_i v_i^2}{\Delta x_i} \quad (26)$$

Finally, the traffic risk map of the straight driving vehicle i is described by MATLAB as shown in Figure 8(b) ($m_i=1500\text{kg}$, $v_i=20\text{m/s}$). The zero value of the equivalent force is set as the white color for convenient analysis. The boundaries of traffic risk influence area are illustrated obviously by arc curves which separated the white areas and deep blue areas in Figure 8(b). In addition, the value of equivalent force decreases progressively with the increase of longitudinal and lateral distances. Moreover, the boundaries of the above influence area will change with the velocity of the vehicle and road conditions based on Eq.(16). It has a time-varying property. Therefore, the traffic risk map is a time-varying map.

MODEL APPLICATION

In this section, the general applications of the model are introduced, including the autonomous driving control and the traffic control. Afterwards, data of real-life driving from the perspective of one driver is used in the new model, and the output results are illustrated.

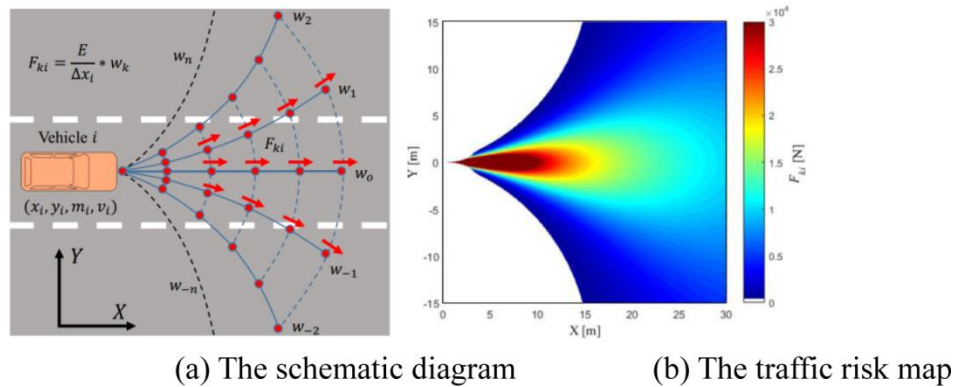


Figure 8. The distribution of equivalent force in traffic safety field.

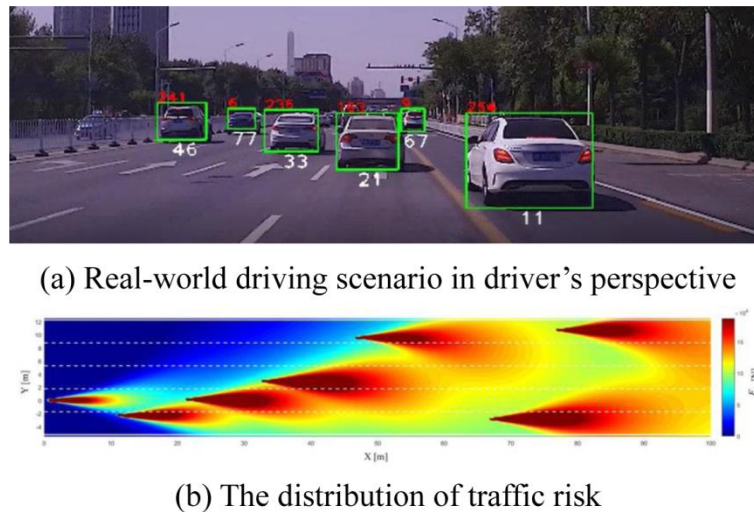


Figure 9. The distribution of equivalent force caused by all detected vehicles.

Application

Based on the traffic safety field model, there are two main categories of applications as follows:

1) Autonomous driving control

According to the new mathematical model, the distribution of traffic risk, indicated in the form of the equivalent force, can be calculated at any time. In this model, any surrounding moving object can be involved, not just the vehicles directly close to the main one. In other words, this model can consider a larger number of potential danger causers, and it can provide a more reliable prediction of coming risks. So based on this model, the autonomous vehicle can recognize the traffic risk more accurately and make its decisions from the perspective of risk maps. Therefore, the autonomous vehicle can act more safely, such as accelerating, braking, or path planning.

2) Traffic control

With the rapid development of connected vehicle technologies, traffic managers have the chance to capture the motion information of all road users in time, including the GPS positions, velocities of vehicles, pedestrians, two-wheelers, and so on. In this instance, the traffic risk in each road of the urban road network can be described by using the traffic safety field method in time. It will be a benefit for traffic managers to regulate the traffic flows according to the

distribution of the traffic risk in time. The traffic environment will become safer and more efficient under a risk and efficiency balance control.

A model application example

A real-world driving scenario from the driver's perspective is shown in Figure 9(a). The front vehicles are detected by the industrial camera of the detection system. The detected information of the front vehicles mainly includes the relative velocity and distance. The real-time distribution of the traffic risk can be calculated as shown in Figure 9(b). In addition, the traffic risk is time-varying. Figure 9 shows only one moment of the driving scenario and its traffic risk distribution in the driving process. Moreover, the autonomous vehicle can make the decision to accelerate, brake, or change lanes in order to keep itself in a safety state.

DISCUSSION

The present study presented the concept of traffic safety field with embedding the equivalent force and established a new traffic risk model. This suggested that traffic safety field can capture the state of traffic risk in the traffic environment. In order to make the traffic more intuitive and reasonable, the traffic risk is described by assigning an equivalent force vector to each point of a map by using this method. The norm of equivalent force vector represents the state of the risk at that point, which is similar to our previous research (Wang et al. 2014, 2015, and 2016). Compared to our present study, the principle of modeling has changed. The greatest improvement of this model is that it omitted a large number of undetermined parameters, which made it more convenient to apply. However, a few problems remain to be solved:

1) The relationship between the risk index and incident severity

We considered that the risk state of the collision process depends on the kinetic energy of colliding objects in this study. It makes the judgment of the risk level nearer/closer to the true situation. In our previous study, the risk level was assessed by using the RDSI (relative driving safety index). However, RDSI is based on a standard driving safety index (DSI^{*}) in a specific dangerous traffic scenario. It means that we must analyze dangerous scenarios as many as possible in order to calculate this index accurately. In the present research, we only use the equivalent force to describe the traffic risk, which is more direct and comprehensible. But we will continue improving the model structure by considering the risk index and incident severity. Traffic participants will be separated into three categories. The first category is normal road users, including the passenger cars, trucks, and buses. The second category is obstacles, including barriers and traffic cones. The third category is vulnerable road users, such as the pedestrians and cyclists. The risk index model will be established according to the equivalent impairment of traffic participants, which is caused by the equivalent force.

2) The real-world data analysis and its application

In this paper, the section of the model application is described in a relatively simple way. The model currently considers only the impacts of moving vehicles, while other factors are not involved, such as road signs, land markings, and pedestrians. However, the applications in autonomous driving and traffic management are complex. The most difficult part for these applications must be the surrounding environment information captured for the autonomous vehicle, and the human-vehicle-environment interactions for traffic control, which can be solved in further research.

CONCLUSION

This paper has described a novel road traffic risk modeling approach by embedding the equivalent force based on the traffic safety field concept. The relationship between the road user and the traffic environment or other road users was established by using mathematical derivations. This relationship indicated that the traffic risk is determined by the motion states of road users, such as the velocity and steering angle of the vehicle, and it is also related to the environmental conditions such as the adhesion coefficient, the rolling resistance coefficient, and the air resistance coefficient. Finally, an accurate traffic risk in the traffic environment caused by a vehicle was calculated by considering the longitudinal and lateral influence range according to real highway experiment statistics data.

As a future plan, the traffic safety field will be described in detail by considering the driver-vehicle-road interaction. The risk level of every road user influenced by the traffic risk equivalent force will be also studied. Some driver assistance algorithms and autonomous driving algorithms can also be developed based on this road risk modeling approach.

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