

Capacitors

$$Q = CV$$

$$i(t) = \frac{dQ}{dt} = C \frac{dV(t)}{dt}$$



$$V(t_1) = \frac{1}{C} \int_{-\infty}^{t_1} i(t) dt = \frac{1}{C} \int_{t_0}^{t_1} i(t) dt + \underbrace{\frac{1}{C} \int_{-\infty}^{t_0} i(t) dt}_{= \frac{1}{C} \int_{t_0}^{t_1} i(t) dt + V(t_0)}$$

$$W_E = \int_{-\infty}^t p(\gamma) d\gamma \quad \leftarrow \quad p = V(t) \left[C \frac{dV(t)}{dt} \right]$$

$$= C \int_{-\infty}^t \frac{dV(\gamma)}{d\gamma} V(\gamma) d\gamma = C \int_{V(-\infty)}^{V(t)} V(\gamma) dV = C \frac{V^2(\gamma)}{2} \Big|_{V(-\infty)}^{V(t)}$$

Assume V discharge

$$W_E = \frac{1}{2} CV^2$$



$$i(t) = C \frac{dV(t)}{dt} \rightarrow C \frac{dV(t)}{dt}$$

Inductors



$$v(t) = L \frac{di(t)}{dt}$$

$$i(t_1) = \frac{1}{L} \int_{-\infty}^{t_1} v(t) dt = \frac{1}{L} \int_{t_0}^{t_1} v(t) dt + \frac{1}{L} \int_{-\infty}^{t_0} v(t) dt$$

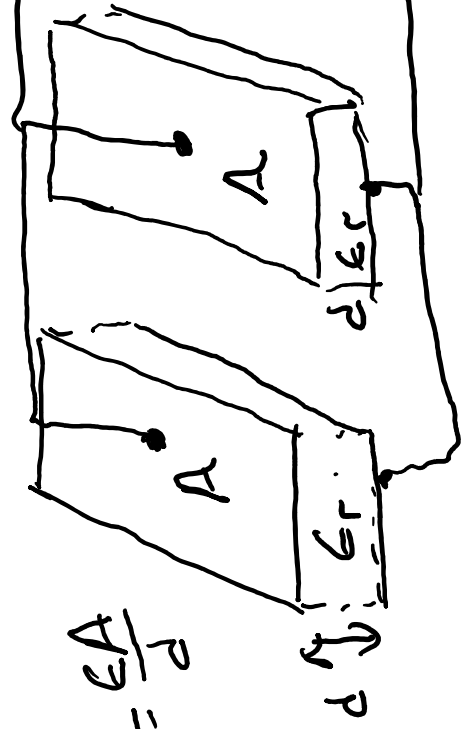
$$= \frac{1}{L} \int_{t_0}^{t_1} v(t) dt + i(t_0)$$

$$W_M = L \frac{i^2(t)}{2} = \frac{1}{2} L i^2$$

→ MAGNETIC

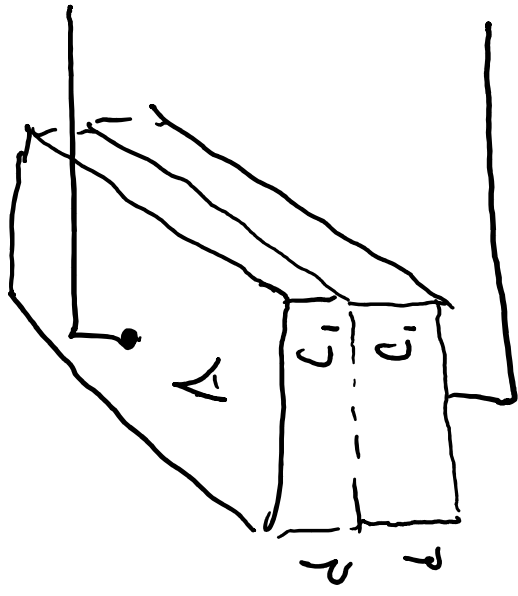
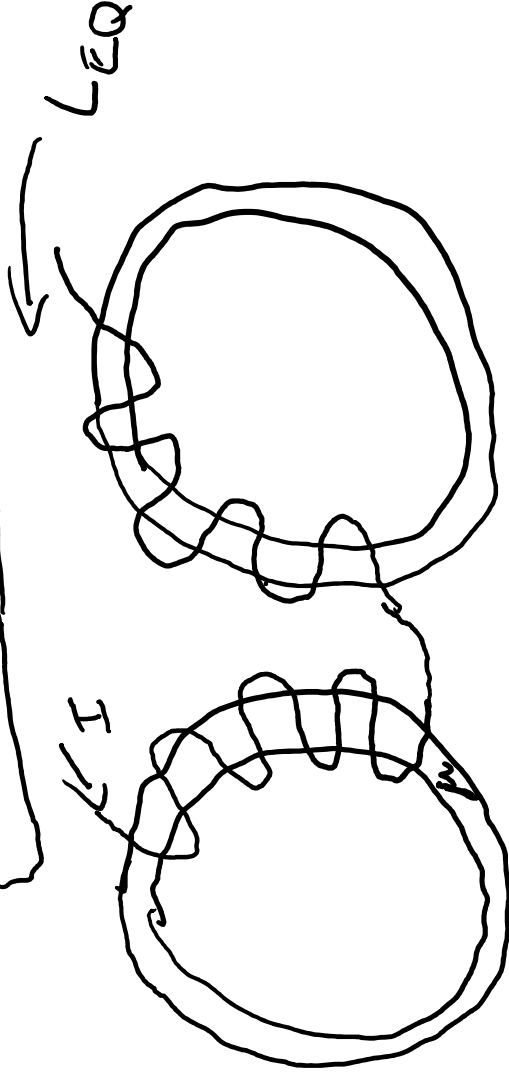
Combinations of C and L

$$C = \frac{\epsilon A}{d}$$

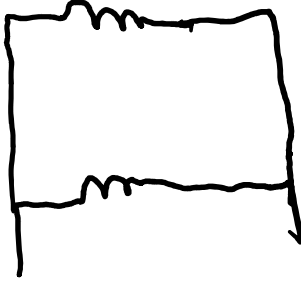


bigger
area
→

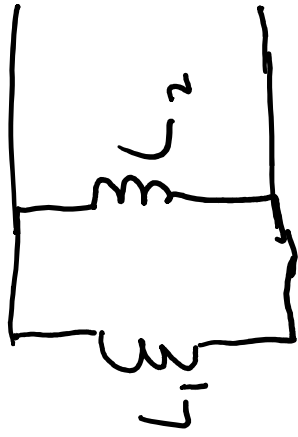
$$C \leftarrow C_{EQ}$$



↑ L
length
increases

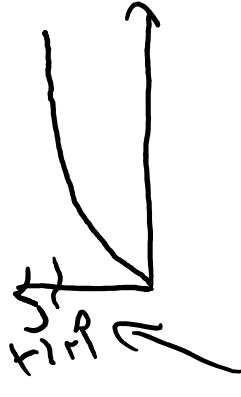


Combinations of C and L



$$L_{EQ} = \frac{L_1 L_2}{L_1 + L_2}$$

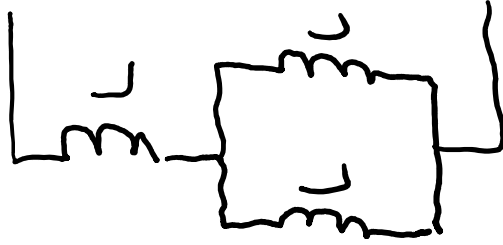
$$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2}$$



$$R_1 \gg R_2$$

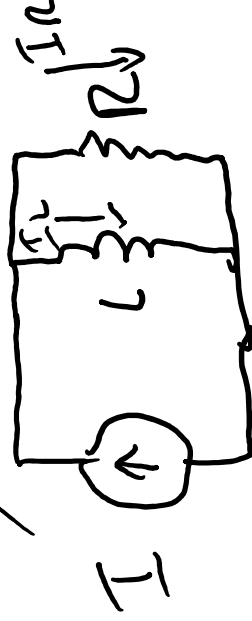
$$R_{EQ} = R_2$$

$$\beta = \sqrt{-1}$$

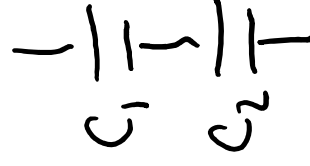


$$L_{EQ} = \frac{3}{2} L$$

$$Z = j\omega L$$



$$Z = \frac{1}{j\omega C}$$

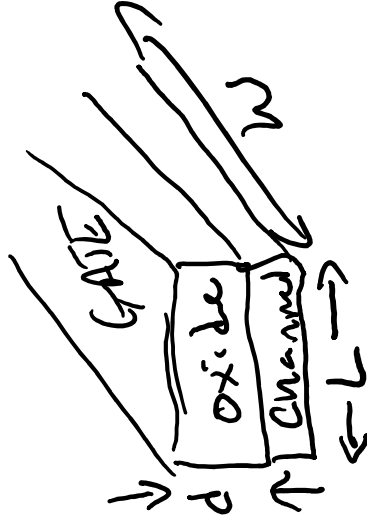
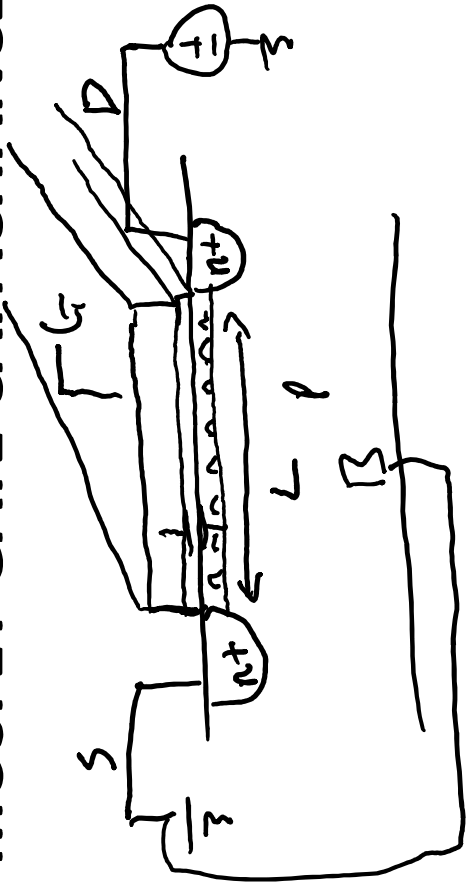


$$C_1 \gg C_2$$

$$C_{EQ} = C_2$$



MOSFET GATE CAPACITANCE

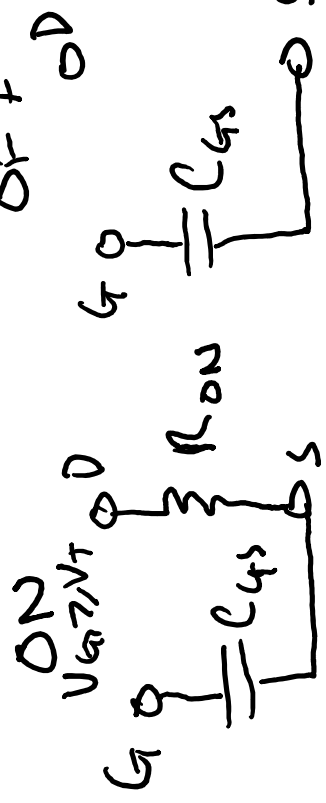


$$i_G(t) = C_{GS} \frac{dV_{GS}(t)}{dt}$$

$$C_{GS} = \frac{\epsilon \times L \cdot W}{d}$$

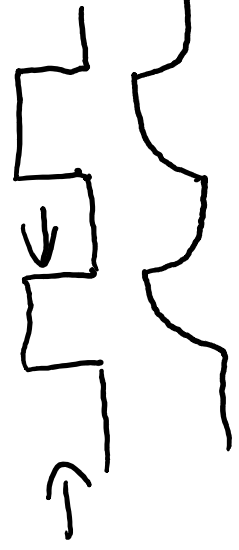
S-R

OFF



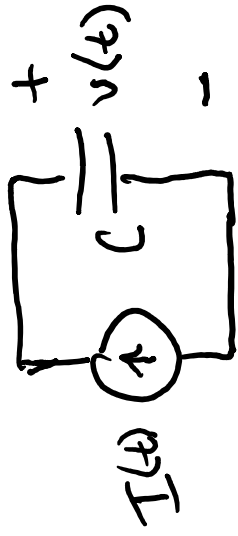
⇒

wire connection

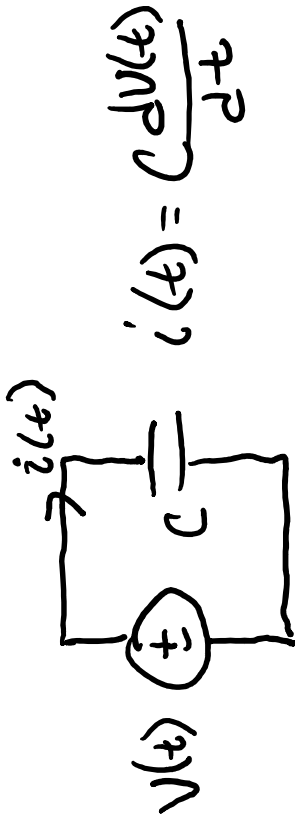


with capacitance

CAPACITOR & INDUCTOR RESPONSES



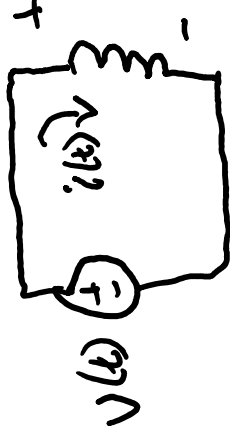
$$v(t) = \frac{1}{C} \int_0^t I(t) dt + v(t=0)$$



$$i(t) = C \frac{dv(t)}{dt}$$



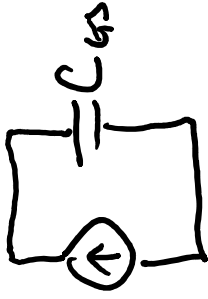
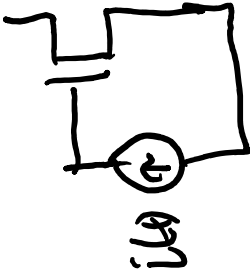
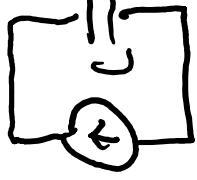
$$v(t) = L \frac{di(t)}{dt}$$



$$i(t) = \frac{1}{L} \int_0^t v(t) dt +$$

STEP INPUT

$$\begin{aligned}
 i(t) &= \frac{1}{C} \int_0^t i(t) dt + V_0 \\
 &= \frac{1}{C} \int_0^t I_0 dt + V_0 = 0 \\
 &= \frac{I_0 t}{C}
 \end{aligned}$$



$V_{GS} \rightarrow 5V$ in $10ns$

What current would be needed

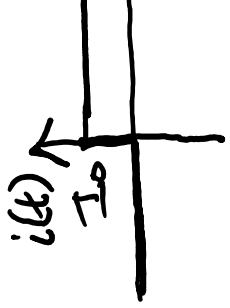
$$C_{GS} = 100 fF = 100 \times 10^{-15} F$$

$$V = \frac{I_0 t}{C} \Rightarrow I_0 = \frac{VC}{t}$$

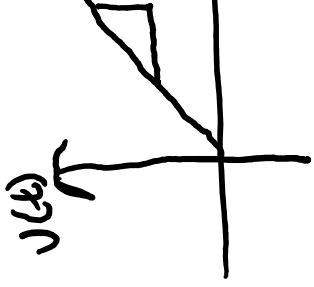
$$= \frac{5 \times 100 \times 10^{-15}}{10 \times 10^{-9}}$$

$$\frac{VC}{t}$$

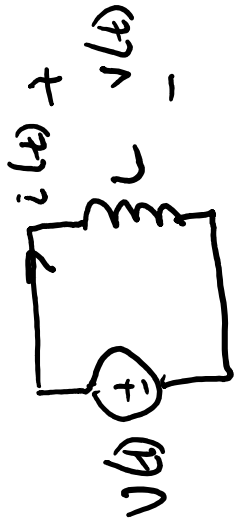
$$i(t) = \begin{cases} 0 & t \leq 0 \\ I_0 & t > 0 \end{cases}$$



$$v(t) = \begin{cases} 0 & t \leq 0 \\ \frac{I_0 t}{C} & t > 0 \end{cases}$$



VOLTAGE STEP FOR INDUCTOR



$$i(t) = \frac{1}{L} \int_0^t v(t) dt + I_0$$

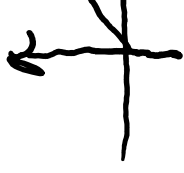
$$= \frac{1}{L} \int_0^t v_0 dt + \cancel{I_0} \rightarrow 0$$

$$= \frac{v_0 t}{L}$$

$$v(t) = \begin{cases} 0 & t \leq 0 \\ v_0 & t > 0 \end{cases}$$



$$i(t) = \begin{cases} 0 & t \leq 0 \\ \frac{v_0 t}{L} & t > 0 \end{cases}$$



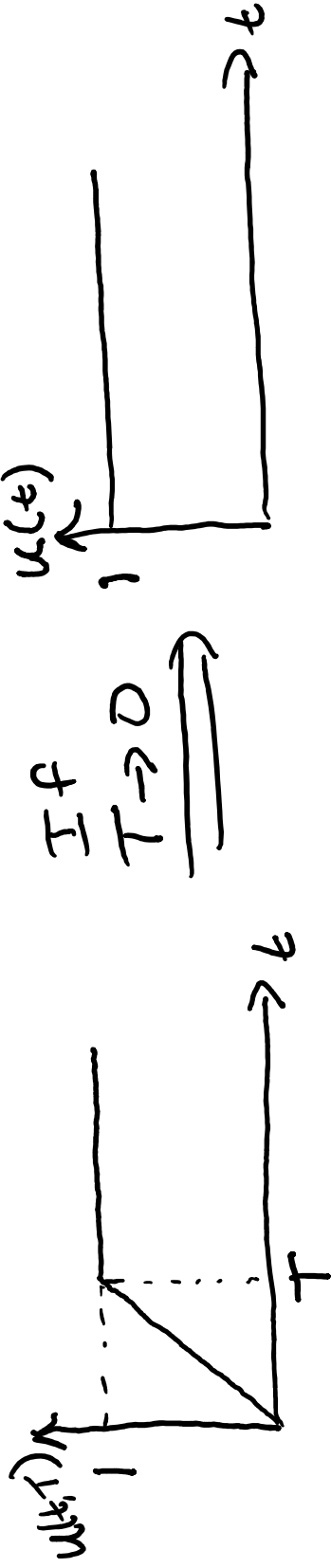
UNIT STEP AND DELTA FUNCTIONS



$$i(t) = C \frac{dv(t)}{dt} \quad \text{what happens at } t=0?$$

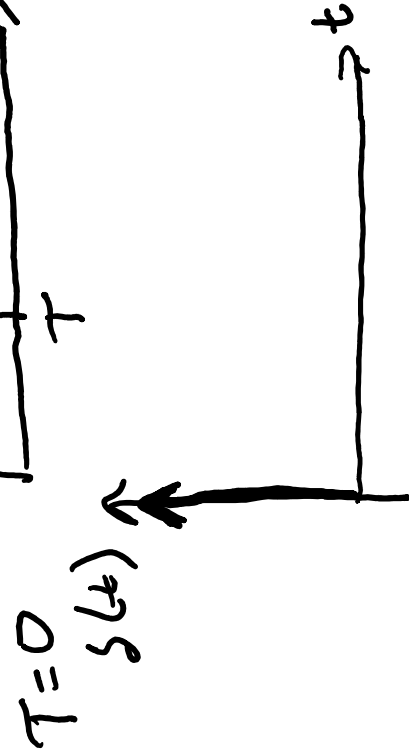
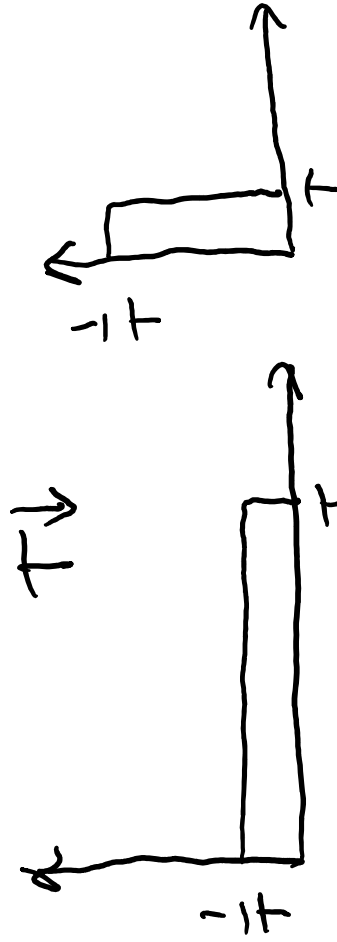
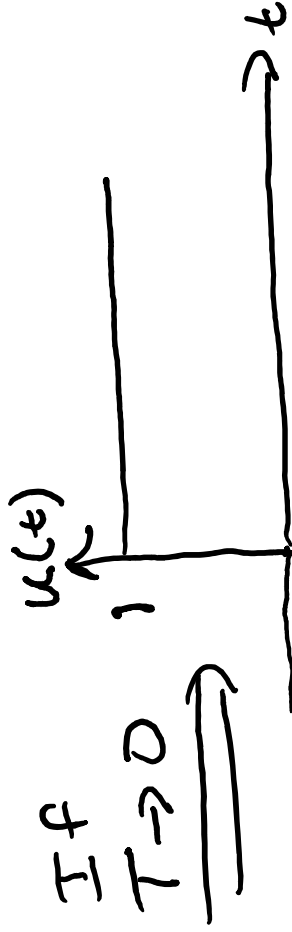
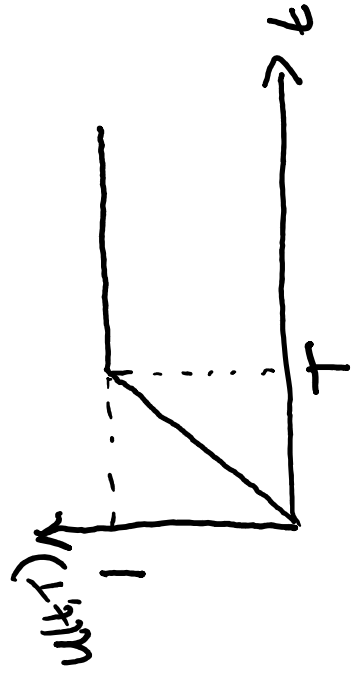
DEFINE RAMPING UNIT STEP FUNCTION

$u(t, T)$ $t \rightarrow$ function of time; T is ramp duration



DEFINE RAMPING UNIT STEP FUNCTION

$u(t, T)$ $t \rightarrow$ function of time; T is ramp duration



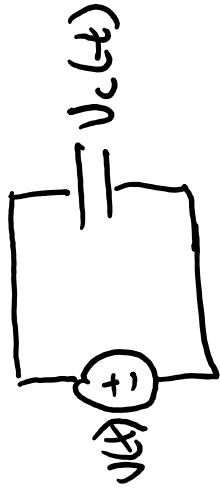
$$\int_{-\infty}^{\infty} s(t) dt = 1 \quad \int_{-\infty}^t s(t) dt = u(t) \Rightarrow s(t)$$

BACK TO VOLTAGE STEP

$$v(t) = V_0 u(t, \tau)$$

$$i(t) = C \frac{dv}{dt}$$

$$s(t, \tau) = d$$



$$Q = CV$$

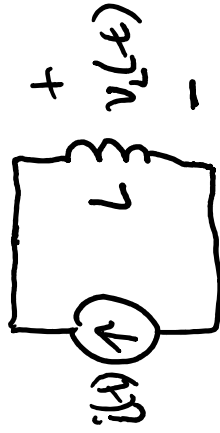
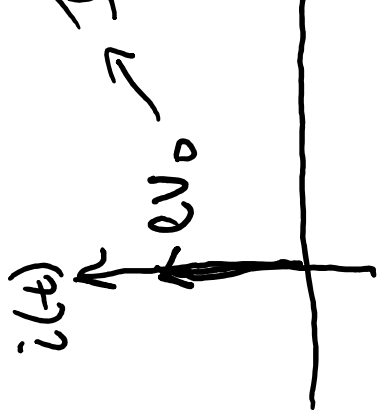
$$\frac{C}{V}$$

$$= CV_0 \frac{du(t, \tau)}{dt}$$

$$= CV_0 s(t, \tau)$$

$$T = 0$$

$$i(t) = CV_0 \underbrace{s(t)}_{\frac{C}{V} \times \frac{1}{s}}$$



$$i(t) = I_0 u(t)$$

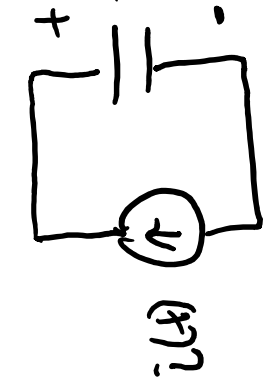
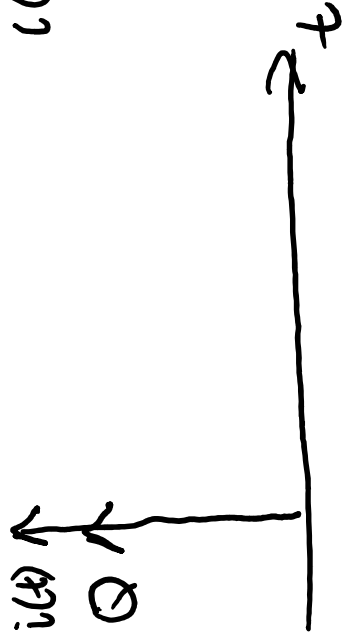
$$v_L(t) = L \frac{di}{dt} = L I_0 \frac{du(t)}{dt} = L I_0 s(t)$$

$$u(t - t_1)$$

IMPULSE INPUT

$$\dot{i}(t) = Q \delta(t) \left[\frac{\text{charge}}{t_{\text{ave}}} \right]$$

$$D \sim \chi^2$$



$$\frac{1}{\int_t^{\infty} p(\tau) d\tau} = \frac{p(t)}{\int_t^{\infty} p(\tau) d\tau}$$

$$\frac{\partial}{\partial c} \psi(t) = \psi(t)$$

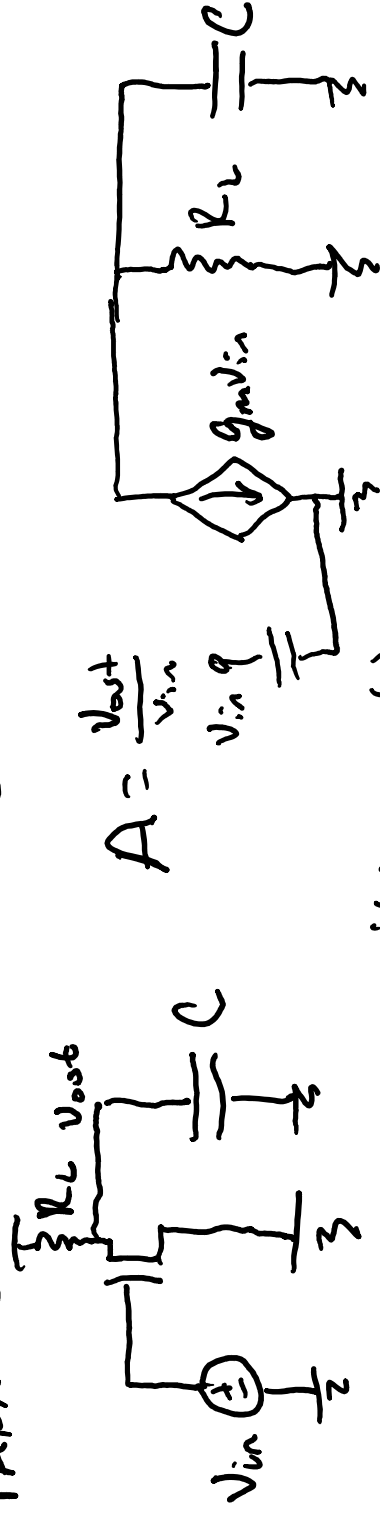
instantaneous voltage jump to Q/C to

CHAP 10: FIRST ORDER TRANSIENTS

ONE ENERGY STORAGE ELEMENT $\rightarrow L, C$



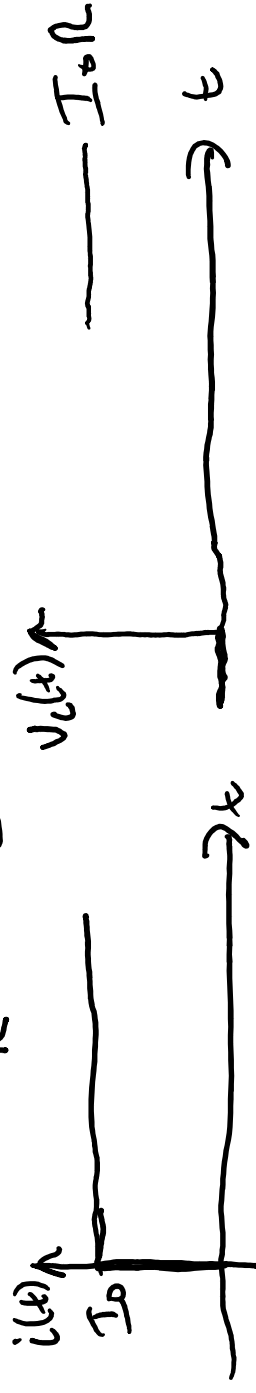
PARALLEL RC \rightarrow SIMPLIFY COMPLICATED CKT INTO NORTON



STEP INPUT $\rightarrow i(t) = I_0 u(t)$

Inhomogeneous
Diff Equation

$$i(t) = \frac{v_c}{R} + C \frac{dv_c}{dt} \Rightarrow \frac{dv_c}{dt} + \frac{v_c}{R_c} = \frac{i(t)}{C}$$



$$\frac{dv_c}{dt} + \frac{v_c}{RC} = \frac{i(t)}{C}$$

METHOD OF HOMOGENEOUS & PARTICULAR SOLUTION

- NATURAL \rightarrow 1) FIND HOMOGEN SOLUTION
RESPONSE
FORCED \rightarrow 2) FIND PARTICULAR SOLUTION
OR
DRIVEN RESPONSE
3) TOTAL SOLUTION = $v_{CH} + v_{CP}$

$$1) \frac{dv_{CH}}{dt} + \frac{v_{CH}}{RC} = 0 \quad \leftarrow \begin{array}{l} \text{driving} \\ \text{function} \\ \text{set to} \\ \text{zero} \end{array}$$

$$v_{CH} = A e^{st}$$

$$\cancel{A} s e^{st} + \frac{\cancel{A} e^{st}}{RC} = 0$$

$$s = -\frac{1}{RC}$$

$$RC = \text{Time constant} = \tau$$