Midterm EXAM

February 14, 2019

SOLUTIONS

Name: ___________________________________

Perm #____________________________________

Lab Section: ______________________________

TA Name__________________________________

You have 75 MINUTES to complete this exam. A one-page note sheet is allowed (front and back), otherwise there should be no books or other materials in your purview. You may use a calculator. Use the backs of pages if you need more room to write. Be sure that your name appears prominently and legibly on the front page. The exam is worth 100 points total.

IMPORTANT POINTS FOR GRADING:

SHOW ALL WORK. ANSWERS GIVEN WITHOUT CLEAR SUPPORTING EVIDENCE OF INDEPENDENT WORK WILL BE IGNORED.

CIRCLE ANSWERS. ANY RESULTS NOT CLEARLY MARKED AS YOUR FINAL ANSWER WILL BE IGNORED.

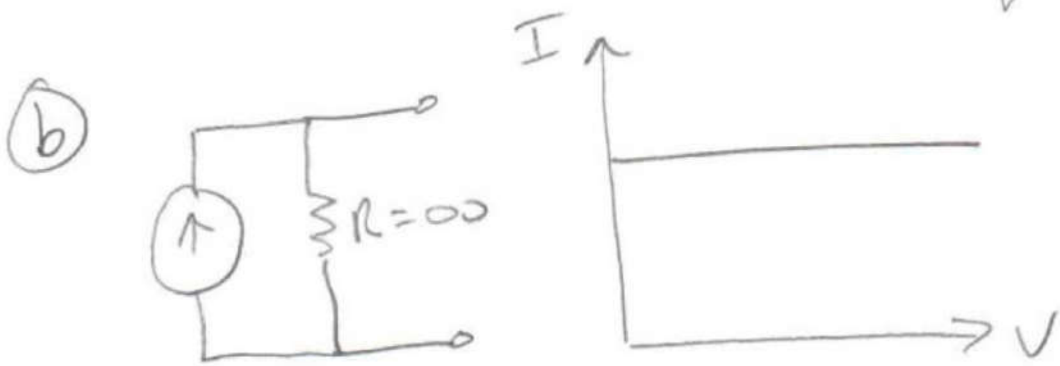
BE NEAT. ILLEGIBLE OR SLOPPY WORK WILL BE IGNORED.

#1 (10 points)	
#2 (20 points)	
#3 (25 points)	
#4 (20 points)	
#5 (25 points)	
Total (100 points)	

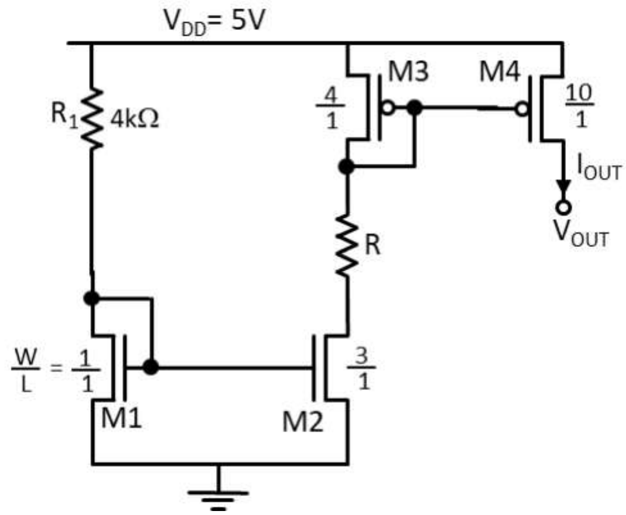
1. (10 points)

(a) Draw the *Thevenin* equivalent circuit for an ideal voltage source. For an ideal source, what is R in the circuit? Sketch the I-V characteristics of the ideal source.

(b) Draw the *Norton* equivalent circuit for an ideal current source. For an ideal source, what is R in the circuit? Sketch the I-V characteristics of the ideal source.



2) (20 points) Consider the current steering circuit shown below. Assume that the V_{OUT} node is connected to another circuit such that I_{OUT} has a path to flow.
$V_{Tn} = |V_{Tp}| = 1V$; $K_n' = 2mA/V^2$, $K_p' = 1mA/V^2$; Remember $K_n = K_n'(W/L)$, $K_p = K_p'(W/L)$.



(a) What is I_{OUT}?
(b) What is the maximum V_{OUT} that can be tolerated for the circuit to work properly?
(c) What is the maximum value of R that can be tolerated for the circuit to function properly?

(a) Consider M1

$V_{GS} = V_{DS} = V_{DD} - I_{D1}R_1$

$V_{GS} = V_{DD} - \frac{K_1}{2}(V_{GS} - V_T)^2 R_1$

$= 5 - \frac{2m}{2}(V_{GS} - 1)^2 4k$

$= 5 - 4(V_{GS} - 1)^2$

$V_{GS} = 5 - 4V_{GS}^2 + 8V_{GS} - 4$

$4V_{GS}^2 - 7V_{GS} - 1 = 0$

$V_{GS} = \frac{7 \pm \sqrt{49+16}}{8}$

$= 1.88V$

$\boxed{I_{D1} = 0.78mA}$

$I_{D2} = 3 \times I_{D1}$

$= 2.34 mA$

$I_{OUT} = \frac{10}{4} I_{D2}$

$\boxed{I_{OUT} = 5.85mA}$

(b) Find V_G for M3

$2.34mA = \frac{K_3}{2}(|V_{GS}| - |V_T|)^2$

$2.34mA = \frac{4m}{2}(|V_{G3} - V_{DD}| - 1)^2$

$\sqrt{(1.17)} + 1V = |V_{G3} - V_{DD}|$

$2.08 = |V_{G3} - V_{DD}|$

$V_{G3} = 5V - 2.08V = 2.92V$

For M4 to stay in SAT

$V_{DD} - V_{OUT} \geq 2.08 - 1$

$\boxed{V_{OUT} \leq 3.92}$

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(c) $V_{DS min}$ for M2

$= V_{GSn} - V_T$

$= 1.88 - 1 = 0.88V$

$V_{GD} = V_{DS}$ for M3

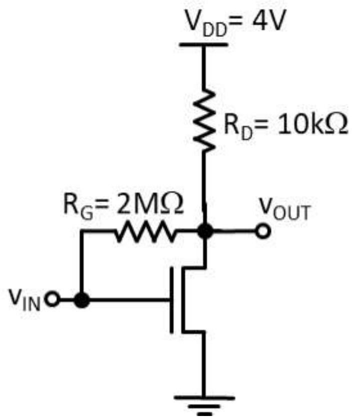
$= 2.08V$

So max voltage across R is:

$5 - 2.08 - 0.88 = 2.04V$

$$\boxed{R = \frac{2.04V}{2.34mA} = 872\Omega}$$

3. (25 points) This circuit uses the two-resistor feedback approach to bias the MOSFET and stabilize its operating point. Assume $K_n = 0.2\text{mA/V}^2$, $V_{TN} = 1\text{V}$.



a. Find the operating point (I_D, V_{DS}).
b. Now a small signal, v_{in}, is injected into the amplifier at the terminal marked v_{IN}. Find the expression for the small-signal gain, $G = \frac{v_{out}}{v_{in}}$, of the circuit in terms of g_m, R_D and R_G.
c. If $R_G \rightarrow \infty$, what does your expression for the gain reduce to?
d. Now plug in the circuit values, what is the % effect of R_G on the small signal gain compared to when $R_G = \infty$?

(a) $V_{OUT} = V_{DD} - \frac{K}{2}(V_{IN} - V_T)^2 R_D$

SINCE NO GATE CURRENT

$V_{IN} = V_{OUT}$

$V_{OUT} = V_{DD} - \frac{K}{2}(V_{OUT} - V_T)^2 R_D$

Substituting:

$V_{OUT} = 4 - (V_{OUT} - 1)^2$

$= 4 - V_{OUT}^2 + 2V_{OUT} - 1$

$V_{OUT}^2 - V_{OUT} - 3 = 0$

$V_{OUT} = \frac{1 \pm \sqrt{1+12}}{2}$

$\boxed{V_{OUT} = 2.3V = V_{DS}}$

$\boxed{I_D = \frac{4-2.3}{10k} = 0.17mA}$

(b)

$\frac{V_{in} - V_{out}}{R_G} = g_m V_{in} + \frac{V_{out}}{R_D}$

$V_{in} R_D - V_{out} R_D = g_m V_{in} R_D R_G + V_{out} R_G$

$V_{out}(R_D + R_G) = -V_{in}(g_m R_D R_G - R_D)$

$G = \frac{V_{out}}{V_{in}} = \frac{-g_m R_D R_G + R_D}{R_D + R_G}$

$G = \frac{-g_m R_D + R_D/R_G}{R_D/R_G + 1}$

(c)

$\frac{V_{out}}{V_{in}} = -g_m R_D$

(d) $g_m = K(V_{IN} - V_T)$

$= 0.2(2.3 - 1)$

$= 0.26 \frac{mA}{V}$

For $R_G = \infty$

$G_T = -g_m R_D = -0.26 \times 10$

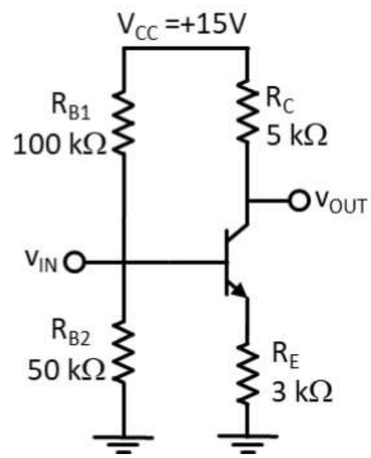
$= -2.6$

For $R_G = 2M$

$$G_T = \frac{-2.6 + \frac{.010}{2}}{\frac{0.01}{2} + 1} = -2.58$$

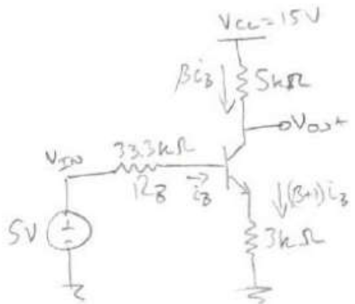
% effect $\Rightarrow \frac{2.58 - 2.6}{2.6} \simeq$ 0.8% reduction in gain

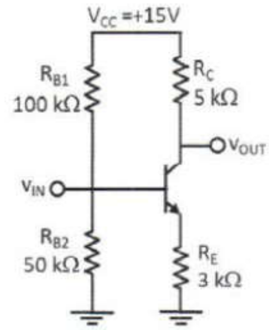
4) (20 points) Draw the *Thevenin* equivalent circuit for the input bias network and find the *Thevenin* equivalent DC Voltage, $V_{IN,EQ}$. (a) Find the voltages at all nodes and the current in all branches. Assume $\beta = 100$. What is the mode of operation for the BJT?



(b) If the BJT is replaced with another that has half the value of β (*i.e.* $\beta = 50$), what is the new value of I_C? Express the change in I_C as a percentage (relative to your answer for part a).

(c) Assume a small signal input voltage, $v_{in,eq}$ is added to the *Thevenin* equivalent DC bias ($V_{IN,EQ}$). Qualitatively, what would you expect the corresponding percentage change in the small-signal voltage gain ($v_{out}/v_{in,eq}$) to be for (b) relative to (a)?





$V_{IN} - i_B R_B - 0.6 - (\beta+1) i_B R_E = 0$

$i_B((\beta+1)R_E + R_B) = V_{IN} - 0.6$

$$i_B = \frac{V_{IN} - 0.6}{(\beta+1)R_E + R_B}$$

$i_B = 13.1\mu A$; $V_B = 4.57V$

$i_C = 1.31mA$; $V_C = 8.45V$

$i_E = 1.32mA$; $V_E = 3.97V$

FORWARD ACTIVE

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$$i_B = \frac{V_{IN} - 0.6}{(\beta+1)R_E + R_B}$$

$i_B = 23.6\mu A$; $V_B = 4.21V$

$i_C = 1.18\ mA$; $V_C = 9.1V$

$i_E = 1.19\ mA$; $V_E = 3.57V$

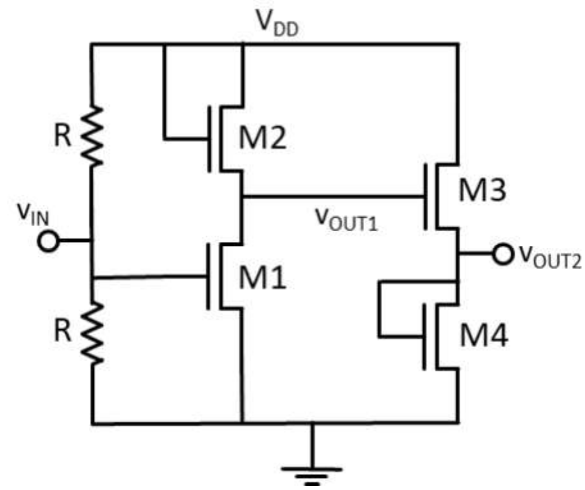
Still in F.A.

$$\frac{\Delta I_C}{I_{C_{\beta=100}}} = \frac{1.31 - 1.18}{1.31} = 9.9\%$$

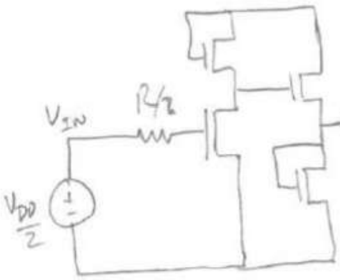
$A = \frac{V_{out}}{V_{in}}$ should decrease by $\sim 10\%$

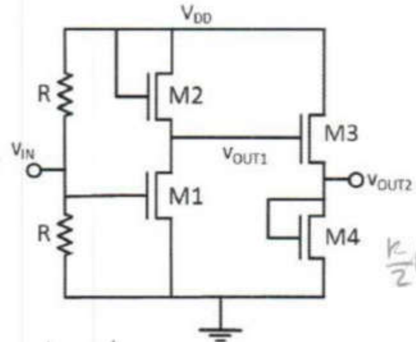
5) (25 points) For the two-stage n-MOSFET amplifier circuit shown below, assume that K and V_T are the same for all MOSFETs.

(a) Find the DC voltages V_{IN}, V_{OUT1} and V_{OUT2} as a function of V_{DD}. *(Hint: Use the Thevenin equivalent for the bias circuit connected to the gate of M1.)*



(b) Now draw the small-signal model for the circuit. Assuming that $K = 1\ mA/V^2$, $V_T = 1V$, and $V_{DD} = 5V$, find the small signal voltage gain $A = v_{out2}/v_{in}$.





Set currents equal for M1 & M2; M3 & M4

$$i_{D1} = \frac{K}{2}(V_{DD}/2 - V_T)^2$$

$$i_{D2} = \frac{K}{2}(V_{DD} - V_{OUT1} - V_T)^2$$

$$i_{D1} = i_{D2}$$

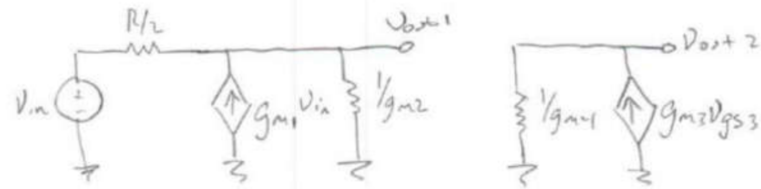
$$\frac{K}{2}\left(\frac{V_{DD}}{2} - V_T\right)^2 = \frac{K}{2}(V_{DD} - V_{OUT1} - V_T)^2$$

$$\boxed{V_{OUT1} = \frac{V_{DD}}{2}}$$

$$i_{D3} = i_{D4}$$

$$\frac{K}{2}(V_{OUT1} - V_{OUT2} - V_T)^2 = \frac{K}{2}(V_{OUT2} - V_T)^2 \Rightarrow \boxed{V_{OUT2} = \frac{V_{OUT1}}{2} = \frac{V_{DD}}{4}}$$

(b) Now draw the small-signal model for the circuit. Assuming that $K = 1\ mA/V^2$, $V_T = 1V$, and $V_{DD} = 5V$, find the small signal voltage gain $A = v_{out2}/v_{in}$.



$$g_{m1} = K(V_{IN} - V_T) = K\left(\frac{V_{DD}}{2} - V_T\right)$$

$$g_{m2} = K(V_{DD} - V_{OUT1} - V_T) = K\left(V_{DD} - \frac{V_{DD}}{2} - V_T\right)$$

$> g_{m1} = g_{m2}$

$$g_{m3} = K(V_{OUT1} - V_{OUT2} - V_T) = K\left(\frac{V_{DD}}{2} - \frac{V_{DD}}{4} - V_T\right)$$

$$g_{m4} = K(V_{OUT2} - V_T) = K\left(\frac{V_{DD}}{4} - V_T\right)$$

$> g_{m3} = g_{m4}$

$$\frac{V_{out1}}{V_{in}} = -\frac{g_{m1}}{g_{m2}} = -1;$$

$$V_{out2} = \frac{g_{m3}}{g_{m4}}(V_{out1} - V_{out2})$$

$$V_{out2}\left(1 + \frac{g_{m3}}{g_{m4}}\right) = \frac{g_{m3}}{g_{m4}} V_{out1} \Rightarrow \frac{V_{out2}}{V_{out1}} = \frac{\frac{g_{m3}}{g_{m4}}}{1 + g_{m3}/g_{m4}} = \frac{1}{2}$$

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$$A = \left(\frac{V_{out1}}{V_{in}}\right)\left(\frac{V_{out2}}{V_{out1}}\right) = -1/2$$