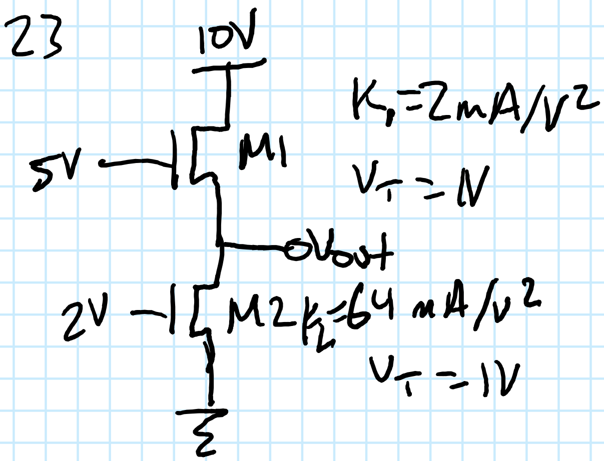
HW3 problem 1

7.23



M1 in SAT

M2 in triode

M1

$$\frac{K_1}{2}(V_{GS1}-V_T)^2 = K_2\left[(V_{GS2}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$\frac{\cancel{2}}{\cancel{2}}\underbrace{(5-V_{out}-1)^2}_{(4-V_{out})^2} = 64\left[(2-1)V_{out} - \frac{V_{out}^2}{2}\right]$$

$$16 - 8V_{out} + V_{out}^2 = 64V_{out} - 32V_{out}^2$$

$$33V_{out}^2 - 72V_{out} + 16 = 0$$

$V_{out} = 0.26$ or $\cancel{1.94}$

Check M1 SAT

$V_{DS1} \geq V_{GS1} - V_T$

$10 - 1.94 \geq 5 - 1.94 - 1$ ✓

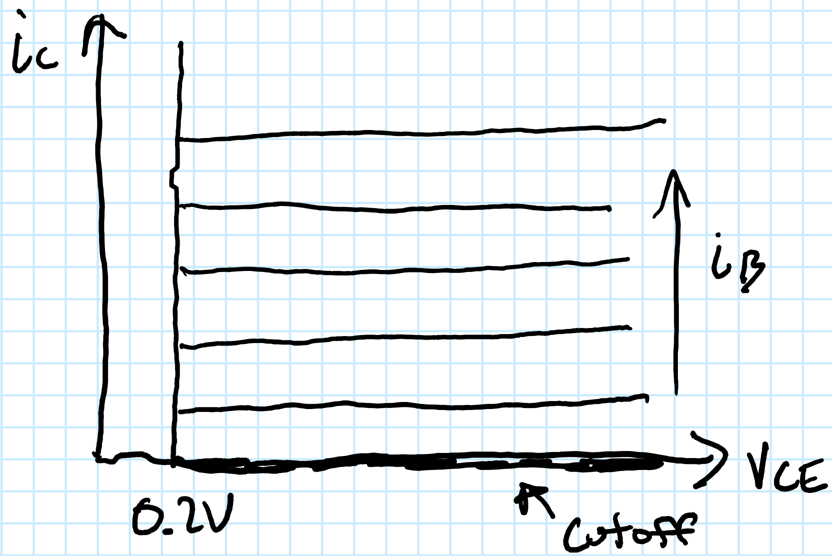
Check M2 Triode

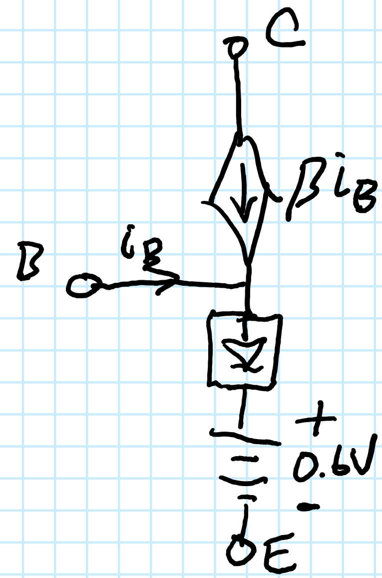
$V_{DS2} < V_{GS2} - V_T$

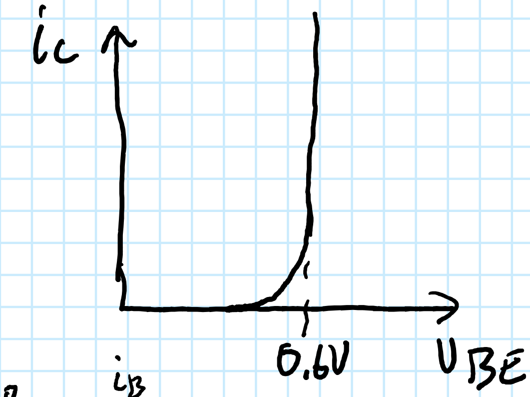
$1.94 < 2 - 1$ NO

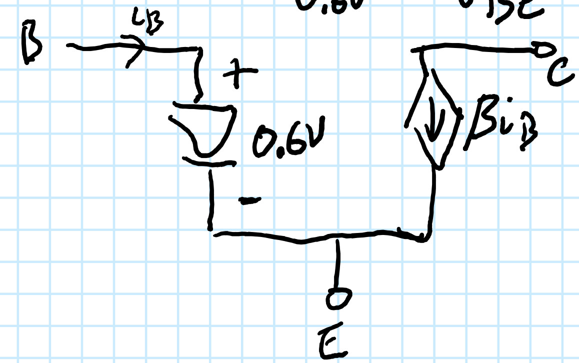
$0.26 < 1$ ✓

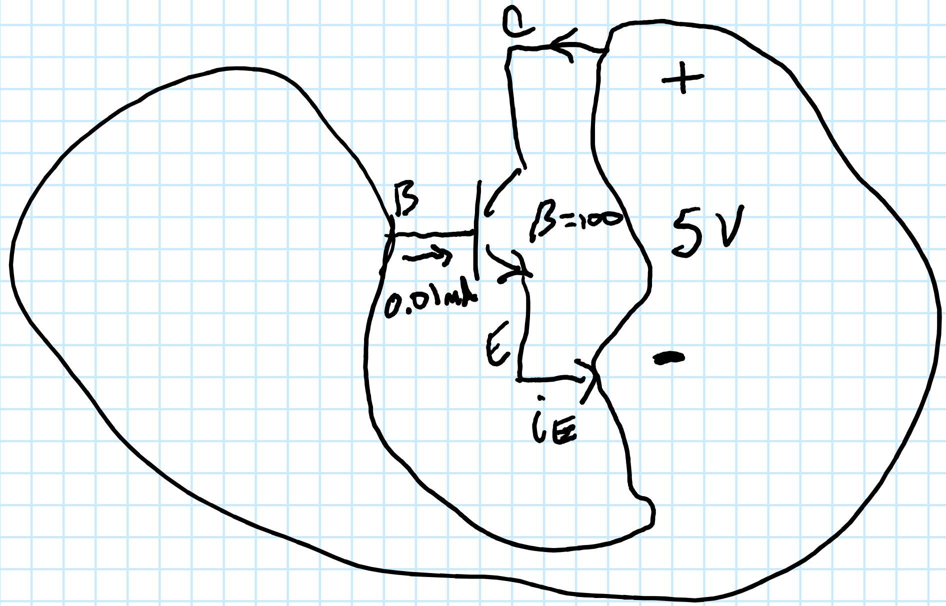
BJT ACTIVE MODEL











Find V_{BE}, i_C, i_E

① MODE?

$i_B > 0$ ✓

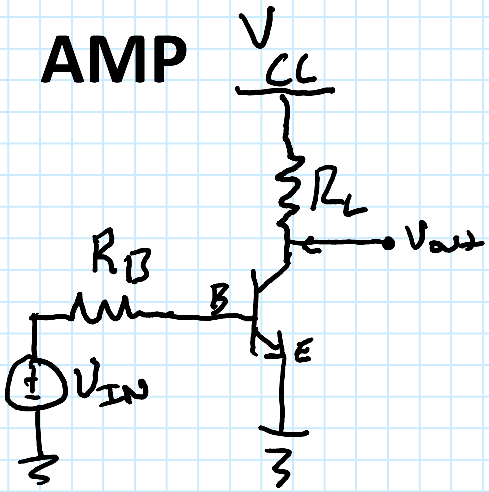
$V_{CE} > 0.2$ ✓

$i_C = \beta i_B = 1mA$

$i_E = (\beta+1) i_B = 1.01mA$

$V_{BE} = 0.6V$

COMMON EMITTER AMP



FIND V_{OUT} vs V_{IN}

Assume Active Region

Find V_{OUT} for $V_{IN} = 1V, 1.1V, 1.2V$

$R_B = 100K\Omega$

$R_L = 10K\Omega$

$\beta = 100$

$V_{CC} = 10V$

MOSFET V_{DD} – high
V_{SS} – low

BJTs V_{CC} – high
V_{EE} – low

$$i_B = \frac{V_{IN} - 0.6}{R_B} = \frac{0.4V}{100K}$$

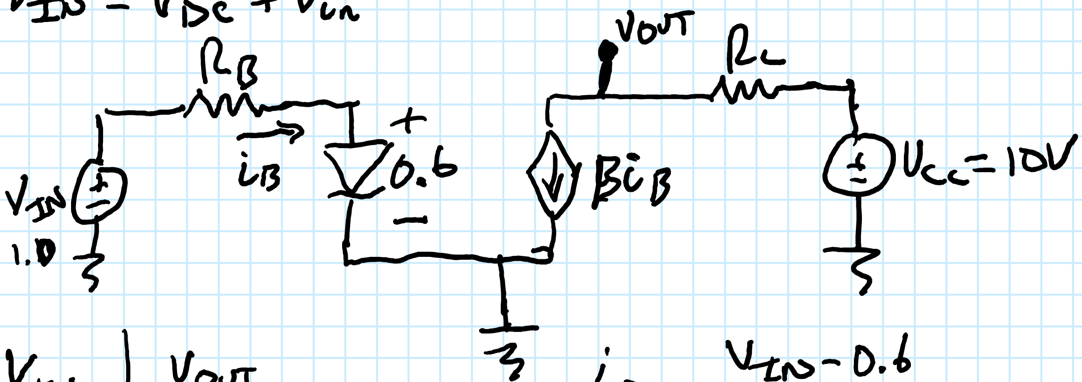
$i_B = 4\mu A$

$i_C = 400\mu A = 0.4mA$

$i_E = 404\mu A$

$$V_{out} = V_{CC} - i_C R_L$$
$$= 10 - 0.4m(10K)$$
$$= 10 - 4 = 6V$$

$V_{IN} = V_{DC} + v_{in}$



V_{IN}	V_{OUT}
1.0	6V
1.1	5V
1.2	4V

$$i_B = \frac{V_{IN} - 0.6}{R_B}$$

$$\frac{V_{CC} - V_{out}}{R_C} = i_C = \beta i_B$$

$$\frac{V_{CC} - V_{out}}{R_C} = \beta \frac{V_{IN} - 0.6}{R_B} \Rightarrow$$

$$V_{OUT} = V_{CC} - \beta \frac{R_L}{R_B}(V_{IN} - 0.6)$$
$$= 10 - 100\left(\frac{10K}{100K}\right)(V_{IN} - 0.6)$$
$$= 10 - 10V_{IN} + 6$$
$$= 16 - 10V_{IN}$$

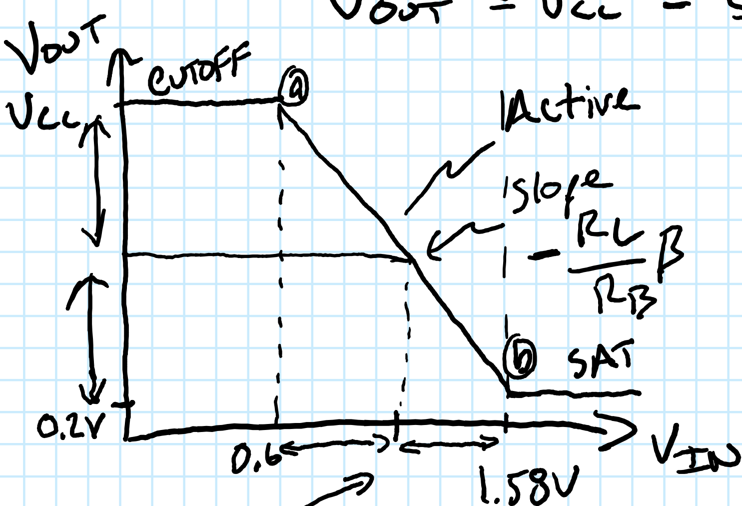
COMMON EMITTER TRANSFER CHARACTERISTICS

LARGE-SIGNAL ANALYSIS

1. TRANSFER CURVE $\rightarrow V_{out}$ vs V_{IN}
2. VALID INPUT & OUTPUT OPERATING VOLTAGES

$$V_{OUT} = V_{CC} - \frac{(V_{IN} - 0.6)}{R_B} R_L \beta$$

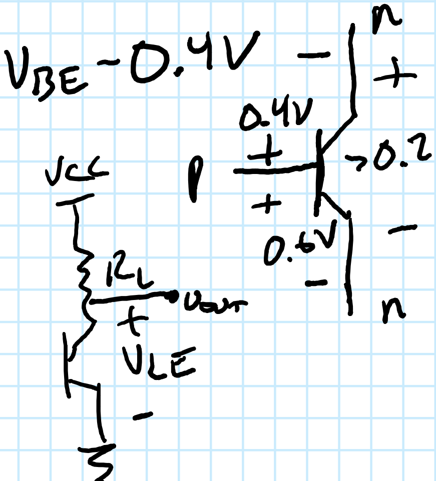
$V_{CB} > -0.4V$



$V_{CE} > 0.2V$

$V_{BE} = 0.6V$

$V_{CE} > V_{BE} - 0.4V$



b $V_{CE} = 0.2V$

$V_{OUT} = V_{CE}$

i_c at b

$$i_c = \frac{V_{CC} - V_{OUT}}{R_L} = \frac{10 - 0.2}{10K} = 980\mu A = \beta i_B = \beta\left(\frac{V_{IN} - 0.6}{R_B}\right)$$

$$V_{IN} = 9.8\mu A \cdot R_B + 0.6 = 1.58V$$

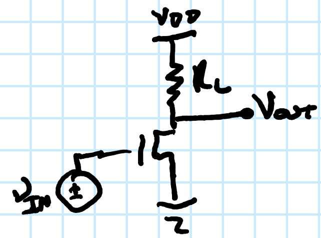
$0.6V < V_{IN} < 1.58V$

CHOOSE OPERATING POINT TO MAXIMIZE V_{IN} SWING

→ Pick middle point of V_{IN} range

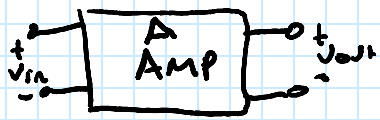
→ linear transfer function means output voltage also in middle of range

CHAPTER 8: SMALL SIGNAL ANALYSIS



$$V_{OUT} = V_{DD} - i_D R_L$$
$$= V_{DD} - \frac{K}{2}(v_{IN} - V_T)^2 R_L$$

$\Rightarrow$ Non-linear



$\Rightarrow V_{OUT} = A v_{IN}$

constant & independent of amplitude of v_{in}

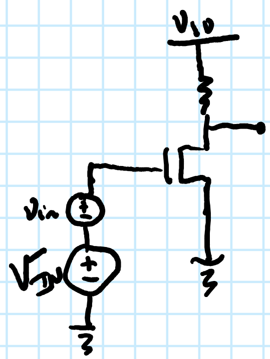
SMALL-SIGNAL

if signals can be represented as small perturbation about an operating point $\Rightarrow$ response of circuit will be approx. linear

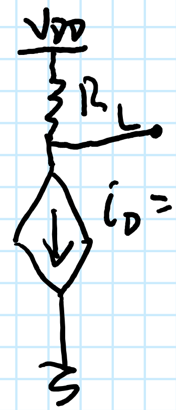
- linear model for a nonlinear device but only valid over small range

LARGE-SIGNAL - valid over large range but complicated (nonlinear)

TOTAL VARIABLE		DC value		s.s.
v_{IN}	=	V_{IN}	+	v_{in}

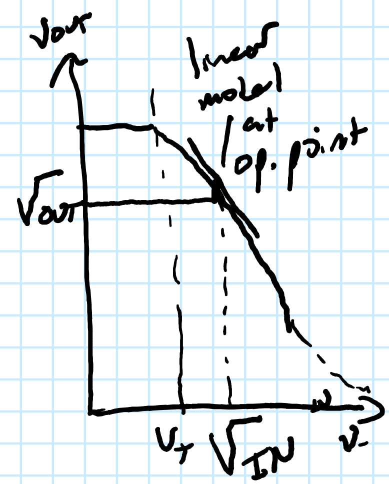


$v_{OUT} = f(V_{IN} + v_{in})$

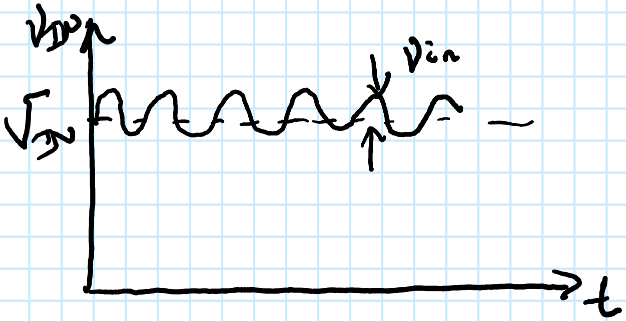


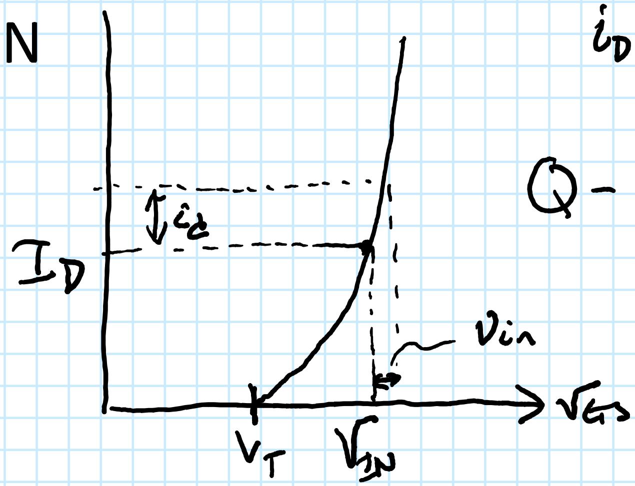
$v_{OUT} = f(V_{IN} + v_{in})$

$$i_D = \frac{K}{2}\left[(V_{IN} + v_{in}) - V_T\right]^2$$



TAYLOR SERIES EXPANSION





$$i_D = \frac{k}{2}\left(v_{GS} - v_T\right)^2$$

Q-point

$y = f(x)$ expand about $x = X_0$

$$y = f(X_0) + \left.\frac{df}{dx}\right|_{X_0}(x - X_0) + \frac{1}{2!}\left.\frac{d^2 f}{dx^2}\right|_{X_0}(x - X_0)^2 + \cdots$$

$y = i_D$ $\quad V_{IN} = X_0 \quad x = V_{IN} + v_{in} \rightarrow x - X_0 = v_{in}$

$= I_D + i_d$

$$i_D = \underbrace{\frac{k}{2}\left(V_{IN} - V_T\right)^2}_{\text{Exact}} + \underbrace{k\left(V_{IN} - V_T\right)v_{in}}_{\text{linear approximation}} + \cancel{\frac{k^2}{2} v_i^2}$$

v_{in} is small compared to V_{IN}

g_m = incremental transconductance

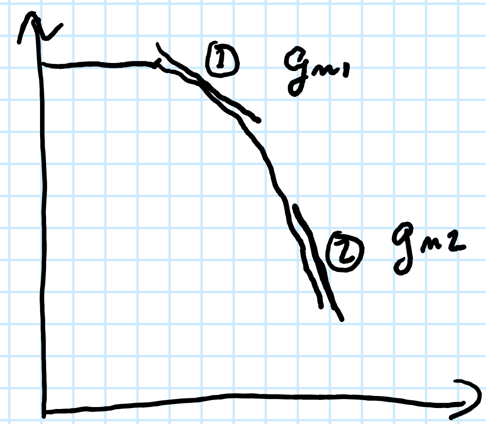
$i_d = g_m v_{in}$

$g_m = k\left(V_{GS} - V_T\right)$

OUTPUT AND GAIN

$$v_{OUT} = \underbrace{V_{OUT}}_{DC} + \underbrace{v_{out}}_{S.S.}$$

$$v_{OUT} = V_{DD} - i_D R_L$$



$$V_{OUT} + v_{out} = V_{DD} - (I_D + i_d) R_L$$

$$= \underbrace{V_{DD} - I_D R_L}_{\substack{V_{OUT} \\ DC}} \underbrace{- i_d R_L}_{S.S.}$$

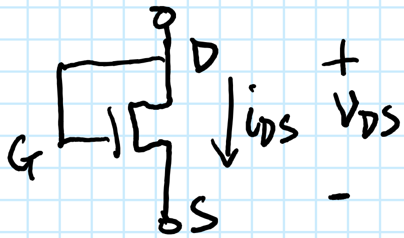
$i_d = g_m v_{in}$

$$v_{out} = -i_d R_L = -g_m R_L v_{in}$$

S.S. GAIN

$$A = \frac{v_{out}}{v_{in}} = -g_m R_L = -\overbrace{K(V_{IN} - V_T)}^{g_m \text{ – constant}} R_L$$

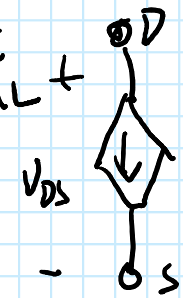
SS EXAMPLE: DIODE CONNECTED MOSFET



$$i_{DS} = K\frac{(v_{GS} - V_T)^2}{2}$$

$$V_{GS} = v_{DS} \Rightarrow i_{DS} = \frac{K}{2}(v_{DS} - V_T)^2 \Rightarrow$$

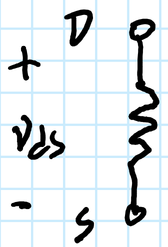
LARGE SIGNAL



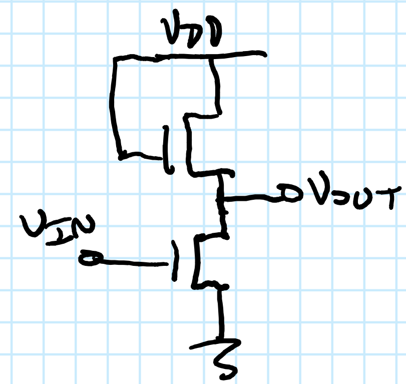
$$i_{ds} = \frac{d\,i_{DS}}{d v_{DS}}\bigg|_{V_{DS}} v_{ds}$$

$$= K(v_{DS} - V_T)\Big|_{V_{DS}} v_{ds} = K(V_{DS} - V_T)\, v_{ds}$$

$$v_{ds} = \frac{i_{ds}}{K(V_{DS} - V_T)}$$ $\Rightarrow$ Small signal acts like a resistor

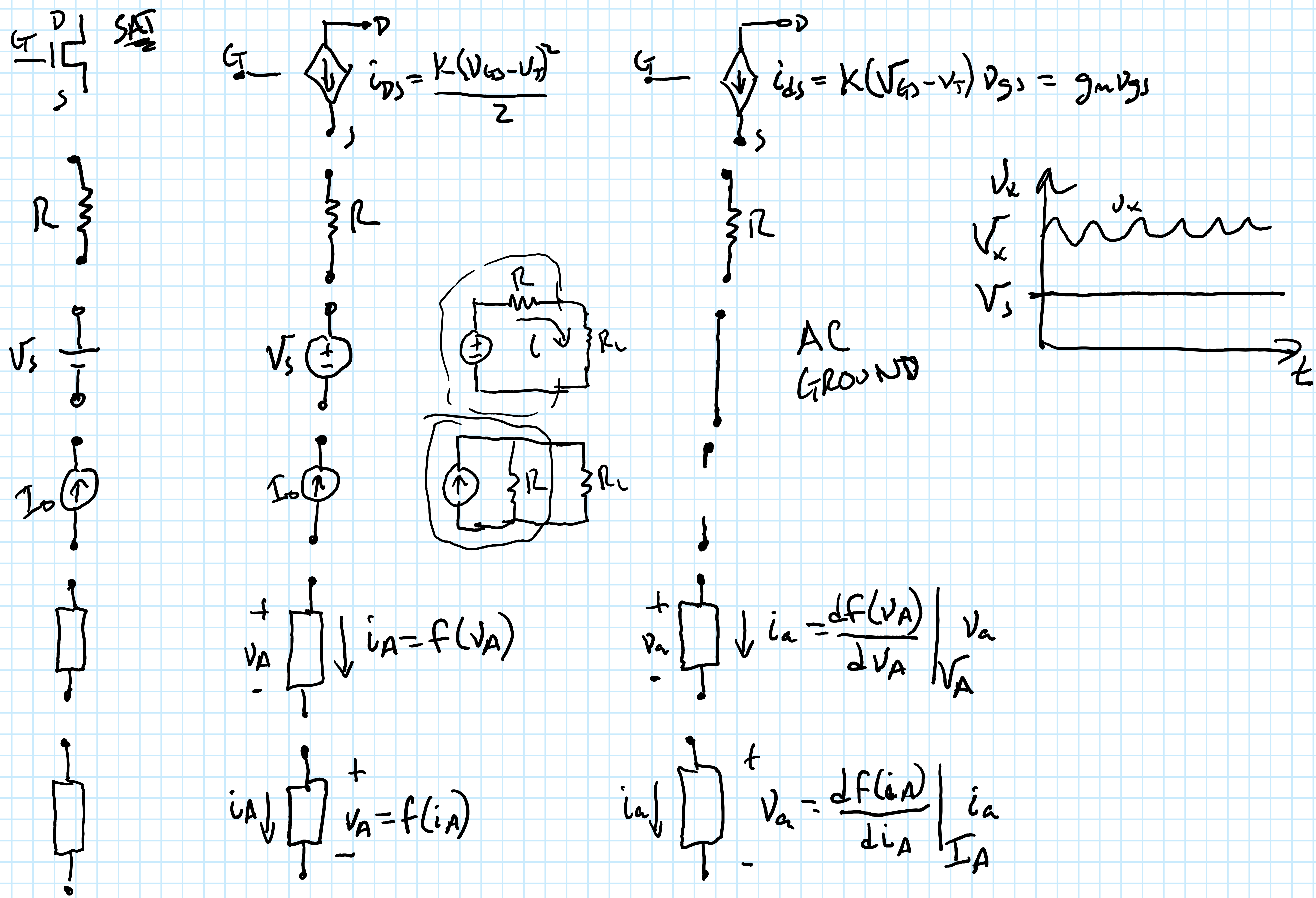


$$r = \frac{1}{K(V_{DS} - V_T)} = \frac{1}{g_m}$$ $\rightarrow$ large resistance in small area



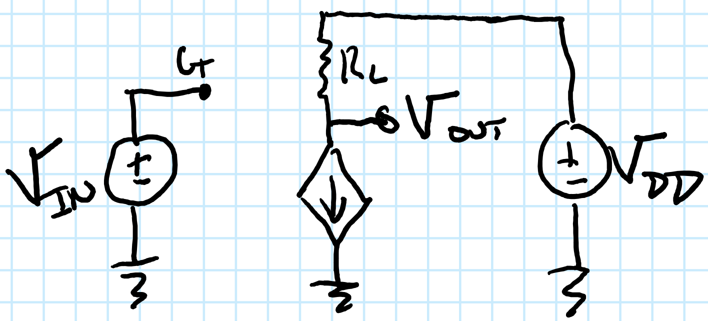
MODELS

LARGE SIGNAL | SMALL SIGNAL



S.S. ANALYSIS OF COMMON SOURCE AMP

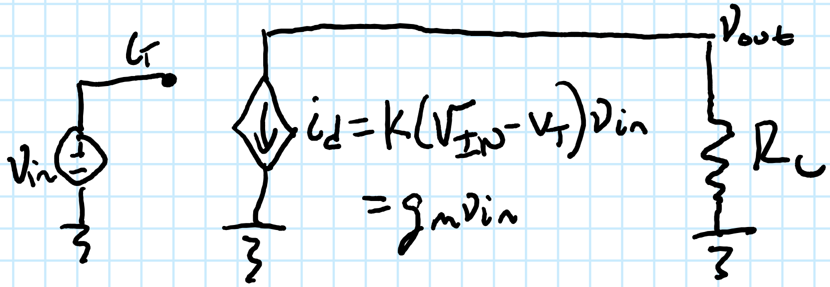
① LARGE-SIGNAL ANALYSIS $\Rightarrow$ DC Operating Point



$$I_D = \frac{K}{2}(V_{IN} - V_T)^2$$

$$V_{OUT} = V_{DD} - I_D R_L = V_{DD} - \frac{K}{2}(V_{IN} - V_T)^2 R_L$$

② SS EQUIVALENT CIRCUIT



$$v_{out} = -i_d R_L$$
$$= -K(V_{IN} - V_T) v_{in} R_L$$

LARGE SIGNAL BIAS POINT DETERMINES S.S. Parameters

$$\boxed{A = \frac{v_{out}}{v_{in}} = -g_m R_L} \Rightarrow \text{S.S. GAIN}$$

$V_{DD} = 10V$ $K = 1mA/V^2$

$R_L = 10k\Omega$ $V_T = 1V$

$V_{IN} = 2V$

what is V_{OUT}

$V_{OUT} = 5V$

What is A?

$$\frac{v_{out}}{v_{in}} = 10^{-3}(2-1)10^4$$
$$= 10$$