ECE 10B HW2 Solutions

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1. a. $I_{D1} = I_{REF} = 25\,\mu A$

$I_{D1} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GSN} - V_{TN})^2$

$I_{D2} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GSN} - V_{TN})^2$

$I_{D3} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_3 (V_{GSN} - V_{TN})^2$

$\frac{I_{D1}}{I_{D2}} = \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}$ $\qquad$ $\frac{I_{D3}}{I_{D1}} = \frac{\left(\frac{W}{L}\right)_3}{\left(\frac{W}{L}\right)_1}$

$\therefore I_{D2} = 2 I_{D1} = 50\,\mu A$ $\qquad$ $I_{D3} = 5 I_{D1} = 125\,\mu A$

$I_{D4} = I_{REF} = 25\,\mu A$

$I_{D4} = \frac{K_p'}{2}\left(\frac{W}{L}\right)_4 (V_{GSP} - V_{TP})^2$ $\qquad$ $I_{D5} = \frac{K_p'}{2}\left(\frac{W}{L}\right)_5 (V_{GSP} - V_{TP})^2$

$\therefore \frac{I_{D5}}{I_{D4}} = \frac{\left(\frac{W}{L}\right)_5}{\left(\frac{W}{L}\right)_4}$ $\qquad$ $\therefore I_{D5} = 25\,\mu A$

b. $I_{D1} = \frac{K_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GSN} - V_{TN})^2$

$\Downarrow$

$25\,\mu A = \frac{25\,\mu A/V^2}{2} \times 5 \times (V_{GSN} - 1)^2$ $\quad$ also $V_{GSN} - V_{TN} > 0$

$\therefore V_{GSN} = 1.63\,V$

$I_{D4} = \frac{K_p'}{2}\left(\frac{W}{L}\right)_4 (V_{GSP} - V_{TP})^2$

$\Downarrow$

$25\,\mu A = \frac{10\,\mu A/V^2}{2} \times 5 \times (V_{GSP} + 1)^2$ $\quad$ also $V_{GSP} - V_{TP} < 0$

$\therefore V_{GSP} = -2\,V$

$$R = \frac{(V_{DD} + V_{GSP}) - (V_{SS} + V_{GSN})}{I_{REF}} = 0.255\,M\Omega$$

2. a. From problem 1 we know that $V_{GSN} = 1.63V$ $V_{GSP} = -2V$

$$V_{SD6} = V_{DD} + V_{GSP} - (V_{SS} + V_{GSN})$$

$$= 5V - 2V + 5V - 1.63V = 6.37V$$

$$V_{GS6} = V_{DS6} = -6.37V$$

$$I_{D6} = I_{REF} = 25\mu A = \frac{K_P'}{2}\left(\frac{W}{L}\right)_6 (V_{GS6} - V_{TP})^2$$

$$\left(\frac{W}{L}\right)_6 = \frac{I_{D6}}{\frac{K_P'}{2}(V_{GS6} - V_{TP})^2} = \frac{25\mu A}{\frac{1}{2} \times 10\mu A/V^2 \times (-6.37V + 1V)^2} = \frac{1}{5.77}$$

b. If M_6 is implemented as a diode-connected nFET

$V_{DS6} = 6.37V$ $V_G = V_D$ $V_{GS6} = 6.37V$

$$\left(\frac{W}{L}\right)_6 = \frac{I_{D6}}{\frac{K_n'}{2}(V_{GS6} - V_{TN})^2} = \frac{25\mu A}{\frac{1}{2} \times 25\mu A/V^2 \times (6.37V - 1V)^2} = \frac{1}{14.42}$$

c. Assume W is fixed, $L_n > L_p$, so nFET would take up the most area.

3. a. $V_{EQ} = \frac{R_1}{R_1 + R_2} V_{GG} = 3V$ $R_{EQ} = \frac{R_1 R_2}{R_1 + R_2} = 210K\Omega$

b. $V_{GS} = V_{EQ} = 3V > V_T$ Assume MOSFET operates in saturation

$$I_D = \frac{K_n}{2}(V_{GS} - V_T)^2 = \frac{25\mu A/V^2}{2} \times (3-1)^2 = 50\mu A$$

$$V_{DS} = V_{DD} - I_D R_D = 10V - 0.05mA \cdot 100K\Omega = 5V$$

$\therefore V_{DS} > V_{GS} - V_T$, the MOSFET indeed operates in saturation and the assumption is correct.

c. Now the new value of K_n is $K_{n1} = 20\mu A/V^2$

$$I_{D1} = \frac{K_{n1}}{2}(V_{GS} - V_T)^2 = \frac{20\mu A/V^2}{2} \times (3V - 1V)^2 = 40\mu A$$

$V_{DS1} = V_{DD} - I_{D1} R_D = 10V - 0.04\,mA \times 100\,K\Omega = 6V$

I_D decreases by $\frac{\Delta I_D}{I_D} = \frac{50\mu A - 40\mu A}{50\mu A} = 20\%$

V_{DS} increases by $\frac{\Delta V_{DS}}{V_{DS}} = \frac{6V - 5V}{5V} = 20\%$

4. a. $V_{EQ} = V_{DD} \cdot \frac{R_1}{R_1 + R_2} = 10V \cdot \frac{1M\Omega}{2.5M\Omega} = 4V$

$R_{EQ} = \frac{R_1 R_2}{R_1 + R_2} = 0.6M\Omega$

b. $I_D = \frac{V_{EQ} - V_{GS}}{R_S} = \frac{K_n}{2}(V_{GS} - V_T)^2$ with the assumption of saturation

plugging the numbers, we have

$$\frac{25}{2}(V_{GS} - 1)^2 \times 10^{-3} = \frac{4 - V_{GS}}{39}$$

$$\Downarrow$$

$V_{GS} = -2.71\,V$ or $V_{GS} = 2.66V$ since $V_{GS} > V_T$, $V_{GS} = 2.66V$

$I_D = \frac{V_{EQ} - V_{GS}}{R_S} = \frac{4V - 2.66V}{39K\Omega} = 34.4\mu A$

$V_{DS} = V_{DD} - I_D R_D - I_D R_S = 6.08V$

$V_{DS} > V_{GS} - V_T$, so the transistor operates in saturation and the assumption is correct.

c. With the new value $K_{n1} = 20\mu A/V^2$, we have

$$\frac{20}{2}(V_{GS1} - 1)^2 \times 10^{-3} = \frac{4 - V_{GS1}}{39}$$

$$\Downarrow$$

$V_{GS1} = -3.34\,V$ or $V_{GS1} = 2.77\,V$ since $V_{GS} > V_T$, $V_{GS} = 2.77V$

$I_{D1} = \frac{V_{EQ} - V_{GS1}}{R_S} = \frac{4V - 2.77V}{39K\Omega} = 31.5\mu A$

$V_{DS1} = V_{DD} - I_{D1} R_D - I_{D1} R_S = 6.41\,V$

I_D decreases by $\frac{\Delta I_D}{I_D} = \frac{34.4\mu A - 31.5\mu A}{34.4\mu A} = 8.4\%$

V_{DS} increases by $\frac{\Delta V_{DS}}{V_{DS}} = \frac{6.41V - 6.08V}{6.08V} = 5.4\%$

5. Since $I_G = 0$, $V_G = V_D$, the MOSFET operates in saturation

$$I_D = \frac{K_n}{2}(V_{GS}-V_T)^2 \qquad I_D = \frac{V_{DD}-V_{DS}}{R_D} = \frac{V_{DD}-V_{GS}}{R_D}$$

$$\therefore \frac{250}{2}(V_{GS}-1)^2 = \frac{3.3-V_{GS}}{10}\times 10^3$$

$$\Downarrow$$

$V_{GS} = 0.81\,V$ or $V_{GS} = 2.01\,V$ since $V_{GS} > V_T$ $V_{GS} = 2.01\,V$

$V_{DS} = V_{GS} = 2.01\,V$ $\qquad I_D = \dfrac{3.3\,V - 2.01\,V}{10\,k\Omega} = 0.129\,mA$

a. Now the new value of K_n is $K_{n1} = 200\,\mu A/V^2$

$$\frac{200}{2}(V_{GS_1}-1)^2 = \frac{3.3-V_{GS_1}}{10}\times 10^3$$

$$\Downarrow$$

$V_{GS_1} = -1.097\,V$ or $V_{GS_1} = 2.097\,V$

To make $V_{GS1} > V_T$, $V_{GS1} = 2.097\,V$ $\qquad V_{DS1} = V_{GS1} = 2.097\,V$

$$I_{D1} = \frac{(3.3-2.097)V}{10\,k\Omega} = 0.120\,mA$$

I_D decreases by $\dfrac{\Delta I_D}{I_D} = \dfrac{0.129\,mA - 0.120\,mA}{0.129\,mA} = 7.0\%$

V_{DS} increases by $\dfrac{\Delta V_{DS}}{V_{DS}} = \dfrac{2.097\,V - 2.01\,V}{2.01\,V} = 4.3\%$

6. In problem 3, the simple biasing scheme is sensitive to the variation in K_n. Since $I_D = \frac{K_n}{2}(V_{GS}-V_T)^2$, $V_{DS} = V_{DD} - I_D R_D$, the variation percentage of I_D and V_{DS} is same as that of K_n with V_{GS} fixed in this case.

The biasing techniques in problem 4 and 5 are less sensitive to variations in device parameters since the operating point is stabilized by negative feedback.

In the case of problem 4, suppose for some reason I_D begins to increase, the voltage drop on R_S would increase, which results in a decrease in V_{GS} since V_{EQ} is fixed. And the decrease in V_{GS} will tend to restore I_D back to its original value.

In the case of problem 5, suppose I_D increases for some reason. As a result, V_{DS} will decrease and so will V_{GS} since $V_{GS} = V_{DS}$. The decrease in V_{GS} will tend to make I_D decrease to its original value.