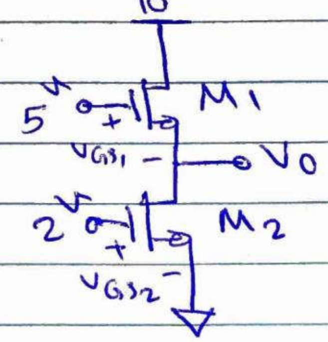
HW #3 Solutions

1. a)



$K = 2^{mA}/_{V^2}$

$V_T = 1^v$

$K = 64^{mA}/_{V^2}$

$V_T = 1^v$

$M_1 = Sat$: $i_{D_1} = K_1/_2 (V_{GS_1} - V_{T_1})^2$

$M_2 = Sat$: $i_{D_2} = K_2/_2 (V_{GS_2} - V_{T_2})^2$

$i_{D_1} = i_{D_2}$: $K_1/_2 (V_{GS_1} - V_{T_1})^2 = \frac{K_2}{2} (V_{GS_2} - V_{T_2})^2$

$$\frac{2^m}{\cancel{2}}((5 - V_0) - 1)^2 = \frac{64^m}{\cancel{2}}((2-0)-1)^2$$

$$(5 - V_0 - 1)^2 = 32(1)^2 \rightarrow V_0^2 + 16 - 8V_0 = 32$$

$$\rightarrow V_0^2 - 8V_0 - 16 = 0 \quad \begin{cases} V_{0_1} = -1.657^v \\ V_{0_2} = 9.657^v \end{cases}$$

* Checking which V_0 is valid :

M_1 & M_2 in saturation $\begin{cases} V_{GS_1} \geqslant V_{T_1} \ \& \ V_{DS_1} > V_{GS_1} - V_{T_1} \\ V_{GS_2} \geqslant V_{T_2} \ \& \ V_{DS_2} > V_{GS_2} - V_{T_2} \end{cases}$

$$\rightarrow \begin{cases} \left.\begin{array}{l} 5 - V_{0_1} = 6.657^v > 1^v \ \checkmark \\ 5 - V_{0_2} = -4.657^v < 1^v \ \times \end{array}\right\} V_{GS_1} \geqslant V_{T_1} \\ 2 - 0 = 2 \geqslant 1 \ \checkmark \quad V_{GS_2} \geqslant V_{T_2} \end{cases}$$

$$\rightarrow \begin{cases} V_{DS_1} = 10 - V_{0_1} = 11.657^v > 5.657^v \ \checkmark \text{ saturation} \end{cases}$$

M1 = triode : $i_{D_1} = K_1\left[(V_{GS_1} - V_{T_1})V_{DS_1} - \frac{V_{DS_1}^2}{2}\right]$

M2 = triode : $i_{D_2} = K_2\left[(V_{GS_2} - V_{T_2})V_{DS_2} - \frac{V_{DS_2}^2}{2}\right]$

$$i_{D_1} = i_{D_2} : K_1\left[(V_{GS_1} - V_{T_1})V_{DS_1} - \frac{V_{DS_1}^2}{2}\right] = K_2\left[(V_{GS_2} - V_{T_2})V_{DS_2} - \frac{V_{DS_2}^2}{2}\right]$$

$$\rightarrow 2\mu\left[(5 - V_o - 1)(10 - V_o) - \frac{(10 - V_o)^2}{2}\right] = 64\mu\left[(2 - 0 - 1)(V_o - 0) - \frac{(V_o - 0)^2}{2}\right]$$

$$\rightarrow (4 - V_o)(10 - V_o) - \frac{(10 - V_o)^2}{2} = 32\left[V_o - \frac{V_o^2}{2}\right]$$

$$(10 - V_o)(\underbrace{\cancel{4 - V_o - 5 + \tfrac{V_o}{2}}}_{-1 - \frac{V_o}{2}}) = 32\left(V_o - \frac{V_o^2}{2}\right)$$

$$-10 - 5V_o + V_o + \frac{V_o^2}{2} = 32V_o - 16V_o^2$$

$$\rightarrow 33V_o^2 - 72V_o - 20 = 0 \quad \begin{cases} V_{o1} = -0.24 \\ V_{o2} = +2.43 \end{cases}$$

M1 & M2 in Triode $\begin{cases} V_{GS_1} > V_{T_1} \ \& \ V_{DS_1} < V_{GS_1} - V_{T_1} \\ V_{GS_2} > V_{T_2} \ \& \ V_{DS_2} < V_{GS_2} - V_{T_2} \end{cases}$

$\rightarrow \begin{cases} 5 - V_{o1} = 5.24 > V_T = 1 \ ✓ \\ 5 - V_{o2} = 2.57 > V_T = 1 \ ✓ \\ 2 - 0 = 2 > 1 \ ✓ \end{cases}$ This is not valid.

$\rightarrow \begin{cases} V_{DS_1} = 10 - V_{o1} = 10.24 > 4.24 \ X \\ V_{DS_1} = 10 - V_{o2} = 7.57 > 1.57 \ X \text{ Saturation} \\ V_{DS_2} = V_{o1} = -0.24 < 1 \ X \\ V_{DS_2} = V_{o2} = 2.43 > 1 \ ✓ \text{ Triode} \end{cases}$

c) M_1 = Triode : $i_{D_1} = k_1\left[(V_{GS_1} - V_{T_1})V_{DS_1} - \frac{V_{DS_1}^2}{2}\right]$

M_2 = Saturation : $i_{D_2} = \frac{k_2}{2}(V_{GS_2} - V_{T_2})^2$

$i_{D_1} = i_{D_2}$:

$$2^{\mu}\left[(5 - V_o - 1)(10 - V_o) - \frac{(10 - V_o)^2}{2}\right]$$

$$= \frac{64^{\mu}}{2}(2-1)^2$$

$$\Rightarrow (4 - V_o)(10 - V_o) - \frac{(10 - V_o)^2}{2} = 16$$

$$\Rightarrow \begin{cases} V_{o_1} = -4.25^{v} \\ V_{o_2} = 12.25^{v} \end{cases}$$

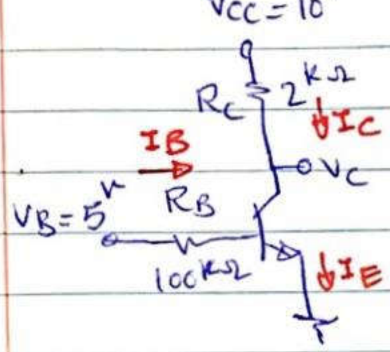
M_1 in Triode & M_2 in Saturation

$$\begin{cases} V_{GS_1} > V_{T_1} \;\&\; V_{DS_1} < V_{GS_1} - V_{T_1} \\ V_{GS_2} > V_{T_2} \;\&\; V_{DS_2} > V_{GS_2} - V_{T_2} \end{cases}$$

$$\rightarrow \begin{cases} 5 - V_{o_1} = 9.25 > 1 \;\checkmark \rightarrow 10 - V_{o_1} = 14.25 > 8.25 \;\times \text{ Saturat} \\ 5 - V_{o_2} = -7.25 < 1 \;\times \\ 2 > 1 \;\checkmark \rightarrow V_{o_1} = -4.25^{v} < 1 \;\times \end{cases}$$

$\Rightarrow$ This is <u>not</u> valid.

2.



$V_{BE} = 0.6^v \rightarrow I_B = \frac{V_B - V_{BE}}{R_B} = \frac{5 - 0.6}{100^k}$

$\rightarrow I_B = 44^{\mu A}$

$I_C = \beta I_B = 100 \times 44^{\mu A} = 4.4^{mA}$

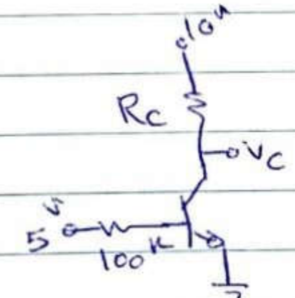
$I_C = (1+\beta) I_B = 4.444^{mA}$

$V_C = V_{CC} - 2^k I_C = 10 - 2^k \times 4.4^{mA} = 1.2^v > 0.2^v \rightarrow$ Active

a. $\beta_{new} = 112 \rightarrow I_B = 44^{\mu A}, \; I_C = 112 \times 44^{\mu A} = 4.928^{mA}$

$V_C = 10 - 2^k I_C = 0.144^v < 0.2^v \rightarrow$ saturation

3.



$i_B = 44^{\mu A}, \quad 50 \leq \beta \leq 150, \; V_{CE} \geq 0.2^v$

$\beta_{min} = 50 : i_C = \beta_{min} i_B = 50 \times 44^{\mu A} = 2.2^{mA}$

$V_C = 10 - 2.2^m \times R_C \geq 0.2^v \rightarrow R_C \leq 4.45^{k\Omega}$

$\beta_{max} = 150 : i_C = 150 \times 44^{\mu A} = 6.6^{mA}$

$V_C = 10 - 6.6^m \times R_C \geq 0.2 \rightarrow R_C \leq 1.48^{k\Omega}$

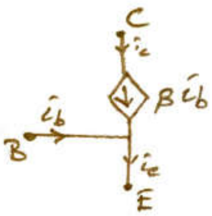
$\Rightarrow \boxed{R_C \leq 1.48^{k\Omega}}$ You can pick a value in this range. $R_C = 1^{k\Omega}$ is a good choice.

a. $\beta_{min} = 50 : V_C = 10 - 2.2^m \times 1^k = 7.8^v$

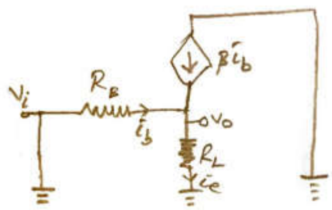
$\Rightarrow 3.4^v \leq V_C \leq 7.8^v$

$\beta_{max} = 150 : V_C = 10 - 6.6^m \times 1^k = 3.4^v$

4) Small-signal model for the BJT:



Small signal model for the common-collector amplifier:



$$i_b = (V_{in} - V_o)/R_B \qquad i_e = i_b + \beta i_b \qquad i_e = \frac{V_o}{R_L}$$

$$\frac{V_o}{R_L} = (\beta+1)\, i_b = (\beta+1)\, \frac{V_{in} - V_{out}}{R_B}$$

$$R_B V_o = R_L(\beta+1)(V_{in} - V_{out})$$

$$\left(R_L(\beta+1) + R_B\right) V_o = R_L(\beta+1) V_i$$

$$\frac{V_o}{V_i} = \frac{R_L(\beta+1)}{R_L(\beta+1) + R_B} = \frac{\frac{R_L}{R_B}(\beta+1)}{\frac{R_L}{R_B}(\beta+1) + 1}$$

$$A = \frac{V_o}{V_{in}} \approx 1 \quad \text{when} \quad \frac{R_L}{R_B}(\beta+1) \gg 1$$

In this circuit base terminal of the transistor serves as the input and the emitter is the output. When $A \approx 1$ the output voltage "follows" the input voltage. In this case the emitter voltage follows the base voltage, hence the circuit is called an <u>emitter follower</u>.

5) a)

Saturation: $I = \frac{K_N}{2}(V_{GS} - V_{TN})^2$

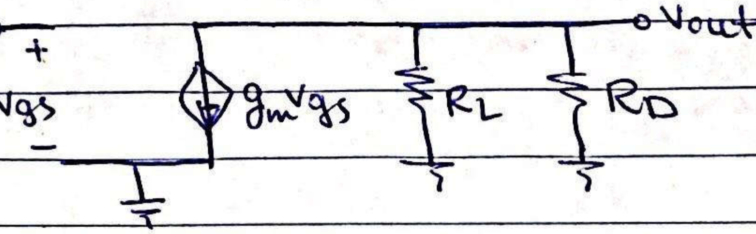
$$I_{RD} = I + I_{R_L}$$

$$\rightarrow \frac{V_{DD} - V_{out}}{R_D} = \frac{K_N}{2}(V_{GS} - V_{TN})^2 + \frac{V_{out}}{R_L}$$

$$V_{GS} = V_{IN} : \quad V_{out}\left(1/R_L + 1/R_D\right) = \frac{V_{DD}}{R_D} - \frac{K_N}{2}(V_{IN} - V_{TN})^2$$

$$\boxed{V_{out} = \frac{R_L R_D}{R_L + R_D}\left(\frac{V_{DD}}{R_D} - \frac{K_N}{2}(V_{IN} - V_{TN})^2\right)}$$

b.



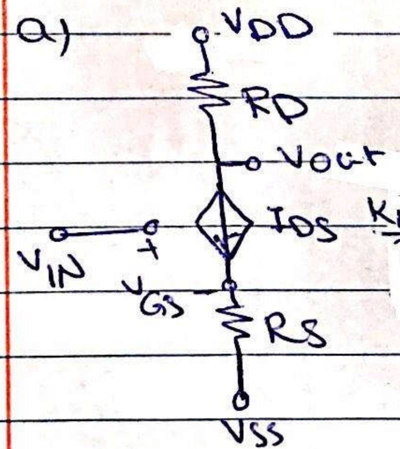
c. $i_{R_D} + i_{R_L} + g_m v_{gs} = 0 \rightarrow \frac{V_{out}}{R_D} + \frac{V_{out}}{R_L} + g_m v_{gs} = 0$

$\rightarrow V_{out}\left(\frac{R_L + R_D}{R_D R_L}\right) = -g_m v_{gs}$, $v_{gs} = v_{in}$

$\rightarrow A = \frac{V_{out}}{V_{in}} = -\frac{R_D R_L}{R_D + R_L} \cdot g_m = -\frac{R_D R_L}{R_D + R_L}\left(K_N(V_{IN} - V_T)\right)$

$g_m = \frac{2 I_D}{\cancel{V_{GS}} - V_T} = K_N(V_{IN} - V_T)$
V_{IN}

6. a)



$\frac{K_N}{2}(V_{IN} - V_S - V_{TN})^2$

$V_{GS} \geq V_T$

$V_{DS} \geq V_{GS} - V_T$

b. $V_{out} = V_{DD} - R_D I_D$, $I_D = \frac{K_N}{2}(V_{GS} - V_{TN})^2$

$V_{GS} = V_{IN} - V_{SS} - R_S I_D$

$\rightarrow I_D = \frac{K_N}{2}(V_{IN} - V_{SS} - R_S I_D - V_{TN})^2$

$\rightarrow \sqrt{I_D} = \pm\sqrt{\frac{K_N}{2}}(V_{IN} - V_{SS} - R_S I_D - V_{TN})$

$\left(\pm\sqrt{\frac{K_N}{2}}\right) R_S I_D + \sqrt{I_D} \pm \sqrt{\frac{K_N}{2}}(V_{SS} + V_{TN} - V_{IN}) = 0$

$$I_D = \frac{-2ab \pm \sqrt{1-4ab} + 1}{2a^2}$$

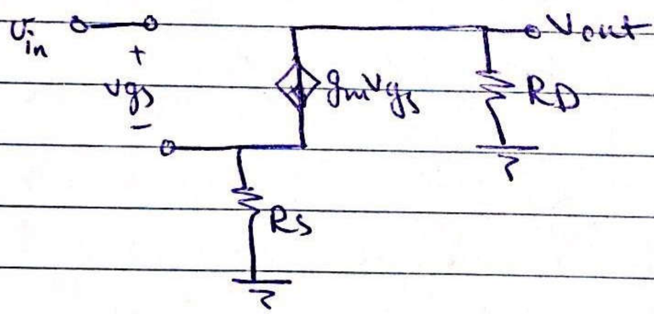
Valid

$$\Rightarrow I_D = \frac{K_N R_S (V_{SS} + V_{TN} - V_{IN}) \pm \sqrt{1 + 2K_N R_S (V_{SS} + V_{TN} - V_{IN})} + 1}{K_N R_S^2}$$

$$V_{out} = V_{DD} - R_D I_D$$

c. $$V_{out}\Big|_{V_{IN}=0} = V_{DD} - R_D \left(\frac{K_N R_S (V_{SS} + V_{TN}) - \sqrt{1 + 2K_N R_S (V_{SS} + V_{TN})} + 1}{K_N R_S^2} \right)$$

d.

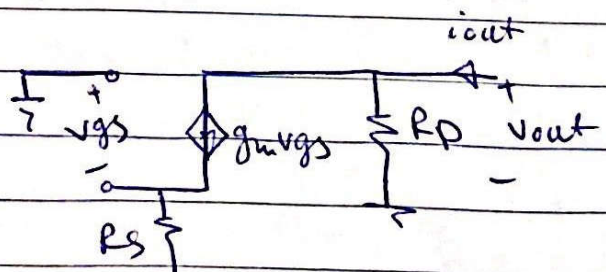


e. $V_{out} = -R_D g_m v_{gs}$, $v_{gs} = v_{in} - g_m R_S v_{gs} \Rightarrow v_{gs} = \dfrac{v_{in}}{1 + g_m R_S}$

$$V_{out} = -R_D g_m \frac{v_{in}}{1 + g_m R_S} \Rightarrow A = \frac{V_{out}}{v_{in}} = -R_D \frac{g_m}{1 + g_m R_S}$$

$$g_m = \frac{2 I_D}{V_{GS} - V_T} = K_N (V_{GS} - V_T)$$

f.

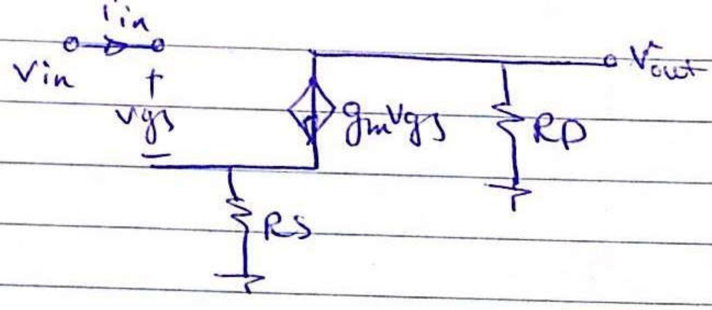


$v_{gs} = -g_m R_S v_{gs} \Rightarrow v_{gs} = 0$

$g_m v_{gs} = 0$

$R_D i_{out} = V_{out} \Rightarrow \boxed{R = \frac{V_{out}}{\ldots}}$

go



$i_{in} = 0 \Rightarrow \frac{v_{in}}{i_{in}} = \infty \Rightarrow \boxed{R_{in} = \infty}$