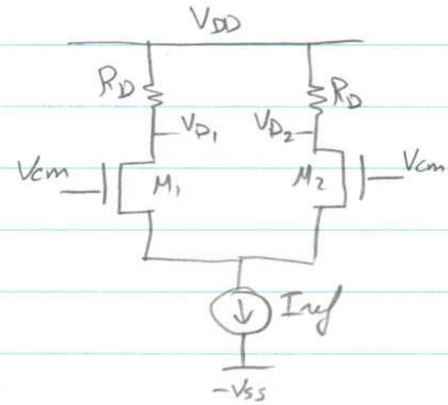
Homework 4
Solution

1)



a) Assume M_1 & M_2 in Sat

$$I_{D_1} = I_{D_2} = \frac{I_{ref}}{2} = \frac{K_n}{2}(V_{GS} - V_T)^2$$

$$(V_{GS} - 0.5)^2 = 0.1, \qquad \boxed{V_{GS} = 0.82V}$$

b) $V_{CM} = 0, \qquad I_{D_1} = I_{D_2} = \frac{I_{ref}}{2} = 0.2mA$

$V_{D_1} = V_{D_2} = V_{DD} - I_{D_1} R_D = 1V$

$V_S = V_G - V_{GS} = 0 - 0.82 = -0.82V$

$V_{DS} = 1.82V \qquad V_{GS} - V_T = 0.32V$

$V_{DS} > V_{GS} - V_T \longrightarrow$ Saturation assumption is valid.

c) $V_{CM} = 1V \qquad I_{D_1} = I_{D_2} = \frac{I_{ref}}{2} = 0.2mA$

$V_{D_1} = V_{D_2} = V_{DD} - I_{D_1} R_D = 1V$

$V_S = V_G - V_{GS} = 1 - 0.82 = 0.18V$

$V_{DS} = 0.82 > V_{GS} - V_T \longrightarrow$ Sat. assumption is valid

d) $V_{CM} = -0.2V$ $\qquad I_{D_1} = I_{D_2} = \frac{I_{ref}}{2} = 0.2mA$

$V_{D_1} = V_{D_2} = V_{DD} - I_{D_1} R_D = 1V$

$V_S = V_G - V_{GS} = -0.2 - 0.82 = -1.02V$

$V_{DS} = 2.02\,V > V_{GS} - V_T \longrightarrow$ still in Sat.

$V_S + V_{SS} = -1.02 + 1.5 = 0.48\,V > 0.4$

(current source operates).

e) $V_{D_1} = V_{D_2} = 1V$

$V_{DS} \geq V_{GS} - V_T \rightarrow V_D \geq V_G - V_T$

$V_G \leq V_D + V_T \quad , \quad V_G \leq 1 + 0.5$

$V_{CM}|_{max} = V_G|_{max} = 1.5\,V.$

f) $V_S + V_{SS} \geq 0.4V$

$V_S = V_G - V_{GS} \quad , \quad V_{GS} = 0.82V$

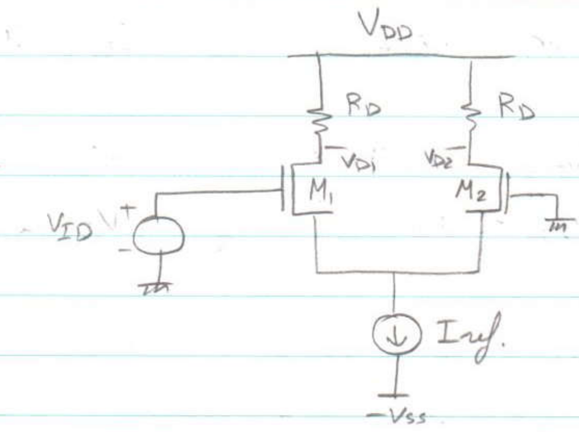
$V_G - V_{GS} + V_{SS} \geq 0.4V$

$V_G \geq 0.4 + 0.82 - 1.5$

$V_G \geq -0.28V$

$V_{cm}|_{min} = V_G|_{min} = -0.28V$

2)



a) M_2 is off $\longrightarrow$ $V_{GS_2} < V_T$ $\boxed{V_{D_2} = V_{DD}}$

$\boxed{V_S > -V_T}$

$I_{D_1} = I_{ref} = 0.4\ mA$

Assume M_1 in Sat. $\longrightarrow$ $I_{D_1} = \frac{K_n}{2}(V_{GS_1} - V_T)^2 = 0.4\ mA$

$V_{GS_1} = 0.95\ V.$ $\qquad$ $V_{D_1} = V_{DD} - I_D R_D$

$V_{G_1} - V_S = 0.95V$ $\qquad$ $V_{D_1} = 0.5V$

$V_{ID} = 0.95 + V_S$ $\qquad$ $V_{DS_1} = V_{D_1} - V_S$

$\boxed{V_{ID} > 0.45V}$ $\qquad$ $\boxed{V_{DS_1} > 1V}$

$V_{DS_1} > V_{GS_1} - V_T$ $\longrightarrow$ Sat. assumption is valid.

b) M_1 is off $\longrightarrow$ $V_{GS_1} < V_T$ $\boxed{V_{D_1} = V_{DD}}$

$\boxed{V_{ID} < V_T + V_S}$

$I_{D_2} = I_{ref} = 0.4\ mA$

Assume M2 in Sat $\longrightarrow$ $I_{D_2} = \frac{K_n}{2}(V_{GS_2} - V_T)^2 = 0.4\,mA$

$V_{GS_2} = 0.95V.$ $\qquad V_S = -0.95V.$

$V_{ID} < V_T + V_S$, $\boxed{V_{ID} < -0.45V}$

$V_{D2} = V_{DD} - I_{D_2} R_D = 0.5V$

$V_{DS_2} = V_{D_2} - V_S = 1.45V > V_{GS} - V_T$ $\leftrightarrows$ Sat. check

c) let $V_{out} = V_{D_2} - V_{D_1}$

in a) $V_{D2} - V_{D_1} = 1.5 - 0.5 = 1V$

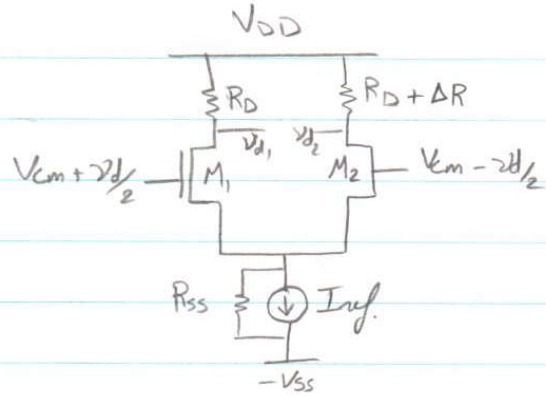
in b) $V_{D_2} - V_{D_1} = 0.5 - 1.5 = -1V$

$-1V \leq V_{out} \leq 1V$

3)

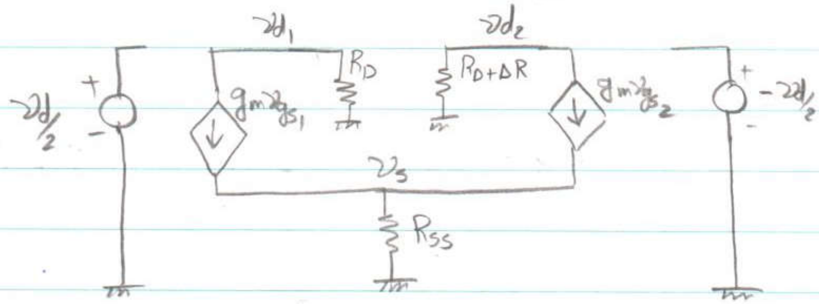
$\Delta R = \pm\ 50\ \Omega$

$g_m = 4\,mA$



* Small Signal Analysis

Differential mode:



$$g_m v_{gs_1} + g_m v_{gs_2} = \frac{v_s}{R_{SS}}$$

$$g_m\left(\cancel{\frac{v_d}{2}} - v_s\right) + g_m\left(\cancel{\frac{-v_d}{2}} - v_s\right) = \frac{v_s}{R_{SS}}$$

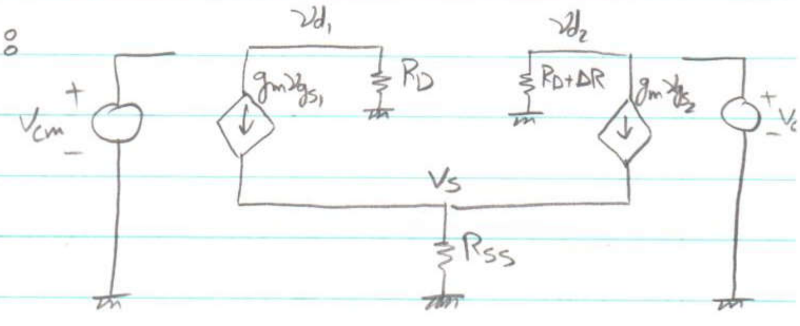
$$-2 g_m v_s = \frac{v_s}{R_{SS}} \longrightarrow \boxed{v_s = 0}$$

$$v_{d_1} = -g_m v_{gs_1} R_D = -g_m \frac{v_d}{2} R_D$$
$$v_{d_2} = -g_m v_{gs_2}(R_D + \Delta R) = g_m \frac{v_d}{2}(R_D + \Delta R)$$

$$v_o = v_{d_2} - v_{d_1} = g_m v_d \left(R_D + \frac{\Delta R}{2}\right)$$

$$A_d = \frac{v_o}{v_d} = g_m\left(R_D + \frac{\Delta R}{2}\right) = \underline{20.1} \text{ or } \underline{19.9}$$

Common mode :



$$v_{gs_1} = v_{gs_2} = V_{cm} - V_s$$

$$V_s = V_{cm} - V_{gs}$$

$$2 g_m v_{gs} = \frac{v_s}{R_{ss}} = \frac{V_{cm} - v_{gs}}{R_{ss}}$$

$$v_{gs}\left(2g_m + \frac{1}{R_{ss}}\right) = \frac{V_{cm}}{R_{ss}}$$

$$v_{gs} = \frac{V_{cm}}{2g_m R_{ss} + 1} \simeq \frac{V_{cm}}{2 g_m R_{ss}}$$

$$g_m v_{gs} \simeq \frac{V_{cm}}{2R_{ss}}$$

$$v_{d_1} = -g_m v_{gs} R_D = -\frac{V_{cm} R_D}{2R_{ss}}$$

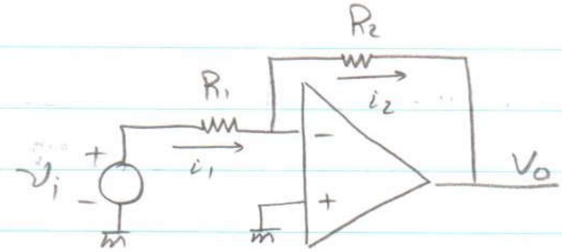
$$v_{d_2} = -g_m v_{gs}(R_D + \Delta R) = -\frac{V_{cm}(R_D + \Delta R)}{2R_{ss}}$$

$$v_o = v_{d_2} - v_{d_1} = \frac{-V_{cm}\,\Delta R}{2R_{ss}}$$

$$A_{cm} = \frac{v_o}{V_{cm}} = -\frac{\Delta R}{2R_{ss}} = \pm 10^{-3} \quad \text{or}$$

$$CMRR = 20 \log |A_d| / |A_{cm}| = 86 \text{ dB}$$

4)



a) Assume $R_{in} = \infty$, $i^+ = i^- = 0$

$$i_1 = i_2$$

$$\frac{v_i - v^-}{R_1} = \frac{v^- - v_o}{R_2}$$

$$v_o = A(v^+ - v^-)$$

$$v_o = -Av^-$$

$$v^- = \frac{v_o}{-A}$$

$$\frac{v_i - \left(\frac{-v_o}{A}\right)}{R_1} = \frac{\left(\frac{-v_o}{A}\right) - v_o}{R_2}$$

$$\frac{v_i}{R_1} = -v_o\left(\frac{1}{AR_1} + \frac{1}{AR_2} + \frac{1}{R_2}\right)$$

$$\frac{v_o}{v_i} = \frac{-AR_2}{R_2 + R_1 + AR_1} = G$$

$$\lim_{A\to\infty} G = -\frac{R_2}{R_1}$$

$$\varepsilon = \frac{|G| - (R_2/R_1)}{R_2/R_1} \times 100\ \%$$

$$v^- = -\frac{v_o}{A} = \frac{Gv_i}{A}$$

$A = 10^3$, $|G| = 90.83$, $\varepsilon = -9.17\ \%$ $v^- = -9.08\ mV$

$A = 10^4$, $|G| = 99.00$, $\varepsilon = -1.00\ \%$ $v^- = -0.99\ mV$

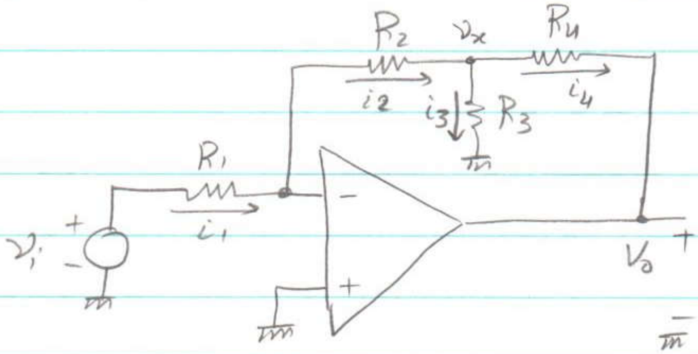
$A = 10^5$, $|G| = 99.90$, $\varepsilon = -0.10\ \%$ $v^- = -0.10\ mV$

b) For $A = 50{,}000$ $\quad |G| = 99.8$

$A = 100{,}000$ $\quad |G| = 99.9$

if A decreased by 50 %, $|G|$ only decrease by 0.1 %

5)



Ideal op. Amp

$A = \infty$, $\quad i^+ = i^- = 0 \qquad R_{in} = \infty \qquad v^+ = v^-$

$$i_1 = i_2 \implies \frac{v_i}{R_1} = \frac{-v_x}{R_2}$$

$$i_2 = i_3 + i_4 \implies \frac{-v_x}{R_2} = \frac{v_x}{R_3} + \frac{v_x - v_0}{R_4}$$

$$v_x = \frac{v_0}{R_4}\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1}$$

$$\frac{v_i}{R_1} = \frac{-v_0}{R_2 R_4}\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1}$$

$$\frac{v_0}{v_i} = \frac{-R_2 R_4}{R_1}\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$$

$$\frac{v_0}{v_i} = -\frac{R_2}{R_1}\left(\frac{R_4}{R_2} + \frac{R_4}{R_3} + 1\right)$$

$$i_{in} = i_1 = \frac{v_i}{R_1} \qquad R_{in} = \frac{v_i}{i_{in}} = R_1$$

$\therefore \quad R_1 = 1\,M\Omega.$

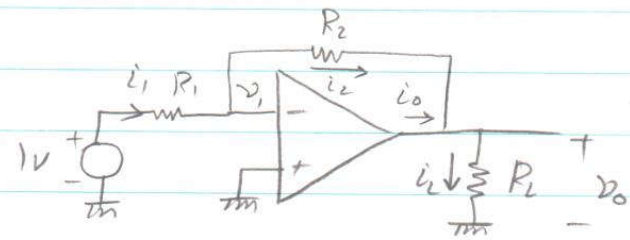
$$|G| = \frac{R_2}{R_1}\left(\frac{R_4}{R_2} + \frac{R_4}{R_3} + 1\right) \qquad \text{let } R_2 = 1M\Omega, \; R_4 = 1M\Omega$$

$$|G| = \left(1 + \frac{10^6}{R_3} + 1\right) = 100$$

$$\therefore \quad R_3 = \frac{10^6}{98} = 10.2\,k\Omega$$

$R_1 = 1M\Omega, \quad R_2 = 1M\Omega, \quad R_3 = 10.2k\Omega, \quad R_4 = 1M\Omega.$

6)



a) ideal op. Amp.

$\therefore \quad v_1 = v^+ = 0$

$$i_1 = i_2 = \frac{1}{R_1} = 1\,mA$$

$$v_o = -\frac{R_2}{R_1}(1) = -10V$$

$$i_L = \frac{v_o}{R_L} = -10\,mA$$

$$i_o = i_L - i_2 = -10 - 1 = -11\,mA$$

b) $\frac{v_o}{v_i} = -10$

$$\frac{i_L}{i_1} = -10$$

$$\frac{P_o}{P_i} = \frac{v_o * i_L}{v_i * i_1}$$

$$\frac{P_o}{P_i} = 100$$