A Principled Incentive Mechanism to Promote Economic Viability of Mini-Grids

Anokhi Mehta
Network Security Lab (NSL), Dept. of ECE
University of Washington - Seattle
Seattle, WA, USA 98195-2500
aamehta@uw.edu

Luyao Niu
Network Security Lab (NSL), Dept. of ECE
University of Washington - Seattle
Seattle, WA, USA 98195-2500
luyaoniu@uw.edu

Dinuka Sahabandu
Network Security Lab (NSL), Dept. of ECE
University of Washington - Seattle
Seattle, WA, USA 98195-2500
sdinuka@uw.edu

June Lukuyu
Dept. of ECE
University of Washington - Seattle
Seattle, WA, USA 98195-2500
jlukuyu@uw.edu

Radha Poovendran
Network Security Lab (NSL), Dept. of ECE
University of Washington - Seattle
Seattle, WA, USA 98195-2500
rp3@uw.edu

***Abstract*—The urgent need for universal electrification and the high cost of grid expansion highlight the crucial role of mini-grids in Sub-Saharan Africa (SSA). These systems harness technologies such as cost-effective renewable energy to provide reliable and affordable electricity to remote, off-grid communities. However, the existing mini-grids lack economic viability in the long run. To address this challenge, mini-grid programs have been developed to stimulate electricity demand by incentivizing the adoption of appliances, particularly for productive purposes. At present, however, there exists no principled analysis to evaluate how incentives for appliance adoption could be structured to benefit customers while contributing to the economic viability of mini-grids. In this paper, we develop a principled incentive mechanism to enhance the economic viability of mini-grids in SSA by stimulating the productive use of electricity. The mini-grid developer offers a range of appliances for rent at affordable rates, coupled with competitive electricity pricing, to stimulate electricity demand from customers. The customers determine which appliances to rent based on the prices announced by the developer and the utility associated with the appliances. We formulate an optimization problem for the customers and mini-grid developers. Leveraging the formulated problem, we show that customers with limited computational resources can efficiently determine the appliances to rent. We further develop an algorithm for the developer to estimate how the customers will react to the electricity price and rental prices, enabling efficient calculation of both price signals. We show that the proposed approach leads to a win-win situation between the developer and customers. Customers create non-negative profit by adopting productive use appliances, and the developer stimulates the electricity demand which is better aligned with the renewable energy generation. We evaluate the proposed approach using a rural village with ten appliances whose load profiles are obtained from a real-world dataset.**

***Index Terms*—Electrification, mini-grid, sustainability**

I. Introduction

Electrification is critically important for empowering individuals and communities in Sub-Saharan Africa (SSA). Access to electricity can significantly improve the quality of life by providing essential services such as clean food, water, healthcare, and lighting [1]. Furthermore, electrification enhances the efficiency of businesses and institutions, thereby bridging economic and educational gaps and ultimately facilitating the breaking of the cycle of poverty. However, the progress toward ensuring access to electricity in SSA, especially in rural areas and informal communities, remains far behind schedule [2].

To accelerate the expansion of electricity access in rural areas, mini-grid technologies have been proposed [3]–[5], where one or more energy sources provide electricity access to customers isolated from the main grid. However, existing mini-grid deployments may lack economic viability [6]–[8]. This is primarily because the deployment of mini-grids is expensive. In contrast, the electricity demand is low, forcing the mini-grid developers to set cost-reflective tariffs to quickly recover their capital costs, which are not affordable to customers, especially low-income households in remote communities [9], [10]. To this end, mini-grid demand stimulation programs have been proposed to incentivize the usage of productive appliances [11], aiming to increase the electricity demand without imposing financial distress. At present, however, no principled analysis exists to facilitate the developer to design such incentives and evaluate how they contribute to the economic viability of mini-grids in a rigorous and scalable manner.

In this paper, we develop a principled incentive mechanism to promote the economic viability of mini-grids. Specifically, the mini-grid developer offers a range of appliances for rent and strategic electricity pricing to stimulate electricity demand. The customers observe the rental and electricity prices and then determine the subset of appliances to rent based on their preferences. To model their interactions, we formulate an optimization problem from the customers' and developer's perspectives. Leveraging the formulated optimization problem, we show that the developed incentive mechanism creates a win-win situation. Customers rent appliances and generate non-negative profits by utilizing productive appliances. On the other hand, the developer stimulates the electricity demand using carefully designed electricity prices and rental prices, enabling the electricity demand to be better aligned with renewable energy generation. To summarize, we make the following contributions in this paper.

- Using our formulated optimization problem, customers can efficiently determine whether to rent an appliance by comparing its marginal gain and loss.
- We develop an algorithm for the developer to evaluate how customers will react to the electricity price and rental prices without knowing the customers' preferences, allowing the developer to compute the electricity price and rental prices efficiently.
- We demonstrate our approach using a rural village with ten appliances whose load profiles are obtained from real-world dataset [12]. We show that the proposed approach promotes the economic viability of the mini-grid.

II. System Model

We consider a mini-grid developer interacts with all customers over finitely many communication rounds $t = 1, \ldots, T$. At each communication round, e.g., a few weeks or months, the developer first announces the electricity price $P_t \in \mathbb{R}^{24}$ for each hour and a rental price $r_{j,t}$ for each appliance j. After receiving the electricity price and rental prices, each customer, i.e., a household connected within the mini-grid, decides which appliance to rent at this communication round. If a customer i decides to rent an appliance j, they place a certain amount of deposit, pays the rent, and earns certain utility when using the appliance. The developer collects the deposits and rents from all customers, and supply the power to customers. Throughout all communication rounds, all customers and developer need to guarantee their profits are non-negative to ensure the economic viability of mini-grid.

A. Customer Model

We denote the set of customers as $\mathcal{H} = \{1, \ldots, n\}$. At each communication round t, each customer $i \in \mathcal{H}$ can rent some appliances from the set $\mathcal{A} = \{1, ..., k\}$. For each appliance $j \in \mathcal{A}$, its power consumption is characterized by a load profile $l_j \in \mathbb{R}^{24}$, specifying its expected hourly power usage.

At communication round t, each customer i determines whether they use each appliance $j \in \mathcal{A}$, given the electricity price P_t and rental price $r_{j,t}$. We represent such a decision using a binary variable $v_{i,j,t}$ defined as

$$v_{i,j,t} = \begin{cases} 1, & \text{if customer } i \text{ uses the appliance } j \\ 0, & \text{otherwise} \end{cases}. \quad (1)$$

For any subset of appliances $\mathcal{V}_{i,t} = \{j \in \mathcal{A} : v_{i,j,t} = 1\} \subseteq \mathcal{A}$ chosen by customer i at communication round t, the aggregated load of all appliances chosen by $\mathcal{V}_{i,t}$ is calculated as $\Lambda_{i,t} = \sum_{j \in \mathcal{A}} v_{i,j,t} l_j$, which specifies the expected hourly power usage of customer h_i. Then the expected cost incurred due to power consumption at each hour for each customer i is represented as $E_{i,t} = P_t^\top \Lambda_{i,t}$, where $P_t^\top$ represents the transpose of P_t. The expected cost incurred by each customer when renting the appliances at communication round t is computed as $Y_{i,t} = \sum_{j=1}^{k} (v_{i,j,t} d_j - v_{i,j,t-1} d_j) + \sum_{j=1}^{k} v_{i,j,t} r_{j,t}$, where d_j is the deposit associated with appliance $j \in \mathcal{A}$ and $r_{j,t}$ is the rent for each communication round. Note that coefficients d_j and $r_{j,t}$ are chosen by the developer, which will be detailed in Section II-B. Here the term $\sum_{j=1}^{k} v_{i,j,t} d_j$ models the total amount of deposit placed by the customer i at communication round t to rent the appliances in $\mathcal{A}$ satisfying $v_{i,j,t} = 1$, the term $-\sum_{j=1}^{k} v_{i,j,t-1} d_j$ captures the returned deposit which were placed by the customer at communication round $t-1$. The last term $\sum_{j=1}^{k} v_{i,j,t} r_{j,t}$ is the total amount of rent that needs to be paid by the customer. Thus the total cost incurred by each customer over T communication rounds is given as

$$C_i = \sum_{t=1}^{T} (E_{i,t} + Y_{i,t}). \quad (2)$$

Each customer $i \in \mathcal{H}$ can obtain utility $u_{i,j}$ by consuming per unit power when using appliance $j \in \mathcal{A}$. Therefore, the expected revenue earned by each customer i is given as

$$R_i = \sum_{t=1}^{T} \sum_{j=1}^{k} v_{i,j,t} u_{i,j} l_j \mathbf{1}_{24}, \quad (3)$$

where $\mathbf{1}_{24} = [1, \ldots, 1]^\top$ with dimension 24. We then have that the expected profit earned by customer i when renting appliances and consuming power can be calculated as $R_i - C_i$.

A selfish and rational customer aims to solve for its *strategy*, i.e., the set of appliances to be rented, as follows:

$$\max_{\{v_{i,j,t}\}_{j,t}} \quad R_i - C_i \quad (4)$$

$$\text{s.t.} \quad v_{i,j,t} \in \{0,1\}, \ \forall j \in \mathcal{A}, \forall t = 1, \ldots, T \quad (5)$$

$$E_{i,t} + Y_{i,t} \leq \sum_{\tau=1}^{t-1} \Big(\sum_{j=1}^{k} v_{i,j,\tau} u_{i,j} l_j \mathbf{1}_{24} - E_{i,\tau} - Y_{i,\tau} \Big), \ \forall t = 1, \ldots, T \quad (6)$$

Constraint (6) defines a budget constraint for each customer $i \in \mathcal{H}$. The left hand side of Eq. (6) represents the cost incurred by customer i at each communication round t, whereas the right hand side of Eq. (6) gives the accumulated profit made by customer up to communication round $t-1$. By satisfying constraint (6) at each communication round t, the customers are guaranteed to earn non-negative profit when renting and using appliances. All customers need to satisfy the budget constraint to avoid financial distress.

B. Developer Model

The objective of the mini-grid developer is to stimulate power consumption from customers in $\mathcal{H}$ by offering appliance rentals, thereby ensuring a balanced aggregated load and power generation. The developer can incentivize the customers by setting electricity price P_t at each communication round t, and specifying $r_{j,t}$ for each appliance $j \in \mathcal{A}$. The developer earns revenue when customers rent appliances and consume energy. The revenue of the developer is computed as $\sum_{i \in \mathcal{H}} C_i$. We denote the power generation profile as $G_t \in \mathbb{R}^{24}$, which specifies the hourly power generation, e.g., the expected renewable energy generation obtained using historical data. Note that the aggregated load $\sum_{i\in\mathcal{H}}\Lambda_{i,t}$ may not necessarily follow the generation profile G_t. Therefore, when the aggregated load exceeds G_t, the developer uses backup generators (e.g., diesel generators) to guarantee load-generation balance and avoid cascading failures in the mini-grid. Consequently, the developer incurs a cost, denoted as $e\left\{\sum_{i\in\mathcal{H}}\Lambda_{i,t}-G_t\right\}_+$, at each communication round t, where operator $\{\cdot\}_+$ denotes $\max\{\cdot,0\}$. Then the developer aims to solve for its strategy, i.e., the $\{P_t\}_t$ and $\{r_{j,t}\}_{j,t}$, as follows:

$$\max_{\{P_t\}_t,\{r_{j,t}\}_{j,t}} \sum_{t=1}^{T}\Bigg[P_t^\top\sum_{i\in\mathcal{H}}\Lambda_{i,t}+\Bigg(\lambda\Bigg\{-G_t+\sum_{i\in\mathcal{H}}\Lambda_{i,t}\Bigg\}_- - e\Bigg\{\sum_{i\in\mathcal{H}}\Lambda_{i,t}l_j-G_t\Bigg\}_+\Bigg)\mathbf{1}_{24}+\sum_{i\in\mathcal{H}}Y_{i,t}\Bigg] \tag{7}$$

$$\text{s.t.}\quad P_t^\top\sum_{i\in\mathcal{H}}\Lambda_{i,t}l_j-e\{\sum_{i\in\mathcal{H}}\Lambda_{i,t}l_j-G_t\}_+\mathbf{1}_{24}+\sum_{i\in\mathcal{H}}Y_{i,t}\geq 0,\ \forall 1,\dots,T \tag{8}$$

where $\{\cdot\}_-$ represents $\min\{\cdot,0\}$ and $\lambda>0$ is a weight parameter. The term $\lambda\left\{-G_t+\sum_{i\in\mathcal{H}}\Lambda_{i,t}\right\}_-\leq 0$ models the loss incurred by the developer due to excess power generation in G_t. As the customers rent and utilize more appliances, this loss will decrease to zero. The term $P_t^\top\sum_{i\in\mathcal{H}}\Lambda_{i,t}\geq 0$ captures the revenue made by the developer when the customers consume power, with unit electricity price being P_t. Finally, $\sum_{i\in\mathcal{H}}Y_{i,t}\geq 0$ is the revenue obtained by the developer when lending appliances. The constraint (8) requires that $\{P_t\}_t$ and $\{r_{j,t}\}_{j,t}$ need to be chosen appropriately so that the developer has non-negative profit for all communication rounds t.

III. Solution Approach

A. Computation of Strategies of Customers

In the following, we compute the strategy of customers. Solving Eq. (4)-(6) is computationally expensive due to the presence of integer variables and coupling over communication rounds. In what follows, we first relax the optimization problem and derive customer strategies based on the relaxed optimization problem.

Our insight of the relaxation is that it suffices to solve Eq. (4)-(6) by ensuring all customers to make a non-negative profit at each communication round. This allows us to decouple the objective function in (4) and constraint (6), leading to the following formulation for each $t=1,\dots,T$

$$\max_{\{v_{i,j,t}\}_j} \sum_{j=1}^{k} v_{i,j,t}u_{i,j}l_j\mathbf{1}_{24}-(E_{i,t}+Y_{i,t}) \tag{9}$$

$$\text{s.t.}\quad v_{i,j,t}\in\{0,1\},\ \forall j\in\mathcal{A} \tag{10}$$

$$\sum_{j=1}^{k} v_{i,j,t}u_{i,j}l_j\mathbf{1}_{24}-(E_{i,t}+Y_{i,t})\geq 0 \tag{11}$$

Compared with the optimization problem in (4)-(6), the formulation in (9)-(11) involves less decision variables and constraints. It further allows us to decouple over all appliances, and thus can be solved efficiently.

For each customer $i\in\mathcal{H}$ and appliance $j\in\mathcal{A}$, Eq. (3) indicates that when $v_{i,j,t}=1$, the marginal revenue at communication round t can be represented as $\beta_{i,j,t}=u_{i,j}l_j\mathbf{1}_{24}$. Using Eq. (2), the marginal cost at communication round t for each customer i and appliance j can be calculated as

$$c_{i,j,t}=\begin{cases} r_{j,t}+P_t^\top l_j, & \text{if } v_{i,j,t-1}=1\\ d_j+r_{j,t}+P_t^\top l_j, & \text{if } v_{i,j,t-1}=0\end{cases} \tag{12}$$

when $v_{i,j,t}=1$. We then have the following result.

Proposition 1. *A selfish and rational customer will choose $v_{i,j,t}=1$ to maximize its profit in Eq.* (4) *if and only if the marginal profit of the customer by choosing appliance j is non-negative, i.e.,*

$$\beta_{i,j,t}-c_{i,j,t}\geq 0. \tag{13}$$

Proof. We prove the 'if' direction by contradiction. Suppose that Eq. (13) does not hold for some $j\in\mathcal{A}$ whereas $v_{i,j,t}=1$ for some customer $i\in\mathcal{H}$ at communication round t. Denote $\mathcal{V}_{i,t}=\{v_{i,j,t}:v_{i,j,t}=1,j\in\mathcal{A}\}$ as the set of appliances that customer i will rent at communication round t, and $\mathcal{V}_{i,-j,t}=\{v_{i,j',t}:v_{i,j',t}=1,j'\neq j\}$ as the set of appliances except a_j that customer i will rent at communication round t. For any give set $\mathcal{V}_{i,t}$, we denote the profit associated with it as $Z(\mathcal{V}_{i,t})$. We then have that by letting $v_{i,j,t}=1$, the profit $Z(\mathcal{V}_{i,-j,t}\cup\{v_{i,j,t}\})<Z(\mathcal{V}_{i,-j,t})$ since the marginal profit of j is negative. Therefore, customer i should choose the binary variable $v_{i,j,t}=0$ to maximize its profit.

We next consider the 'only if' direction. We assume that $v_{i,j,t}=1$ for some appliance $j\in\mathcal{A}$ whereas Eq. (13) does not hold. Since customer i aims at maximizing the profit, we have that $v_{i,j,t}$ should be chosen as zero. Otherwise $Z(\mathcal{V}_{i,-j,t}\cup\{v_{i,j,t}\})<Z(\mathcal{V}_{i,-j,t})$, leading to contradiction to our hypothesis. Combining the arguments above leads to the proposition. □

Given Proposition 1, we have that a selfish and rational customer will verify whether the marginal profit of each appliance is non-negative or not, and set $v_{i,j,t}=1$ for those appliances with non-negative marginal profit.

B. Computation of the Strategy of Developer

In this subsection, we compute the strategy for the developer, i.e., the electricity price P_t and rental prices $\{r_{j,t}\}_j$ at each communication round t. The developer's strategy depends on the decisions made by all customers. In practice, however, it is intractable for the developer to compute the strategies of all customers since the developer does not know the customers' utilities. In what follows, we identify a submodularity property of the utility function of the developer. We then leverage the submodularity property to efficiently estimate the customers' strategies, and hence compute the strategy for the developer.

1) Background: In the following, we present preliminary background on submodularity and hybrid submodularity. Let $\mathcal{V}$ be a finite set. We denote the powerset as $2^{\mathcal{V}}$. We present the definition of submodularity as follows.

Definition 1 (Submodularity [13])**.** *Let $\mathcal{V}$ be a finite set. A function $f: 2^{\mathcal{V}} \to \mathbb{R}$ is submodular if $f(\mathcal{S} \cup \{v\}) - f(\mathcal{S}) \geq f(\mathcal{T} \cup \{v\}) - f(\mathcal{T})$ holds for any $\mathcal{S} \subseteq \mathcal{T} \subseteq \mathcal{V}$ and $v \in \mathcal{V} \setminus \mathcal{T}$.*

The submodularity presented in Definition 1 models the diminishing return property of function f whose input argument is a discrete finite set. In what follows, we present the definition of hybrid submodularity, which extends Definition 1 to incorporate scenarios where the input arguments of function f contain both continuous and discrete variables.

Definition 2 (Hybrid Submodularity [14])**.** *Let $\mathcal{V}$ be a finite set and $\mathcal{P} \subset \mathbb{R}^n$. Let $\mathcal{F}(\mathcal{P})$ be the collection of finite subsets of $\mathcal{P}$. A function $f: 2^{\mathcal{V}} \times \mathcal{F}(\mathcal{P}) \to \mathbb{R}$ is hybrid submodular if, for any $\mathcal{S} \subseteq \mathcal{T} \subseteq \mathcal{V}$ and $\Lambda \subseteq \Lambda' \subseteq \mathcal{P}$, the following properties hold:*

1) *For any $j \in \mathcal{V} \setminus \mathcal{T}$, we have $f(\mathcal{S} \cup \{j\}, \Lambda) - f(\mathcal{S}, \Lambda) \geq f(\mathcal{T} \cup \{j\}, \Lambda') - f(\mathcal{T}, \Lambda')$.*
2) *For any $P \in \mathcal{P} \setminus \Lambda'$, we have $f(\mathcal{S}, \Lambda \cup \{P\}) - f(\mathcal{S}, \Lambda) \geq f(\mathcal{T}, \Lambda' \cup \{\lambda\}) - f(\mathcal{T}, \Lambda')$.*

2) Computation of the Strategy: Based on Eq. (7), we observe that the developer is penalized when the power generation and aggregated load are imbalanced. We define the following value function to model the utility of the developer for any given set of appliances $\mathcal{V}_t = \cup_{i \in \mathcal{H}} \mathcal{V}_{i,t}$ as below

$$W(\mathcal{V}_t) = \lambda\{-G_t + \sum_{v_{i,j,t} \in \mathcal{V}_t} v_{i,j,t} l_j\}_- + P_t \odot \sum_{v_{i,j,t} \in \mathcal{V}_t} v_{i,j,t} l_j - e_t\{\sum_{v_{i,j,t} \in \mathcal{V}_t} v_{i,j,t} l_j - G_t\}_+ + \frac{1}{24}\sum_{i \in \mathcal{H}} Y_{i,t}\mathbf{1}_{24}, \quad (14)$$

where $\odot$ represents element-wise multiplication. We characterize the vector-valued function in Eq. (14) as follows.

Proposition 2. *Each entry of the value function defined in Eq. (14) is submodular in $\mathcal{V}_t$.*

Proof. We prove the proposition using Definition 1. We consider two sets $\mathcal{V}_t \subseteq \mathcal{V}_t'$, and let $v_{i,j,t} \notin \mathcal{V}_t'$. We have that We have the following

$$\begin{aligned}
&W(\mathcal{V}_t' \cup \{v_{i,j,t}\}) - W(\mathcal{V}_t') - (W(\mathcal{V}_t' \cup \{v_{i,j,t}\}) - W(\mathcal{V}_t)) \\
&= \left\{-G_t + \sum_{\mathcal{V}_t' \cup \{v_{i,j,t}\}} v_{i,j,t} l_j\right\}_- \lambda + P_t \odot \sum_{\mathcal{V}_t' \cup \{v_{i,j,t}\}} v_{i,j,t} l_j \\
&\quad - e_t \left\{\sum_{\mathcal{V}_t' \cup \{v_{i,j,t}\}} v_{i,j,t} l_j - G_t\right\}_+ + \frac{1}{24}\sum_{i \in \mathcal{H}} Y_{i,t} \\
&\quad - \left(\left\{-G_t + \sum_{\mathcal{V}_t'} v_{i,j,t} l_j\right\}_- \lambda + P_t \odot \sum_{\mathcal{V}_t'} v_{i,j,t} l_j \right. \\
&\quad \left. - e_t \left\{\sum_{\mathcal{V}_t'} v_{i,j,t} l_j - G_t\right\}_+ + \frac{1}{24}\sum_{i \in \mathcal{H}} Y_{i,t} \right) \\
&\quad - \left\{ \left\{-G_t + \sum_{\mathcal{V}_t \cup \{v_{i,j,t}\}} v_{i,j,t} l_j\right\}_- \lambda + P_t \odot \sum_{\mathcal{V}_t \cup \{v_{i,j,t}\}} v_{i,j,t} l_j \right. \\
&\quad - e_t \left\{\sum_{\mathcal{V}_t \cup \{v_{i,j,t}\}} v_{i,j,t} l_j - G_t\right\}_+ + \frac{1}{24}\sum_{i \in \mathcal{H}} Y_{i,t} \\
&\qquad - \left\{-G_t + \sum_{\mathcal{V}_t} v_{i,j,t} l_j\right\}_- \lambda + P_t \odot \sum_{\mathcal{V}_t} v_{i,j,t} l_j \\
&\quad \left. - e_t \left\{\sum_{\mathcal{V}_t} v_{i,j,t} l_j - G_t\right\}_+ + \frac{1}{24}\sum_{i \in \mathcal{H}} Y_{i,t} \right\}.
\end{aligned}$$

We then divide the discussion into three settings.

Setting I: $-G_t + \sum_{\mathcal{V}_t'} v_{i,j,t} l_j \leq 0$. In this setting, we have that $-G_t + \sum_{\mathcal{V}_t} v_{i,j,t} l_j \leq 0$ also holds since $\mathcal{V}_t \subseteq \mathcal{V}_t'$. Furthermore, we have

$$\begin{aligned}
&\left\{-G_t + \sum_{\mathcal{V}_t' \cup \{v_{i,j,t}\}} v_{i,j,t} l_j\right\}_- - \left\{-G_t + \sum_{\mathcal{V}_t'} v_{i,j,t} l_j\right\}_- \\
&\quad \leq \left\{-G_t + \sum_{\mathcal{V}_t \cup \{v_{i,j,t}\}} v_{i,j,t} l_j\right\}_- - \left\{-G_t + \sum_{\mathcal{V}_t} v_{i,j,t} l_j\right\}_-,
\end{aligned} \quad (15)$$

$$\sum_{\mathcal{V}_t' \cup \{v_{i,j,t}\}} v_{i,j,t} l_j - \sum_{\mathcal{V}_t'} v_{i,j,t} l_j = \sum_{\mathcal{V}_t \cup \{v_{i,j,t}\}} v_{i,j,t} l_j - \sum_{\mathcal{V}_t} v_{i,j,t} l_j, \quad (16)$$

$$\begin{aligned}
&- \left\{\sum_{\mathcal{V}_t' \cup \{v_{i,j,t}\}} v_{i,j,t} l_j - G_t\right\}_+ + \left\{\sum_{\mathcal{V}_t'} v_{i,j,t} l_j - G_t\right\}_+ \\
&\quad = - \left\{\sum_{\mathcal{V}_t \cup \{v_{i,j,t}\}} v_{i,j,t} l_j - G_t\right\}_+ + \left\{\sum_{\mathcal{V}_t} v_{i,j,t} l_j - G_t\right\}_+.
\end{aligned} \quad (17)$$

Therefore, $W(\mathcal{V}_t' \cup \{v_{i,j,t}\}) - W(\mathcal{V}_t') - (W(\mathcal{V}_t' \cup \{v_{i,j,t}\}) - W(\mathcal{V}_t)) \leq 0$ under this setting, and thus $W(\mathcal{V}_t)$ is submodular in $\mathcal{V}_t$.

Setting II: $-G_t + \sum_{\mathcal{V}_t} v_{i,j,t}l_j \geq 0$. Note that in this setting, we have that $-G_t + \sum_{\mathcal{V}_t'} v_{i,j,t}l_j \geq 0$ holds. The proof in this setting is similar to setting I, and is omitted.

Setting III: $-G_t + \sum_{\mathcal{V}_t} v_{i,j,t}l_j \leq 0$ and $-G_t + \sum_{\mathcal{V}_t'} v_{i,j,t}l_j \geq 0$. In this case, we have that

$$\begin{aligned}&\left\{-G_t + \sum_{\mathcal{V}_t'\cup\{v_{i,j,t}\}} v_{i,j,t}l_j\right\}_- - \left\{-G_t + \sum_{\mathcal{V}_t'} v_{i,j,t}l_j\right\}_- \\ &\quad < \left\{-G_t + \sum_{\mathcal{V}_t\cup\{v_{i,j,t}\}} v_{i,j,t}l_j\right\}_- - \left\{-G_t + \sum_{\mathcal{V}_t} v_{i,j,t}l_j\right\}_-,\end{aligned} \tag{18}$$

$$\sum_{\mathcal{V}_t'\cup\{v_{i,j,t}\}} v_{i,j,t}l_j - \sum_{\mathcal{V}_t'} v_{i,j,t}l_j = \sum_{\mathcal{V}_t\cup\{v_{i,j,t}\}} v_{i,j,t}l_j - \sum_{\mathcal{V}_t} v_{i,j,t}l_j, \tag{19}$$

$$\begin{aligned}&-\left\{\sum_{\mathcal{V}_t'\cup\{v_{i,j,t}\}} v_{i,j,t}l_j - G_t\right\}_+ + \left\{\sum_{\mathcal{V}_t'} v_{i,j,t}l_j - G_t\right\}_+ \\ &\quad \leq -\left\{\sum_{\mathcal{V}_t\cup\{v_{i,j,t}\}} v_{i,j,t}l_j - G_t\right\}_+ + \left\{\sum_{\mathcal{V}_t} v_{i,j,t}l_j - G_t\right\}_+.\end{aligned} \tag{20}$$

Hence, we have that $W(\mathcal{V}_t' \cup \{v_{i,j,t}\}) - W(\mathcal{V}_t') - (W(\mathcal{V}_t' \cup \{v_{i,j,t}\}) - W(\mathcal{V}_t)) \leq 0$.

Combining the arguments above yields the proposition. □

Given Proposition 2, we have the following corollary, which follows from the fact that linear combination of submodular functions with non-negative weights is submodular.

Corollary 1. *The real-valued function $W(\mathcal{V}_t)^\top \mathbf{1}_{24}$ is submodular in $\mathcal{V}_t$ at each communication round t.*

Using Proposition 2 and Corollary 1, we further have the following hybrid submodularity result.

Theorem 1. *The real-valued function $W(\mathcal{V}_t)^\top \mathbf{1}_{24}$ is hybrid submodular in $\mathcal{V}_t$ and $P_t, \{r_{j,t}\}_{j\in\mathcal{A}}$ for all t.*

Proof. We note that the vector-valued function $W(\mathcal{V}_t)$ is linear in continuous variable P_t. Then the theorem follows from Proposition 1, Corollary 1, and Definition 2. □

Using Theorem 1, we develop a local search algorithm for the developer to estimate how the customers will react to electricity price and rental prices, and thus approximately solve for these prices at each communication round t, as shown in Algorithm 1. At each communication round $t = 1, \dots, T$, the developer first initializes $\mathcal{V}_t$, P_t, and $\{r_{j,t}\}_j$. Then the developer conducts a local search by letting $v'_{i,j,t} \notin \mathcal{V}_t$ be one. The developer then solves for P_t and $\{r_{j,t}\}_j$ using the following optimization problem

$$\max_{P_t, r_{j,t}} \quad W(\mathcal{V}_t)^\top \mathbf{1}_{24} \tag{21}$$

$$\begin{aligned}\text{s.t.} \quad & P_t^\top \sum_{i\in\mathcal{H}} v_{i,j,t}l_j - \Big[e\Big\{\sum_{i\in\mathcal{H}} v_{i,j,t}l_j - G_t\Big\}_+ \\ &\qquad + \sum_{i\in\mathcal{H}} Y_{i,t}\Big]\mathbf{1}_{24} \geq 0\end{aligned} \tag{22}$$

$$\sum_{j=1}^{k} v_{i,j,t}u_{i,j}l_j\mathbf{1}_{24} - E_{i,t} - Y_{i,t} \geq 0 \tag{23}$$

If $W(\mathcal{V}_t \cup \{v'_{i,j,t}\})^\top \mathbf{1}_{24} > (1+\epsilon)W(\mathcal{V}_t)^\top \mathbf{1}_{24}$, then set $\mathcal{V}_t$ is updated as $\mathcal{V}_t \cup \{v'_{i,j,t}\}$. Otherwise, the set $\mathcal{V}_t$ will remain the same. Then developer then sets some $v_{i,j,t} \in \mathcal{V}_t$ to be zero, and re-evaluates if $W(\mathcal{V}_t \setminus \{v_{i,j,t}\})^\top \mathbf{1}_{24} > (1+\epsilon)W(\mathcal{V}_t)^\top \mathbf{1}_{24}$. If the utility can be improved, then the set set $\mathcal{V}_t$ is updated as $\mathcal{V}_t \setminus \{v_{i,j,t}\}$. The process continues until $\mathcal{V}_t$ cannot get updated further. We finally remark that by satisfying constraints Eq. (22) and (23), the mini-grid is sustainable since the customers and developer obtain non-negative utilities in this process.

Algorithm 1 Local search algorithm to compute P_t and $\{r_{j,t}\}_{j\in\mathcal{A}}$ for the developer at communication round t

Input: G_t, $\Lambda_{i,t-1}$ for all $i \in \mathcal{H}$, $v_{i,j,t-1}$, d_j, l_j, $u_{i,j}$ for all $j \in \mathcal{A}$, λ, and e
Output: P_t and $r_{j,t}$ for all $j \in \mathcal{A}$
Initialize: $\mathcal{V}_t$, P_t, and $\{r_{j,t}\}_{j\in\mathcal{A}}$
$Flag \leftarrow 1$
while $Flag = 1$ **do**
  $Flag \leftarrow 0$
  for $v'_{i,j,t} \notin \mathcal{V}_t$ **do**
    Given $\mathcal{V}_t$, compute P_t and $\{r_{j,t}\}_{j\in\mathcal{A}}$ using Eq. (21) - (23)
    if $W(\mathcal{V}_t \cup \{v'_{i,j,t}\}\})^\top \mathbf{1}_{24} > (1+\epsilon)W(\mathcal{V}_t)^\top \mathbf{1}_{24}$ **then**
      $\mathcal{V}_t \leftarrow \mathcal{V}_t \cup \{v'_{i,j,t}\}$
      $Flag \leftarrow 1$
    end if
  end for
  for $v_{i,j,t} \in \mathcal{V}_t$ **do**
    Given $\mathcal{V}_t$, compute P_t and $\{r_{j,t}\}_{j\in\mathcal{A}}$ using Eq. (21) - (23)
    if $W(\mathcal{V}_t \setminus \{v_{i,j,t}\}\})^\top \mathbf{1}_{24} > (1+\epsilon)W(\mathcal{V}_t)^\top \mathbf{1}_{24}$ **then**
      $\mathcal{V}_t \leftarrow \mathcal{V}_t \setminus \{v_{i,j,t}\}$
      $Flag \leftarrow 1$
      break
    end if
  end for
end while
Return P_t and $r_{j,t}$ for all $j \in \mathcal{A}$

IV. EXPERIMENT

We consider a solar mini-grid established by a private sector developer. The mini-grid serves 20 customers (i.e., households) within a rural village. The developer interacts with customers over six communication rounds, each corresponding to two months. The mini-grid developer offers 10 appliances, including fridges, laptops, rice mills, mobile charging banks, radios, rice cookers, sewing machines, tablets, hand power tools, and washing machines that the customers can rent. These appliances cover various uses, from entertainment devices like TVs to essential appliances like rice cookers and productive tools like rice mills. Each appliance's load profile and operation constraints are obtained from [12]. The utility $u_{i,j}$ of each appliance j and customer i is generated following a uniform distribution within range $[0, 500]$. The generation profile G_t is modeled as a bell curve following a Gaussian distribution, with its peak value of 20kW occurring at noon. Parameters λ and e are chosen as 0.001 and 1, respectively, since the unit cost of using a diesel generator is generally higher than that of storing renewable energy generation.

We first summarize the numbers of each type of appliance that the customers rent. At communication round 6, the numbers of customers that rent fridges, laptops, rice mills, mobile charging banks, radios, rice cookers, sewing machines, tablets, hand power tools, and washing machines are 19, 20, 20, 17, 17, 20, 19, 9, 20, and 19, respectively. We observe that the customers are willing to rent some appliances based on their utilities, and thus, the mini-grid developer successfully stimulates the electricity demand of these customers.

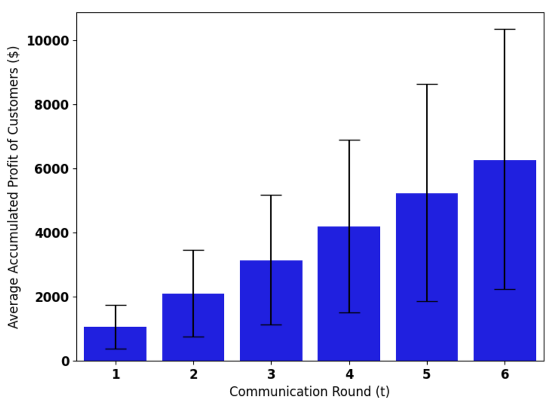

Fig. 1. Average of customers' accumulated profit. The error bars are the minimum and maximum accumulated profit at each communication round. The customers create positive profits for all communication rounds by renting productive appliances.

Fig. 1 presents the accumulated profit averaged over all customers. We observe that the accumulated profit is monotone increasing with respect to the communication rounds. Therefore, the customers earn positive profits by renting appliances and consuming energy. Even in the worst case, the customer can earn a profit of $2234.38 after six communication rounds. We finally present the accumulated profit of the developer in Table I. We observe that the developer obtains positive profit at all communication rounds. Therefore, our proposed approach promotes the economic viability of the mini-grid by stimulating the productive use of electricity.

TABLE I
THIS TABLE PRESENTS THE ACCUMULATED PROFIT OF DEVELOPER AT EACH COMMUNICATION ROUND. THE MINI-GRID DEVELOPER EARNS POSITIVE PROFITS AT ALL COMMUNICATION ROUNDS BY STIMULATING THE PRODUCTIVE USE OF ELECTRICITY, WHICH ENHANCES THE ECONOMIC VIABILITY OF THE MINI-GRID.

Comm. Round (t)	1	2	3	4	5	6
Accum. Profit ($)	4917.06	39686.98	74836.21	110263.05	146352.02	181457.67

V. CONCLUSION

In this paper, we developed a principled incentive mechanism to promote the economic viability of mini-grids in Sub-Saharan Africa. We considered that a mini-grid developer stimulated electricity demand from customers by offering appliance rentals. We formulated two optimization problems to model the interaction between the mini-grid developer and customers. At each communication round, we showed that the customers could decide which appliance to rent by comparing the marginal gain and loss associated with the appliance. For the developer, we proved that its utility satisfied a hybrid submodularity property. Leveraging this property, we showed that the developer could efficiently estimate what appliances would be rented by customers. We then developed a heuristic algorithm to guide the developer to determine electricity price and rental prices of appliances at each communication round. We evaluated our developed incentive mechanism using a rural village with ten appliances whose load profiles are obtained from real-world dataset. We showed that the proposed approach promoted the economic viability of the mini-grid. Our future work will incorporate more factors including customers' income and cognitive uncertainty into the analysis.

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