

Optimal L_q Norm Regularization for Sparse Reflectivity Inversion

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SUMMARY

In order to directly obtain the reflectivity series from seismic data, we develop an advanced reflectivity inversion method. The choice between L_1 and L_2 norm minimization is always difficult for the conventional reflectivity inversion methods, which also require the selection of regularization terms. The proposed method adopts L_q norm ($0 < q < 1$) minimization and optimal regularization parameter selection. In the beginning, we discuss the validity of the non-convex constraint, and the advantage of the L_q norm constraint. Then we briefly introduce the optimal regularization parameter selection based on cross-validation. The performance of the proposed method in the synthetic examples with different noise levels shows that the inverted results are reasonable in the cases with moderate noises. In the end, a field example from the Barnett Shale proves the effectiveness of the proposed method.

INTRODUCTION

Geoscientists have spent everlasting effort in improving the resolution of seismic data to better facilitate seismic interpretation. Based on the convolution model, a great variety of inversion methods have been proposed to eliminate the smearing effects associated with the seismic wavelet. Among many inversion methods, reflectivity inversion is one that can reproduce reflectivity series from seismic data with improved resolution without a priori knowledge from well logs. The reflectivity inversion is especially helpful for locating the thin-bed reservoir (Chopra et al., 2007). Chai et al. (2015) used reflectivity inversion to investigate the attenuated medium property. The output reflectivity shows up as a spiky series of reflection coefficients at the layer boundaries, which can be beneficial for the seismic interpretation (Li et al., 2016b; Zhao et al., 2017). Not only on time domain migrated data, Zhang et al. (2016) applied the reflectivity inversion on the depth domain seismic data to increase the vertical resolution. Besides, seismic attributes can also be directly calculated from the inverted reflectivity, which leads to attributes with more details (Zhang and Zhang, 2015).

The least-squares solution (L_2 norm) minimizes the root-mean square (RMS) error between forward modeling and observed data. Because seismic data are usually band-limited and noisy, the conventional least-squares solution could lead to poor results, as it matches the inverted results with the noise as well. In some studies, sparsity (L_1 norm) is a common constraint, which assumes the reflectivity coefficients only locate at a small portion of the whole seismic series (Zhang and Castagna, 2011; Chai et al., 2014). As it can generate robust results, the sparse inversion becomes more and more popular, but how to determine the type of normalization terms and choose the regularization parameters are rarely discussed.

This paper specifically focuses on using L_q norm ($0 < q < 1$) minimization and auto regularization parameter selection for sparse reflectivity inversion. We demonstrate the effectiveness of the proposed method with synthetic and field data examples.

THEORY

L_q Norm Reflectivity Inversion

As an attempt to perform an optimal sparse reflectivity inversion, we propose an approach via L_q norm regularized optimization. Regularized optimization techniques are widely applied and discussed in statistics and signal processing communities (Donoho and Johnstone, 1994; Donoho, 1995; Tibshirani, 1996; Efron et al., 2004). To characterize the sparsity of a signal, the natural choice is through L_0 norm regularization, which essentially sets a certain number of small estimated coefficients to zero.

Alternatively, a family of soft thresholding regularizations was introduced that preserve better empirical performances and/or theoretical guarantees. For example, the L_1 norm regularization, i.e. Lasso regression (Tibshirani, 1996), which possesses the sparsity of the resulting estimators. As a natural generalization of L_0 regularization, the L_q regularizations came into the sight of Frank and Friedman (1993) named as bridge regression, in which $0 < q \leq 2$. The bridge regression builds the connection between L_0 regularization and L_2 regularization (i.e. ridge regression or Tikhonov regularization), including the Lasso (L_1) regression as special case. It is known that the convex L_q regularization with $q > 1$ does not hold the sparsity property, whereas the Lasso L_1 regularization provides the biased estimation.

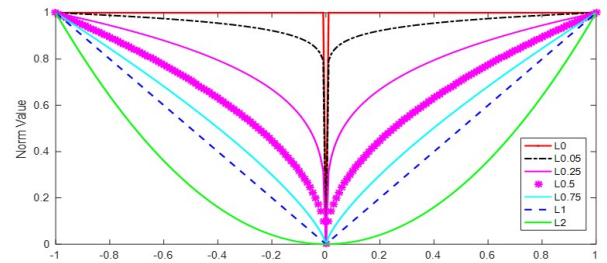


Figure 1: Contours of constant value of $\sum_j |r_j|^q$ for given values of $q = 0, 0.05, 0.25, 0.5, 0.75, 1$ and 2 .

In reflectivity inversion study, we focus on the L_q regularization with $0 < q < 1$, which is a concave regularization that will produce the sparse resulting estimation. Similarity, the Cauchy norm and Geman norm also produce the sparse estimation through a concave regularization (Wang et al., 2016). The L_q regularization with $0 < q < 1$ is used here due to the

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flexible choice of q so that we can adapt different sparsity requirement of the reflectivity series, which is shown in Figure 1.

The recorded seismic data are assumed to be band-limited, which can be modeled as a convolution of source wavelet $w(t)$, which has a limited spectral bandwidth, and the reflectivity series $r(t)$, which has a white spectrum:

$$s(t) = w(t) * r(t) + \varepsilon(t), \quad (1)$$

where $\varepsilon(t)$ is the additive noise.

The convolution model in equation (1) can be written in a matrix form realized at time point $t = 1, 2, \dots, M$ with wavelet kernel matrix. Using the property of convolution operation, we can rewrite equation (1) in a linear form

$$\mathbf{s} = \mathbf{W}\mathbf{r} + \varepsilon, \quad (2)$$

where $\mathbf{s} = [s_1, \dots, s_M]^T$ is the observed data vector, \mathbf{W} is the wavelet kernel matrix, $\mathbf{r} = [r_1, \dots, r_M]^T$ is the reflectivity series, and ε is the corresponding noise vector. We estimate the wavelet by the inverse Fourier transform of the power spectrum of seismic data (Zhang et al., 2016).

The convolution model for seismic data assumes that the subsurface consists of a series of horizontal layers with impedance, which can generate reflections at the boundaries between adjacent layers. Because seismic data are band-limited with background noise, the solutions for equation (2) could be nonunique, which necessitates a constraint to find the best solution. With the assumption that the reflectivity series is sparse, we need to put sparse constraint on the inverse of equation (2), which can be formulated as

$$\phi(\mathbf{r}, \lambda) = \|\mathbf{s} - \mathbf{W}\mathbf{r}\|^2 + \lambda \|\mathbf{r}\|_q, \quad (3)$$

where $\|\mathbf{r}\|_q = \left(\sum_{j=1}^M |r_j|^q\right)^{1/q}$ and λ is the regularization parameter. For given λ , the solution $\hat{\mathbf{r}} = \arg \min_{\mathbf{r}} \phi(\mathbf{r}, \lambda)$ can be solved through coordinate-wise optimization method, e.g. Iteratively Re-weighted Least Squares (IRLS), or Cyclic Descent (CD) (Marjanovic and Solo, 2014).

Regularization Parameter Selection

The regularization parameter λ controls the sparsity of the reflectivity series and the selection of optimal sparsity that is essential. Since only the observed data \mathbf{s} and kernel matrix \mathbf{W} are available, fivefold cross-validation (CV) is used here for estimating the prediction error and deciding the optimal regularization parameter.

The procedure of fivefold cross-validation is roughly as follows: First, we split the data into five roughly equal size parts; Second, for the k th part setting ($k = 1, 2, \dots, 5$) as test data, we fit the model in equation (3) using the other four parts as training data and calculate the cross-validation prediction error based on the estimator $\hat{\mathbf{r}}^{(-k)}(\lambda)$ for each λ ; Then, we perform this for all the parts and combine the five prediction errors together to obtain the cross-validation estimate

$$CV(\lambda) = \frac{1}{5} \sum_{k=1}^5 \left(\mathbf{s}_k - \mathbf{W}_k \hat{\mathbf{r}}^{(-k)}(\lambda) \right)^2. \quad (4)$$

For given range of λ , say $0 < \lambda < \infty$ or $\lambda \in [a, b]$ for some fixed constants a, b , the chosen optimal $\hat{\lambda}$ is the one that minimizes $CV(\lambda)$. The CV method can be viewed as a grid search approach.

SYNTHETIC EXAMPLES

We first test the proposed method on a simple synthetic model as shown in Figure 2. The synthetic model consists of five horizontal layers with varying thickness to represent thin layers, thick layers, and thin interbedded layers. The corresponding reflectivity section at each interface is shown in Table 1, (following Wang et al. (2016)).

Table 1: Synthetic reflectivity inversion example design with locations and true values. The inverted results from different noise situations: noise-free, 10 dB, 3 dB, 0 dB are also listed.

Reflectivity Depth (ms)	True Value	Noise Free	10 dB Noise	3 dB Noise	0 dB Noise
100	0.3	0.3	0.321	0.256	0.142
110	0.2	0.2	0.219	0.117	0.101
200	0.2	0.2	0.197	0.125	0.342
212	-0.3	-0.3	-0.299	-0.127	-0.341
224	0.2	0.2	0.178	0.092	0.088
300	0.3	0.3	0.309	0.146	0.164
314	-0.2	-0.2	-0.206	-0.118	-0.071
400	0.2	0.2	0.182	0.140	0.164
416	-0.3	-0.3	-0.195	-0.164	-0.135
500	0.3	0.3	0.304	0.189	0.309
518	0.2	0.2	0.202	0.187	0.234

Based on the convolution model, we obtain the seismic data by convolving the reflectivity profile with a Ricker wavelet with 35Hz dominant frequency. The synthetic seismic trace is shown in Figure 2(b). Figure 2(c) displays the inverted result from the proposed method. The inverted result is identical to the true values, and the values can be found from the second and third columns in the Table 1.

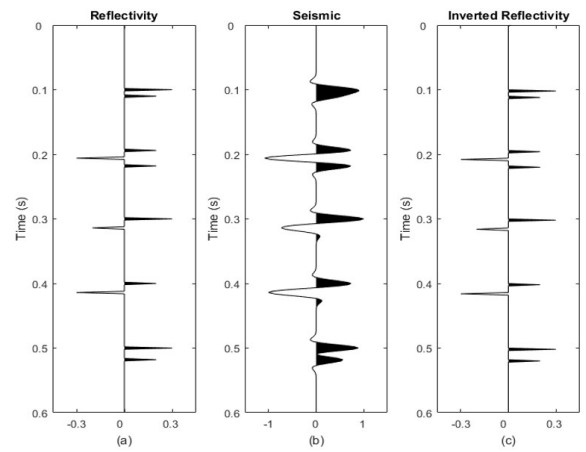


Figure 2: (a) Reflectivity, (b) Synthetic seismic and (c) Inverted results. The reflectivity locations can be found in Table 1.

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To test the robustness of the proposed method, we add random noise of different levels (10 dB, 3 dB, and 0 dB) to the seismic trace. (The signal noise ratios of 10 dB, 3 dB, and 0 dB are 10, 2 and 1, respectively.) Figure 3 shows the noisy traces with their corresponding inverted results. From the plots and the values in Table 1, we observe the inverted results are very stable and reflect the major reflections even at really noisy situations, which is brought from the L_q optimization. (Note that we only list the inverted results at the original reflectivity locations, there could be inaccurate inverted results at other locations, which could be observed from the plots in Figure 3.)

FIELD APPLICATION

We apply the proposed method on a field dataset from the Barnett Shale, Fort Worth Basin, Texas. Figure 4 display a vertical section of the field seismic data. The target exploration horizons in this area include Upper Barnett Limestone, Upper Barnett Shale, and Lower Barnett Shale. The Forestburg Unconformity is the bottom of Upper Barnett, and the Viola Limestone is the bottom of Lower Barnett. The horizons are plotted in Figure 4 and Figure 5. Figure 5 shows the inverted reflectivity series. We can identify thin layers. And compared with manually picked horizons, the inverted results show a good correspondence. It is obvious that the reflectivity results show higher resolution than the seismic profile with more details, which is beneficial and required by the interpreters.

We also validate our inversion results with well log data from a well tied with seismic data. We display the original seismic amplitude, impedance from well log, reflectivity from well log, and reflectivity inverted from the proposed method in Figure 6. The analysis window is defined by the top of Upper Barnett Limestone and the top of Viola Limestone. The well location and analysis window is denoted as the red wiggle in Figure 4. Figure 6 shows the zoom-in details in the analysis window. Note that the inverted results show good correspondence with the "true answers" from the well logs, and the difference is expected as the proposed reflectivity inversion is independent from any well information and it has the same vertical sampling rate with the seismic data while that of well log is higher.

DISCUSSIONS

In this paper, we propose a reflectivity inversion method independent from well data. In other words, the inversion of reflection coefficients from seismic amplitudes is completely data driven, which can be promising for machine learning based interpretation. Seismic facies analysis is an important area for quantitative interpretation, and is always based on well log information. Li et al. (2016a) proposed an adaptive signal decomposition method for seismic sequence stratigraphy interpretation, which is also applied in Zhao et al. (2017) as a constraint of the pattern recognition method to assist facies analysis. Here, a robust and accurate reflectivity inversion approach will be a good candidate attribute for the machine learning approaches in automatic facies analysis.

Our proposed method is based on the convolution model, which means that the seismic amplitude is the convolution result of

the reflectivity and the wavelet. The seismic wavelet extraction we use in the study assumes that reflectivity is random and thus the power spectra of the wavelet and seismic data are equal. We understand that the spectrum of the reflectivity may not be absolutely random, which could result in inaccurate wavelet estimation. We also assume a constant wavelet traveling along the depth. However, many wave propagation effects, such as dispersion and attenuation, would change the spectrum of the seismic signal, so a non-stationary wavelet extraction method will be considered in the future research.

The non-convex constraints have drawn a lot of attentions, because it satisfies the mathematical assumptions of seismic reflectivity series. We discuss several non-convex constraints and adaptively adopt the L_q ($0 < q < 1$) norm constraint, and because the estimation of q value is fully data-driven, we believe the inverted results are relatively optimal. However, we also notice that there are some mismatches between the inverted results and well logs, because the reflectivity series in the field is not strictly mathematically sparse. In the future, based a sparse initial model, we would propose a hybrid reflectivity inversion method to meet the actual conditions.

CONCLUSIONS

In this paper, we introduce a novel sparse reflectivity inversion method with L_q norm constraint and optimal regularization term selection. The synthetic examples verify that the proposed method can eliminate the effects of wavelet effectively and can obtain the stable reflectivity series even with moderate random Gaussian noises. Furthermore, the derived reflectivity shows high correspondence with the reflectivity calculated from well logs.

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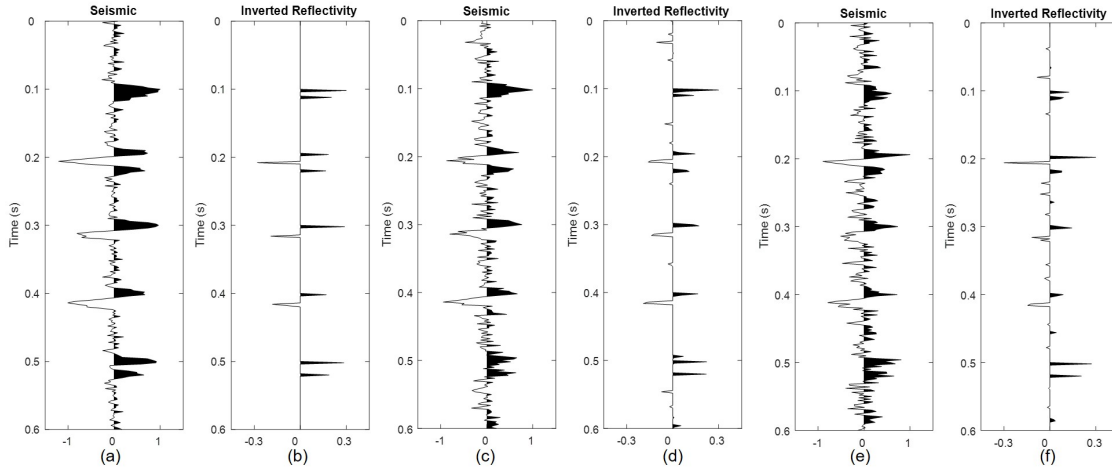


Figure 3: Noisy synthetic seismic traces and inverted reflectivity series: (a) and (b) are 10 dB , (c) and (d) are 3 dB , (e) and (f) are 0 dB .

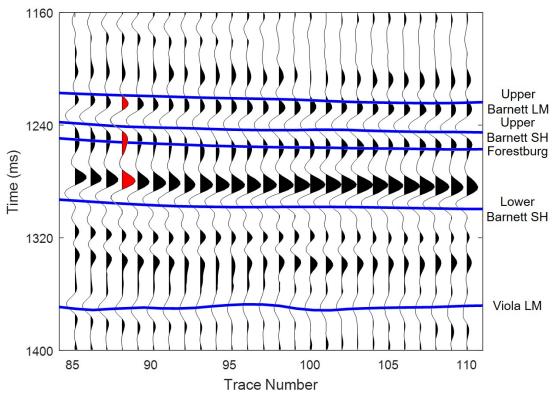


Figure 4: Field seismic data from Barnett shale, Fort Worth basin, Texas. Horizons are also annotated.

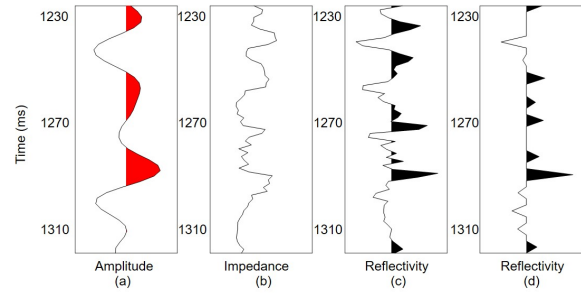


Figure 6: Comparison between the inverted reflectivity and the results from the well logs: (a) The extracted seismic trace (denoted by the red plot in Figure 4), (b) Impedance curve from well logs, (c) Reflectivity series calculated from the impedance log, (d) Inverted reflectivity from the proposed method.

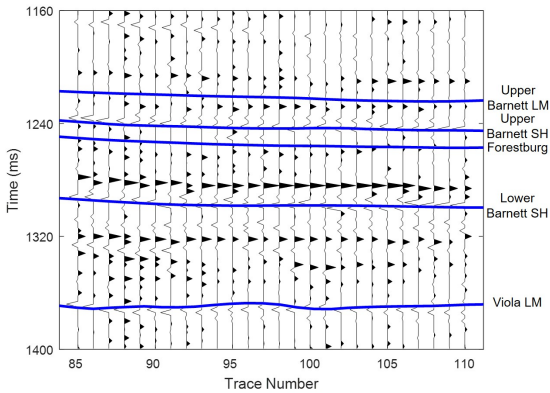


Figure 5: Inverted reflectivity series of the data in Figure 4.

REFERENCES

- Chai, X., S. Wang, J. Wei, J. Li, and H. Yin, 2015, Reflectivity inversion for attenuated seismic data: Physical modeling and field data experiments: *Geophysics*, **81**, T11–T24.
- Chai, X., S. Wang, S. Yuan, J. Zhao, L. Sun, and X. Wei, 2014, Sparse reflectivity inversion for nonstationary seismic data: *Geophysics*, **79**, V93–V105.
- Chopra, S., J. Castagna, Y. Xu, and R. Tonn, 2007, Thin-bed reflectivity inversion and seismic interpretation, *in* SEG Technical Program Expanded Abstracts 2007: Society of Exploration Geophysicists, 1923–1927.
- Donoho, D. L., 1995, De-noising by soft-thresholding: *IEEE transactions on information theory*, **41**, 613–627.
- Donoho, D. L., and I. M. Johnstone, 1994, Ideal spatial adaptation by wavelet shrinkage: *biometrika*, 425–455.
- Efron, B., T. Hastie, I. Johnstone, and R. Tibshirani, 2004, Least angle regression: *The Annals of statistics*, **32**, 407–499.
- Frank, L. E., and J. H. Friedman, 1993, A statistical view of some chemometrics regression tools: *Technometrics*, **35**, 109–135.
- Li, F., T. Zhao, X. Qi, K. Marfurt, and B. Zhang, 2016a, Lateral consistency preserved variational mode decomposition (vmd), *in* SEG Technical Program Expanded Abstracts 2016: Society of Exploration Geophysicists, 1717–1721.
- Li, F., H. Zhou, T. Zhao, and K. J. Marfurt, 2016b, Unconventional reservoir characterization based on spectrally corrected seismic attenuation estimation: *JOURNAL OF SEISMIC EXPLORATION*, **25**, 447–461.
- Marjanovic, G., and V. Solo, 2014, $l_{-}\{q\}$ sparsity penalized linear regression with cyclic descent: *IEEE Transactions on Signal Processing*, **62**, 1464–1475.
- Tibshirani, R., 1996, Regression shrinkage and selection via the lasso: *Journal of the Royal Statistical Society. Series B (Methodological)*, 267–288.
- Wang, B., Z. Guo, X. Chen, and W. Lu, 2016, Nonstationary sparse-reflectivity inversion using nonconvex constraint in frequency domain, *in* SEG Technical Program Expanded Abstracts 2016: Society of Exploration Geophysicists, 3741–3745.
- Zhang, R., and J. Castagna, 2011, Seismic sparse-layer reflectivity inversion using basis pursuit decomposition: *Geophysics*, **76**, R147–R158.
- Zhang, R., and B. Zhang, 2015, Seismic reflectivity attributes, *in* SEG Technical Program Expanded Abstracts 2015: Society of Exploration Geophysicists, 2868–2871.
- Zhang, R., K. Zhang, and J. E. Alekhue, 2016, Depth-domain seismic reflectivity inversion with compressed sensing technique: *Interpretation*, **5**, T1–T9.
- Zhao, T., F. Li, and K. J. Marfurt, 2017, Constraining self-organizing map facies analysis with stratigraphy: An approach to increase the credibility in automatic seismic facies classification: *Interpretation*, **5**, T163–T171.