國立臺灣科技大學108學年度碩士班招生試題

系所組別：資訊工程系碩士班

科　　目：計算機數學

(總分為100分)

1. [12%] Let T be a tree and denoted by $T = (V, E)$ in which V denotes the vertex set and E denotes the edge set. Let $|S|$ denote the cardinality of the set S. We now remove one edge from the tree T, and it leads to two separated trees $T_1 = (V_1, E_1)$ and $T_2 = (V_2, E_2)$ with $|E_1| = 2|V_2|$. Let $|E_2| = a|V| + b$. Please calculate the values of a and b.

2. [12%] Let $gcd(930, 1280) = 930m + 1280n$. Please calculate the values of m and n. Here, "gcd" denotes "greatest common divisor."

3. [13%] Let G be a group which is an algebraic structure. Let H and K be the subgroups of G with $H \subset K \subset G$. Let $|F|$ denote the cardinality of the group F. If $|G| = 186$ and $|H| = 31$, please calculate all possible values of $|K|$.

4. [13%] Given 5 matrices M_1, M_2, M_3, M_4 and M_5, we now want to perform the matrix-multiplication-chain operation for $M_1 \times M_2 \times M_3 \times M_4 \times M_5$, and suppose the association law is allowable for the above operation. For example, $M_1 \times M_2 \times M_3$ can be performed by two ways, namely $((M_1 \times M_2) \times M_3)$ and $(M_1 \times (M_2 \times M_3))$. Please calculate the number of ways to perform the matrix-multiplication-chain operation for $M_1 \times M_2 \times M_3 \times M_4 \times M_5$.

5. [10%] Let $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}$, find an orthonormal basis for the column space of A.

6. [16%] Let $A = \begin{bmatrix} 13 & 0 & -5 \\ 0 & 8 & 0 \\ -5 & 0 & 13 \end{bmatrix}$.

   (a) (2%) Find the eigenvalues of A.

   (b) (4%) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix.

   (c) (4%) Find a matrix B such that $B^2 = A$.

   (d) (6%) If $C = I + A + A^2/2! + A^3/3! + \cdots$, where I is an identity matrix of order 3, find the matrix C.

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7. [10%] Given an input image whose constituting pixels with integer coordinates represented by the Cartesian product $[0,700] \times [0,500]$, it will be transformed to new coordinates represented by $[-1,1] \times [-1,1]$ for a certain purpose as Figure 1 shows. Please find the transformation matrix to achieve such a normalization process.

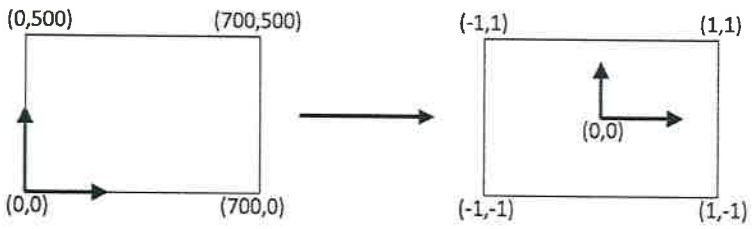

Figure 1

8. [14%] In the applications of image processing, there are several different transformation ways on a two-dimensional plane as shown in Figure 2, where the affine transformation is a function between affine spaces which preserves points, straight lines and planes. The formula of this transformation can be expressed below:

$$\begin{cases} x' = ax + by + c \\ y' = dx + ey + f \end{cases}$$

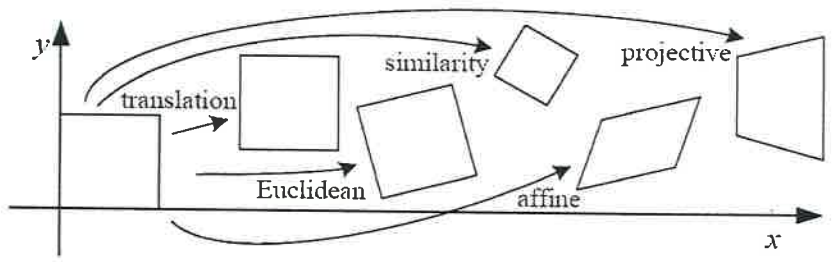

Figure 2

Consider the cost function of the affine transformation for n points of a map as:

$$C(a,b,c,d,e,f) = \sum_{i=1}^{n} \left(r_{x_i}(a,b,c,d,e,f)^2 + r_{y_i}(a,b,c,d,e,f)^2 \right)$$

where the residuals of each point (x_i, y_i) for x-coordinate and y-coordinate are respectively stated as follows.

$$r_{x_i}(a,b,c,d,e,f) = (ax_i + by_i + c) - x_i'$$
$$r_{y_i}(a,b,c,d,e,f) = (dx_i + ey_i + f) - y_i'$$

(a) (7%) Please write down the affine transformation in matrix form for n points.

(b) (7%) Based on the above matrix form, if we want to minimize the cost function, please find the least squares solution.