

CPSC 449

Assignment 2

Question 4

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❑ Question 4

- We want to prove the following:
Or $(\text{match } x \text{ } xs) = \text{elem } x \text{ } xs$
 - To prove that a property $P(ys)$ holds for all finite lists ys , do the following
 1. Base case.
 - a. Prove that $P([])$
 2. Induction step.
 - a. Assume that $P(xs)$.
 - b. Prove that $P(x:xs)$, using the induction hypothesis.
 - Base case
or $(\text{match } x \text{ } []) = \text{elem } x \text{ } []$ (base)
 - Induction step
Assuming the induction hypothesis:
or $(\text{match } x \text{ } xs) = \text{elem } x \text{ } xs$ (hyp)
Prove that:
or $(\text{match } x \text{ } (x:xs)) = \text{elem } x \text{ } (x:xs)$ (ind)

Base case

- Want:
Or $(\text{match } x \text{ } []) = \text{elem } x \text{ } []$
 1. Left-hand side:
or $(\text{match } x \text{ } [])$
 $= \text{or } ([]) \quad \text{by (match.1)}$
 $= \text{False} \quad \text{by (or.1)}$
 2. Right-hand side:
 $\text{elem } x \text{ } []$
 $= \text{False} \quad \text{by (elem.1)}$

Induction Step

- Assume:
or $(\text{match } x \text{ } xs) = \text{elem } x \text{ } xs$ (hyp)
Prove:
Or $(\text{match } x \text{ } (x:xs)) = \text{elem } x \text{ } (x:xs)$ (ind)
 1. Left-hand side:
or $(\text{match } x \text{ } (x:xs))$
 $= \text{or } ((x == x) : (\text{match } x \text{ } xs)) \quad \text{(match.2)}$
 $= (x == x) \parallel (\text{or } (\text{match } x \text{ } xs)) \quad \text{(or.2)}$
 $= \text{True} \parallel \text{elem } x \text{ } xs \quad \text{by (hyp)}$
 2. Right-Hand side:
 $\text{elem } x \text{ } (x:xs)$
 $= (x == x) \parallel (\text{elem } x \text{ } xs) \quad \text{(elem.2)}$
 $= \text{True} \parallel \text{elem } x \text{ } xs$

Both proof goals have been discharged. The theorem is established.