中国科学技术大学

UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA

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14.4. 位相调制光出射光场

$$E_{出} = A\cos(\omega t + \delta \sin\omega_m t)$$

平方律探测器

$$\frac{1}{T}\int_0^T E_{出}^2\,dt = \frac{1}{T}\int_0^T \frac{A^2}{2}\left[1+\cos(2\omega t + 2\delta\sin\omega_m t)\right]dt$$

$$= \frac{1}{2}A^2$$

14.6. 电场沿 <111> 方向.

$$\Gamma = \frac{\sqrt{3}\pi n_o^3 r_{41}}{\lambda}\left(\frac{VL}{d}\right)$$

$$\therefore V_m = \frac{\Gamma_m \lambda d}{\sqrt{3}\pi n_o^3 r_{41} l}$$

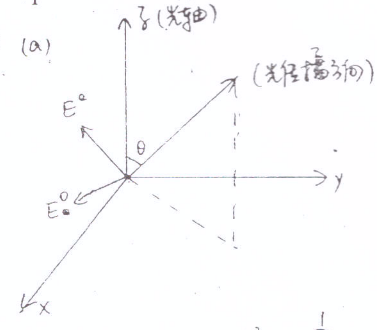
$$\frac{\Delta\omega}{2\pi} = \frac{1}{2\pi R_L C}.$$

$$C = \varepsilon\frac{A}{d}.$$

$$\therefore R_L = \frac{1}{\Delta\omega C} = \frac{d}{2\pi\Delta\nu\,\varepsilon A}$$

$$P = \frac{V_m^2}{2R_L} = \frac{\Gamma_m^2 \lambda^2 d A \varepsilon \Delta\nu}{3\pi n_o^6 r_{41}^2 l^2}$$

14.7 (a)



$$n_e(\theta) = \left(\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}\right)^{-\frac{1}{2}}$$

$$= n_o\left(1 - \sin^2\theta + \frac{n_o^2}{n_e^2}\sin^2\theta\right)^{-\frac{1}{2}}$$

$$\doteq n_o\left(1 - \theta^2 + \frac{n_o^2}{n_e^2}\theta^2\right)^{-\frac{1}{2}}$$

$$\doteq n_o\left(1 + \frac{1}{2}\theta^2 - \frac{n_o^2}{2n_e^2}\theta^2\right)$$

$$\therefore n_o - n_e(\theta) = \frac{n_o\theta^2}{2}\left(\frac{n_o^2}{n_e^2} - 1\right)$$

$$\Delta\Gamma = \frac{\omega l}{c}\left[n_o - n_e(\theta)\right] = \frac{\omega l n_o \theta^2}{2c}\left(\frac{n_o^2}{n_e^2} - 1\right)$$

(b).

$$\Delta\Gamma < \frac{\pi}{4}$$

$$\frac{\omega}{2c} n_o l\left(\frac{n_o^2}{n_e^2} - 1\right)\theta^2 < \frac{\pi}{4}$$

$$\theta < \left[\frac{2\pi c}{4\omega n_o l\left(\frac{n_o^2}{n_e^2} - 1\right)}\right]^{1/2}$$

$$= \left[\frac{\lambda}{4 n_o l\left(\frac{n_o^2}{n_e^2} - 1\right)}\right]^{1/2}$$

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14.9. X射线衍射发生在分立的原子平面上，它可被理想化为无限薄的薄片，而声波的扰动则是正弦曲线型的，因此应该看作正弦型光栅。

其透过率函数为 $t(x)=1+\cos\frac{2\pi x}{d}$

$$\tilde{U}(\theta)\propto\int_{-\frac{d}{2}}^{\frac{d}{2}}\left(1+\cos\frac{2\pi x}{d}\right)\exp(-ikx\sin\theta)\,dx$$

$$=\int_{-\frac{d}{2}}^{\frac{d}{2}}\left(1+\frac{1}{2}e^{i2\pi x/d}+\frac{1}{2}e^{-i2\pi x/d}\right)e^{-ikx\sin\theta}dx$$

$$\propto\frac{\sin\beta}{\beta}+\frac{1}{2}\frac{\sin(\beta-\pi)}{\beta-\pi}+\frac{1}{2}\frac{\sin(\beta+\pi)}{\beta+\pi}$$

$$\beta=\frac{\pi d\sin\theta}{\lambda}$$

总复振幅 $\propto\tilde{U}(\theta)\cdot\tilde{N}(\theta)$

$$\tilde{N}(\theta)=\frac{\sin N\beta}{\sin\beta}$$

$\tilde{N}(\theta)$ 和 $\tilde{U}(\theta)$ 相乘的结果 除了 0、±1 三级主极大

$\tilde{N}(\theta)$ 其它主极大都与 $\tilde{U}(\theta)$ 零点重合

即正弦型光栅只有 0级和1级衍射角.

14.10.

$$\begin{cases}\dfrac{dE_i}{dr}=i\eta E_d e^{i\Delta k\cdot r} & ①\\[2ex] \dfrac{dE_d}{dr}=i\eta E_i e^{-i\Delta k\cdot r} & ②\end{cases}$$

$\cancel{E_i(0)}$ $E_d(0)=0$.

由① $E_d=\frac{1}{i\eta}\frac{dE_i}{dr}e^{-i\Delta k\cdot r}$

$$\frac{dE_d}{dr}=\frac{1}{i\eta}e^{-i\Delta k\cdot r}\left[\frac{d^2E_i}{dr^2}+\frac{dE_i}{dr}(-i\Delta k)\right]$$

代入②得. $\frac{d^2E_i}{dr^2}-i\Delta k\frac{dE_i}{dr}+\eta^2E_i=0$.

$$x^2-i\Delta kx+\eta^2=0$$

$$x_{1,2}=i\left(\frac{\Delta k}{2}\pm\sqrt{\left(\frac{\Delta k}{2}\right)^2+\eta^2}\right)\quad\left(设\ \Delta=\sqrt{\left(\frac{\Delta k}{2}\right)^2+\eta^2}\right.$$

$$E_i=Ae^{x_1r}+Be^{x_2r}\quad(A+B=E_i(0))$$

代入①式

$$\frac{dE_i}{dr}=A\left(\frac{\Delta k}{2}+\Delta\right)e^{i(\frac{\Delta k}{2}+\Delta)r}+B\left(\frac{\Delta k}{2}-\Delta\right)e^{i(\frac{\Delta k}{2}-\Delta)r}$$

$$=i\eta E_d e^{i\Delta k\cdot r}$$

$$E_d=\frac{A\left(\frac{\Delta k}{2}+\Delta\right)}{\eta}e^{i(-\frac{\Delta k}{2}+\Delta)r}+\frac{B\left(\frac{\Delta k}{2}-\Delta\right)}{\eta}e^{i(-\frac{\Delta k}{2}-\Delta)r}$$

$E_d(0)=0$

$$\therefore\ A\left(\frac{\Delta k}{2}+\Delta\right)+B\left(\frac{\Delta k}{2}-\Delta\right)=0.$$

又 $A+B=E_i(0)$

解得 $A=\dfrac{\Delta-\frac{\Delta k}{2}}{2\Delta}E_i(0)$

$$B=\frac{\Delta+\frac{\Delta k}{2}}{2\Delta}E_i(0)$$

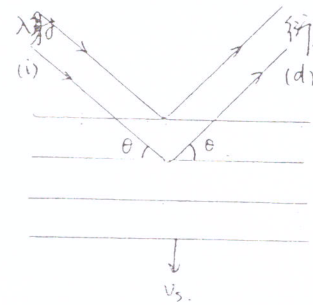
$$E_d = \frac{\Delta^2-\left(\frac{\Delta k}{2}\right)^2}{2\Delta\eta} E_i(0)\left[e^{i\left(-\frac{\Delta k}{2}+\Delta\right)r} - e^{i\left(-\frac{\Delta k}{2}-\Delta\right)r}\right]$$

$$= \frac{\eta E_i(0)}{2\sqrt{\left(\frac{\Delta k}{2}\right)^2+\eta^2}} e^{-i\frac{\Delta k}{2}r}\left[e^{i\Delta r}-e^{-i\Delta r}\right]$$

$$= \frac{\eta E_i(0)}{\sqrt{\left(\frac{\Delta k}{2}\right)^2+\eta^2}} e^{-i\frac{\Delta k}{2}r}\cdot i\sin\left(\sqrt{\left(\frac{\Delta k}{2}\right)^2+\eta^2}\cdot r\right)$$

$\frac{|E_d|^2}{|E_i(0)|^2}$ 最大值为 $\frac{\eta^2}{\eta^2+\left(\frac{\Delta k}{2}\right)^2}$

14.11.



声波作为观察者，由多普勒理论其感受到的入射光频率为

$$f_i = \frac{v+v_s\sin\theta}{v} f$$

衍射光频率 $f_d = \frac{v-v_s\sin\theta}{v} f$

$$\therefore f_i = f_d + \frac{2v_s\sin\theta}{v} f$$

由 14.9-12 $\sin\theta = \frac{\lambda}{2\lambda_s n}$

$$\frac{2v_s\sin\theta}{v} f = \frac{v_s\lambda}{v\cdot\lambda_s n} f$$

$f_s = v_s/\lambda_s$

$\lambda = c/f$

$v = c/n$

$$= f_s$$

$$\therefore f_i = f_d + f_s$$

即 $\omega_i = \omega_d + \omega_s$