Artificial Intelligence, Spring 2017

Homework 2 – Search

Xinglu Wang *3140102282* *ISEE 1403, ZJU*

1 Problem 1

Formulate the problem as a search problem, $M = \{0, 1, 2, 3\}$ denotes the number of missionaries on the left side of river, $C = \{0, 1, 2, 3\}$ denotes cannibals on the left side, $B = \{L, R\}$ denotes which side boat parks.

- Start state: $(3, 3, L)$
- Goal Test: Is state == $(0, 0, R)$
- State Space: All possible tuple (M, C, B), where $(M = C) \wedge (M = 0) \wedge (M = 3)$
- Successive function:
  - action: from (M, C, B) to (M', C', B'), satisfying

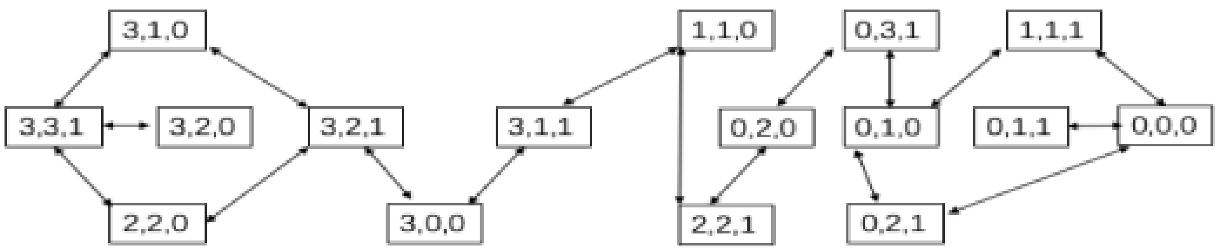
$$M' = M - 1 \vee M' = M - 2 \vee C' = C - 1 \vee C' = C - 2 \vee \begin{cases} C' = & C - 1 \\ M' = & M - 1 \end{cases}$$

$$B' = \begin{cases} L, & B = R \\ R, & B = L \end{cases}$$

$$(M' = C') \wedge (M' = 0) \wedge (M' = 3)$$

  - costs: always 1.

The complete state space:



2 Problem 2

- a). If all costs is 1, the cheapest solution is exactly the shallowest solution, i.e. $g(n) = depth(n)$. So uniform-cost search will become bread-first search.

- b). For best-first search, if $f(n) = -depth(n)$, then it will choose the deepest from frontier first and become depth-first search

- c). If the heuristic function of A* is $h(n) = 0$, then it will become uniform-cost search.

3 Problem 3

a). branching factor $b = 4$.

b). Since a step means $|x|$ or $|y|$ increase by 1, the final state satisfies $|x| + |y| = k \wedge |x| \leq k \wedge |y| \leq k$, which has $4k$ distinct states.

c). Notation: frontier = set of nodes in frontier; | frontier | = number of nodes in frontier; Acc. | frontier | = number of nodes expanded. The -1 in tableau means starting point is not counted in while target point is counted in .

depth	frontier	| frontier |	Acc. | frontier |
$d = 0$	ϕ, ($\{(0,0)\}$ is not counted in)	1-1	$1-1$
$d = 1$	$\{(0,1),(1,0),(-1,0),(0,-1)\}$	4	4
$d = 2$	$\{(-1,1),(1,1),(0,0),(0,2),...,...,...\}$	16	$4+16$
...	...	...	
$d = k$	...	4^k	$\sum_{n=1}^{k} 4^k$

Therefore, Acc. | frontier | $= \sum_{n=1}^{k} 4^k = (4^{k+1} - 4)/3 = (4^{x+y+1} - 4)/3$

d). Similarly,

depth	frontier	| frontier |	Acc. | frontier |
$d = 0$	ϕ	0	0
$d = 1$	$\{(0,1),(1,0),(-1,0),(0,-1)\}$	4	4
$d = 2$	$\{(x,y) \in \mathbb{R}^2; \|x\| + \|y\| = 2 \wedge \|x\| \leq 2 \wedge \|y\| \leq 2\}$	8	$4+8$
...	...	...	
$d = k$	$\{(x,y) \in \mathbb{R}^2; \|x\| + \|y\| = k \wedge \|x\| \leq k \wedge \|y\| \leq k\}$	$4k$	$\sum_{n=1}^{k} 4k$

Therefore, Acc. | frontier | $= \sum_{n=1}^{k} 4k = 2k(k+1) = 2(x+y)(x+y+1)$

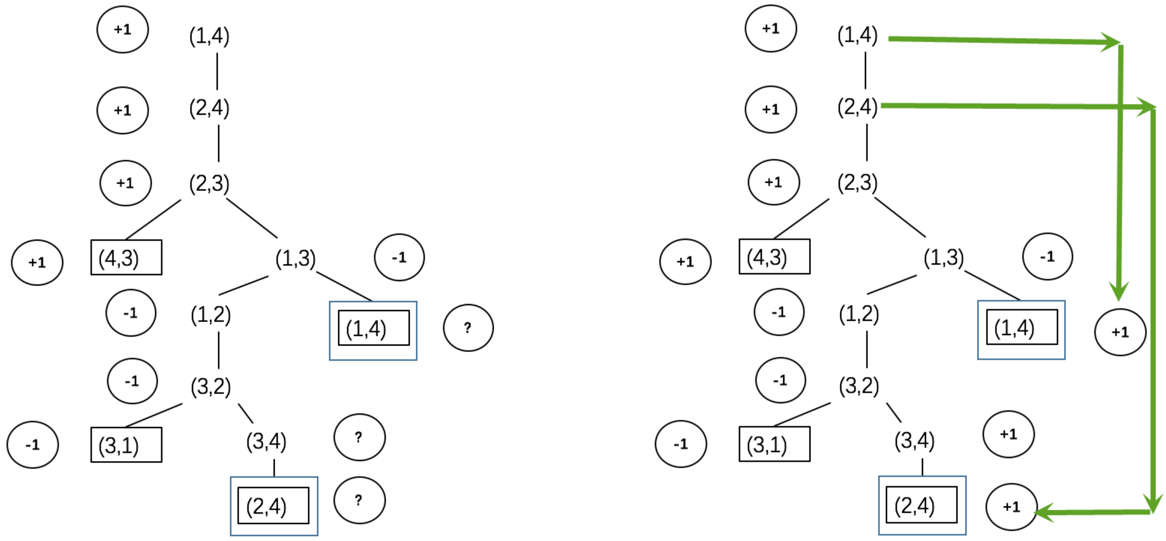
e). Yes, $h(n) = h^*(n)$ in this case and is admissible.

f). A^* will search the nodes 'towards' destination, i.e. $\{(a,b) \in \mathbb{R}^2; 0 \leq a \leq x \wedge 0 \leq b \leq y\}$. Totally, $(x+1)(y+1) - 1$

g). Yes, $h(n) \leq h^*(n)$, since some links are removed.

h). No, $\exists n, h(n) > h^*(n)$, if current node n is one of the ends of the added link.

4 Problem 4



Left: figure a for problem a. **Right**: figure b for problem b with backed-up values.

b. Ref. to fig. b. Use back-up, we can get optimal value of all the parent nodes. As for "?" values, we copy the value from visited node. It is right because for player A, $max(+1, ?) = +1$; for player B $min(-1, ?) = -1$ and thus we can prune the infinite branch.

c. Minmax algorithm fails because it use deep-first tree search and can go into infinite loop. The modified algorithm in *b.* can not always give optimal decision for all games with loop. For example, if this 4-square game becomes a 2×2 matrix game, initialled as

A	
	B

, and we define

A reaching	getting value of
(0,0)	1
(0,1)	2
(1,0)	3
(1,1)	4

and B of the opposite value -1,-2,-3,-4.

Then we cannot decide the value of $max(+1, ?)$, $max(+2, ?)$, $min(-1, ?)$ and so on and especially $min(?, ?)$ and $max(?, ?)$. Therefore, just to copy from appeared value will fail.

d. First consider n is odd.

Proposition 4.1. *If n is odd, then there exists a optimal strategy for B to win.*

Proof. **1).** For n = 3, B will definitely win.

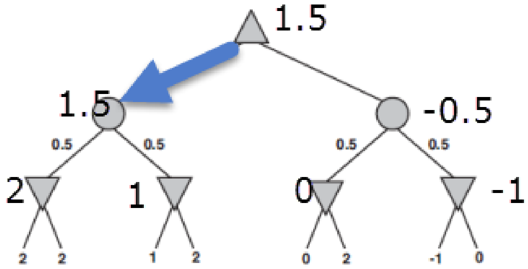
2). Assume n-2 is odd, and there exists a optimal strategy for B to win.

3). For n > 3, after first round, $(S_A, S_B) = (2, n-1)$, the size of square is n-2. According to *2)*, there exists a strategy, s.t. $S_B = 2 \wedge S_A \leq n-2$ after the round of B. Next round of A, $S_A \leq n-1$ then next round of B, B can choose move left (s.t. $S_B = 1$) as its strategy and win. By induction, B will win when n is odd. □

Similarly, if n is even, then there exists a optimal strategy for A to win.

5 Problem 5

a. Up Triangle means max player, and Down Triangle means min player. Note that circle means *stochastic* strategy set. Use back-up, we get



b. Given leaves 1-6, we need to evaluate x_7 and x_8. If $\min(x_7, x_8) > 3$ then the best move for max player won't be left. Given leaves 1-7, we do not need to evaluate x_8. $0.5\min(-1, x_8) + 0.5 * 0 \leq -0.5$, the optimal move for max player keeps left.

c. The value range for left-hand chance node: $[0, 2]$

d. After evaluating x_5 the value range for right-hand chance node is [-2,0], definitely smaller than 1.5. Thus x_6, x_7, x_8 can be pruned.