Homework #1
Introduction to Algorithms/Algorithms 1
600.363/463
Spring 2013

Due on: Tuesday, February 5th, 5pm
Late submissions: will NOT be accepted
Format: Please start each problem on a new page.
Where to submit: On blackboard, under student assessment.
Otherwise, please bring your solutions to the lecture.

February 11, 2013

1 Problem 1 (20 points)

1.1 (10 points)

For each statement below explain if it is true or false and prove your answer. Be as precise as you can. The base of $\log$ is 2 unless stated otherwise.

1. $7^{39}(n + 343n^2) = \Theta(n^2)$
2. $2^n = \Theta(e^{n+\sqrt{n}})$
3. $e^n = \Theta(2^{(3n)})$
4. $e^n = \Theta(2^{(n+3)})$
5. $\log_2(n^7) = O(\log_e(n^{10000000000}))$
6. $\arctan(n) = O(n)$
7. Let f, g be positive functions. Then $f(n) + g(n) = \Theta(\min(f(n), g(n)))$
8. $n^{\log\log(n)} = \Omega(\big(\log(n^{100})\big)^{1.000001})$
9. Let f be a positive function. Then $f(n) + o(f(n)) = \Theta(f(n))$.

1.2 (10 points)

1. Prove that
$$\sum_{i=\sqrt{n}}^{n} \frac{1}{i^3} = O(\frac{1}{\sqrt{n}}).$$

2 Problem 2(20 Points)

2.1 (10 points)

Prove by induction that $4^n + 6n + 8$ is divisible by 9 for all $n >= 1$.

2.2 (10 points)

1. Let A, B, C, D be sets. Prove that
$$(A \cap B) \cup (C \cap D) = (A \cup C) \cap (B \cup C) \cap (A \cup D) \cap (B \cup D).$$

2. There are 10 cookies of different colors in the jar. What is the number of different ways to divide the cookies between Alice and Bob? What if the cookies are of the same color (i.e., identical)?

3. We have 50 balls. Each ball, independently and randomly, is placed into one of 5 bins. What is the probability that there are no empty bins at the end of our experiment?

4. There are 25 students in the class. Prove that there are always at least three students whose birthdays are in the same month.