Homework 3 Solutions

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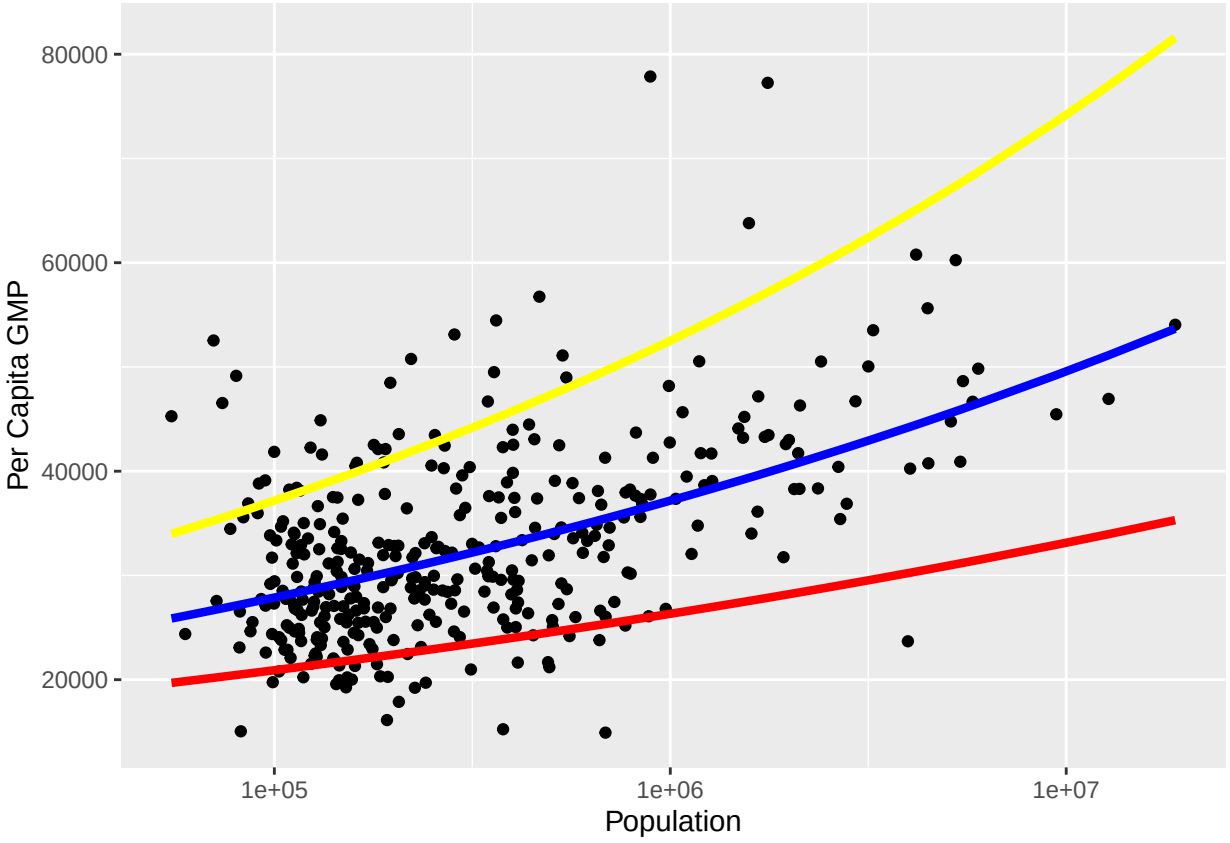
1

```
gmp <- read.table("data/gmp.dat")
gmp$pop <- round(gmp$gmp/gmp$pcgmp)
nlmfit1 <- 6611*gmp$pop^(1/8)
nlmfit2 <- 6611*gmp$pop^(0.1)
nlmfit3 <- 6611*gmp$pop^(0.15)
gmp %>% ggplot() + geom_point(aes(x = pop, y = pcgmp)) +
  labs(x = "Population", y = "Per Capita GMP", color = "Parameter") +
  scale_x_log10() +
  geom_line(aes(x = pop, y = nlmfit1), col = "blue", size = 1.5) +
  geom_line(aes(x = pop, y = nlmfit2), col = "red", size = 1.5) +
  geom_line(aes(x = pop, y = nlmfit3), col = "yellow", size = 1.5)
```



2

```
mse <- function(x, N = gmp$pop, Y = gmp$pcgmp){
  return(sum((Y - x[1]*N^x[2])^2)/length(Y))
}
mse(c(6611,0.15))
```

```
## [1] 207057513
```

```
mse(c(5000,0.10))
```

```
## [1] 298459914
```

3

```
nlm(mse, c(y0=6611,a=1/8))
```

```
## $minimum
## [1] 61857060
##
## $estimate
## [1] 6611.0000000    0.1263177
##
## $gradient
## [1] 50.048639 -9.976327
##
## $code
## [1] 2
##
## $iterations
## [1] 3
```

```
nlm(mse, c(y0=6611,a=0.1))
```

```
## $minimum
## [1] 61857060
##
## $estimate
## [1] 6611.0000003    0.1263177
##
## $gradient
## [1]   50.04683 -166.46087
##
## $code
## [1] 2
##
## $iterations
## [1] 6
```

```
nlm(mse, c(y0=6611,a=0.15))
```

```
## $minimum
## [1] 61857060
##
## $estimate
## [1] 6610.9999997    0.1263182
```

```
##
## $gradient
## [1]   51.76354 -210.18952
##
## $code
## [1] 2
##
## $iterations
## [1] 7
```

`minimum` represents the value of the estimated minimum of `mse`, while `estimate` represents the point at which `minimum` is obtained. For `a = 1/8`, we obtained a minimum `61857060` at point `(6611.00, 0.126377)`. For `a = 0.1`, we obtained a minimum `61857060` at point `(6611.00, 0.126377)`. For `a = 0.15`, we obtained a minimum `61857060` at point `(6610.9999997, 0.1263182)`.

4

```
plm <- function(y0, a, N = gmp$pop, Y = gmp$pcgmp){
  nlm1 <- nlm(mse, c(y0,a))
  return (list(nlm1$estimate, nlm1$minimum))
}
plm(y0=6611, a=0.15)
```

```
## [[1]]
## [1] 6610.9999997    0.1263182
##
## [[2]]
## [1] 61857060
```

```
plm(y0=5000, a= 0.10)
```

```
## [[1]]
## [1] 5000.0000008    0.1475913
##
## [[2]]
## [1] 62521484
```

Both results differ probably because the large difference in the initial guess. The initial guess 5000 and 0.10 results in a lower MSE.

5

(a)

```
mean(gmp$pcgmp)
```

```
## [1] 32922.53
```

```
sd(gmp$pcgmp)/sqrt(length(gmp$pcgmp))
```

```
## [1] 481.9195
```

(b)

```
except_i <- function(i){
  return (mean(gmp$pcgmp[-i]))
```

```
}
```

(c)

```
jackknifed.means <- vector(length = 366)
real_mean <- mean(gmp$pcgmp)

data_size <- function(the_data) {
  if (is.null(dim(the_data))) { n <- length(the_data) }
  else { n <- nrow(the_data) }
}

scale_and_sqrt_vars <- function(jackknife.ests,n) {
  var.of.reestimates <- apply(jackknife.ests,1,var)
  jackknife.var <- ((n-1)^2/n)* var.of.reestimates
  jackknife.stderr <- sqrt(jackknife.var)
  return(jackknife.stderr)
}

jackknife <- function(the_data) {
  n <- data_size(the_data)
  jackknife.ests <- matrix(sapply(1:n, except_i), ncol=n)
  return(scale_and_sqrt_vars(jackknife.ests,n))
}

jackknifed.means <- jackknife(gmp$pcgmp)
all.equal(mean(jackknifed.means), sd(gmp$pcgmp)/sqrt(length(gmp$pcgmp)))
```

```
## [1] TRUE
```

8

```
plm.jackknife <- function(y0, a, N = gmp$pop, Y = gmp$pcgmp) {
  n <- data_size(N)
  jackknife.ests <- matrix(0,nrow=2,ncol=n)
  for (omit in 1:n) {
    new.coefs <- plm(y0,a,N[-omit],Y[-omit])[[1]]
    jackknife.ests[,omit] <- new.coefs
  }
  return(scale_and_sqrt_vars(jackknife.ests,n))
}
(jackknife.parameters1 <- plm.jackknife(y0=6611, a=0.15))
```

```
## [1] 0 0
```

```
(jackknife.parameters2 <- plm.jackknife(y0=5000, a=0.10))
```

```
## [1] 0 0
```

6

```
gmp2013 <- read.table("data/gmp-2013.dat", header = T)
gmp2013$pop <- round(gmp2013$gmp/gmp2013$pcgmp)
plm(6611,1/8,gmp2013$pop,gmp2013$pcgmp)[[1]]
```

```
## [1] 6611.0000000    0.1263177
```

```
plm.jackknife(6611,1/8,gmp2013$pop,gmp2013$pcgmp)
```

```
## [1] 0 0
```

```
plm(5000,0.10,gmp2013$pop,gmp2013$pcgmp)[[1]]
```

```
## [1] 5000.0000008    0.1475913
```

```
plm.jackknife(5000,0.10,gmp2013$pop,gmp2013$pcgmp)
```

```
## [1] 0 0
```