Numerical Analysis Project #2

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1 1-D Multigrid Method

We set the tolerance of residual norm to be 10^{-16}, and the maximum number of iterations to be 20.

1.1 Dirichlet Boundary Condition

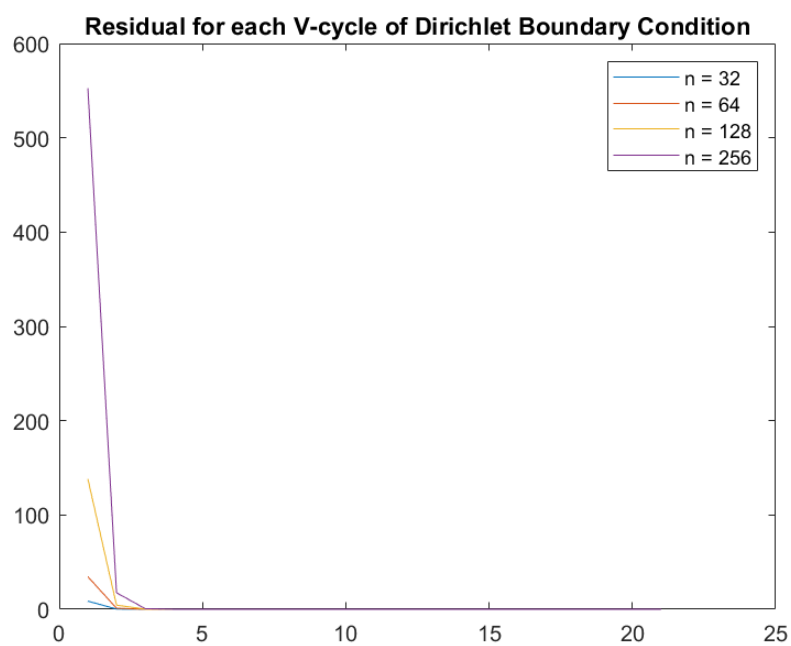

Figure 1: Residuals of each V-cycle for Dirichlet boundary condition

As we can see from the graph, the larger the n, the larger is the residual from FMG. But this quickly dampens out by repeatedly applying V-cycles. For n = 32, the residual norm reaches a value lower than 10^{-12}. For n = 64, it reaches a value lower than 10^{-11}. For n = 128, n = 256, it reaches a value lower than 10^{-10}.

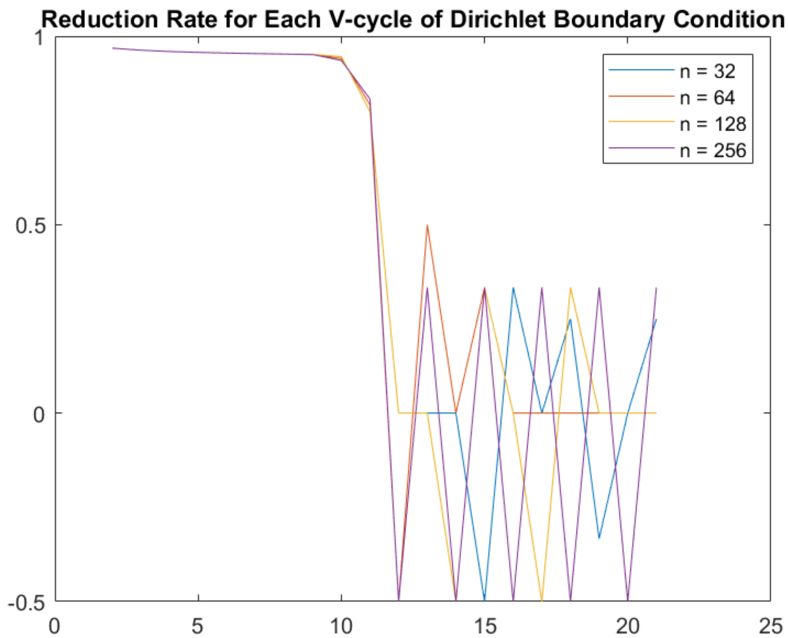

Figure 2: Reduction rate of each V-cycle for Dirichlet boundary condition

The reduction rate for each n has a few differences. For n = 32, the reduction rate is roughly the same for the first 10 iterations, i.e. until the residual norm smaller than 10^{-12}. Starting from the 11^{th} iteration, the reduction rate oscillates from values near 0 to a minus value (increase in residual).

We see similar results for n = 64, but with a more stable reduction rate near the tail of the graph, which is zero. The norm of the residual manages to reach to smaller than 10^{-10}. The same can also be said for n = 128. For n = 256, we obtain similar results to the previous ones, but with wilder oscillations of reduction rate after the 10^{th} iteration, i.e. when the residual norm is smaller than 10^{-10}.

For the maximum norm of the error vector, we get 4.94147×10^{-5} for n = 32, 1.23644×10^{-5} for n = 64, 3.09088×10^{-6} for n = 128, and 7.72706×10^{-7} for n = 256. This checks out with Theorem 9.4 in the notes.

1.2 Pure Neumann Boundary Condition

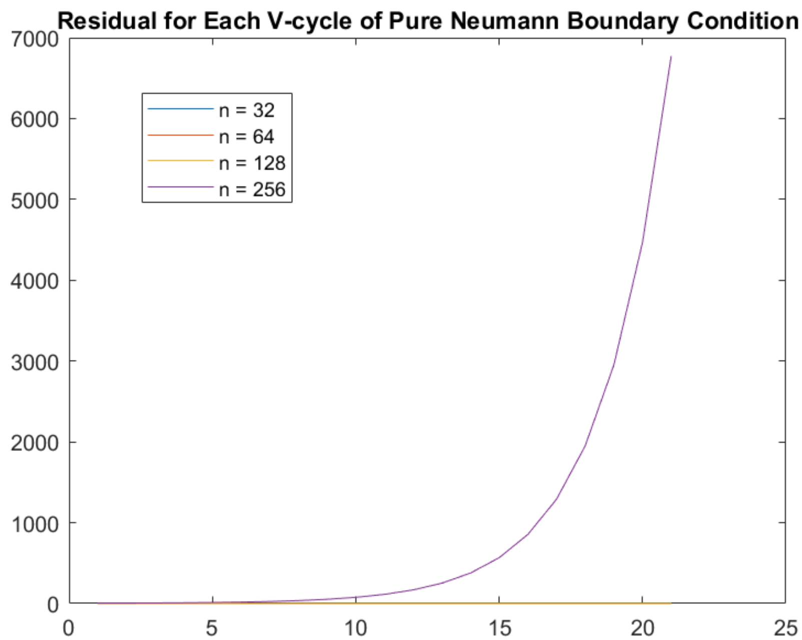

Figure 3: Residuals of each V-cycle for pure Neumann boundary condition

For n = 32, the residual reaches a value lower than 10^{-9}. For n = 64, the residuals can only reach to a value lower than 10^{-3}. For n = 128, the residual is more or less the same throughout the cycles, reaching a value higher than the residual of the FMG, and for n = 256, the residual diverges. We can conclude that pure Neumann boundary condition converges only when n is small.

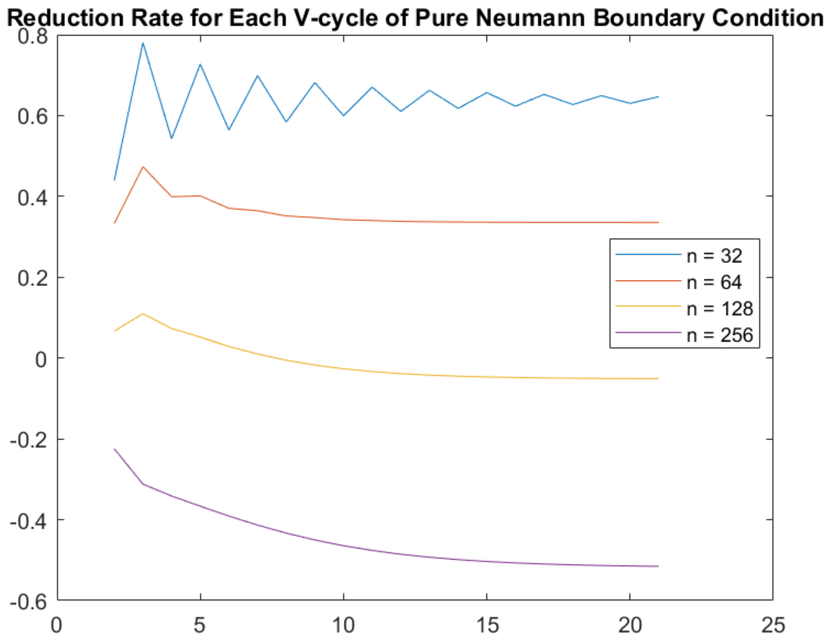

Figure 4: Reduction rate of each V-cycle for pure Neumann boundary condition

For n = 32, we see a good reduction rate throughout the V-cycles. Similar results are obtained for n = 64 and n = 128, a spike at the few first cycles but eventually reduces slowly. For n = 256, we see wild oscillations throughout the V-cycles, implying divergence of the residual.

For the maximum norm of the error vector, we obtain 2.31977 for n = 32, 2.31931 for n = 64, 2.31982 for n = 128, and 1.93439 for n = 256.

1.3 Left Neumann Boundary Condition

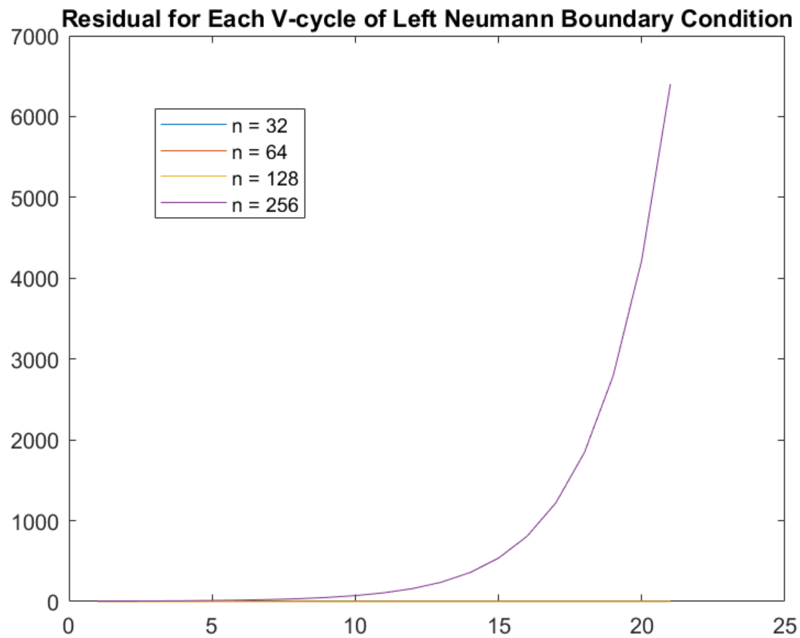

Figure 5: Residuals of each V-cycle for left Neumann boundary condition

For n = 32, the residual is able to reach a value lower than 10^{-9}, and 10^{-4} for n = 64. We obtain similar results as in the pure Neumann boundary condition for n = 128 and n = 256.

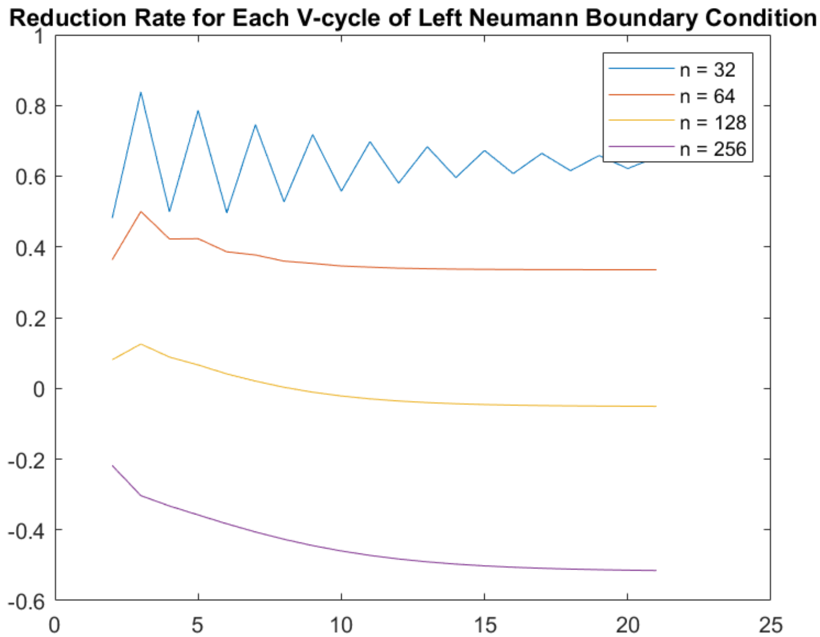

Figure 6: Reduction rate of each V-cycle for left Neumann boundary condition

The results are similar to that of pure Neumann boundary condition.

For the maximum norm of error, we obtain a value of 2.31679 for n = 32, 2.3185 for n = 64, 2.31877 for n = 128, and 1.95584 for n = 256.

1.4 Right Neumann Boundary Condition

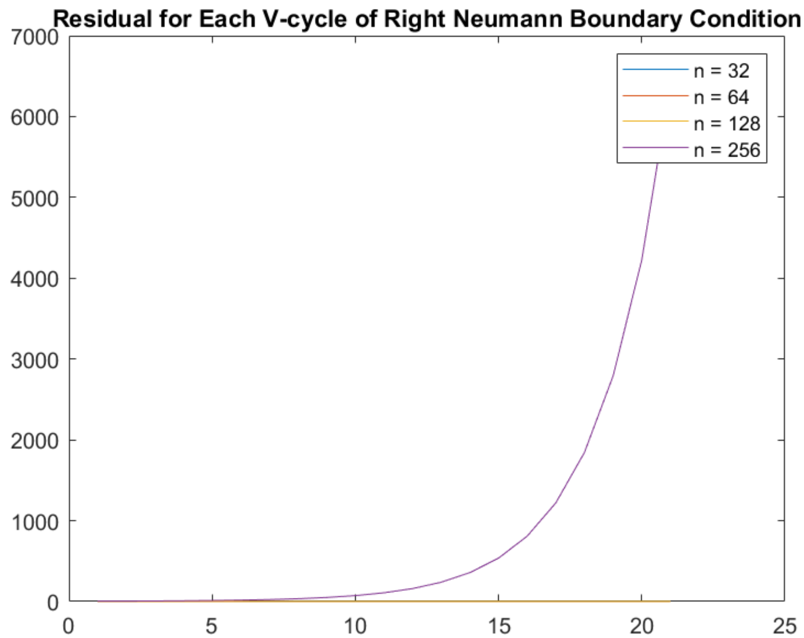

Figure 7: Residuals of each V-cycle for right Neumann boundary condition

For n = 32, the residual is able to reach a value lower than 10^{-9}, and lower than 10^{-3} for n = 64. The results for n = 128 and n = 256 are similar to that of pure and left Neumann boundary conditions.

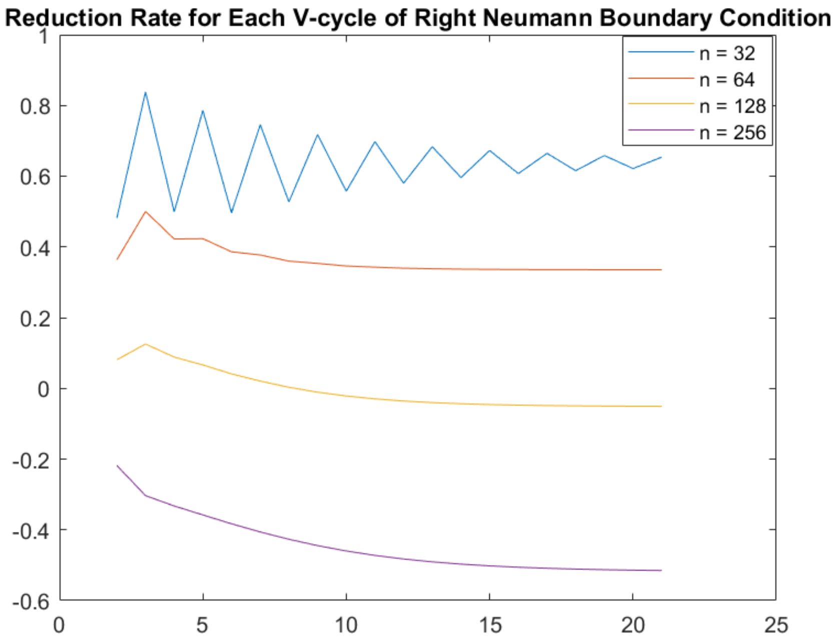

Figure 8: Reduction rate of each V-cycle for right Neumann boundary condition

Results of reduction rate are similar to that of pure and left Neumann boundary conditions.

The maximum norm of error is 2.31977 for n = 32, 2.31931 for n = 64, 2.31982 for n = 128, and 1.9344 for n = 256.