影像處理 LAB3

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4-1 Two-Dimensional Fast Fourier Transform

做法說明

這部分請說明整個 filtering 的流程，包含 input 影像的前處理到 Output filtered 結果的過程。另外要講解 myDFT2()和 myIDFT2()的作法，有寫 myFFT2()的可以再說明 myFFT2()和 myIDFT2()的作法。

The process of image frequency domain filtering:

1) Padding zero. Due to the periodicity of the image while being processed by Fourier transform, it is necessary to pad the image with 0s, for the sake of eliminating the wraparound error, which occurs when the convolution mask slides over the image and the values out of the target period are counted. It is made by assigning the value at (2M, 2N) 0, and then the matrix resizes itself automatically thanks to the feature of MATLAB.

2) Centralization. As is discussed in the lecture, the lowest frequency sinusoid wave has the highest magnitude, which is to say, F(0, 0) is the visual center of the frequency picture. Besides, the spectrum of frequency has the feature of central symmetry to the origin (0, 0). Thus it is intuitive to move the brightest dot to the center. And since the formula of the Fourier transformation, all that it needs to do for the transition is to multiply each pixel with $(-1)^{x+y}$, which can be implemented within two `for` loops.

$$F(u,v) \Rightarrow F(u-u0, v-v0)$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi\left[\frac{(u-u0)x}{M}+\frac{(v-v0)y}{N}\right]}$$

$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} [f(x,y)e^{-j2\pi\left(\frac{u_0 x}{M}+\frac{v_0 y}{N}\right)}]e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

When u0 = M/2 and v0 = N/2, the coefficient become (-1)^(x+y).

3) Do the Fourier transformation on the image got from 2).

a) Discrete Fourier Transformation Version.

First, Fourier transformation is a separable one, which means we can do the transformation on rows first and then move to columns. So the first loop is included to transform every rows. And the second loop is done to transform every columns of the output from the previous one. Thus all that needs to be done is about 1-dimensional discrete Fourier transformation. It is implemented by brute force. By the formula

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M}$$

totally, it can be done within two `for` loops, one for M values and the other for the summation. The time complexity is $O(M^2)$. To be specific, there are M^2 complex multiplication (without the expansion of Euler formula) and M*(M-1) complex addition overall.

b) Fast Fourier Transformation Version.

TODO

4) Generate the Gaussian lowpass filter. Use the definition of 1-D frequency domain Gaussian filter, it can be easily implemented.

$$H(u,v) = e^{-\frac{D(u,v)^2}{2\sigma^2}}$$

In the function getGLPF(), the cutoff frequency D0 and the size of the filter should be identified before getting the filter. Replace the variance of the Gaussian formula, which controls the width of the bell, with the given cutoff D0, to control the range of frequency to be reserved after convolution. To make the origin of the filter in the center, the D(u, v) is the distance between (u, v) and (M/2, N/2).

5) Filter in the frequency domain. Simply use the array multiplication to get the filtered frequency, since

$$f(x,y) * h(x,y) \leftrightarrow F(u,v)H(u,v)$$

, which can be done in a single statement.

6) Transform back to the spatial domain. It is done by the function myIDFT2(), which is implemented with the help of myDFT2(), since

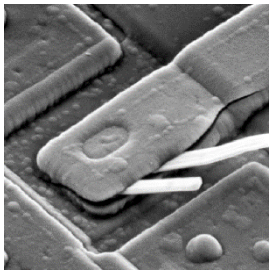
$$F^*(u,v) \overset{DFT}{\Longrightarrow} MNf^*(x,y) \overset{*\frac{1}{MN}}{\Longrightarrow} f^*(x,y) \overset{*}{\Rightarrow} f(x,y)$$

. It can also be implemented within one statement.

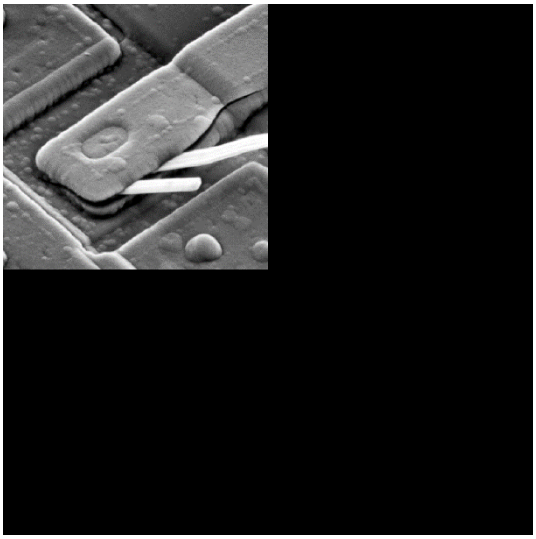
```
output = conj(myDFT2(conj(input))/(M*N));
```

7) Correct the coefficient for centralization and extract the image out of paddings. For we have multiplied a coefficient to centralize the frequency domain $(-1)^{x+y}$. Then re-multiply the same coefficient to get the real image back.

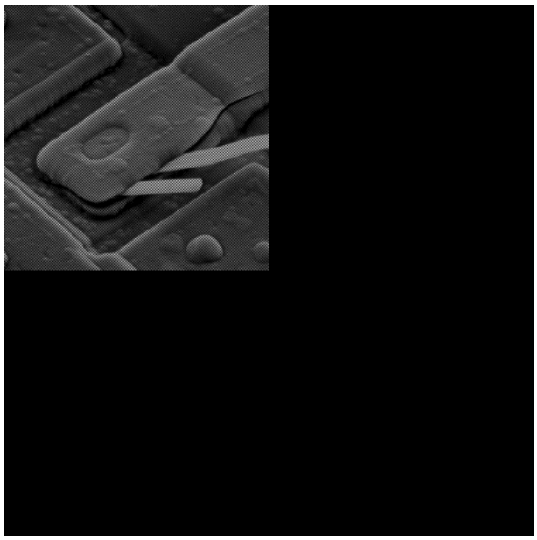
結果圖片



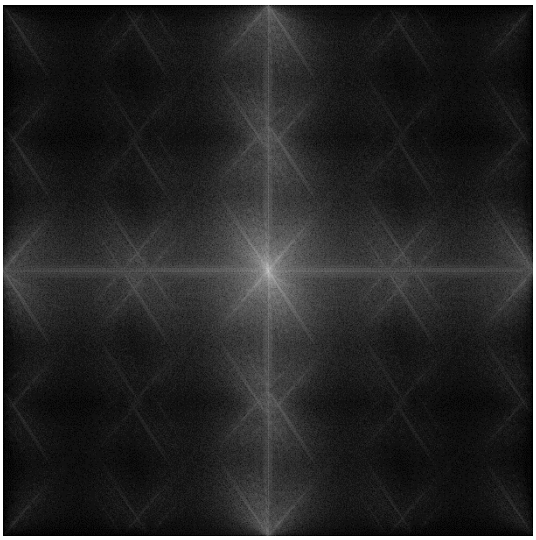
(a)



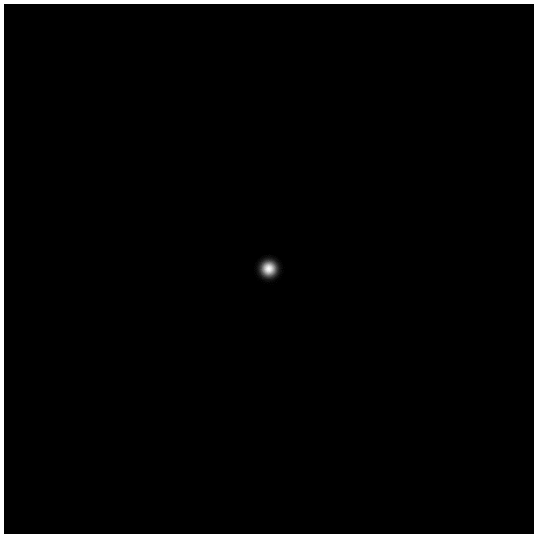
(b)



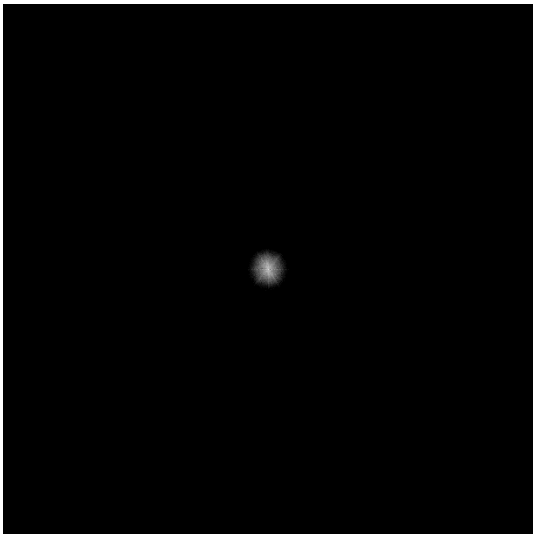
(c)



(d)

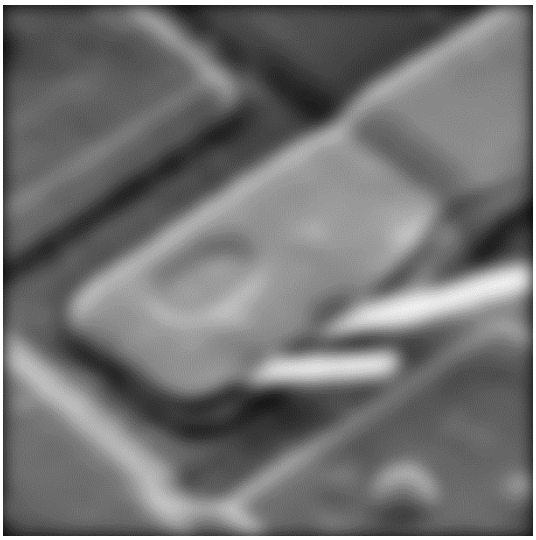


(e)



(f)

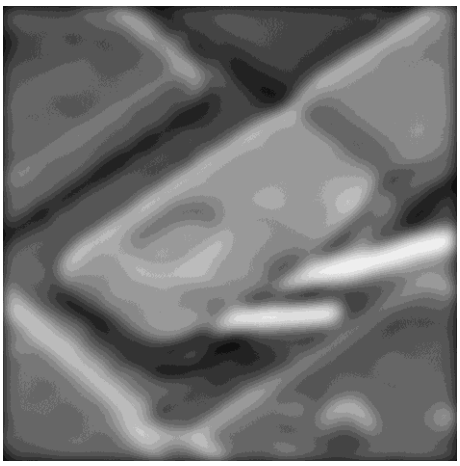
(g)

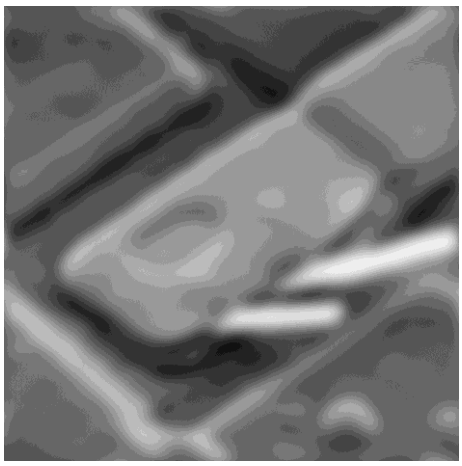


(h)

分析以及討論

1. Whether padding or not. Since the Discrete Fourier transform is applied, the function be transformed is inherently periodic. Thus without padding, the image actually itself is repeated while we are filtering in the frequency domain. That is to say, if it is reflected to the spatial domain, the near-by period will account into the computation when we compute the transformed value. Following are the final result images with (left) and without (right) the padding zero.



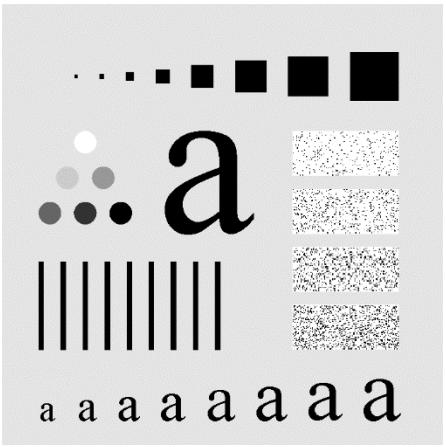


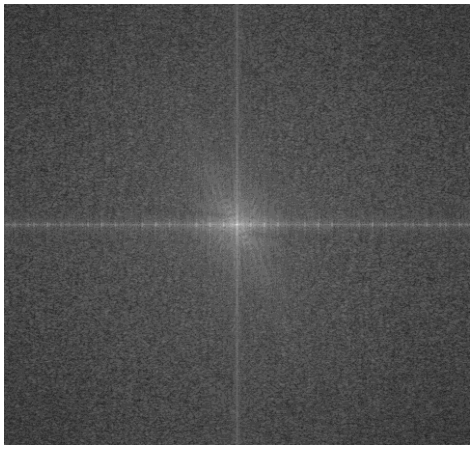
And the difference in the border of the 2 result images can be distinguished easily. With the padding, the borders tend to be darker for the values accounted, when the mask is out of the bound of the image, is zero. And without padding the value around the image is just the value of pixel on the other side (due to the periodicity). And in this image the pairs of opposite sides are in the similar intensity, and hence in the right one the borders of the image are brighter.

2. The mechanism and complexity of FFT.

4-2 Fourier Spectrum and Average Value

結果圖片





(a) (b)

分析以及討論

1. Mean value. To compare between the mean value computed from the image and the information of the center of the spectrum, I printed them both as following.

```
>> sum(sum(image))/(688*688)

ans =

    0.8130
```

```
>> spectrum(688:689,688:689)

ans =

   1.0e+05 *

    1.4520    2.4370
    2.3967    3.8483

>> ans/(688*688)

ans =

    0.3068    0.5148
    0.5063    0.8130
```

For the size of spectrum are even number (due to padding), I print the center 4*4 matrix of it. We can easily get that the mean value get from the point at (689, 689) is identical to the average value got directly from the image. That is the formula

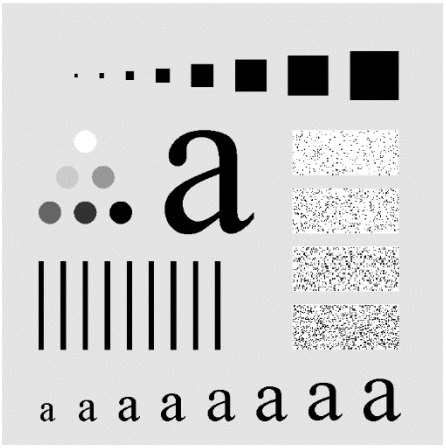
$$F(0,0) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$$

, which is again showed from the fact that the center is moved to (1+688, 1+688) as we multiply the coefficient (-1)^(x+y).

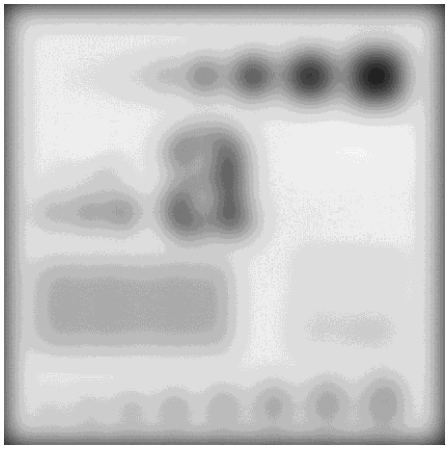
2. The distribution of the spectrum. As computing the mean value of the image, I notice an interesting phenomenon, which is the closer to the center of the spectrum image, the brighter the pixel is. Theoretically speaking, it means that the wave of the intensity changing with position is almost at low frequency. From the image, we can identify the smooth change of the intensity. What's more, there are two clear lines perpendicular to each other in the spectrum, which can be reflected to the origin image in spatial domain. The components in the image basically are square ones with intensity changing in two direction, horizontal and vertical. Furthermore, we can clearly identify the up and down of the horizontal line of the spectrum, which reveal the relatively large part of vertical lines in the origin image. Recall the transform of the box function, there will be decreasing waves in the frequency domain.

4-3 Lowpass Filtering

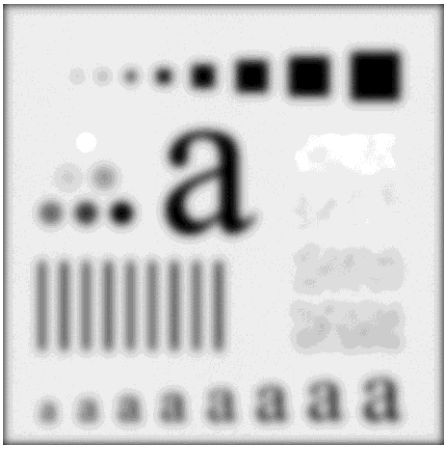
結果圖片



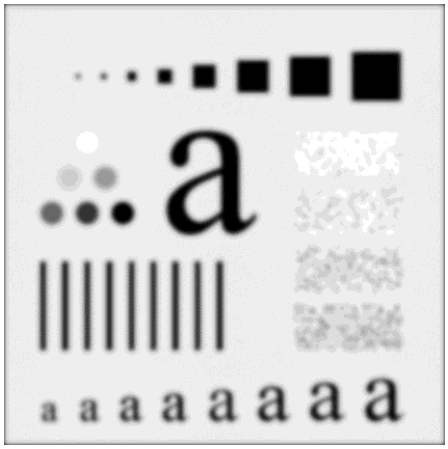
(a)



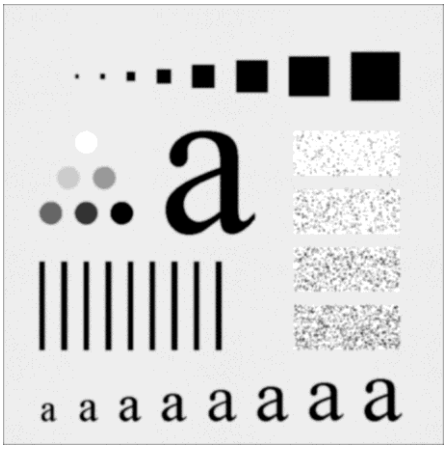
(b)



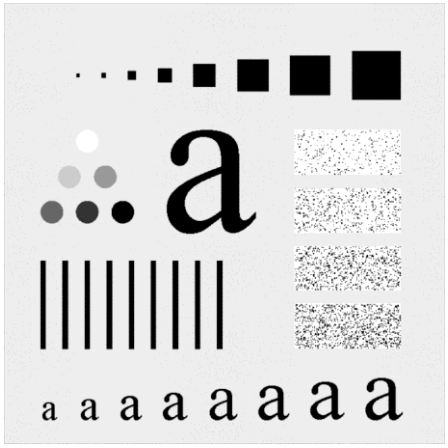
(c)



(d)



(e)



(f)

分析以及討論

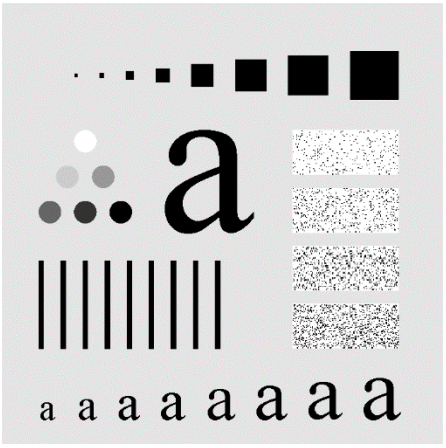
1. The facility of Gaussian filter. In this part of assignment, I just smell the magic and the flexibility of the Gaussian formula. To control the proportion that you

want the low frequency to get passed, just change the variance D0. For the magic, it is about the effect of the Gaussian lowpass filter, which will cause the image blurred in a pleasant way, instead of the ideal one which will cause ring artifacts.

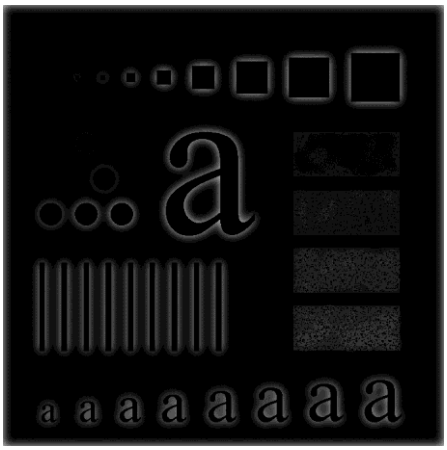
2. The different final images got from different D0. With the increase of the range of low frequency getting passed, the image become clearer for more waves with high frequency get passed. And the radius of cutoff at 460 (fig (f), 460/688 = 66.8%) can reserve enough features of the image that the difference cannot be identified by human eyes. That reminds me of considering to compress the image in frequency domain.

4-4 Highpass Filtering

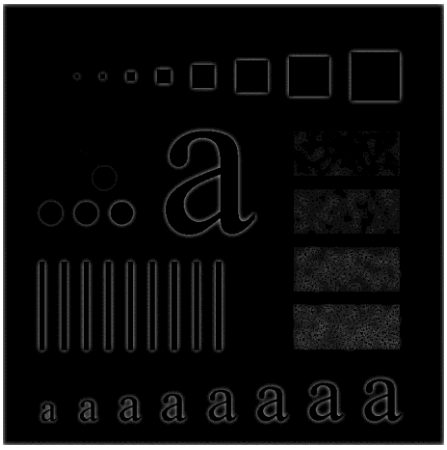
結果圖片



(a)



(b)



(c)

(d)

分析以及討論

1. The different results got from different range of low-frequency compressed. We can see with the range of low frequency compressed increasing, the change of intensity which will be reserved need a higher frequency. So we can see in (b) the edge (high-frequency) tends to have more relative smooth changes (the edges of six circles can be seen at least), while in (d) the edge are almost have very high frequency, which means fast change of intensity according to position (the top circles' edge cannot be seen due to the smooth change of intensity)

2. Draw the filter in spatial domain.