Development and Evaluation of a learned Biped Walking Trajectory based on 3D Points on the NAO Robotic System

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Introduction

The NAO humanoid robot

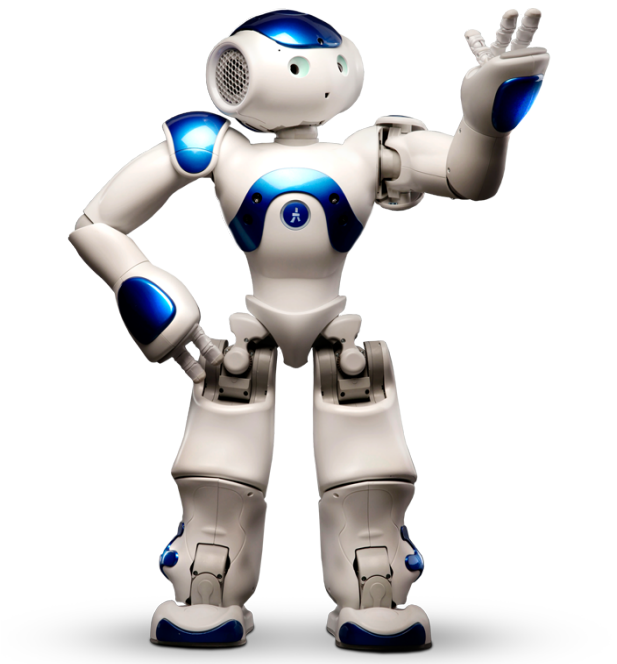

Figure: NAO[1]

[1] SoftBank Robotics. https://www.softbankrobotics.com/us/nao.

Gait generation

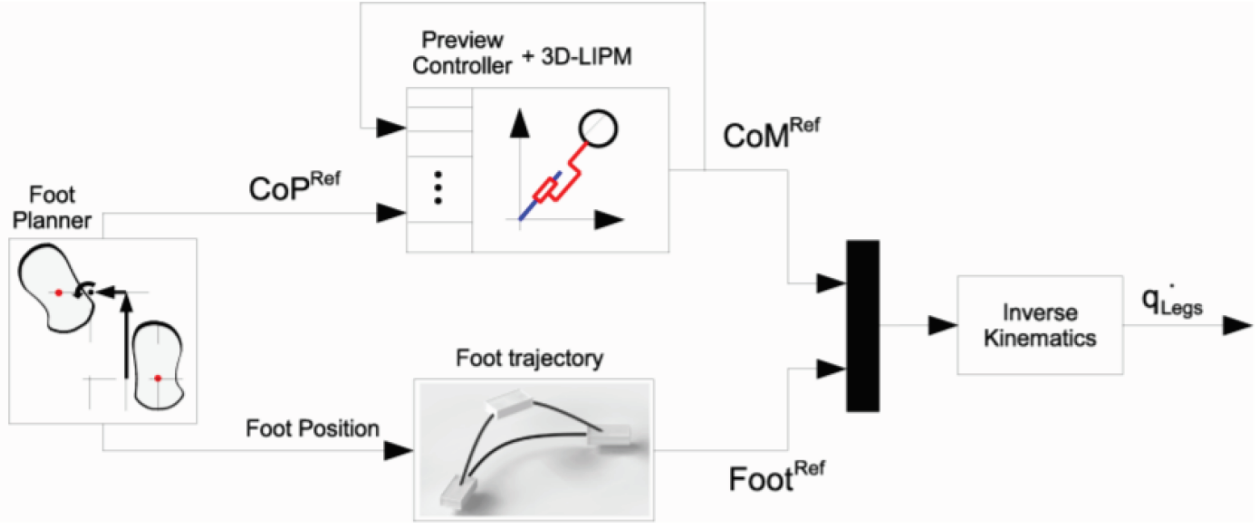

Figure: Process of gait generation[2]

[2] David Gouaillier, Cyrille Collette, and Chris Kilner. "Omni-directional closed-loop walk for NAO". In: *Humanoid Robots (Humanoids), 2010 10th IEEE-RAS International Conference on Humanoid Robots*. IEEE. 2010, pp. 448–454.

Motivation

- To compare existing biped walking models
- To replace the biped walking model
- To perform self-improvement with reinforcement learning

Simulation environment

A biped multibody system

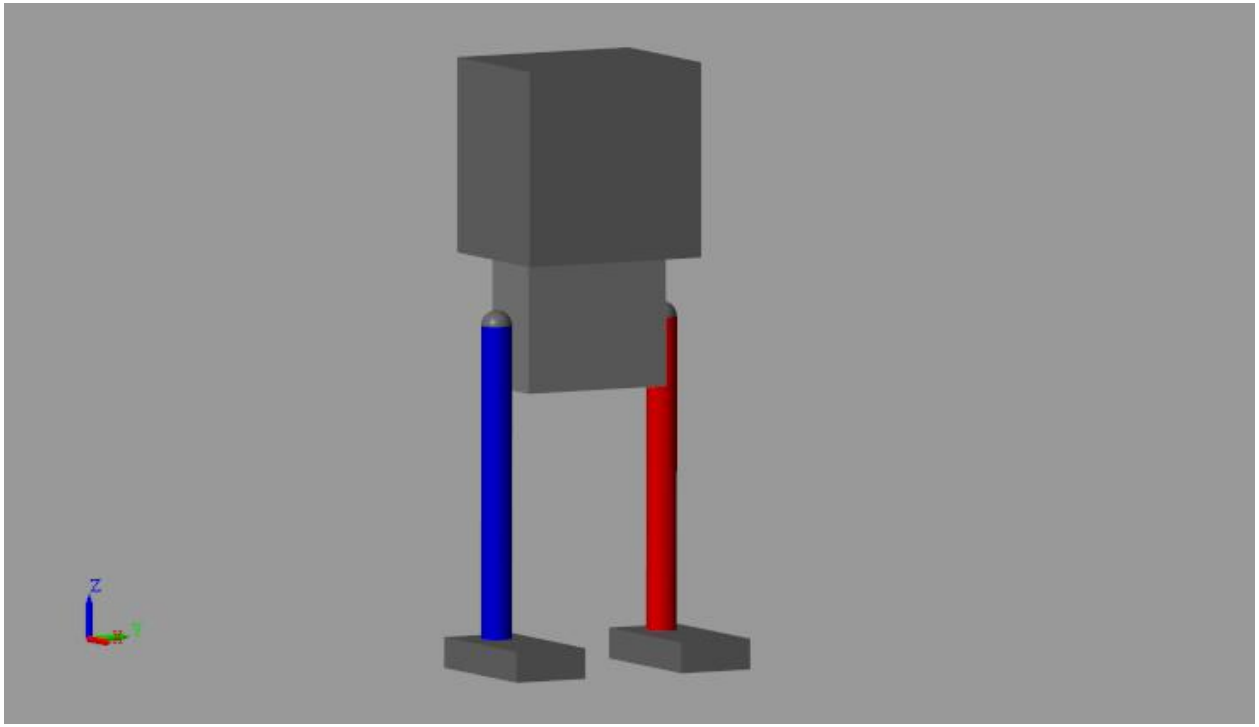

Figure: A biped multibody system

Simulation environment

Foot-to-ground forces

- Pressure: a spring system[3]

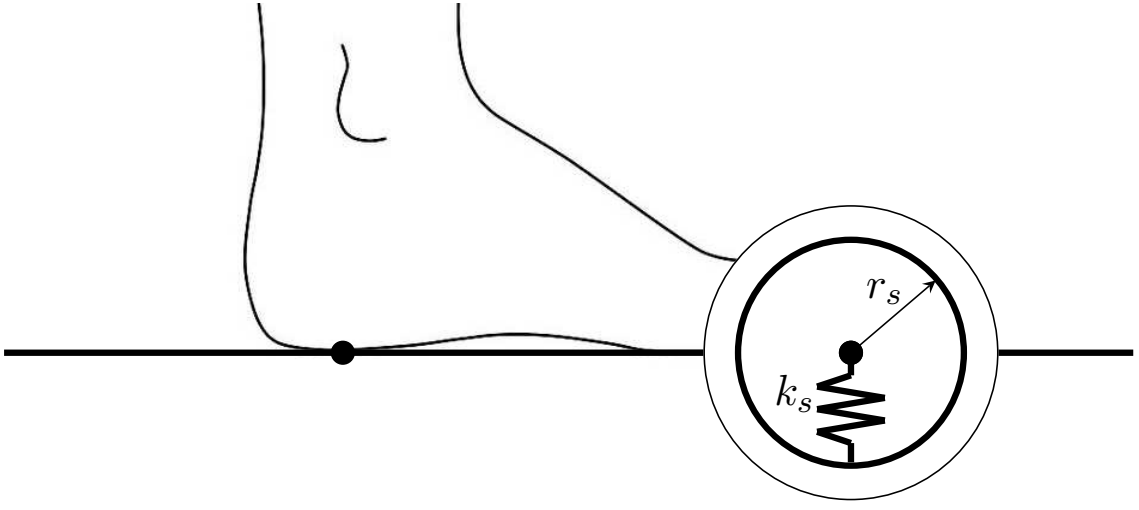

Figure: Foot pressure model

$$k_s = \frac{m_r \cdot g}{r_s}$$

[3] Steve Miller. *Simscape Multibody Contact Forces Library*. 2018. URL: https://www.mathworks.com/matlabcentral/fileexchange/47417-

Simulation environment

Foot-to-ground forces

- Pressure: a spring system
- Friction: to determine μ_s andμ_k by tuning
  - Walking speed
  - Elapse time
  - $\mu_s = 1000$ and $\mu_k = 1000$

Walking with models

Previous works

- A zero-moment point (ZMP) previewing gait[3]
- A 3D linear inverted pendulum model (LIPM) gait[4]
- Walk2014 from UNSW[5]

[3] Shuuji Kajita et al. "Biped walking pattern generation by using preview control of zero-moment point". In: *ICRA*. Vol. 3. 2003, pp. 1620–1626.

[4] Colin Graf and Thomas Röfer. "A Closed-loop 3D-LIPM Gait for the RoboCup Standard Platform League Humanoid". In: *Proceedings of the Fifth Workshop on Humanoid Soccer Robots in conjunction with the 2010 IEEE-RAS International Conference on Humanoid Robots*. Ed. by Enrico Pagello et al. Nashville, TN, USA, 2010.

[5] Bernhard Hengst. *runswift walk2014 report robocup standard platform league*. Tech. rep. Technical report, The University of New South Wales, 2014.

A ZMP previewing gait

The zero-moment point

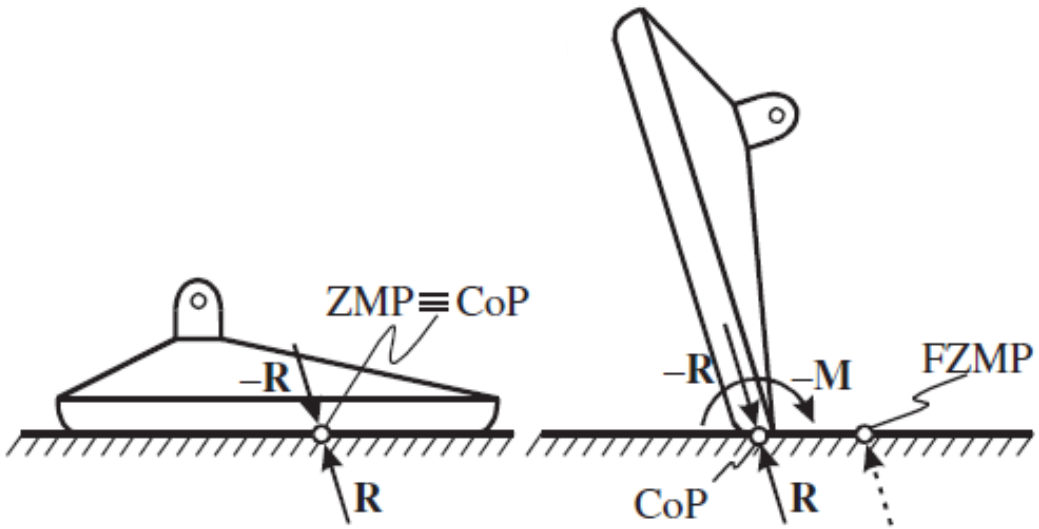

Figure: The zero-moment point[6]

[6] Miomir Vukobratović and Branislav Borovac. "Zero-moment point - thirty five years of its life". In: *International journal of humanoid robotics* 1.01 (2004), pp. 157–173.

A ZMP previewing gait

Cart-table model (1)

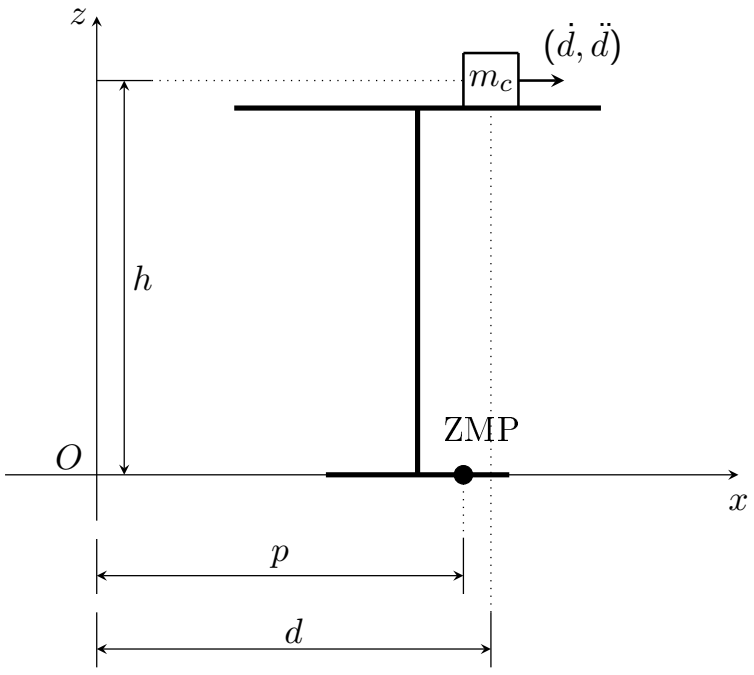

Figure: Cart-table model

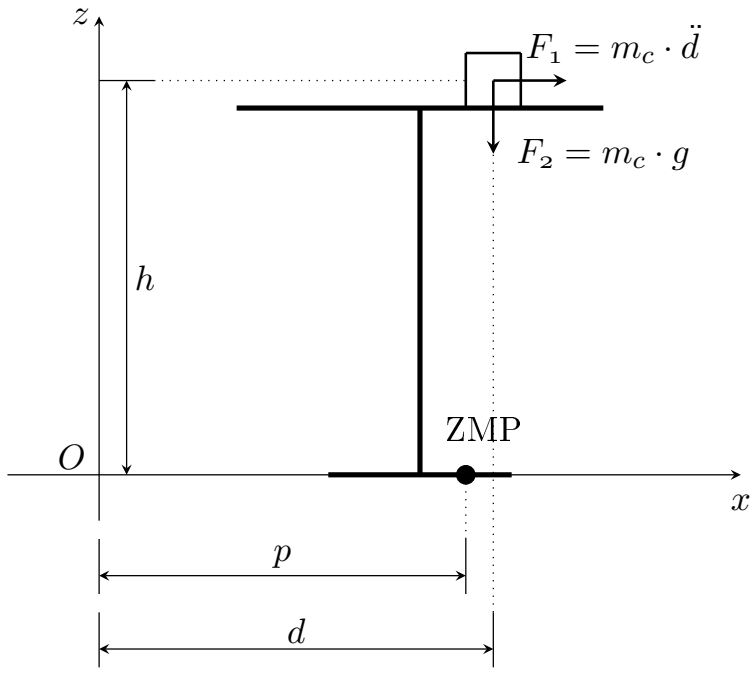

Figure: Force diagram

$$m_c \cdot \ddot{d} \cdot (d - p) - m_c \cdot g \cdot h = 0$$

A ZMP previewing gait

Cart-table model (2)

- State vector: $\begin{bmatrix} d & \dot{d} & \ddot{d} \end{bmatrix}^T$
- Input: jerk
- Output: position of ZMP
- Continuous-time state-space representation:

$$\begin{bmatrix} \dot{d} \\ \ddot{d} \\ \dddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \\ \ddot{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$p = \begin{bmatrix} 1 & 0 & -\frac{h}{g} \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \\ \ddot{d} \end{bmatrix}$$

A ZMP previewing gait

Cart-table model (2)

- State vector: $\begin{bmatrix} d(k) & v(k) & a(k) \end{bmatrix}^T$
- Input: jerk $j(k)$
- Output: position of ZMP $p(k)$
- Discrete-time state-space representation (sampling time T_s):

$$\begin{bmatrix} v(k+1) \\ a(k+1) \\ j(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_s & \frac{T_s^2}{2} \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d(k) \\ v(k) \\ a(k) \end{bmatrix} + \begin{bmatrix} \frac{T_s^3}{6} \\ \frac{T_s^2}{2} \\ T_s \end{bmatrix} u(k)$$

$$p(k) = \begin{bmatrix} 1 & 0 & -\frac{h}{g} \end{bmatrix} \begin{bmatrix} d(k) \\ v(k) \\ a(k) \end{bmatrix}$$

A ZMP previewing gait

Preview controller (1)

- Why preview controller?
- The optimal preview controller:

$$u(k) = G_e \sum_{i=0}^{k} e(k) - G_x x(k) - \sum_{l=1}^{N} G_r(l) r(k+l)$$

- The cost function:

$$J = \sum_{i=k}^{\infty} \left[Q_e e^2(i) + \Delta \boldsymbol{x}^T(i) Q_x \Delta \boldsymbol{x}(i) + R \Delta u^2(i) \right]$$

A ZMP previewing gait

Preview controller (2)

- Limitations:

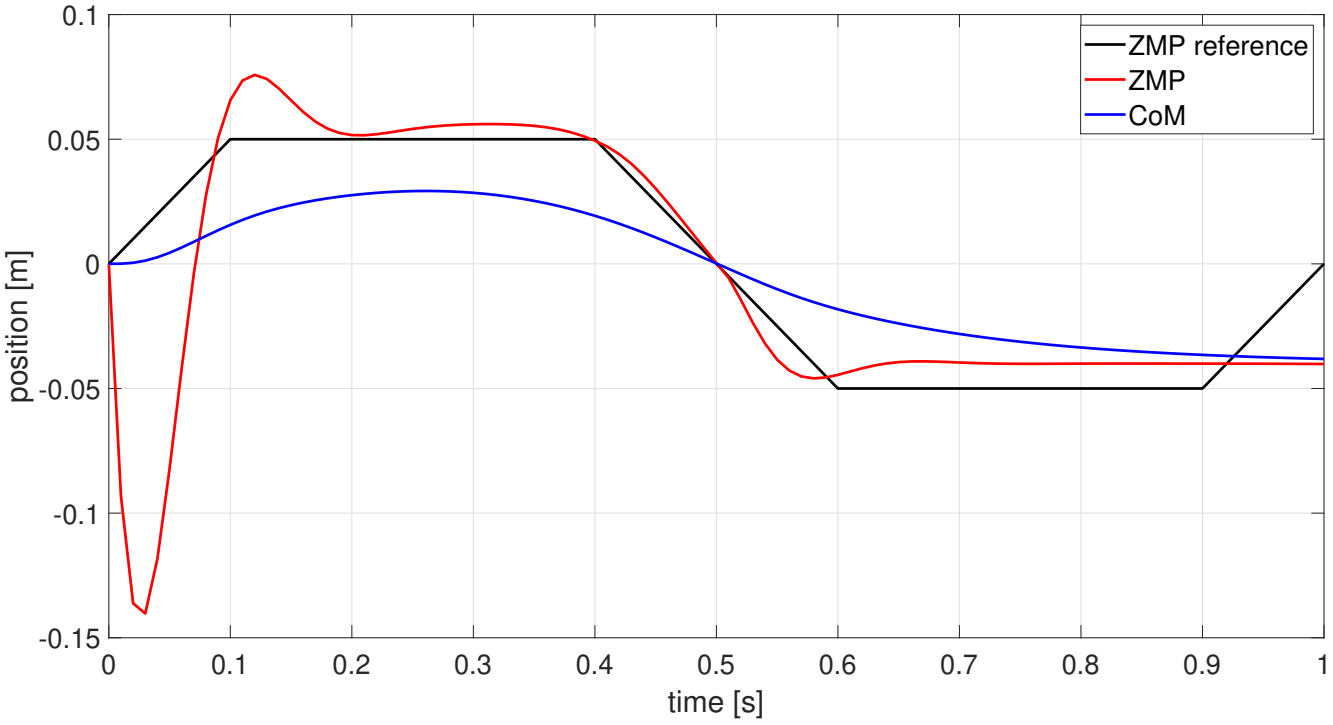

Figure: Limitations when using preview controller

A ZMP previewing gait

Preview controller (3)

- First strategy: to reserve some space, $Q_e = 50500$, $Q_x = 0$, $R = 0.59$ and $N = 35$

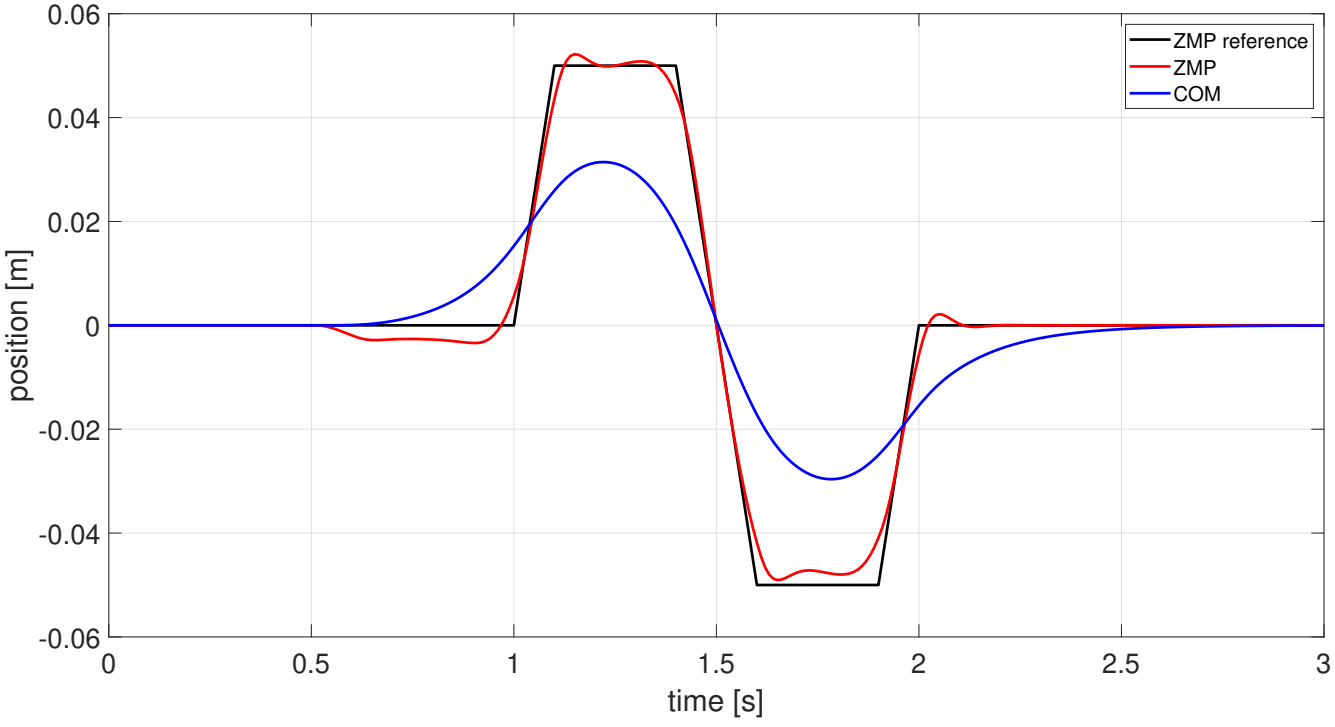

Figure: Reserving space for the preview controller

A ZMP previewing gait

Preview controller (3)

- First strategy: to reserve some space
- Second strategy: to plan ZMP reference in time horizon much longer than simulation time

A 3D LIPM gait

3D LIPM

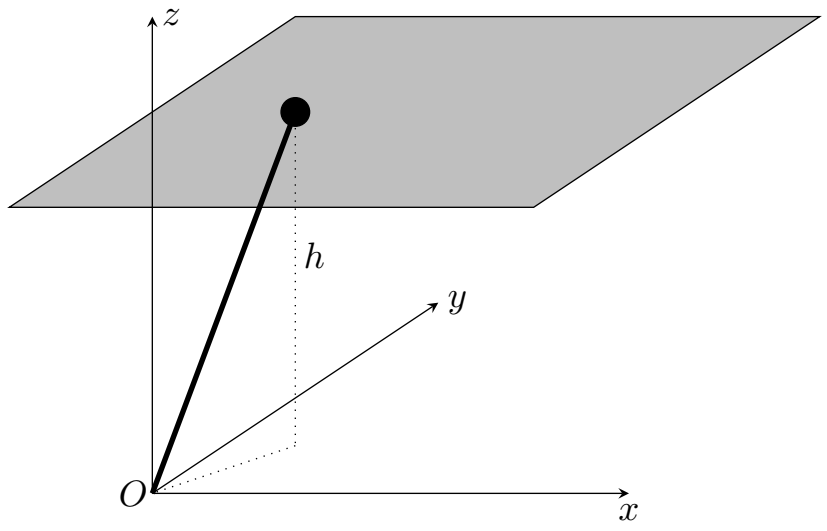

Figure: 3D inverted pendulum

$$x(t) = x_0 \cdot \cosh(k_c \cdot t) + \frac{\dot{x}_0 \cdot \sinh(k_c \cdot t)}{k_c} \quad , \; k_c = \sqrt{\frac{g}{h}}$$

$$\dot{x}(t) = k_c \cdot x_0 \cdot \sinh(k_c \cdot t) + \dot{x}_0 \cdot \cosh(k_c \cdot t)$$

A 3D LIPM gait

Calculation of step duration (1)

- Calculate the step duration iteratively
- Some time points:
  - t_e: end point of the current single support phase
  - t_b: start point of the next single support phase
  - t_0: inflection point of pendulum motion, $\dot{x}_0 = 0$

A 3D LIPM gait

Calculation of step duration (1)

- Calculate the step duration iteratively
- Some time points: t_e, t_b and t_0
- Some coordinate systems:

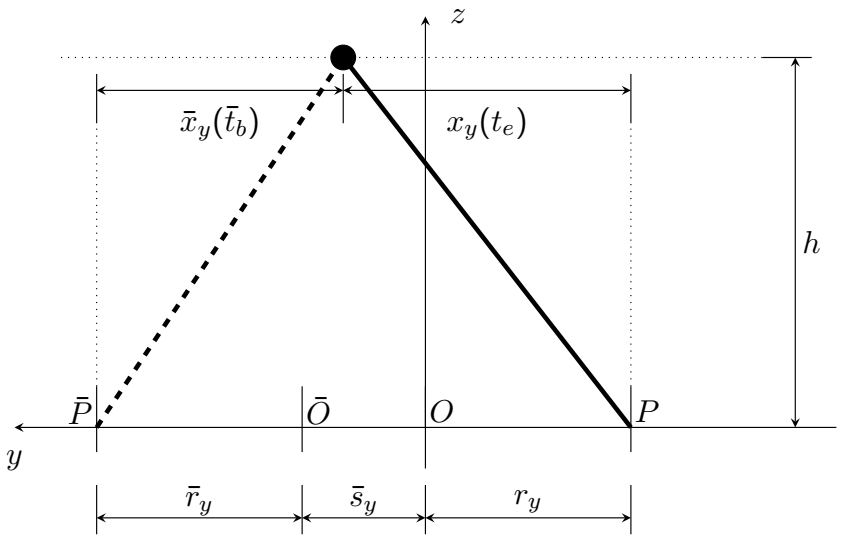

Figure: Two facing inverted pendulum on the $y-z$ plane

$$\bar{r}_y + \bar{s}_y + r_y = \bar{x}_y(\bar{t}_b) + x_y(t_e)$$
$$\dot{x}_y(t_e) = \dot{\bar{x}}_y(\bar{t}_b)$$

A 3D LIPM gait

Calculation of step duration (2)

Algorithm 1 Computing the step duration

1: $t_e \leftarrow$ Initial guess
2: **repeat**
3: $\quad \bar{t}_b \leftarrow \frac{1}{k_c} \cdot \operatorname{arcsinh}\left(\frac{x_{0_y} \cdot \sinh(k_c \cdot t_e)}{\bar{x}_{0_y}}\right)$
4: $\quad \dot{x}_y(t_e) \leftarrow k_c \cdot x_{0_y} \cdot \sinh(k_c \cdot t_e)$
5: $\quad \Delta t_e \leftarrow \frac{\bar{r}_y + \bar{s}_y + r_y - (\bar{x}_y(\bar{t}_b) + x_y(t_e))}{2 \cdot \dot{x}_y(t_e)}$
6: $\quad t_e \leftarrow t_e + \Delta t_e$
7: **until** $|\Delta t_e| < \varepsilon$
8: $\bar{t}_b \leftarrow \frac{1}{k_c} \cdot \operatorname{arcsinh}\left(\frac{x_{0_y} \cdot \sinh(k_c \cdot t_e)}{\bar{x}_{0_y}}\right)$

Walk2014

- One of the most popular gait nowadays in SPL
- Inverted-pendulum-like dynamics
- Not model-based but measurement-based

Walk2014

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- Inverted-pendulum-like dynamics
- Not model-based but measurement-based

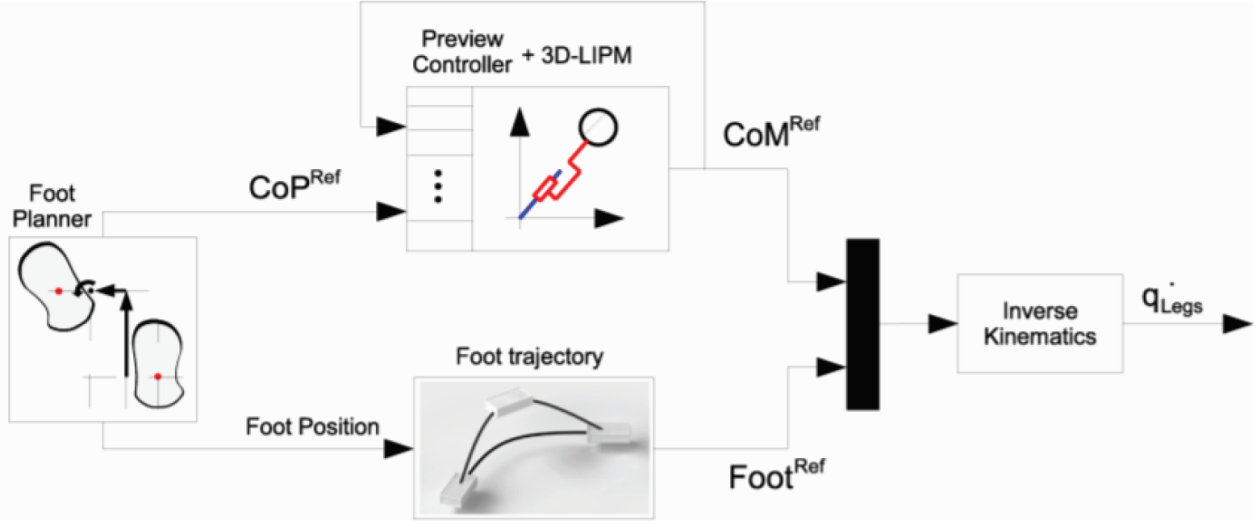


Figure: Process of gait generation

Walk2014

- One of the most popular gait nowadays in SPL
- Inverted-pendulum-like dynamics
- Not model-based but measurement-based

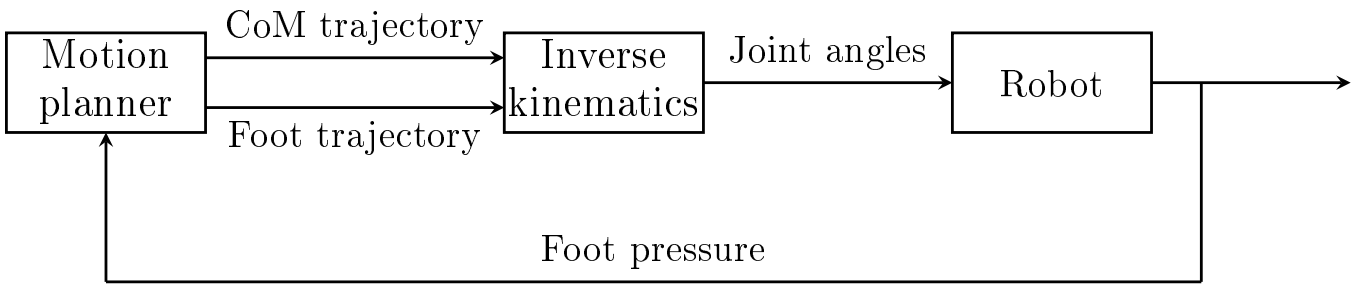


Figure: Process of gait generation for Walk2014

Walk2014

Measurement-based control

- Logic control:

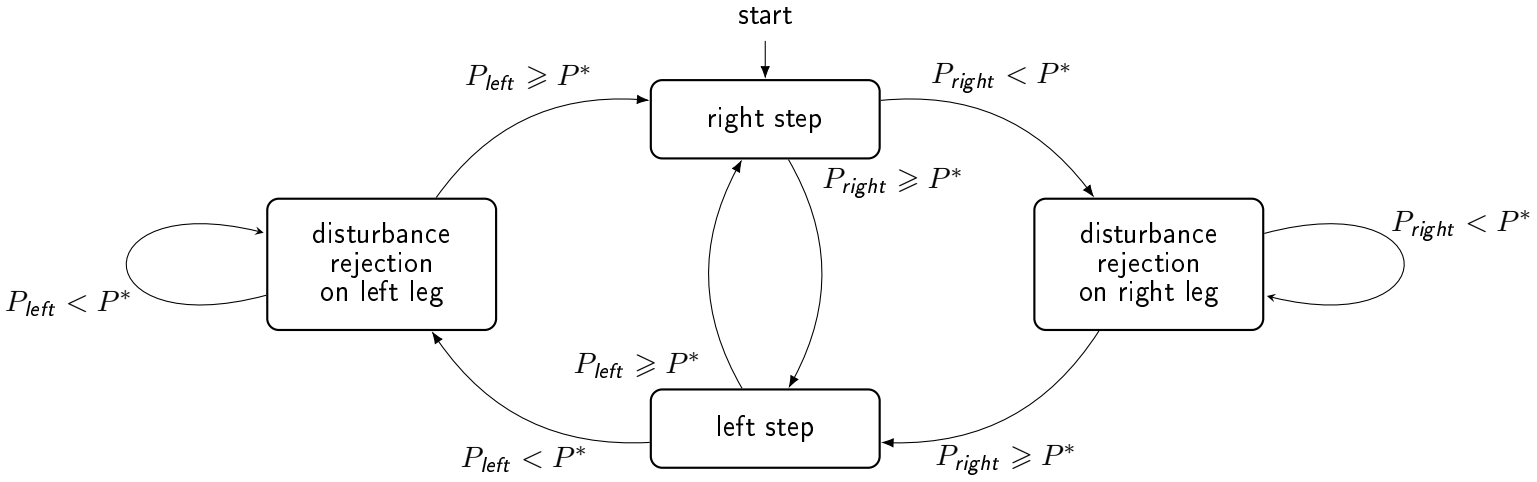


Figure: Logic control of Walk2014

- Pressure filtering:

$$G(s) = \frac{\omega_c}{s + \omega_c}$$

Walk2014

Inverted-pendulum-like dynamics

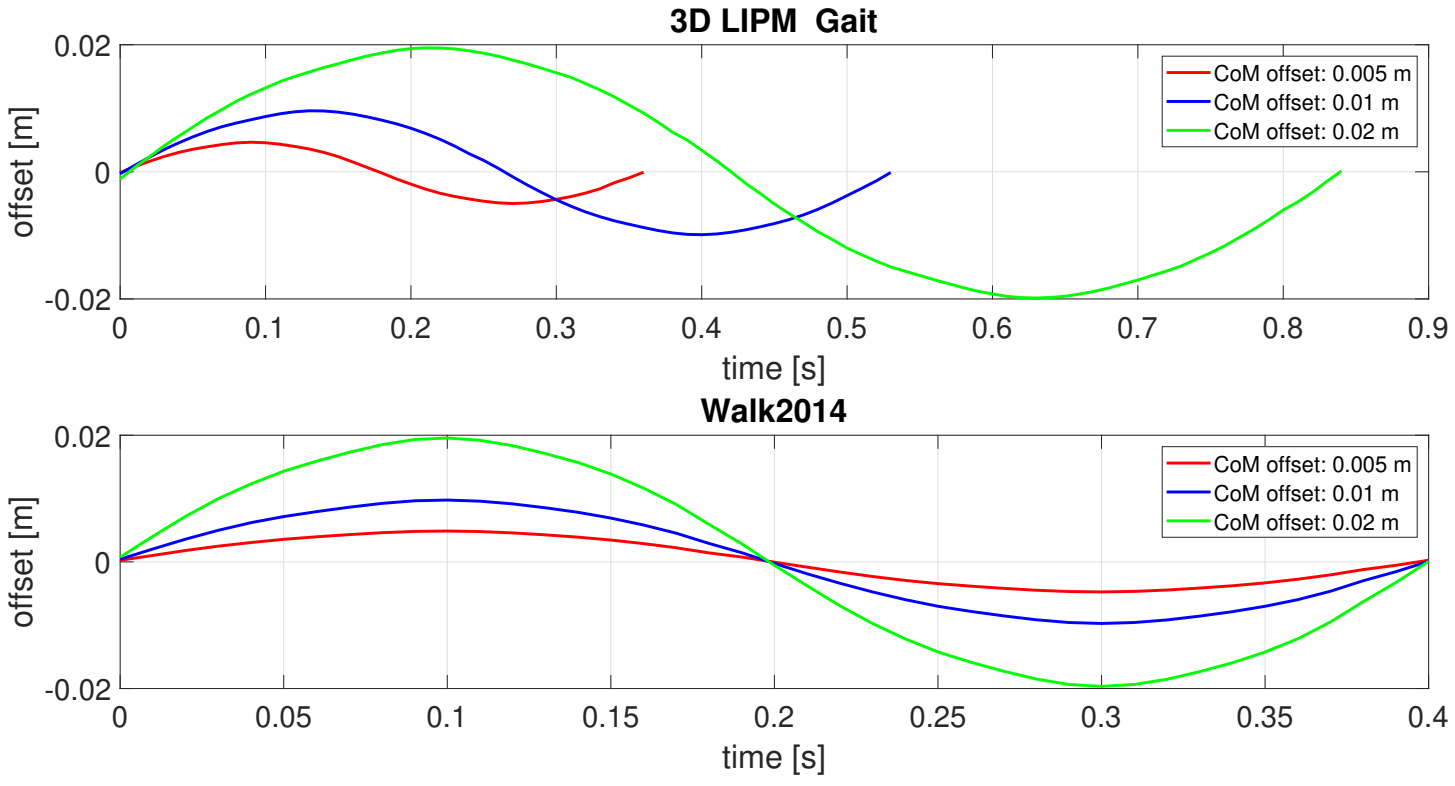


Figure: CoM swinging offset vs. step duration

Simulation results and conclusion

Performance indices

- Walking speed
- Elapse time
- CoM swinging offset
- Ability of disturbance (pushing) rejection

Simulation results and conclusion

Walking speed

Table: Average forward walking speed of different gaits

Gait	Speed [cm/s]
ZMP Previewing Gait	25.73
3D LIPM Gait	32.50
Walk2014	32.77

Table: Average sideward walking speed of different gaits

Gait	Speed [cm/s]
ZMP Previewing Gait	7.30
3D LIPM Gait	9.92
Walk2014	10.43

Simulation results and conclusion

Elapse time

Table: Elapse time for simulating different gaits

Gait	Time [s]
ZMP Previewing Gait	84.29
3D LIPM Gait	59.84
Walk2014	49.52

Simulation results and conclusion

CoM swinging offset

Table: CoM swinging offset of different gaits (step duration 0.2 s)

Gait	Offset [m]
ZMP Previewing Gait	5.8×10^{-3}
3D LIPM Gait	6.6×10^{-3}
Walk2014	flexible

Simulation results and conclusion

Disturbance rejection

Table: Time for disturbance (pushing) rejection of different gaits

Gait	Time [s]
ZMP Previewing Gait	slow because of calculation
3D LIPM Gait	1.01
Walk2014	0.93

Walking with movement primitives

- Preparation of movement primitives for walking
- Dynamic movement primitives (DMPs)[7]
- Robot execution
- Improvement with reinforcement learning

[7] Stefan Schaal. "Dynamic movement primitives-a framework for motor control in humans and humanoid robotics". In: *Adaptive motion of animals and machines*. Springer, 2006, pp. 261–280.

Preparation of movement primitives

The Kinect sensor

Figure: The Kinect sensor[8]

[8] URL: https://en.wikipedia.org/wiki/Kinect.

Preparation of movement primitives

The Kinect sensor

Preparation of movement primitives

Joint angle sequences

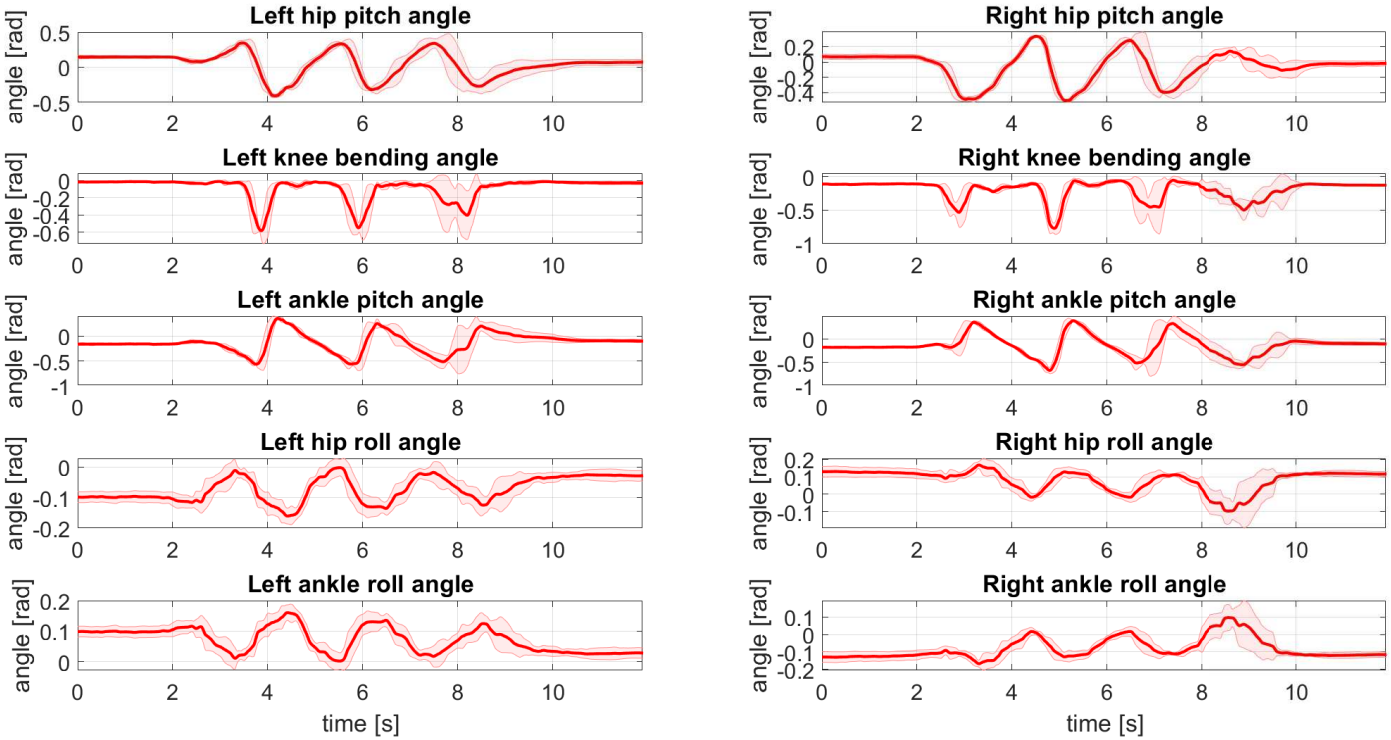


Figure: Joint angle sequences of a straight walking process

Preparation of movement primitives

Joint angle sequences

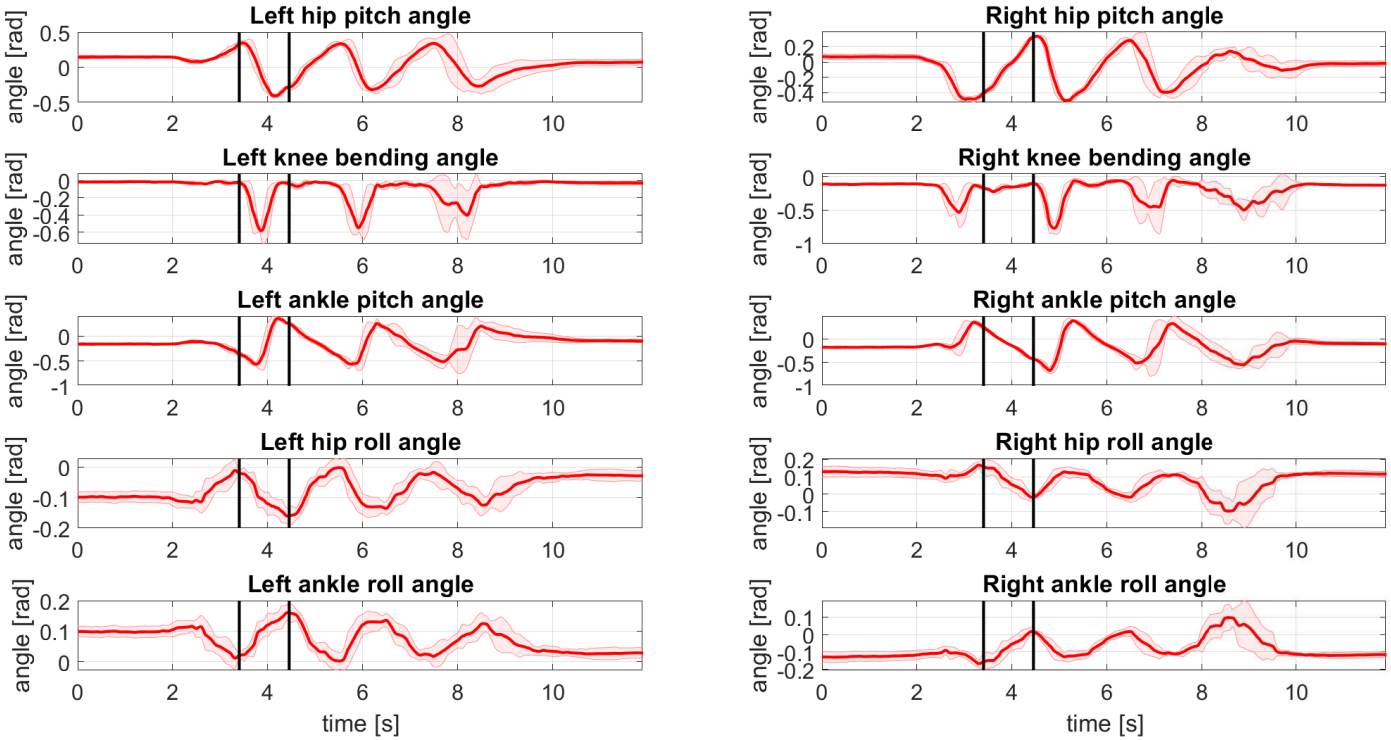


Figure: Joint angle sequences of a straight walking process

Preparation of movement primitives

Joint angle sequences

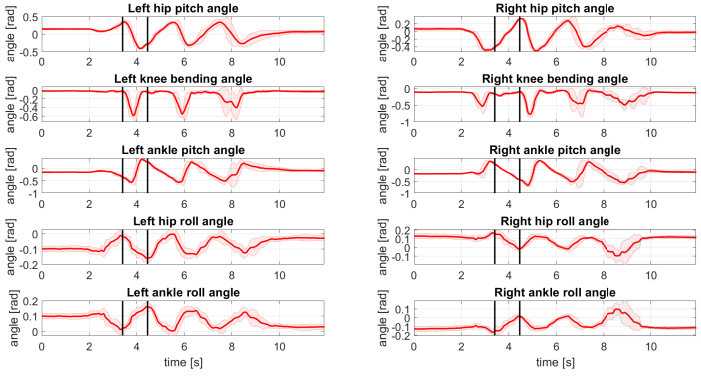


Figure: Joint angle sequences of a straight walking process

Dynamic movement primitives

Basics

- A canonical system: to remove explicit time dependence

$$\dot{x}(t) = -\alpha_x \cdot \frac{1}{\tau} \cdot x(t)$$

  - Faster execution: $0 < \tau < 1$
  - Slower execution: $\tau > 1$

- A second-order system: critically damped, $\alpha_y = 4 \cdot \beta_y$

$$\tau^2 \ddot{y}(t) = \alpha_y \left(\beta_y \left(y_g - y(t)\right) - \tau y(t)\right) + f(t)$$

- A forcing term:

$$f(x) = \frac{\sum_{i=1}^{N_{bf}} w_i \Psi_i(x)}{\sum_{i=1}^{N_{bf}} \Psi_i(x)} x(y_g - y_0)$$

Dynamic movement primitives

Imitation learning (1)

- Movement primitive from human: $\begin{bmatrix} \boldsymbol{y}_D & \dot{\boldsymbol{y}}_D & \ddot{\boldsymbol{y}}_D \end{bmatrix}^T$
- The weights:

$$w_i = \frac{\boldsymbol{s}^T \Gamma_i \boldsymbol{f}_G}{\boldsymbol{s}^T \Gamma_i \boldsymbol{s}}$$

$$\boldsymbol{s} = \begin{bmatrix} x_1 \left(y_{D_T} - y_{D_1}\right) \\ \vdots \\ x_T \left(y_{D_T} - y_{D_1}\right) \end{bmatrix}, \Gamma_i = \begin{bmatrix} \Psi_{i_1} & & 0 \\ & \ddots & \\ 0 & & \Psi_{i_T} \end{bmatrix}, \boldsymbol{f}_G = \begin{bmatrix} f_{G_1} \\ \vdots \\ f_{G_T} \end{bmatrix}$$

$$\boldsymbol{f}_G = \ddot{\boldsymbol{y}}_D - \alpha_y \left(\beta_y (y_{D_T} - \boldsymbol{y}_D) - \dot{\boldsymbol{y}}_D\right)$$

Dynamic movement primitives

Imitation learning (1)

- The weights:

$$w_i = \frac{\boldsymbol{s}^T \Gamma_i \boldsymbol{f}_G}{\boldsymbol{s}^T \Gamma_i \boldsymbol{s}}$$

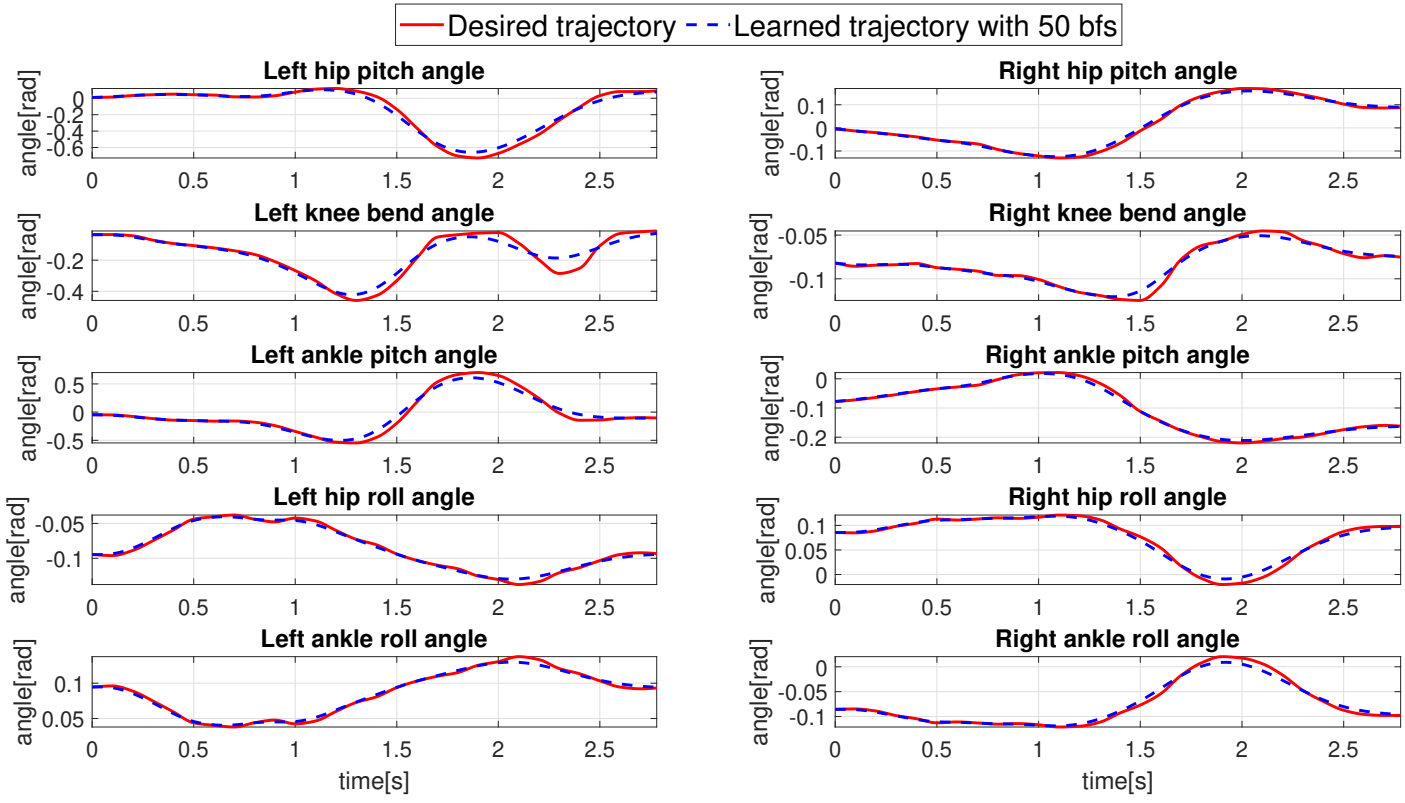
- The regeneration process:

Algorithm 2 Regenerating a given curve

Initializing: $y_{R_1} = y_{D_1}$, $\dot{y}_{R_1} = 0$, $\ddot{y}_{R_1} = 0$
for $j = 2$ to T **do**
  $\ddot{y}_{R_{j-1}} = \frac{1}{\tau^2}\left(\alpha_y\left(\beta_y(y_{D_T} - y_{R_{j-1}}) - \tau\dot{y}_{R_{j-1}}\right) - f(x_{j-1})\right)$
  $\dot{y}_{R_j} = \dot{y}_{R_{j-1}} + \ddot{y}_{R_{j-1}} \cdot \Delta t$
  $y_{R_j} = y_{R_{j-1}} + \dot{y}_{R_{j-1}} \cdot \Delta t$
end for

Dynamic movement primitives

Imitation learning (2)



Dynamic movement primitives

Temporal scaling

- Temporal scaling factor: τ
  - Faster execution: $0 < \tau < 1$
  - Slower execution: $\tau > 1$
- Theoretical bound: instability comes from the explicit Euler method

$$\frac{\alpha_y \cdot \Delta t}{4} < \tau$$

- Physical execution bound: achieved by trial and error

$$0.5 \leqslant \tau$$

Dynamic movement primitives

Primitives concatenation (1)

Reproduction and concatenation of movement primitives

Algorithm 3 Reproducing and concatenating movements

1: From the second primitive on: initializing y_{R_1} with the end value of the last primitive and $\dot{y}_{R_1} = 0$, $\ddot{y}_{R_1} = 0$
2: **for** $j = 2$ to T **do**
3: $\quad \ddot{y}_{R_{j-1}} = \frac{1}{\tau^2}\left(\alpha_y\left(\beta_y(y_{D_T} - y_{R_{j-1}}) - \tau\dot{y}_{R_{j-1}}\right) - f(x_{j-1})\right)$
4: $\quad \dot{y}_{R_j} = \dot{y}_{R_{j-1}} + \ddot{y}_{R_{j-1}} \cdot \Delta t$
5: $\quad y_{R_j} = y_{R_{j-1}} + \dot{y}_{R_{j-1}} \cdot \Delta t$
6: **end for**

Dynamic movement primitives

Primitives concatenation (2)

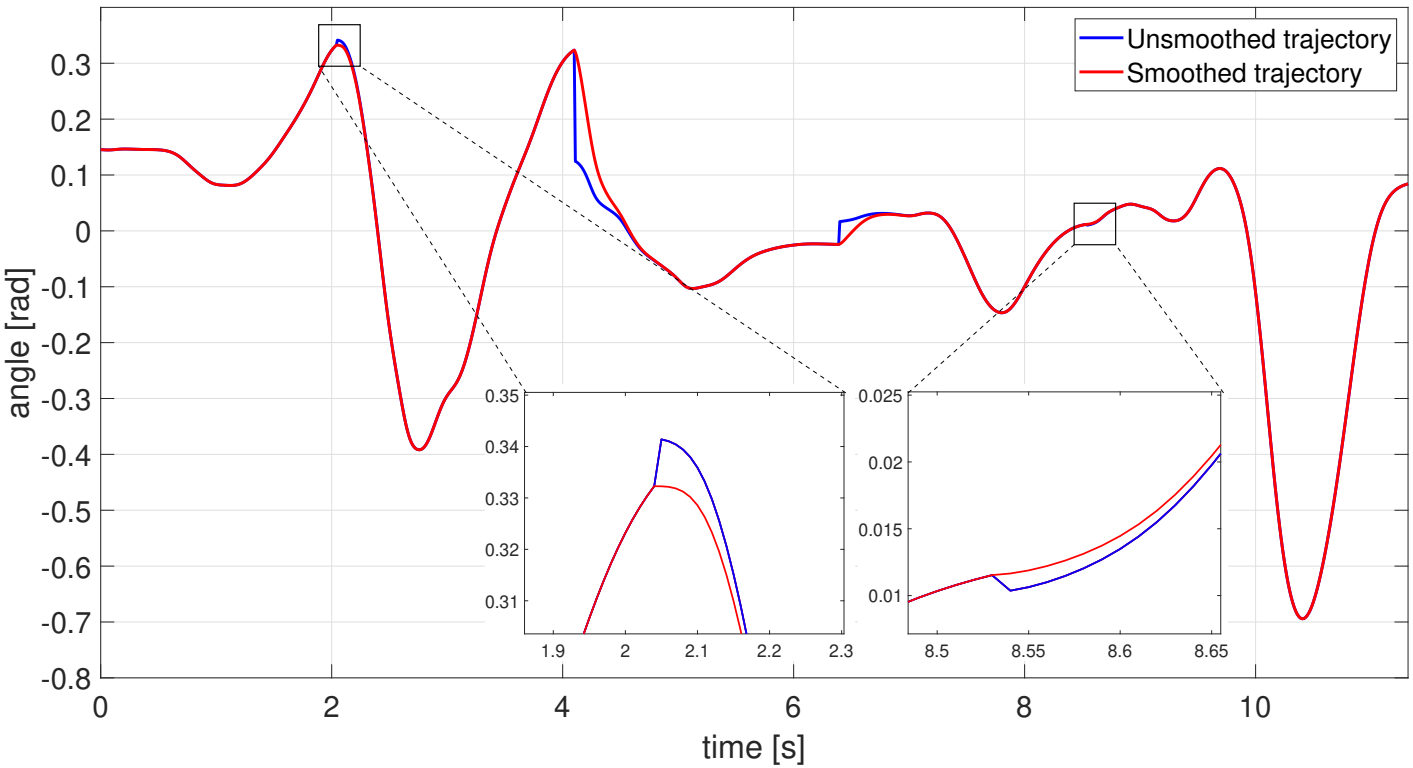


Figure: Regeneration and concatenation of several movement primitives

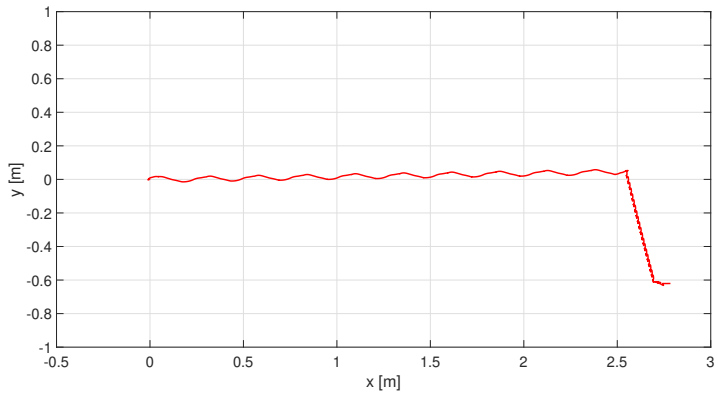
Robot execution

Animation and simulation results

- Model: Walk2014
- A scenario: 10 straight walking steps, 20 sideward walking steps and a kick

Robot execution

Animation and simulation results



CoM trajectory (top view)

Primitive improvement

Reinforcement learning

- Reinforcement learning: at time step t
  - Environment state: $s_t \in \mathcal{S}$
  - Action: $a_t \in \mathcal{A}$
  - Policy: $\pi(s_t)$
  - Reward: $R_{t+1} \in \mathcal{R}$ or R
- Policy learning by Weighting Exploration with the Returns (PoWER)[8]

[8] Jens Kober and Jan Peters. "Learning motor primitives for robotics". In: *Robotics and Automation, 2009. ICRA'09. IEEE International Conference on*. IEEE. 2009, pp. 2112–2118.

Primitive improvement

Reward functions

- Stability: $R_{stability} = \begin{cases} 0 & \text{if stable} \\ -10000 & \text{otherwise} \end{cases}$
- Joint angles: $R_{joint} = \begin{cases} -10000 & \text{if beyond limits} \\ 0 & \text{otherwise} \end{cases}$
- Speed: $R_{sideward} = k_{sideward} \cdot d_{sideward}$
- Elimination of undesired movements:

$$R_{x_{offset}} = \begin{cases} 0 & \text{if } |x_f - x_0| \leqslant 0.01 \\ -k_{x_{offset}} \cdot |x_f - x_0| & \text{otherwise} \end{cases}$$

$$R_{y_{offset}} = \begin{cases} 0 & \text{if } |y_f - y_0| \leqslant 0.01 \\ -k_{y_{offset}} \cdot |y_f - y_0| & \text{otherwise} \end{cases}$$

Primitive improvement

PoWER

- At k^{th} rollout, perform a single rollout with stochastic exploration on weights: $w'_k = w_k + \phi$, $\phi \sim \mathcal{N}\left(\mu, \sigma^2\right)$
- Evaluate this rollout with reward functions
- Store best rollouts in an importance sampler with capacity M
- After k^{th} rollout, update the weights: $w_{k+1} = w_k + \frac{\sum_{i=1}^{M} \phi_i \cdot R_i}{\sum_{i=1}^{M} R_i}$
- Until $w_{k+1} \approx w_k$

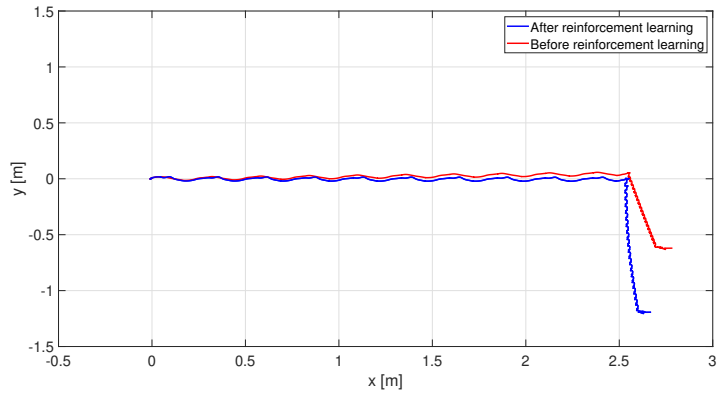
Primitive improvement

Animation and simulation results

- The same scenario: 10 straight walking steps, 20 sideward walking step and a kick

Primitive improvement

Animation and simulation results



CoM trajectories (top view)

Conclusion and future work

Conclusion and future work

- Best biped walking model: Walk2014
- The walking model can be partially replaced by primitives

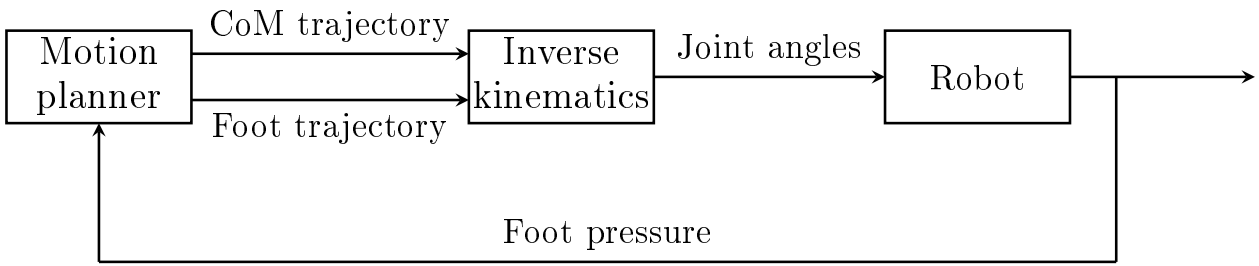


Figure: Process of gait generation for Walk2014

The End

Thank you very much for your attention!

Simulation data of friction coefficient

Table: Walking distance in 10 s of a simple gait with different friction coefficients

	$\mu_k = 1^*$	$\mu_k = 10$	$\mu_k = 100$	$\mu_k = 1000$	$\mu_k = 10000$
$\mu_s = 1^{**}$	0.250***	0.465	0.522	0.530	0.530
$\mu_s = 10$	0.448	0.975	1.088	1.090	1.090
$\mu_s = 100$	0.468	1.297	1.299	1.299	1.299
$\mu_s = 1000$	0.473	1.326	1.330	1.331	1.331
$\mu_s = 10000$	0.394	1.330	1.330	1.331	1.331

*The kinetic friction coefficient μ_k is dimensionless.

**The static friction coefficient μ_s is dimensionless.

***The measured data in this table are length values with unit [m].

Simulation data of friction coefficient

Table: Elapse time of a simple gait with different friction coefficients

	$\mu_k = 1^{*}$	$\mu_k = 10$	$\mu_k = 100$	$\mu_k = 1000$	$\mu_k = 10000$
$\mu_s = 1^{**}$	23.39***	24.80	26.59	35.36	51.10
$\mu_s = 10$	24.20	23.31	26.19	27.73	30.67
$\mu_s = 100$	24.76	25.59	25.03	26.14	27.55
$\mu_s = 1000$	26.60	27.01	26.28	26.98	28.15
$\mu_s = 10000$	29.93	29.09	27.77	28.94	29.27

*The kinetic friction coefficient μ_k is dimensionless.

**The static friction coefficient μ_s is dimensionless.

***The measured data in this table are time values with unit [s].

Explanation of coordinate system of 3D LIPM

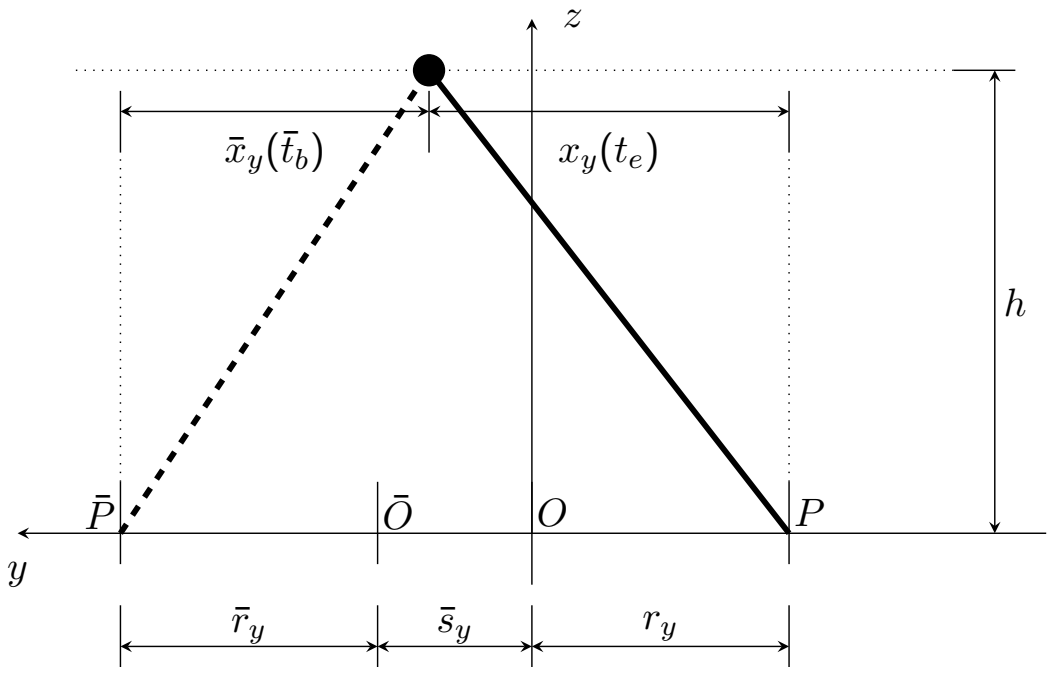


Figure: Two facing inverted pendulum on the $y - z$ plane

$$\bar{r}_y + \bar{s}_y + r_y = \bar{x}_y(\bar{t}_b) + x_y(t_e)$$
$$\dot{x}_y(t_e) = \dot{\bar{x}}_y(\bar{t}_b)$$

Forward and sideward trajectories of three gaits

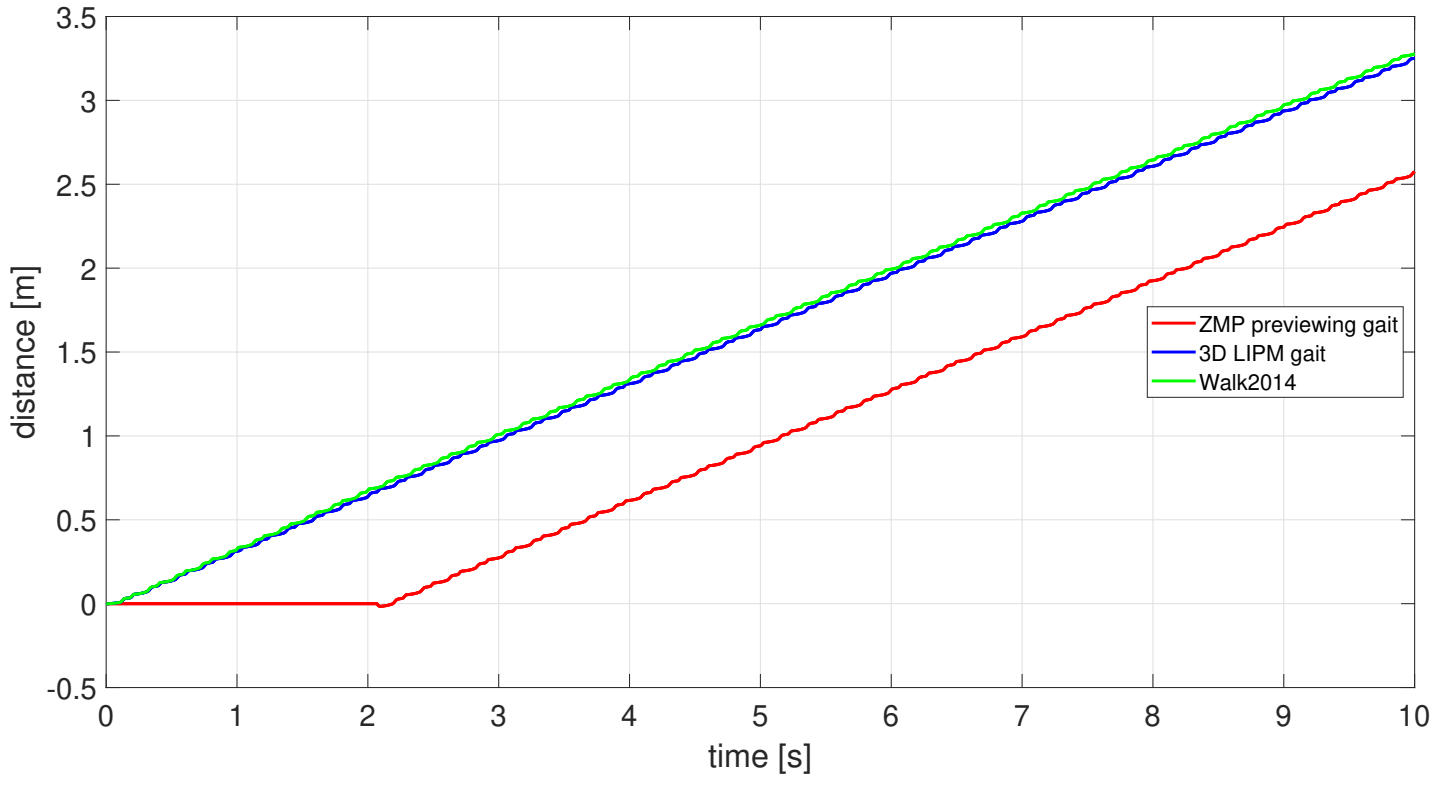


Figure: CoM trajectory of three gaits during forward walking

Forward and sideward trajectories of three gaits

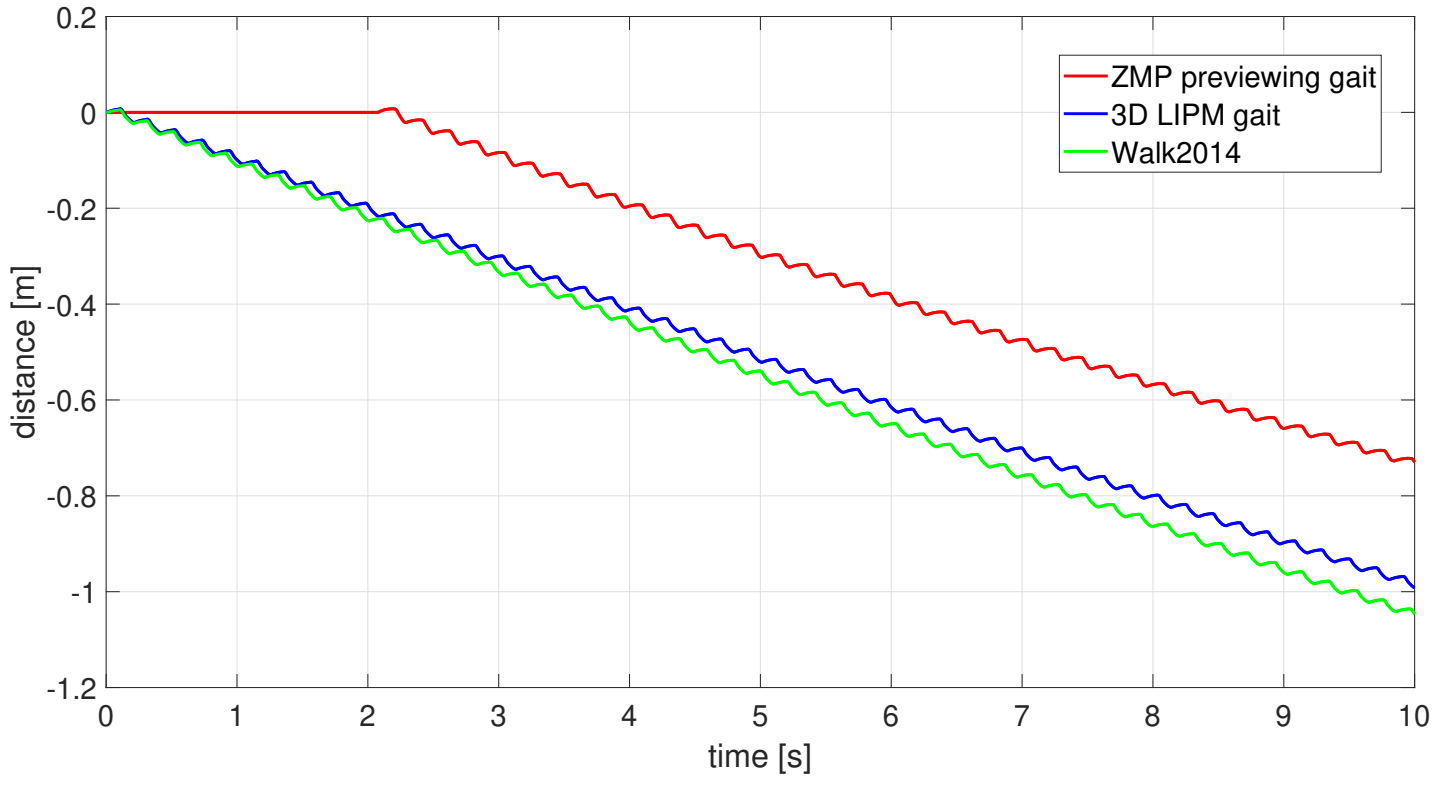
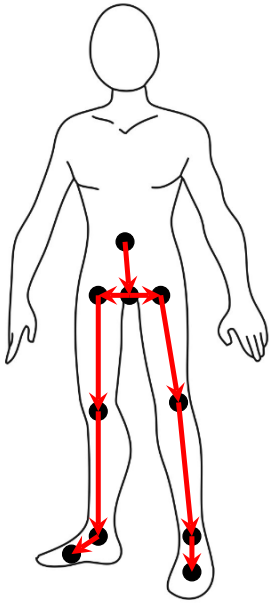


Figure: CoM trajectory of three gaits during sideward walking

Mapping from 3D points to joint angles

- Direction vectors:



$$\boldsymbol{v}_{knee2ankle} = \begin{bmatrix} x_{ankle} - x_{knee} \\ y_{ankle} - y_{knee} \\ z_{ankle} - z_{knee} \end{bmatrix}$$

- Joint angle calculation: Euclidean dot product formula

$$\cos\theta = \frac{\boldsymbol{v}_1 \cdot \boldsymbol{v}_2}{\|\boldsymbol{v}_1\|\|\boldsymbol{v}_2\|}$$

Imitation learning

Calculating the weights

- A given curve: $\begin{bmatrix} \boldsymbol{y}_D & \dot{\boldsymbol{y}}_D & \ddot{\boldsymbol{y}}_D \end{bmatrix}^T$
- In finite time horizon: $[1, 2, \cdots, T]$
- A goal forcing term:

$$\boldsymbol{f}_G = \ddot{\boldsymbol{y}}_D - \alpha_y \left(\beta_y (y_{D_T} - \boldsymbol{y}_D) - \dot{\boldsymbol{y}}_D\right)$$

Imitation learning

Calculating the weights

- A given curve: $\begin{bmatrix} \boldsymbol{y}_D & \dot{\boldsymbol{y}}_D & \ddot{\boldsymbol{y}}_D \end{bmatrix}^T$
- In finite time horizon: $[1, 2, \cdots, T]$
- A goal forcing term: $\boldsymbol{f}_G$
- Locally Weighted Regression (LWR):

$$w_i = \frac{\boldsymbol{s}^T \Gamma_i \boldsymbol{f}_G}{\boldsymbol{s}^T \Gamma_i \boldsymbol{s}}$$

Imitation learning

Calculating the weights

- A given curve: $\begin{bmatrix} \boldsymbol{y}_D & \dot{\boldsymbol{y}}_D & \ddot{\boldsymbol{y}}_D \end{bmatrix}^T$
- In finite time horizon: $[1, 2, \cdots, T]$
- A goal forcing term: $\boldsymbol{f}_G$
- Locally Weighted Regression (LWR): w_i
- The cost function for each Gaussian function Ψ_i

$$J_i = \sum_{j=1}^{T} \Psi_i(j) \left[f_G(j) - w_i \left(y_{D_T} - y_D(j) \right) x(j) \right]^2$$

Imitation learning

Calculating the weights

- A given curve: $\begin{bmatrix} \boldsymbol{y}_D & \dot{\boldsymbol{y}}_D & \ddot{\boldsymbol{y}}_D \end{bmatrix}^T$
- In finite time horizon: $[1, 2, \cdots, T]$
- A goal forcing term: $\boldsymbol{f}_G$
- Locally Weighted Regression (LWR): $w_i = \frac{\boldsymbol{s}^T \Gamma_i \boldsymbol{f}_G}{\boldsymbol{s}^T \Gamma_i \boldsymbol{s}}$

$$\boldsymbol{s} = \begin{bmatrix} x_1 \left(y_{D_T} - y_{D_1}\right) \\ \vdots \\ x_T \left(y_{D_T} - y_{D_1}\right) \end{bmatrix}, \Gamma_i = \begin{bmatrix} \Psi_{i_1} & & 0 \\ & \ddots & \\ 0 & & \Psi_{i_T} \end{bmatrix}, \boldsymbol{f}_G = \begin{bmatrix} f_{G_1} \\ \vdots \\ f_{G_T} \end{bmatrix}$$

Imitation learning

Calculating the forcing term

- The calculated weights:

$$w_i = \frac{\boldsymbol{s}^T \Gamma_i \boldsymbol{f}_G}{\boldsymbol{s}^T \Gamma_i \boldsymbol{s}}$$

- The forcing term:

$$f(x) = \frac{\sum_{i=1}^{N_{bf}} w_i \Psi_i(x)}{\sum_{i=1}^{N_{bf}} \Psi_i(x)} x(y_g - y_0)$$

Ψ_i: Gaussian function $\Psi_i(x) = e^{-h_i(x-c_i)^2}$

w_i: weight of Gaussian function Ψ_i

N_{bf}: number of Gaussian functions

x: the canonical system

$(y_g - y_0)$: a spatial scaling term

Imitation learning

Regenerating a given curve

- A curve to be regenerated: $\begin{bmatrix} \boldsymbol{y}_R & \dot{\boldsymbol{y}}_R & \ddot{\boldsymbol{y}}_R \end{bmatrix}^T$
- Regeneration of a given curve: explicit Euler method with step size Δt

Algorithm 2 Regenerating a given curve

Initializing: $y_{R_1} = y_{D_1}$, $\dot{y}_{R_1} = 0$, $\ddot{y}_{R_1} = 0$
for $j = 2$ to T **do**
  $\ddot{y}_{R_{j-1}} = \frac{1}{\tau^2}\left(\alpha_y\left(\beta_y(y_{D_T} - y_{R_{j-1}}) - \tau\dot{y}_{R_{j-1}}\right) - f(x_{j-1})\right)$
  $\dot{y}_{R_j} = \dot{y}_{R_{j-1}} + \ddot{y}_{R_{j-1}} \cdot \Delta t$
  $y_{R_j} = y_{R_{j-1}} + \dot{y}_{R_{j-1}} \cdot \Delta t$
end for

Temporal scaling and stability analysis

- Temporal scaling factor: τ
  - Faster execution: $0 < \tau < 1$
  - Slower execution: $\tau > 1$
- Instability comes from the explicit Euler method:

$$\ddot{y}_{j+1} = \frac{1}{\tau^2}\alpha_y\left(\beta_y(y_g - y_j) - \tau\dot{y}_j\right)$$
$$\dot{y}_{j+1} = \dot{y}_j + \ddot{y}_j \cdot \Delta t$$
$$y_{j+1} = y_j + \dot{y}_j \cdot \Delta t$$

$$\begin{bmatrix} y_{j+1} \\ \dot{y}_{j+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ -\frac{\alpha_y \cdot \beta_y \cdot \Delta t}{\tau^2} & 1 - \frac{\alpha_y \cdot \Delta t}{\tau} \end{bmatrix} \begin{bmatrix} y_j \\ \dot{y}_j \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\alpha_y \cdot \beta_y \cdot \Delta t}{\tau^2} \cdot y_g \end{bmatrix}$$

- Theoretical bound:

$$\frac{\alpha_y \cdot \Delta t}{4} < \tau$$