Homework 1 Report

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We use Dynare 4.6.2 for this homework. Dynare 4.6 has changed the usage of some functions, for example, *stoch_simul*, and thus the code may not work for a lower version.

```
addpath('/Applications/Dynare/4.6.2/matlab'); % add dynare path
```

Exercise 4.10

Before starting Q1, we first replicate the right panel of table 4.2 in the textbook to ensure our model, variable definitions, and solution method are consistent with the textbook.

```
dynare Q410.mod;
print_table()    % exactly replicate table 4.2 using textbook calibration
```

```
Results of table 4.2:

Variable  sig_x       rho1       rho2
   y       3.08       0.62       1.00
   c       2.71       0.78       0.84
   i       9.04       0.07       0.67
   h       2.12       0.62       1.00
tb_y       1.78       0.51      -0.04
ca_y       1.45       0.32       0.05
```

Q1. Calibrate the EDEIR Model for Canada 1960-2011

```
%{
NOTE:
    1. moments to match: [std(y),autocor(y),std(i),std(tb/y)]
    2. target moment values: [3.71%, 0.86, 10.31%, 1.72%]
    3. pars to calibrate: rho, eta, phi, psi_1
    4. method: min distance
    5. solver: fminunc/BFGS Quasi-Newton
    6. init guess: param = [0.42 0.0129 0.028 0.000742]
%}

param_init = [0.42 0.0129 0.028 0.000742];
param_est  = fminunc(@(x)m_dist(x), param_init);
```

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```
fprintf('Estimation results:\n');
fprintf('\n');
fprintf('rho   = %.6f\n', param_est(1));
fprintf('eta   = %.6f\n', param_est(2));
fprintf('phi   = %.6f\n', param_est(3));
fprintf('psi_1 = %.6f\n', param_est(4));
```

```
Estimation results:

rho   = 0.621901
eta   = 0.010536
phi   = 0.018179
psi_1 = 0.026189
```

Thus, we get a higher ρ and ψ_1 but a lower η and ϕ.

Q2. Compute the Theoretical Second Moments

We employ the estimated parameters to re-solve the model. Dynare automatically reports the theoretical moments for 1st order solutions.

```
% Q2 Compute theoretical second moments

set_param_value('rho',  param_est(1));
set_param_value('eta',  param_est(2));
set_param_value('phi',  param_est(3));
set_param_value('psi_1',param_est(4));

[info,oo_,options_,M_] = stoch_simul(M_,options_,oo_,var_list_);

print_table()
```

```
Results of table 4.2:

Variable  sig_x       rho1       rho2
   y       3.71       0.84       1.00
   c       3.18       0.89       0.98
   i      10.31       0.16       0.64
   h       2.55       0.84       1.00
tb_y       1.73       0.04      -0.12
ca_y       1.66       0.04      -0.07
```

Q3. Comment

- The model does a good job in generating the observed standard deviation of output y, investment i, and trade-balance-to-output ratio $\frac{tb}{y}$.
  But this is not surprising because that is how we calibrate the model using the SMM.
- The model does a poor job in explaining the autocorrelation of i and $\frac{tb}{y}$, which are much less correlated than in the data.
- The model also does a poor job in explaining the correlation between y and $\frac{tb}{y}$. Its sign is the opposite of the one in the data.

Q4. TFP Shock and Output Volatility

```
% Q4 compute std(ln A)
sd_list  = sqrt(diag(oo_.var));
sd_A     = sd_list(strcmp('A',M_.endo_names))*100;
sd_y     = sd_list(strcmp('y',M_.endo_names))*100;
sd_A_old = 100*sqrt(0.0129^2/(1-0.42^2));

fprintf('Unconditional SD(ln(A)) = %.4f\n', sd_A);
fprintf('Old value:                 %.4f\n', sd_A_old);

fprintf('Unconditional SD(y)     = %.4f\n', sd_y);
fprintf('Old value:                 %.4f\n', 3.08);
```

```
Unconditional SD(ln(A)) = 1.3455
Old value:                1.4214
Unconditional SD(y)     = 3.7130
Old value:                3.0800
```

From the results above we can see that

- the volatility of TFP shock has actually **decreased**;
- On the other hand, the volatility of output has **increased**.

This means the business cycle relies less on the TFP shock. However, its internal amplification and propagation mechanisms have become stronger than in the past. The latter effect outweighs the previous effect, and as a result, the overall output volatility still increases.

1 Exercise 5.2

Q1. Equilibrium Conditions

There are 21 variables: $C_t, h_t, A_t, I_t, Z_t, X_t, D_t, R_t, g_t, \widetilde{D}_t, A_t^T, A_t^N, Y_t^T, Y_t^N, K_t^T, K_t^N, h_t^T, h_t^N, I_t^T, I_t^N, \lambda_t$. Accordingly, there are 21 equations for the equilibrium:

$$A_t = C_t + I_t \tag{1}$$

$$A_t = \left[\eta \left(A_t^T\right)^{1-\frac{1}{\mu}} + (1-\eta)\left(A_t^N\right)^{1-\frac{1}{\mu}}\right]^{1/(1-\frac{1}{\mu})} \tag{2}$$

$$Y_t^T = Z_t \left(K_t^T\right)^{1-\alpha_T} \left(X_t h_t^T\right)^{\alpha_T} \tag{3}$$

$$Y_t^N = Z_t \left(K_t^N\right)^{1-\alpha_N} \left(X_t h_t^N\right)^{\alpha_N} \tag{4}$$

$$K_{t+1}^T = (1-\delta)K_t^T + I_t^T - \frac{\phi}{2}\left(\frac{K_{t+1}^T}{K_t^T} - g\right)^2 K_t^T \tag{5}$$

$$K_{t+1}^N = (1-\delta)K_t^N + I_t^N - \frac{\phi}{2}\left(\frac{K_{t+1}^N}{K_t^N} - g\right)^2 K_t^N \tag{6}$$

$$I_t = I_t^T + I_t^N \tag{7}$$

$$Y_t^N = A_t^N \tag{8}$$

$$h_t = h_t^T + h_t^N \tag{9}$$

$$\frac{D_{t+1}}{R_t} = D_t + A_t^T - Y_t^T \tag{10}$$

$$R_t = R^* + \psi\left[e^{\widetilde{D}_{t+1}/X_t - \bar{d}} - 1\right] \tag{11}$$

$$\widetilde{D}_t = D_t \tag{12}$$

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \sigma_Z \varepsilon_t^Z \tag{13}$$

$$\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \sigma_g \varepsilon_t^g \tag{14}$$

$$g_t = \frac{X_t}{X_{t-1}} \tag{15}$$

$$\lambda_t = \beta^t \left[C_t^\gamma (1-h_t)^{1-\gamma}\right]^{-\sigma} (1-h_t)^{1-\gamma} \gamma C_t^{\gamma-1} \tag{16}$$

$$\beta^t \left[C_t^\gamma (1-h_t)^{1-\gamma}\right]^{-\sigma} (1-h_t)^{-\gamma} (1-\gamma) C_t^\gamma = \lambda_t A_t^{\frac{1}{\mu}} \eta \left(A_t^T\right)^{-\frac{1}{\mu}} \alpha_T Z_t X_t \left(\frac{K_t^T}{X_t h_t^T}\right)^{1-\alpha_T} \tag{17}$$

$$\beta^t \left[C_t^\gamma (1-h_t)^{1-\gamma}\right]^{-\sigma} (1-h_t)^{-\gamma} (1-\gamma) C_t^\gamma = \lambda_t A_t^{\frac{1}{\mu}} (1-\eta) \left(A_t^N\right)^{-\frac{1}{\mu}} \alpha_N Z_t X_t \left(\frac{K_t^N}{X_t h_t^N}\right)^{1-\alpha_N} \tag{18}$$

$$\lambda_t A_t^{\frac{1}{\mu}} \eta \left(A_t^T\right)^{-1/\mu} \frac{1}{R_t} = E_t \lambda_{t+1} A_{t+1}^{\frac{1}{\mu}} \eta \left(A_{t+1}^T\right)^{-1/\mu} \tag{19}$$

$$\lambda_t \left(1 + \phi\left(\frac{K_{t+1}^T}{K_t^T} - g\right)\right) = E_t \lambda_{t+1} \left\{ A_{t+1}^{\frac{1}{\mu}} \eta \left(A_{t+1}^T\right)^{-\frac{1}{\mu}} (1-\alpha_T) \frac{Y_{t+1}^T}{K_{t+1}^T} + 1 - \delta + \phi\left(\frac{K_{t+2}^T}{K_{t+1}^T} - g\right)\frac{K_{t+2}^T}{K_{t+1}^T} - \frac{\phi}{2}\left(\frac{K_{t+2}^T}{K_{t+1}^T} - g\right)^2 \right\} \tag{20}$$

$$\lambda_t \left(1 + \phi\left(\frac{K_{t+1}^N}{K_t^N} - g\right)\right) = E_t \lambda_{t+1} \Bigg\{ A_{t+1}^{\frac{1}{\mu}} (1-\eta) \left(A_{t+1}^N\right)^{-\frac{1}{\mu}} (1-\alpha_N) \frac{Y_{t+1}^N}{K_{t+1}^N} + 1 - \delta + \phi\left(\frac{K_{t+2}^N}{K_{t+1}^N} - g\right)\frac{K_{t+2}^N}{K_{t+1}^N} - \frac{\phi}{2}\left(\frac{K_{t+2}^N}{K_{t+1}^N} - g\right)^2 \tag{21}$$

Q2. Prices

From the FOCs, it is easy to derive

$$p_t^N = \frac{1-\eta}{\eta}\left(\frac{A_t^N}{A_t^T}\right)^{-\frac{1}{\mu}},$$
$$p_t = \left[\eta\left(A_t^T\right)^{1-\frac{1}{\mu}} + (1-\eta)\left(A_t^N\right)^{1-\frac{1}{\mu}}\right]^{\frac{1}{1-\mu}} \eta^{-1}\left(A_t^T\right)^{1/\mu}.$$

Both of them are stationary variables. This is because their values only depend on the ratio, $\frac{A_t^N}{A_t^T}$, which is a constant along the BGP.

Q3. Stationary Form of the Equilibrium Conditions

Define stationary variables as follows:

$$\hat{c}_t \equiv \frac{C_t}{X_{t-1}}, \hat{a}_t \equiv \frac{A_t}{X_{t-1}}, \hat{\imath}_t \equiv \frac{I_t}{X_{t-1}}, \hat{d}_t \equiv \frac{D_t}{X_{t-1}}, \hat{r}_t \equiv \frac{R_t}{X_{t-1}}, \hat{a}_t^T \equiv \frac{A_t^T}{X_{t-1}}, \hat{a}_t^N \equiv \frac{A_t^N}{X_{t-1}}, \widehat{y}_t^T \equiv \frac{Y_t^T}{X_{t-1}},$$
$$\hat{y}_t^N \equiv \frac{Y_t^N}{X_{t-1}}, \hat{k}_t^T \equiv \frac{K_t^T}{X_{t-1}}, \hat{k}_t^N \equiv \frac{K_t^N}{X_{t-1}}, \hat{\imath}_t^T \equiv \frac{I_t^T}{X_{t-1}}, \hat{l}_t^N \equiv \frac{I_t^N}{X_{t-1}}, \hat{\lambda}_t \equiv X_{t-1}^{1+(\sigma-1)\gamma}\lambda_t.$$

Then, the stationary form equilibrium conditions can be written as follows:

$$\hat{a}_t = \hat{c}_t + \hat{\imath}_t \tag{22}$$

$$\hat{a}_t = \left[\eta\left(\hat{a}_t^T\right)^{1-1/\mu} + (1-\eta)\left(\hat{a}_t^N\right)^{1-1/\mu}\right]^{1/(1-1/\mu)} \tag{23}$$

$$\hat{y}_t^T = Z_t\left(\hat{k}_t^T\right)^{1-\alpha_T}\left(g_t h_t^T\right)^{\alpha_T} \tag{24}$$

$$\hat{y}_t^N = Z_t\left(\hat{k}_t^N\right)^{1-\alpha_N}\left(g_t h_t^N\right)^{\alpha_N} \tag{25}$$

$$\hat{k}_{t+1}^T = \frac{(1-\delta)\hat{k}_t^T + \hat{\imath}_t^T - \frac{\phi}{2}\left(\frac{\hat{k}_{t+1}^T}{\hat{k}_t^T}g_t - g\right)^2\hat{k}_t^T}{g_t} \tag{26}$$

$$\hat{k}_{t+1}^N = \frac{(1-\delta)\hat{k}_t^N + \hat{\imath}_t^N - \frac{\phi}{2}\left(\frac{\hat{k}_{t+1}^N}{\hat{k}_t^N}g_t - g\right)^2\hat{k}_t^N}{g_t} \tag{27}$$

$$\hat{\imath}_t = \hat{\imath}_t^T + \hat{\imath}_t^N \tag{28}$$

$$\hat{y}_t^N = \hat{a}_t^N \tag{29}$$

$$h_t = h_t^T + h_t^N \tag{30}$$

$$\frac{\hat{d}_{t+1}g_t}{R_t} = \hat{d}_t + \hat{a}_t^T - \hat{y}_t^T \tag{31}$$

$$R_t = R^* + \psi\left[e^{\hat{d}_{t+1}-\bar{d}} - 1\right] \tag{32}$$

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \sigma_Z \varepsilon_t^Z \tag{33}$$

$$\ln\left(g_t/g\right) = \rho_g \ln\left(g_{t-1}/g\right) + \sigma_g \varepsilon_t^g \tag{34}$$

$$g_t = \frac{X_t}{X_{t-1}} \tag{35}$$

$$(1-\gamma)\hat{c}_t = \gamma\left(1-h_t\right)\hat{a}_t^{1/\mu}\eta\left(\hat{a}_t^T\right)^{-1/\mu}\alpha_T Z_t g_t\left(\frac{\hat{k}_t^T}{g_t n_t^T}\right)^{1-\alpha_T} \tag{36}$$

$$(1-\gamma)\hat{c}_t = \gamma\left(1-h_t\right)\hat{a}_t^{1/\mu}(1-\eta)\left(\hat{a}_t^N\right)^{-1/\mu}\alpha_N Z_t g_t\left(\frac{\hat{k}_t^N}{g_t h_t^N}\right)^{1-\alpha_N} \tag{37}$$

$$\left(1-h_t\right)^{(1-\gamma)(1-\sigma)}\gamma\hat{c}_t^{\gamma-1-\sigma\gamma}\hat{a}_t^{1/\mu}\eta\left(\hat{a}_t^T\right)^{-1/\mu} = E_t\beta R_t g_t^{-\sigma\gamma}\left(1-h_{t+1}\right)^{(1-\gamma)(1-\sigma)}\gamma\hat{c}_{t+1}^{\gamma-1-\sigma\gamma}g_t^{\gamma-1}\hat{a}_{t+1}^{1/\mu}\eta\left(\hat{a}_{t+1}^T\right)^{-1/\mu} \tag{38}$$

$$\left[\hat{c}_t^\gamma\left(1-h_t\right)^{1-\gamma}\right]^{-\sigma}\left(1-h_t\right)^{1-\gamma}\gamma\hat{c}_t^{\gamma-1}\left(1+\phi\left(\frac{\hat{k}_{t+1}^T}{\hat{k}_t^T}g_t-g\right)\right) = E_t\beta\left[\hat{c}_{t+1}^\gamma g_t^\gamma\left(1-h_{t+1}\right)^{1-\gamma}\right]^{-\sigma}\left(1-h_{t+1}\right)^{1-\gamma}\gamma\hat{c}_{t+1}^{\gamma-1}g_t^{\gamma-1}$$
$$\left\{\eta\left(\frac{\hat{a}_{t+1}}{\hat{a}_{t+1}^T}\right)^{\frac{1}{\mu}}\left(1-\alpha_T\right)\frac{\hat{a}_{t+1}^T}{\hat{k}_{t+1}^T}+1-\delta+\phi\left(\frac{\hat{k}_{t+2}^T}{\hat{k}_{t+1}^T}g_{t+1}-g\right)\frac{\hat{k}_{t+2}^T}{\hat{k}_{t+1}^T}g_{t+1}-\frac{\phi}{2}\left(\frac{\hat{k}_{t+2}^T}{\hat{k}_{t+1}^T}g_{t+1}-g\right)^2\right\} \tag{39}$$

$$\left[\hat{c}_t^\gamma\left(1-h_t\right)^{1-\gamma}\right]^{-\sigma}\left(1-h_t\right)^{1-\gamma}\gamma\hat{c}_t^{\gamma-1}\left(1+\phi\left(\frac{\hat{k}_{t+1}^N}{\hat{k}_t^N}g_t-g\right)\right) = E_t\beta\left[\hat{c}_{t+1}^\gamma g_t^\gamma\left(1-h_{t+1}\right)^{1-\gamma}\right]^{-\sigma}\left(1-h_{t+1}\right)^{1-\gamma}\gamma\hat{c}_{t+1}^{\gamma-1}g_t^{\gamma-1}$$
$$\left\{(1-\eta)\left(\frac{\hat{a}_{t+1}}{\hat{a}_{t+1}^N}\right)^{\frac{1}{\mu}}\left(1-\alpha_N\right)\frac{\hat{a}_{t+1}^N}{\hat{k}_{t+1}^N}+1-\delta+\phi\left(\frac{\hat{k}_{t+2}^T}{\hat{k}_{t+1}^N}g_{t+1}-g\right)\frac{\hat{k}_{t+2}^T}{\hat{k}_{t+1}^N}g_{t+1}-\frac{\phi}{2}\left(\frac{\hat{k}_{t+2}^N}{\hat{k}_{t+1}^N}g_{t+1}-g\right)^2\right\} \tag{40}$$

Q4. Steady State

```
% calibrate the model and saves the steady state
param.beta       = 0.98;
param.gamma      = 0.36;
param.d_bar      = 0.10;
param.psi        = 0.001;
param.alpha_T    = 0.40;
param.alpha_N    = 0.80;
param.delta      = 0.05;
param.g_bar      = 1.0066;
param.phi        = 1.37;
param.miu        = 0.44;
param.sigma      = 2;
% share paremeter of tradable goods; to be calibrated endogenously
param.yeta       = 0.5;
% solve for the steady state with eta = 0.5, an initial guess
% x_ss0 provides an initial guess for the steady state
x0    = [0,0,0.1,1,1,1,1,1,1,1,1,1,1,1,1,1,1];
x_ss0 = fsolve(@(x)ss_model(x,param), x0);
% find the eta
eeta = fsolve(@(x)calibrate_eta(x,param,x_ss0), param.yeta);
% update the parameters and the steady state vals
[~, param, x_ss] = calibrate_eta(eeta, param, x_ss0);
```

We find $\eta = 0.349$. Passing this η into Dynare, the steady state is reported in Dynare as follows:

```
dynare Ex502.mod
```

```
STEADY-STATE RESULTS:

c            0.224174
h            0.32947
a            0.295923
i            0.0717491
d            0.1
r            1.02958
a_T          0.296406
a_N          0.295665
y_T          0.298638
y_N          0.295665
k_T          0.78296
k_N          0.484693
i_T          0.0443155
i_N          0.0274336
h_T          0.0698872
h_N          0.259583
lambda       3.55303
z            1
g            1.0066
p            2.87583
p_N          1.87583
tb           0.00077606
```

Q5. The Law of Motion

Dynare also reports the transition function for all variables. The following output has been trimmed. Note that the model is solved without taking logs.

```
POLICY AND TRANSITION FUNCTIONS
                        z            d          k_T            k_N
Constant         1.000000     0.100000     0.782960       0.484693
z(-1)            0.950000     0.207817     0.193557       0.111752
d(-1)                   0     1.020164     0.010874       0.000190
k_T(-1)                 0    -0.390975     0.886212      -0.011684
k_N(-1)                 0     0.072941     0.130435       0.847934
ez               0.005300     0.001159     0.001080       0.000623
eg                      0     0.003594    -0.016149      -0.008560
```

Thus, for a state vector $[z, d, k^T, k^N]'$, we have

$$h_x = \begin{bmatrix} 0.95 & 0 & 0 & 0 \\ 0.20 & 1.02 & -0.39 & 0.07 \\ 0.19 & 0.01 & 0.88 & 0.13 \\ 0.11 & 0.00 & -0.01 & 0.84 \end{bmatrix},$$

and its eigenvalues are

$$\begin{bmatrix} 0.8883 + 0.0155i & 0.8883 - 0.0155i & 0.9776 & 0.9500 \end{bmatrix},$$

all of which falls within the unit circle. So the system is stable.

Q6. Statitics

Since ΔY_t, ΔC_t, and tby_t are all themselves stationary, Dynare can directly report their moments and correlations by carefully defining them in the mod file:

```
THEORETICAL MOMENTS
VARIABLE     STD. DEV.
delta_c        0.0173
delta_y        0.0159

MATRIX OF CORRELATIONS
Variables     delta_y
delta_c        0.9581
delta_y        1.0000
tby           -0.3035
```

Q7. Moments

For moments of other non-stationary variables, Dynare does not report them directly. I follow the procedure describe in the textbook to calculate them.

```
% retrieve the simulated sequence
y_tilde = data(strcmp('logY',M_.endo_names),:)
c_tilde = data(strcmp('logC',M_.endo_names),:)
i_tilde = data(strcmp('logI',M_.endo_names),:)
g       = data(strcmp('g'   ,M_.endo_names),:)

% add the trend back
logX = cumsum(log(g))
logC = c_tilde(2:end) + logX(1:end-1)
logY = y_tilde(2:end) + logX(1:end-1)
logI = i_tilde(2:end) + logX(1:end-1)

% use hpfilter to detrend
[~,c] = hpfilter(logC,1600)
[~,y] = hpfilter(logY,1600)
[~,i] = hpfilter(logI,1600)

burnin=1000;
c = c((burnin+1):end)
y = y((burnin+1):end)
i = i((burnin+1):end)
tby = tby((burnin+1):end)
```

```
% report selected moments in table 5.2
std(y)*100
std(c)/std(y)
std(i)/std(y)

corr(y(2:end),y(1:end-1))
corr(y,c)
corr(y,i)
```

```
2.1253
1.0677
2.6207
0.7535
0.9519
0.9625
```

Q8. One Sector Model

Let the tradable sector's share parameter $\eta \to 1$. Then the nontradable sector will become negligible, and the model will be same as the one in Section 5.2.
Since direct setting $\eta = 1$ will cause some numerical issues in Dynare, we instead remove all model equations relate to the nontradable sector. This yields the same model equations as the model in Sector 5.2.
Thus, what we do below is essentially replicate the results of Table 5.2.

	std(ΔC_t)	std(ΔY_t)	corr(ΔY_t, tby_t)
One Sector Model	1.78	1.42	-0.58
Two Sector Model	1.73	1.59	-0.30

Statistics	Data	Origin Model	One Sector	Two Sector
$\sigma(y)$	2.40	2.13	2.16	2.12
$\sigma(c)/\sigma(y)$	1.26	1.10	1.08	1.06
$\sigma(i)/\sigma(y)$	4.15	3.83	3.91	2.62
$\rho(y)$	0.83	0.82	0.82	0.75
$\rho(y, c)$	0.82	0.91	0.92	0.95
$\rho(y, i)$	0.91	0.80	0.78	0.96

Comparing the 3rd column and the 4th column, we can see that our procedure generates almost the same results.

Q9. Comments

The most obvious differences we can find from the above table are

- $\sigma(i)/\sigma(y)$ is much lower in the two sector model;
- corr(ΔY_t, tby_t) is much higher in the two sector model.

Here is my explanation:

- When the GDP growth rate is high, the economy usually needs more investment.
- For an open economy, it will borrow from the international market. This is why corr(ΔY_t, tby_t) is negative.
- The introduction of nontradable sector makes it harder for the economy to pay back its debt, since part of the output is nontradable.
- Thus, the economy has less incentive to borrow from the international market to finance investment.
- This logic leads to the two main differences listed above.

Exercise 6.5

Q1. The EDEIR Model

```
%---------------- EDEIR ------------------
% replicate column 4 of table 4.4
% make sure the results generated by the new model is consistent with the old results

dynare EDEIR
```

```
THEORETICAL MOMENTS
VARIABLE         MEAN  STD. DEV.   VARIANCE
y              0.3964     0.0308     0.0010
c              0.1106     0.0271     0.0007
i             -1.0795     0.0904     0.0082
h              0.0074     0.0212     0.0004
tb_y           0.0200     0.0178     0.0003
ca_y           0.0000     0.0145     0.0002

MATRIX OF CORRELATIONS
Variables          y       c       i       h    tb_y    ca_y
y             1.0000  0.8440  0.6688  1.0000 -0.0435  0.0503
c             0.8440  1.0000  0.5177  0.8440 -0.3114  0.0654
i             0.6688  0.5177  1.0000  0.6688 -0.6178 -0.7068
h             1.0000  0.8440  0.6688  1.0000 -0.0435  0.0503
tb_y         -0.0435 -0.3114 -0.6178 -0.0435  1.0000  0.8318
ca_y          0.0503  0.0654 -0.7068  0.0503  0.8318  1.0000

COEFFICIENTS OF AUTOCORRELATION
Order          1       2       3       4       5
y         0.6170  0.3603  0.2066  0.1201  0.0733
c         0.7822  0.6367  0.5493  0.4996  0.4721
i         0.0686 -0.1379 -0.1363 -0.0935 -0.0553
h         0.6170  0.3603  0.2066  0.1201  0.0733
tb_y      0.5086  0.3484  0.3028  0.2938  0.2945
ca_y      0.3220  0.0875  0.0130 -0.0067 -0.0096
Total computing time : 0h00m01s
```

We can see that the moments reported in Dynare are basically the same as in table 4.4.
Now we turn off the TFP shock and turn on the interest rate shock.

```
% turn off productivity shock and turn on interest rate shock
set_param_value('eta', 0);
set_param_value('sigma_mu', 0.012);
% re-run the model
options_.irf = 20;
[info,oo_,options_,M_]=stoch_simul(M_,options_,oo_,var_list_);
```

```
THEORETICAL MOMENTS
VARIABLE         MEAN  STD. DEV.   VARIANCE
y              0.3964     0.1560     0.0243
c              0.1106     0.1092     0.0119
i             -1.0795     0.9933     0.9866
h              0.0074     0.1072     0.0115
```

tb_y	0.0200	0.2417	0.0584
ca_y	0.0000	0.2377	0.0565

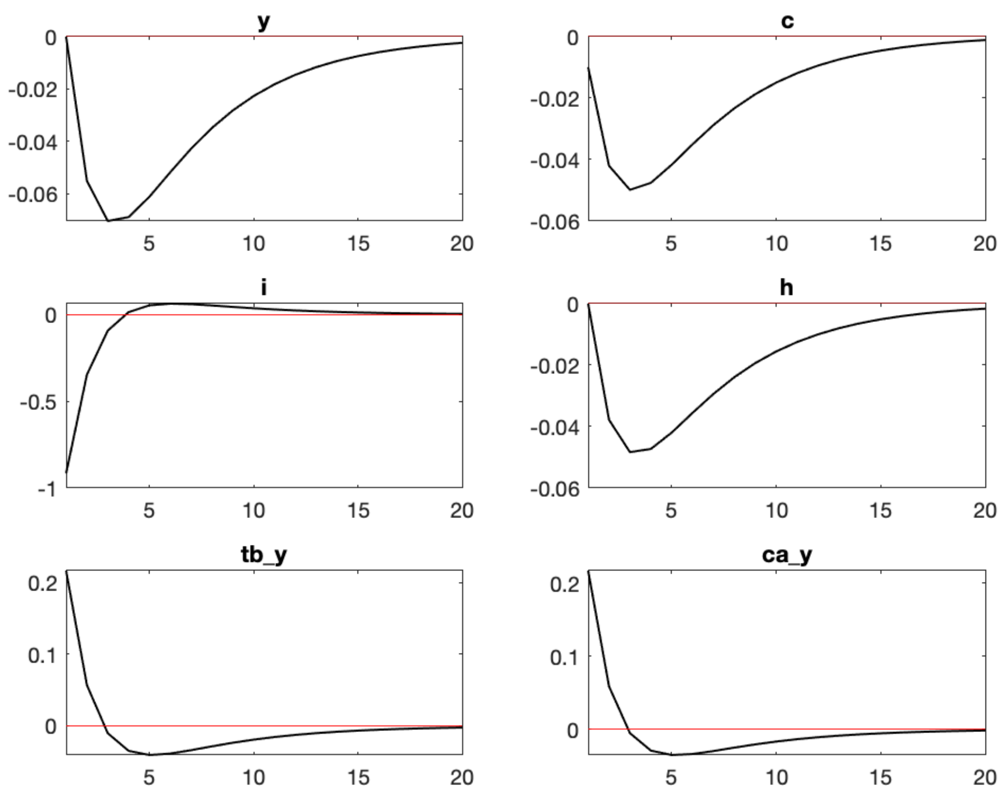
MATRIX OF CORRELATIONS

Variables	y	c	i	h	tb_y	ca_y
y	1.0000	0.9932	0.0635	1.0000	0.2357	0.1869
c	0.9932	1.0000	0.1636	0.9932	0.1351	0.0867
i	0.0635	0.1636	1.0000	0.0635	-0.9547	-0.9684
h	1.0000	0.9932	0.0635	1.0000	0.2357	0.1869
tb_y	0.2357	0.1351	-0.9547	0.2357	1.0000	0.9985
ca_y	0.1869	0.0867	-0.9684	0.1869	0.9985	1.0000

COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
y	0.9243	0.7986	0.6670	0.5468	0.4435
c	0.9411	0.8189	0.6852	0.5610	0.4535
i	0.3708	0.0914	-0.0249	-0.0668	-0.0759
h	0.9243	0.7986	0.6670	0.5468	0.4435
tb_y	0.3330	0.0432	-0.0720	-0.1085	-0.1111
ca_y	0.3197	0.0368	-0.0752	-0.1102	-0.1120

The impulse response functions are shown as follows:



Intuition for the IRFs

- A rise in interest rate makes borrowing more costly (note that the steady state debt level is positive), thus the economy consumes less and saves more to pay back the debt. Thus, c drops but $\frac{tb}{y}$ and $\frac{ca}{y}$ increase.

- Investment further drops because of the high interest rate.
- Due to GHH utility, there is no wealth effect in labor supply. Because $i \downarrow \rightarrow K \downarrow \rightarrow MPL = w \downarrow$, employment h also drops. Meanwhile, y drops.

Q2. The IDF Model

```
% replicate column 1 of table 4.4
% make sure the results generated by the new model is consistent with the old results

dynare IDF
```

```
THEORETICAL MOMENTS
VARIABLE         MEAN  STD. DEV.   VARIANCE
y              0.3964     0.0307     0.0009
c              0.1106     0.0235     0.0006
i             -1.0795     0.0910     0.0083
h              0.0074     0.0211     0.0004
tb_y           0.0200     0.0155     0.0002
ca_y           0.0000     0.0146     0.0002

MATRIX OF CORRELATIONS
Variables         y       c       i       h    tb_y    ca_y
y            1.0000  0.9376  0.6581  1.0000 -0.0122  0.0263
c            0.9376  1.0000  0.5581  0.9376 -0.0701  0.0623
i            0.6581  0.5581  1.0000  0.6581 -0.7025 -0.7301
h            1.0000  0.9376  0.6581  1.0000 -0.0122  0.0263
tb_y        -0.0122 -0.0701 -0.7025 -0.0122  1.0000  0.9609
ca_y         0.0263  0.0623 -0.7301  0.0263  0.9609  1.0000

COEFFICIENTS OF AUTOCORRELATION
Order          1       2       3       4       5
y         0.6122  0.3505  0.1925  0.1029  0.0539
c         0.6988  0.4959  0.3725  0.3010  0.2604
i         0.0700 -0.1405 -0.1415 -0.0995 -0.0612
h         0.6122  0.3505  0.1925  0.1029  0.0539
tb_y      0.3268  0.1113  0.0531  0.0440  0.0473
ca_y      0.3022  0.0657 -0.0061 -0.0228 -0.0232
Total computing time : 0h00m01s
Note: warning(s) encountered in MATLAB/Octave code
```

We can see that the moments reported in Dynare are basically the same as in table 4.4.
Now we turn off the TFP shock and turn on the interest rate shock.

```
% turn off productivity shock and turn on interest rate shock
set_param_value('sigma_tfp', 0);
set_param_value('sigma_mu',  0.012);

% re-run the model
options_.irf = 20;
```

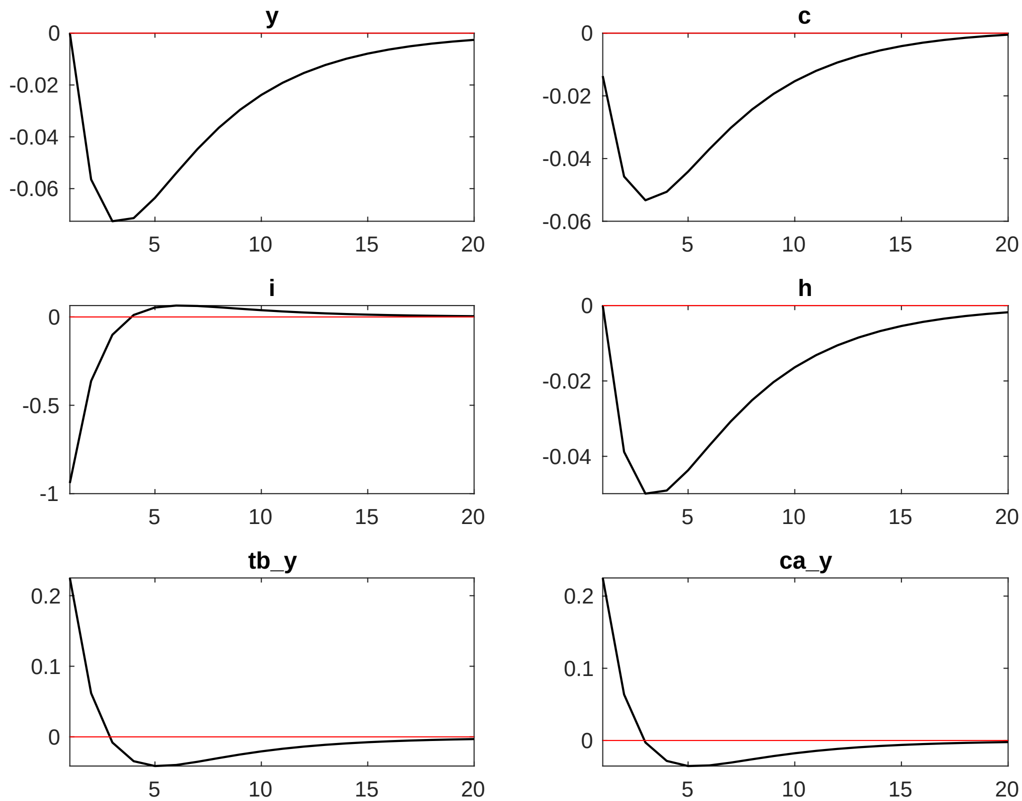
```
[info,oo_,options_,M_]=stoch_simul(M_,options_,oo_,var_list_);
```

```
THEORETICAL MOMENTS
VARIABLE         MEAN  STD. DEV.   VARIANCE
y              0.3964     0.1622     0.0263
c              0.1106     0.1162     0.0135
i             -1.0795     1.0224     1.0454
h              0.0074     0.1115     0.0124
tb_y           0.0200     0.2513     0.0632
ca_y           0.0000     0.2470     0.0610

MATRIX OF CORRELATIONS
Variables          y       c       i       h    tb_y    ca_y
y             1.0000  0.9884  0.0684  1.0000  0.2254  0.1724
c             0.9884  1.0000  0.2005  0.9884  0.0912  0.0388
i             0.0684  0.2005  1.0000  0.0684 -0.9562 -0.9709
h             1.0000  0.9884  0.0684  1.0000  0.2254  0.1724
tb_y          0.2254  0.0912 -0.9562  0.2254  1.0000  0.9983
ca_y          0.1724  0.0388 -0.9709  0.1724  0.9983  1.0000

COEFFICIENTS OF AUTOCORRELATION
Order          1       2       3       4       5
y         0.9259  0.8018  0.6712  0.5513  0.4481
c         0.9436  0.8205  0.6843  0.5570  0.4464
i         0.3781  0.0976 -0.0214 -0.0654 -0.0758
h         0.9259  0.8018  0.6712  0.5513  0.4481
tb_y      0.3422  0.0515 -0.0666 -0.1056 -0.1099
ca_y      0.3282  0.0438 -0.0712 -0.1086 -0.1119
```

The impulse response functions are shown as follows:



These IRFs are very similar to the ones in Q1. The intuitions are the same as in Q1.

Q3. Comparisons

In terms of IRFs, the two models generate almost exactly the same IRFs. As for statistic moments, we summarize earlier findings in a table below. The results are very similar.

	IDF	EDEIR
Volatilities		
std(y)	16.2	15.6
std(c)	11.6	10.9
std(i)	102.2	99.3
std(h)	11.2	10.7
std($\frac{tb}{y}$)	25.1	24.2
std($\frac{ca}{y}$)	24.7	23.8
Serial Correlations		
corr(y_t, y_{t-1})	0.93	0.92
corr(c_t, c_{t-1})	0.94	0.94
corr(i_t, i_{t-1})	0.38	0.37
corr(h_t, h_{t-1})	0.93	0.92
corr($\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}}$)	0.34	0.33
corr($\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}$)	0.33	0.32
Correlations with Output		
corr(c_t, y_t)	0.99	0.99
corr(i_t, y_t)	0.07	0.06
corr(h_t, y_t)	1.00	1.00
corr($\frac{tb_t}{y_t}, y_t$)	0.23	0.24
corr($\frac{ca_t}{y_t}, y_t$)	0.17	0.19