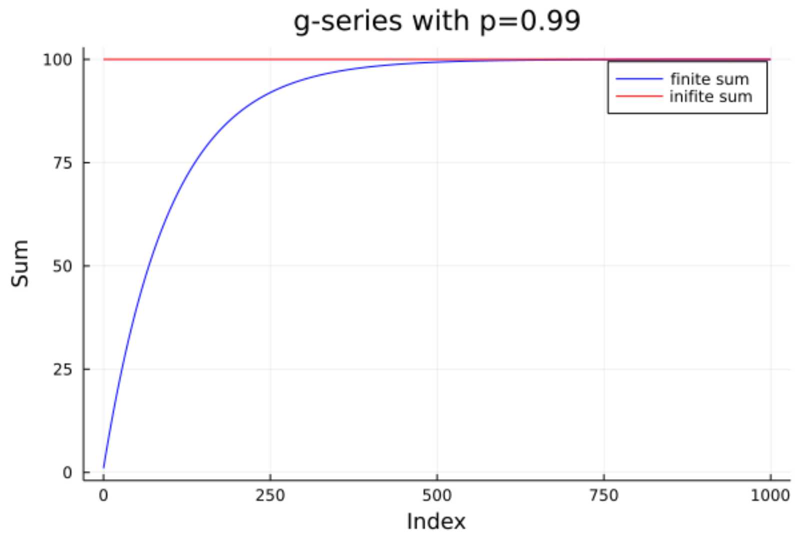
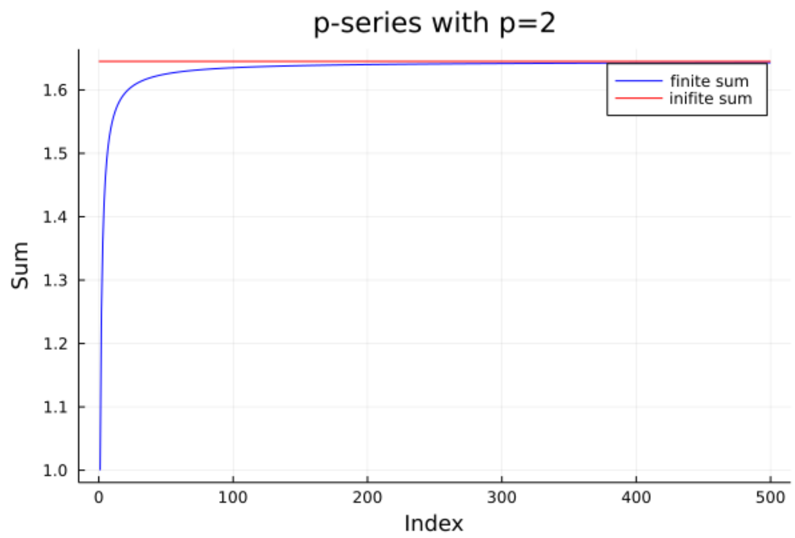
Convergence of Infinite Series

MAT2319 – Introduction to Julia

September 16, 2021

We'll look at two series that converge to a finite value when they are summed.





1. Create a new Julia script a new notebook (either in Pluto or in Jupyter), save it as `seriesConvergence.jl`.

2. Consider a geometric series

$$G = \sum_{k=0}^{\infty} p^k.$$

   We need to define the value of p and the values of k.

   (a) $p = 0.99$

   (b) k is a vector containing the integers from 0 to 1000, inclusive.

3. Calculate each term in the g-series (before summation)

4. Calculate the value of the infinite series. We know that

$$G = \frac{1}{1-p}.$$

5. Plot the value of the infinite series. Plot a horizontal red line that has x value 0 and maximum value in k (use `max`), and the y value is constant at G.

6. On the same plot, plot the value of the infinite series for all values of k. Plot the cumulative sum versus k. The cumulative sum of a vector is a vector of the same size, where the value of each element is equal to the sum of all elements to the left of its in the original vector (use `cumsum`). Use blue color when plotting.

7. Label the x and y axes and give the figure a title. Also create a legend and label the first line "*infinite sum*", and the second line "*finite sum*".

8. Run the script and note that the finite sum of 1000 elements comes very closely to the value of the infinite sum.

9. Next, we'll do the similar thing for another series, the p-series:

$$P = \sum_{n=1}^{\infty} \frac{1}{n^p}.$$

   At the bottom of the same script, initialize new variables

   (a) $p = 2$

   (b) n is a vector containing all the integers from 1 to 500, inclusive.

10. Calculate each term in the p-series (before summation)

11. Calculate the value of the infinite series. We know that

$$P = \frac{\pi^2}{6}.$$

12. Plot a figure to see the convergence of the p-series as in the g-series.