Lindsey Vanderlyn

Problem Set 10

Problem 1: Verify with the Laplace Transform that $y(t) = (1-e^{-t})\,u(t)$ is the step response of $\dot{y} + y = x$. Be sure to indicate regions of convergence in the s plane.

Step 1: Take Laplace

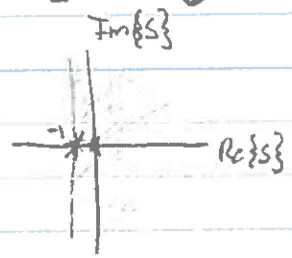
$$\mathcal{L}(\dot{y} + y = x)$$

$$sY(s) + Y(s) = X(s)$$

$$Y(s)(s+1) = X(s)$$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{1}{(s+1)}}$$

if $Re\{s\} > -1$



Verify impulse response

$$\dot{y}e^{t} + ye^{t} = xe^{t}$$

$$ye^{t} = \int_{-\infty}^{\infty} x(\tau)e^{t-\tau}\,d\tau$$

$$y\cancel{e^{t}} = \cancel{e^{t}}\int_{-\infty}^{\infty} x(\tau)\cdot e^{-\tau}\,d\tau$$

$$y = x(t) * \boxed{e^{-t}}$$

$h(t)$

Step 2: $H(s) = \frac{1}{s+1}$ $\mathcal{L}\,u(t) = \frac{1}{s} = U(s)$

Multiply by impulsetrain

$$H(s)(U(s)) = \left(\frac{1}{s+1}\right)\left(\frac{1}{s}\right) \rightarrow \frac{A}{s} + \frac{B}{s+1}$$

Step 3: Use partial fractions

$$\frac{A + sB}{s+1} = \frac{1}{s(s+1)}$$

if $s=0$

$\frac{A}{1} = 1$ $A = 1$

if $(s+1) = 0$

$s = -1$

$B = -1$

Step 4: Take inverse Laplace

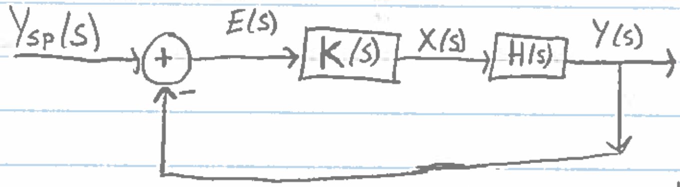
$$\mathcal{L}^{-1}\left(\frac{1}{s} + \frac{-1}{s+1}\right)$$

$$u(t) + (-e^{-t}(u(t)))$$

$$\boxed{(1 - e^{-t})\,u(t)}$$

Problem 2: Find the DC gain of this system $(y(s)/y_{sp}(s))$

Part A. if you use an integral controller $K(s) = k_I/s$ for any $H(s)$. Does it depend on the value of k_I?



Start with just X

$X = ((Y_{sp} - HX)K)$

$y(s) = XH$

$X(s) = EK$

$E(s) = Y_{sp} - XH$

$K(s) = k_I/s$

$K = \frac{X}{Y_{sp} - HX}$

$Y_{sp} - HX = \frac{X}{K}$

$Y_{sp} = \frac{X}{K} + HX$

$\frac{Y_{sp}}{X} = \frac{1}{K} + H = \frac{1+KH}{K}$ convert then to y

$\frac{X}{Y_{sp}} = \frac{K}{1+KH} \rightarrow \frac{Y}{Y_{sp}} = \frac{KH}{1+KH}$

$Y = XH$

DC gain is $\lim_{s\to 0} \frac{Y}{Y_{sp}}$

$$\boxed{\lim_{s\to 0} \frac{k_I/s\, H}{1 + k_I/s\, H} = 1 \text{ b/c}}$$

goes to $\frac{\infty}{\infty}$ take deriv and 1 dissapears leaving $\frac{(k_I/s)' H}{(k_I/s)' H} \rightarrow 1$

Part B. Assume $H(s) = \frac{1/\tau}{s+1}$ find poles assuming $K \gg 1/\tau$.

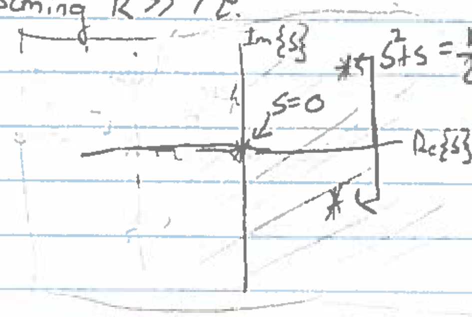
$\frac{Y}{Y_{sp}} = \frac{(K_I/s)\left(\frac{1/\tau}{s+1/\tau}\right)}{1 + (K_I/s)\left(\frac{1/\tau}{s+1/\tau}\right)}$

+ solve for when $s^2 + s/\tau = 0$ $\tau \neq 0$

and $\frac{k_I/\tau}{s^2 + s/\tau} = -1$ boundary

$\frac{Y}{Y_{sp}} = \frac{K_I/\tau}{s^2 + s/\tau + \frac{k_I}{\tau}}$

$s^2 + s = \frac{k_I}{\tau}$



Problem 3:

* * *
* See back *
* for graphs *
* * * *

Analazye the behavior of the systems listed below with a Bode plot, a pole-zero map and the step response. For each system, note the relationship between the three plots: order of the system, number of poles and zeros, real or complex poles, oscillations and so forth. Hand in a couple of sentences for each system describing its behavior and notable characteristics.

A) $\frac{s}{s+1}$

Comments:

I believe this system is first order because the step response is a line. There is only one pole which falls on -1 (only on the real axis) and the system has no zeros. In general, it seems not to have any strange behavior and seems to not change phase, but will converge.

B) $\frac{s}{s^2+100s+1}$

Comments:

I believe that this system is at least second order because the step response looks like it is the right half of a parobola. There are two poles in this case, both occurring on the real axis at -100 and 0 respectively and there doesn't really seem to be much oscillation. The bode plot seems to show a linear drop in amplitude (on a log log scale) and a constant drop in phase, but it should converge.

C) $\frac{s}{s^2+s+1}$

Comments:

I believe this is also a second order system. The bode plot doesn't show any large changes in phase, but the poles are interesting. They are both at $^-0.5$ on the real plane, but contain an imaginary part of $^+0.8$ and $^-0.8$ respectively, meaning convergence w/ oscillation

D) $\frac{s}{s^2+0.1s+1}$

Comments:

I believe this one is also second order based on the shape which matches the previous two plots. The poles in this one are again interesting, however because they are at $0.1+j$ and $0.1-j$, again mirrored about the real axis. The bode plot remains similar to ~~that~~ those in B and C, but this means oscillation w/o convergence.

E) $\frac{s^2-0.01s+1}{s^2+0.01s+1}$

Comments:

I'm not sure what order of system this is because the step response is a straight line... On another note, the behavior of the bode plot here is pretty interesting, it appears that the magnitude remains constant, but the phase changes. This may imply some form of oscillation, but I'm not sure. On another note, this is the first one to have 0's (at $\pm j$) and has two poles at $^-0.1 \pm j$, meaning oscillation with convergence

F) $\frac{s^2+0.1s+1}{s^2+0.11s+1}$

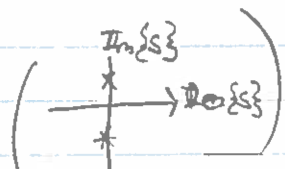
Comments:

This one again has a step response which's a straight line so I'm not sure exactly what the order of a system like this is. The bode plot, however is the most interesting showing an increase in magnitude paired with a decrease in phase, implying again some form of oscillation. The poles are again mirrored about the real axis and nearly overlaping the zeros, so oscillation is a thing.

Problem 4:

Stabalize the system:

$$\frac{1}{s^2+0.01s+1}$$



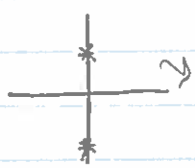
A) See the attachment at the end for the step response

B) $H(s) = \dfrac{\left(\frac{1}{s^2-0.01s+1}\right)\cdot K}{1+\left(\frac{1}{s^2-0.01s+1}\right)K} \rightarrow \dfrac{K}{(s^2-0.01s+1)+K}$

K No converging b/c poles are only imaginary

1) If we set $K = 100000$

Poles look



they are still only imaginary so no converging :(

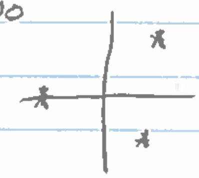
2) If we set $K = 0.0001$

Proportional control is insufficient for this case :(

C. Integral control

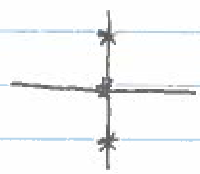
$$H(s) = \frac{(K_I/s)\left(\frac{1}{s^2-0.01s+1}\right)}{1+(K_I/s)\left(\frac{1}{s^2-0.01s+1}\right)} \rightarrow \frac{K_I}{s^3-0.01s^2+s+K_I}$$

1) if we set K_I to be 1000000



Poles on right side = no convergence

2) if we set K_I to be 0.0001



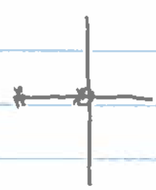
→ Poles not on left side = no convergence

Integral control can not cause convergence :(

D. Derivative control

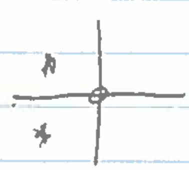
$$H(s) = \frac{(Kd\cdot s)\left(\frac{1}{s^2-0.01s+1}\right)}{1+(Kd\cdot s)\left(\frac{1}{s^2-0.01s+1}\right)} \rightarrow \frac{Kd\cdot s}{s^2-0.01s+1+Kds}$$

1) if we set Kd to be 100000



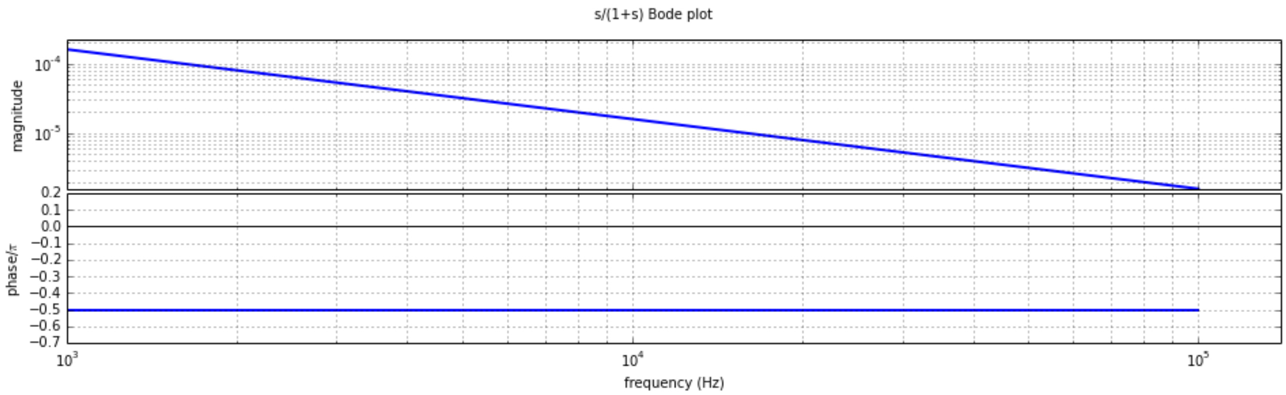
→ Poles are on the left; it converges :)

2) if we set Kd to be 1

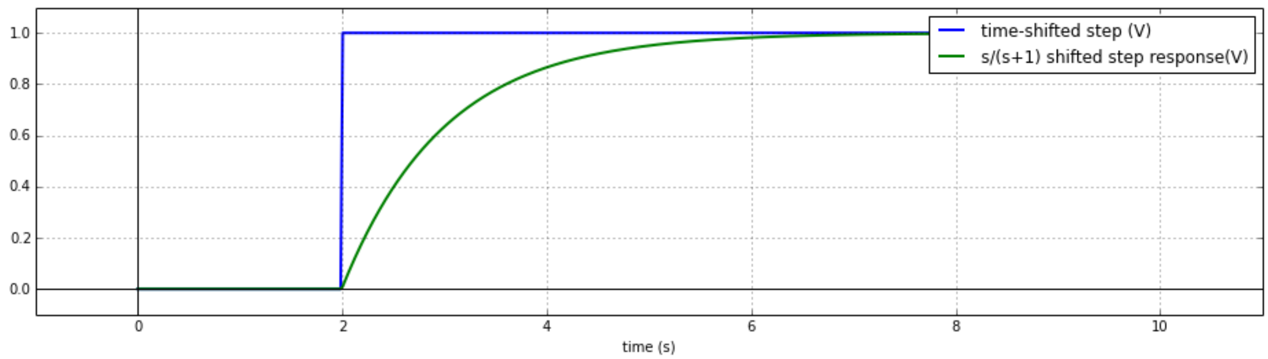


Yay!

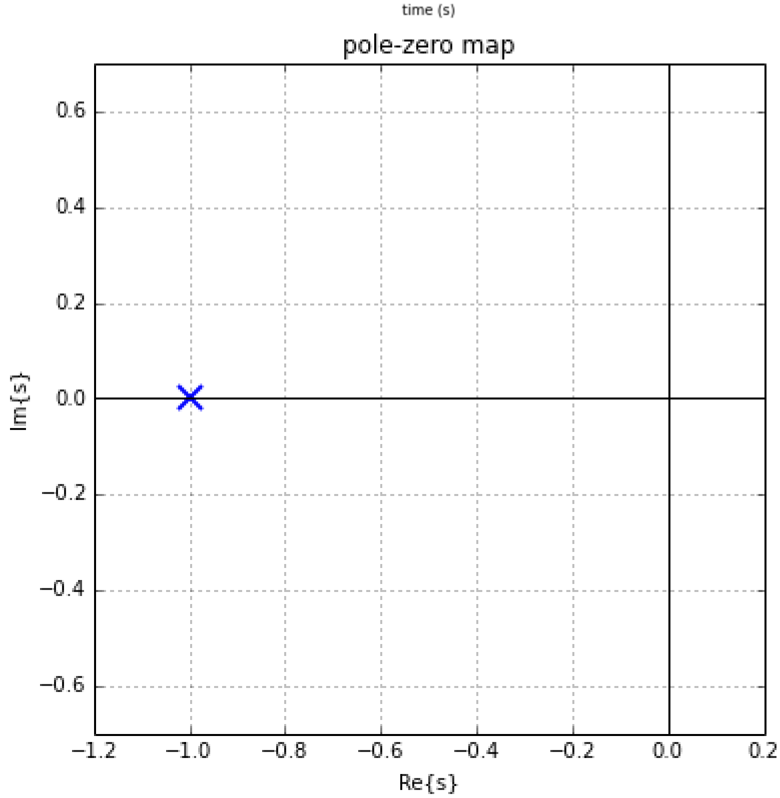
Part A:



s/(1+s) Bode plot
magnitude
phase/π
frequency (Hz)

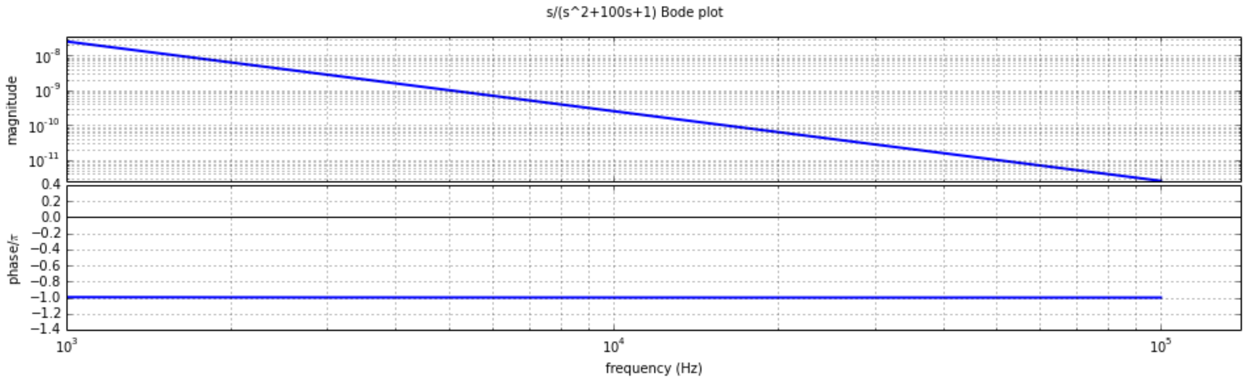


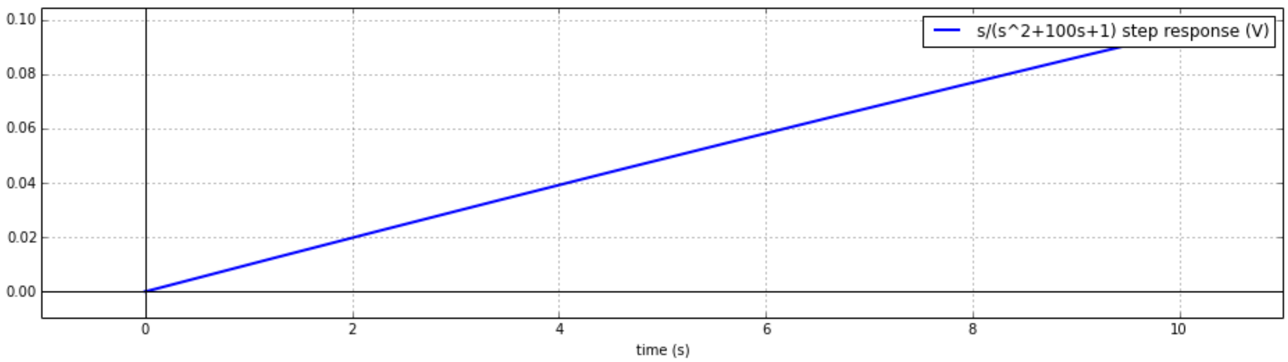
time-shifted step (V)
s/(s+1) shifted step response(V)
time (s)

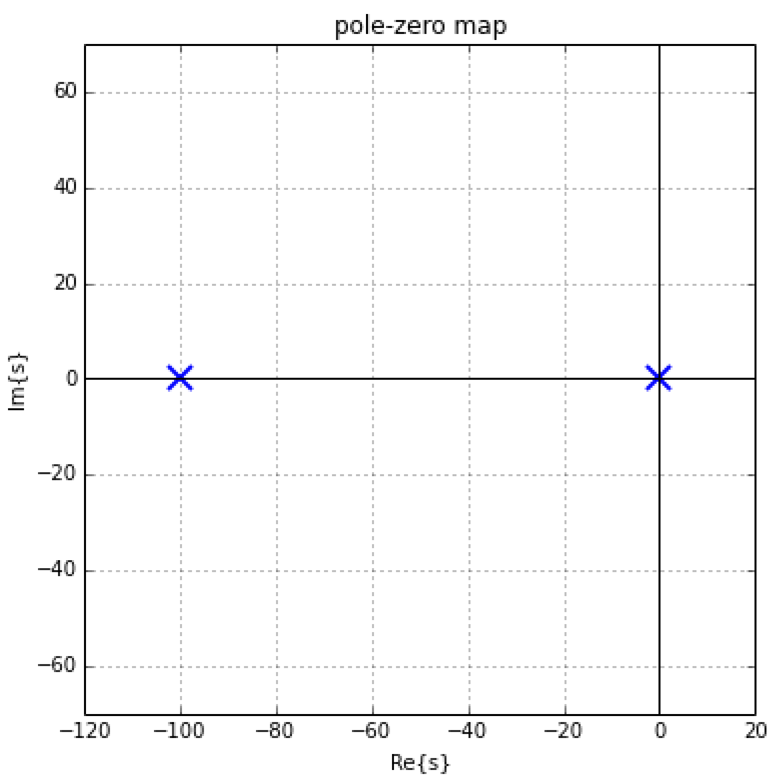


pole-zero map
Im{s}
Re{s}

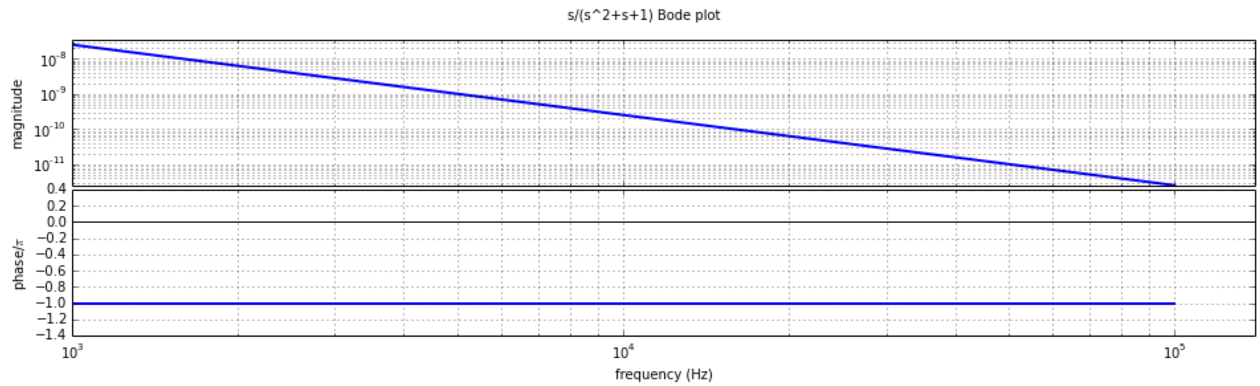
Part B:

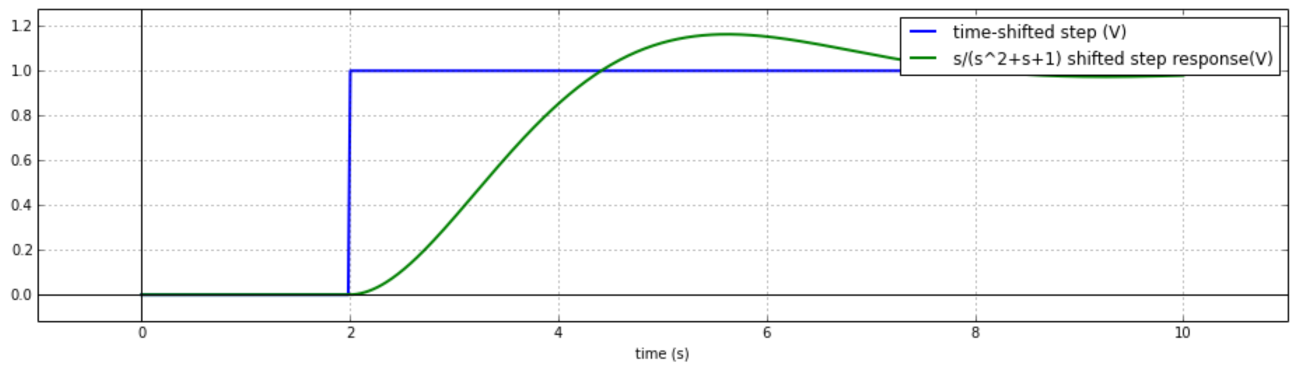

s/(s^2+100s+1) Bode plot
magnitude
phase/π
frequency (Hz)

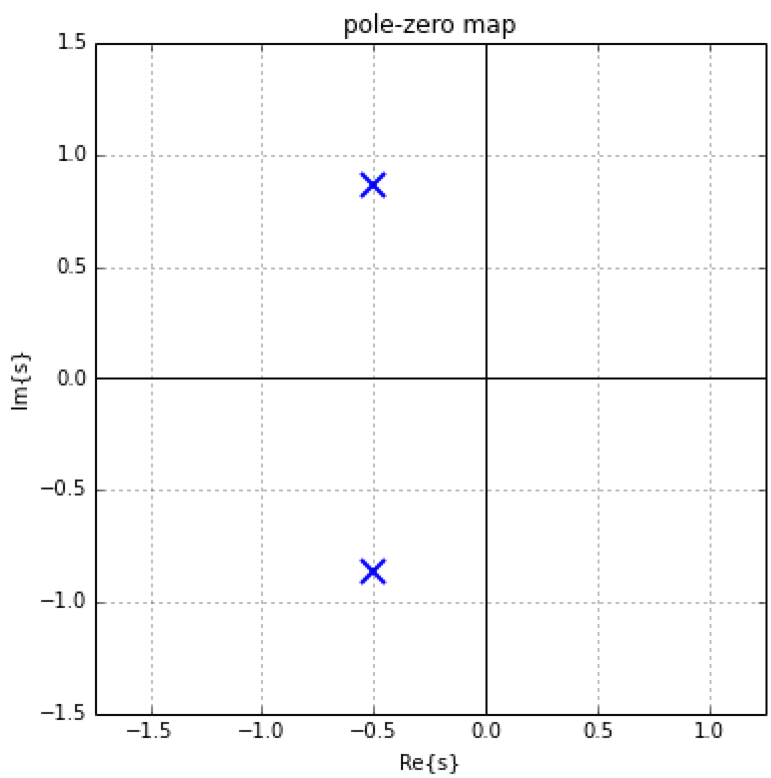

s/(s^2+100s+1) step response (V)
time (s)


pole-zero map
Im{s}
Re{s}

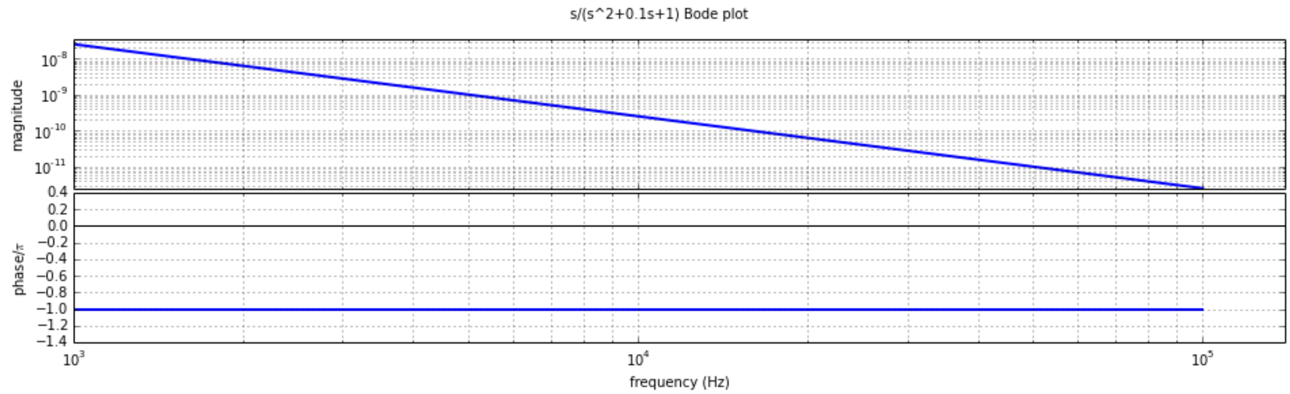
Part C:


s/(s^2+s+1) Bode plot
magnitude
phase/π
frequency (Hz)

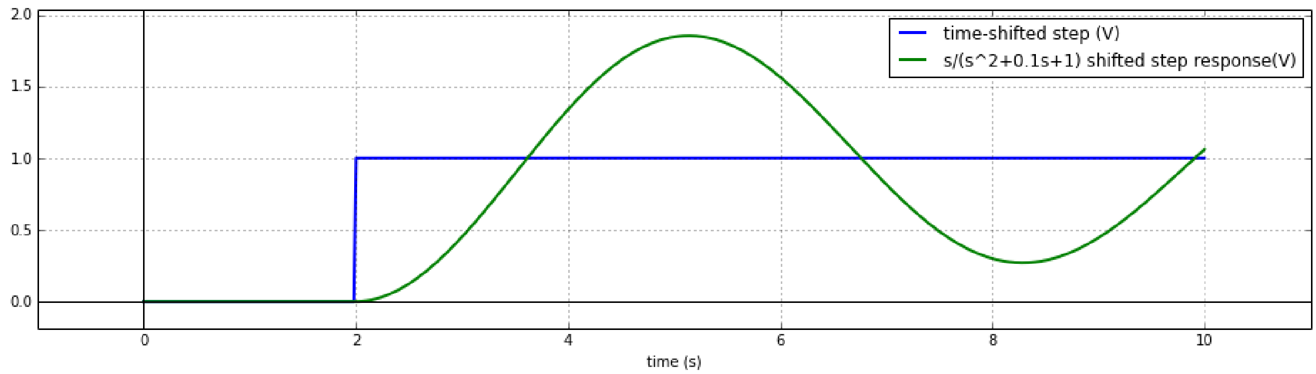

time-shifted step (V)
s/(s^2+s+1) shifted step response(V)
time (s)


pole-zero map
Im{s}
Re{s}

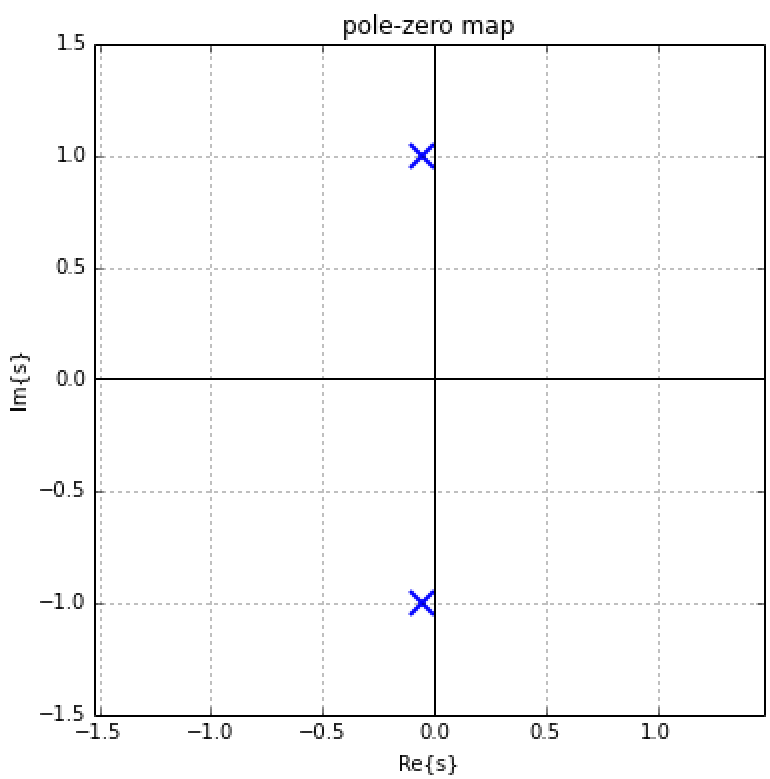
Part D:



s/(s^2+0.1s+1) Bode plot
magnitude
phase/π
frequency (Hz)

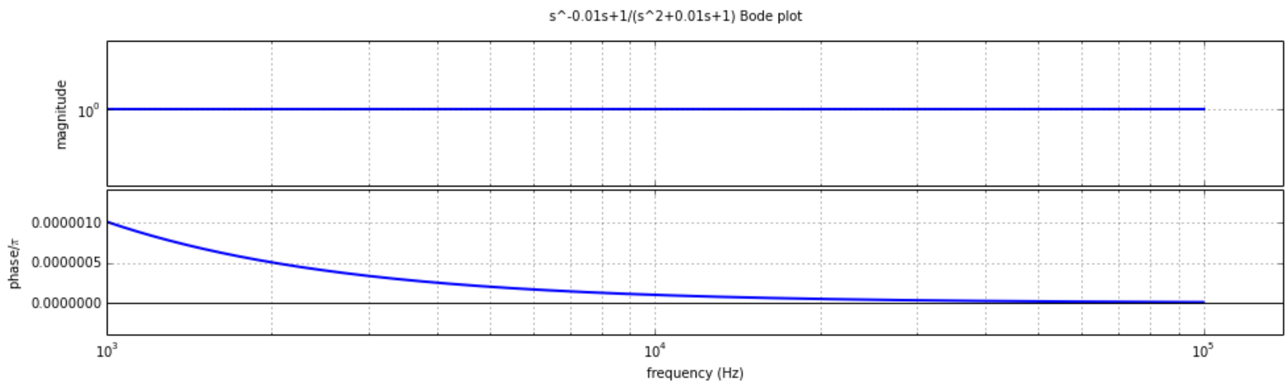


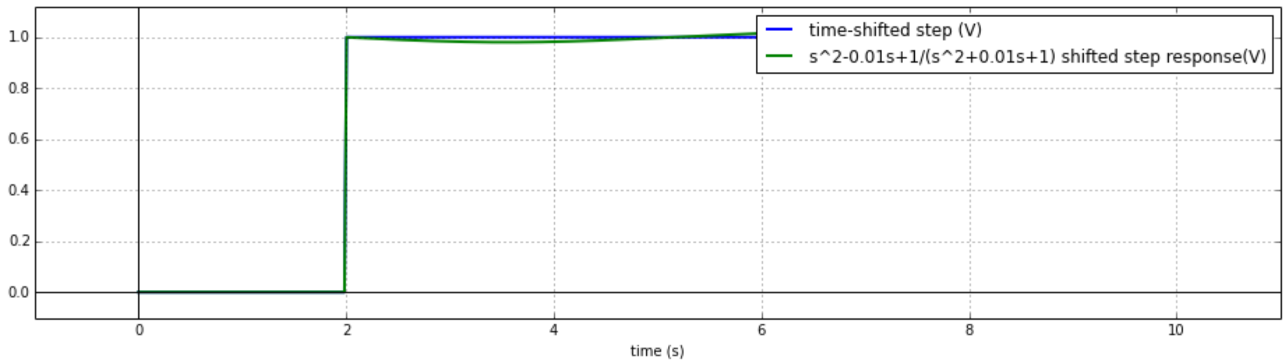
time-shifted step (V)
s/(s^2+0.1s+1) shifted step response(V)
time (s)

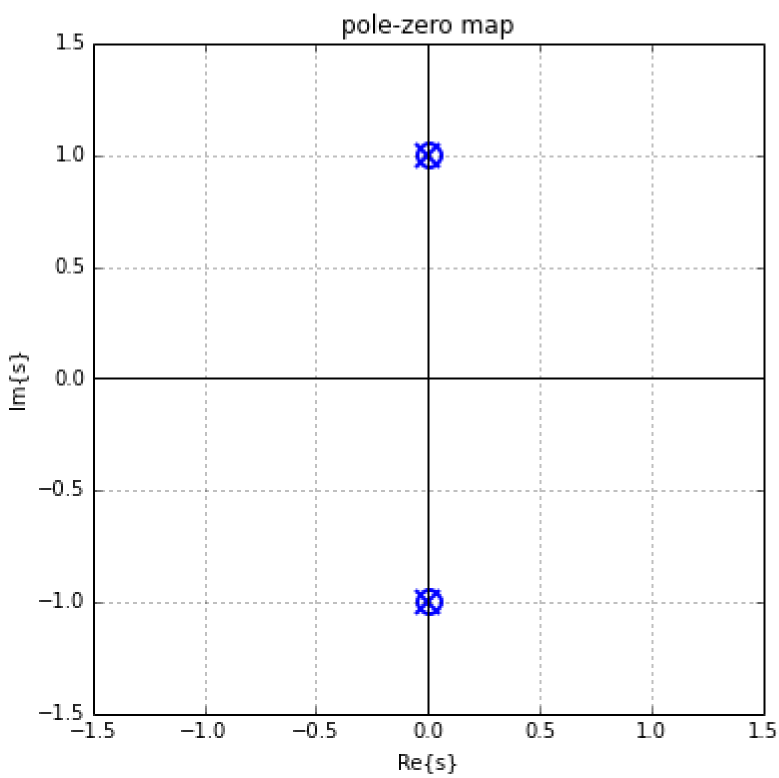


pole-zero map
Im{s}
Re{s}

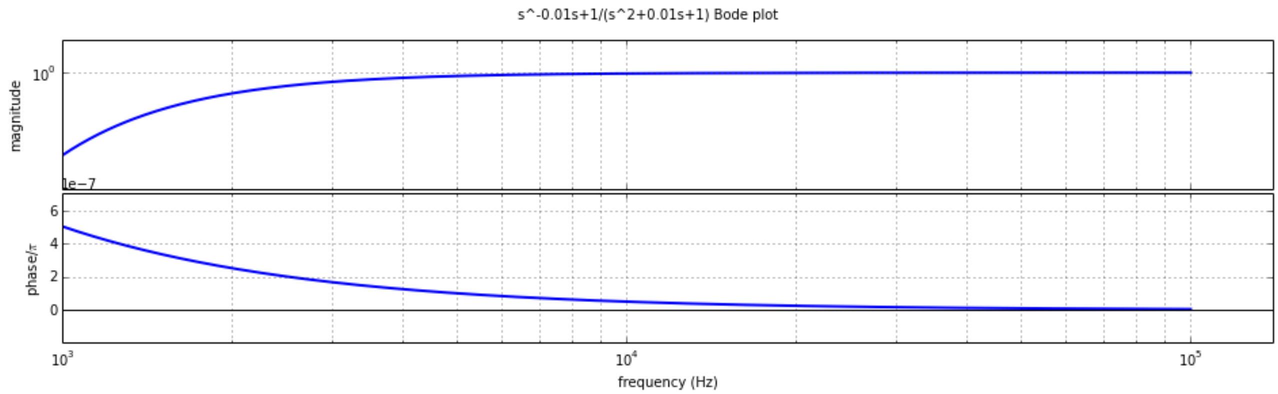
Part E:

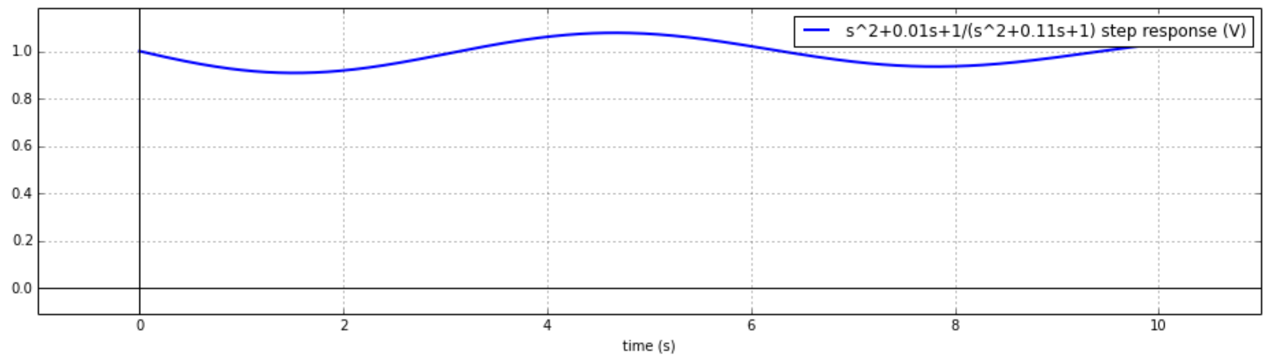

s^-0.01s+1/(s^2+0.01s+1) Bode plot
magnitude
phase/π
frequency (Hz)

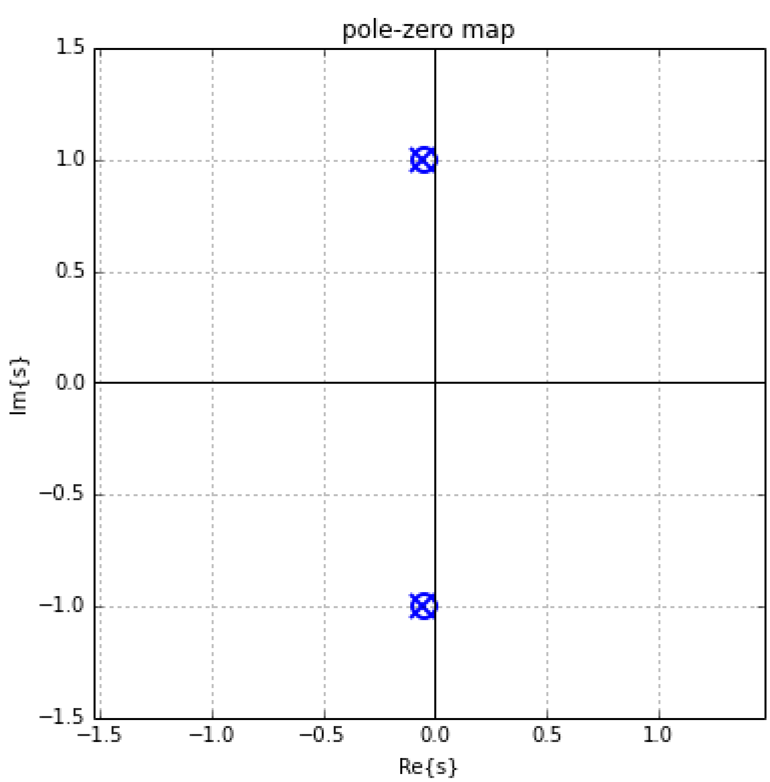

time-shifted step (V)
s^2-0.01s+1/(s^2+0.01s+1) shifted step response(V)
time (s)


pole-zero map
Im{s}
Re{s}

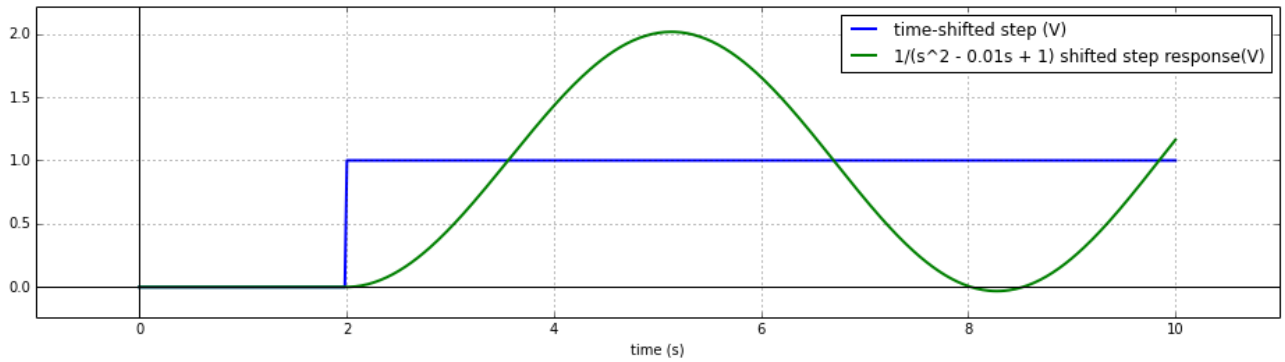
Part F:

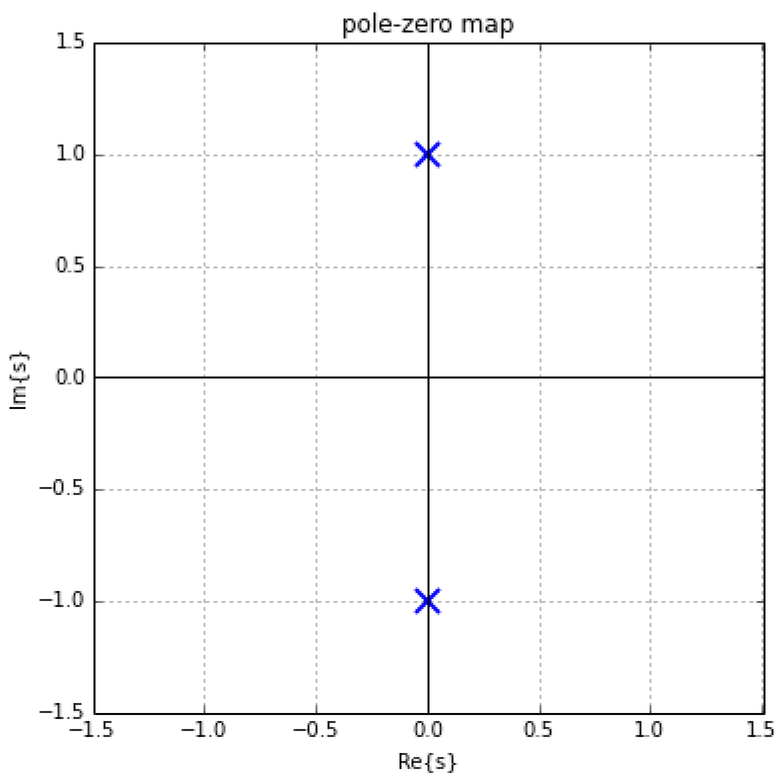

s^-0.01s+1/(s^2+0.01s+1) Bode plot
magnitude
phase/π
1e−7
frequency (Hz)


s^2+0.01s+1/(s^2+0.11s+1) step response (V)
time (s)


pole-zero map
Im{s}
Re{s}

Problem 4A:


time-shifted step (V)
1/(s^2 - 0.01s + 1) shifted step response(V)
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
time (s)


pole-zero map
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Im{s}
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
Re{s}