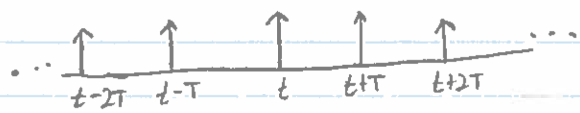
SigSys PS07

1) Consider a train of unit impulses seperated by T time units, given by the following expression:

$$P(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$

a. Sketch a representation of $p(t)$



b. Find the Fourier Series representation of p w/infinite terms

$$\hat{p}_\infty(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T}kt}$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$c_k = \int_{-T/2}^{\sim 0} 0 \cdot e^{-j\frac{2\pi}{T}kt}\, dt \; + \qquad x(t) = \begin{cases} 1 & \text{if } t=kT \\ 0 & \text{if not} \end{cases}$$

$$\int_{\sim 0}^{\sim 0} 1 \cdot e^{-j\frac{2\pi}{T}kt}\, dt \; + \quad \int_{0}^{+T/2} 0 \cdot e^{j\frac{2\pi}{T}kt}\, dt$$

$$c_k = \frac{1}{T}\int_{\sim 0}^{\sim 0} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

Using the picking property we can evaluate this to be 1 b/c the width of the impulse is infintesimaly small

$$\boxed{\hat{P}_\infty(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{j\frac{2\pi}{T}kt}}$$

c. Let a function $x(t)$ be represented as a fourier series with an infinite number of terms as follows

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\omega_0 kt}$$

if $\omega = \omega_0 k$; $X(\omega) = \dfrac{c_k \cdot k \cdot 2\pi}{\omega}$

d. Using your answer from the previous two parts, find $P(\omega)$

$$X(\omega) = \frac{C_K \cdot K \cdot 2\pi}{\omega} = C_K \cdot T$$

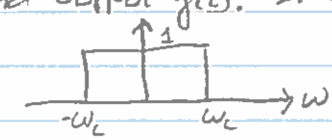
$$C_K = \frac{1}{T} \qquad X(\omega) = \frac{1}{T}\cdot T = \boxed{1 \text{ for all frequencies}}$$

e. $P(\omega)$

K 1 for all frequencies

Changing T will change the spacing on the impulse train b/n successive impulses, but will not have any effect on the $P(\omega)$ function. This makes sense b/c an impulse is not really dependent on the period, unlike other functions where as period increases the frequency changes

2) Consider an LTI system with an impulse response $h(t)$, input signal $x(t)$ and output $y(t)$. It is known that $H(\omega)$ is the following:



a. find $h(t)$

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(t)\, e^{j\omega t}\, d\omega \qquad H(t) = \begin{cases} 1 \text{ if } -\omega_c < t < \omega_c \\ 0 \text{ if otherwise} \end{cases}$$

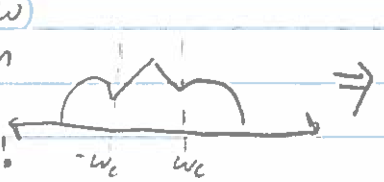
$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi j t}\left[e^{j\omega_c t} - e^{-j\omega_c t}\right]$$

$$\boxed{h(t) = \frac{1}{\pi t}\cdot \sin(\omega_c t)}$$

b. $X(\omega)$

*convolution in frequency domain is multiplication!



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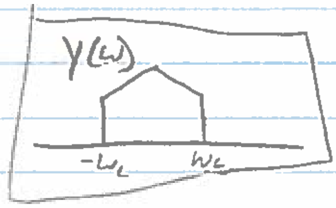
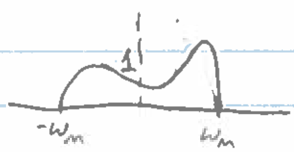


c. This LTI system is known as an ideal low pass filter because it completely kills all of the frequencies above the cut off rather than having any period where there is only a partial cut off

d. See attachment at the end (first is 0.75 cuttoff, then 1.75)

3) Consider a signal $x(t)$ which is band-limited to the range $[-\omega_m, \omega_m]$. In other words, $X(\omega) = 0$ for $\omega < -\omega_m$ and $\omega > \omega_m$. Suppose that $X(\omega)$ is given in figur 4. Let $y(t) = x(t) \cdot \cos(\omega_c t)$, where $\omega_c \gg \omega_m$

$X(\omega)$

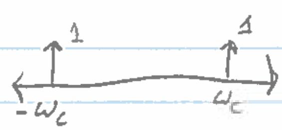


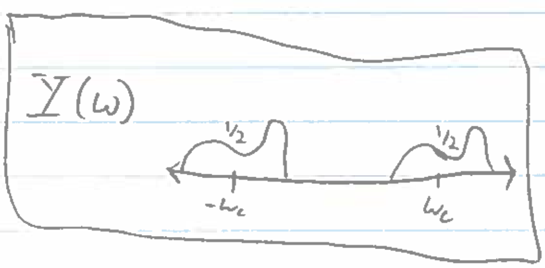
$y(t) = x(t)\, f(t)$ $\rightarrow$ $Y(\omega) = X(\omega) * F(\omega) \cdot \frac{1}{2\pi}$

$f(t) = \cos(\omega_c t)$

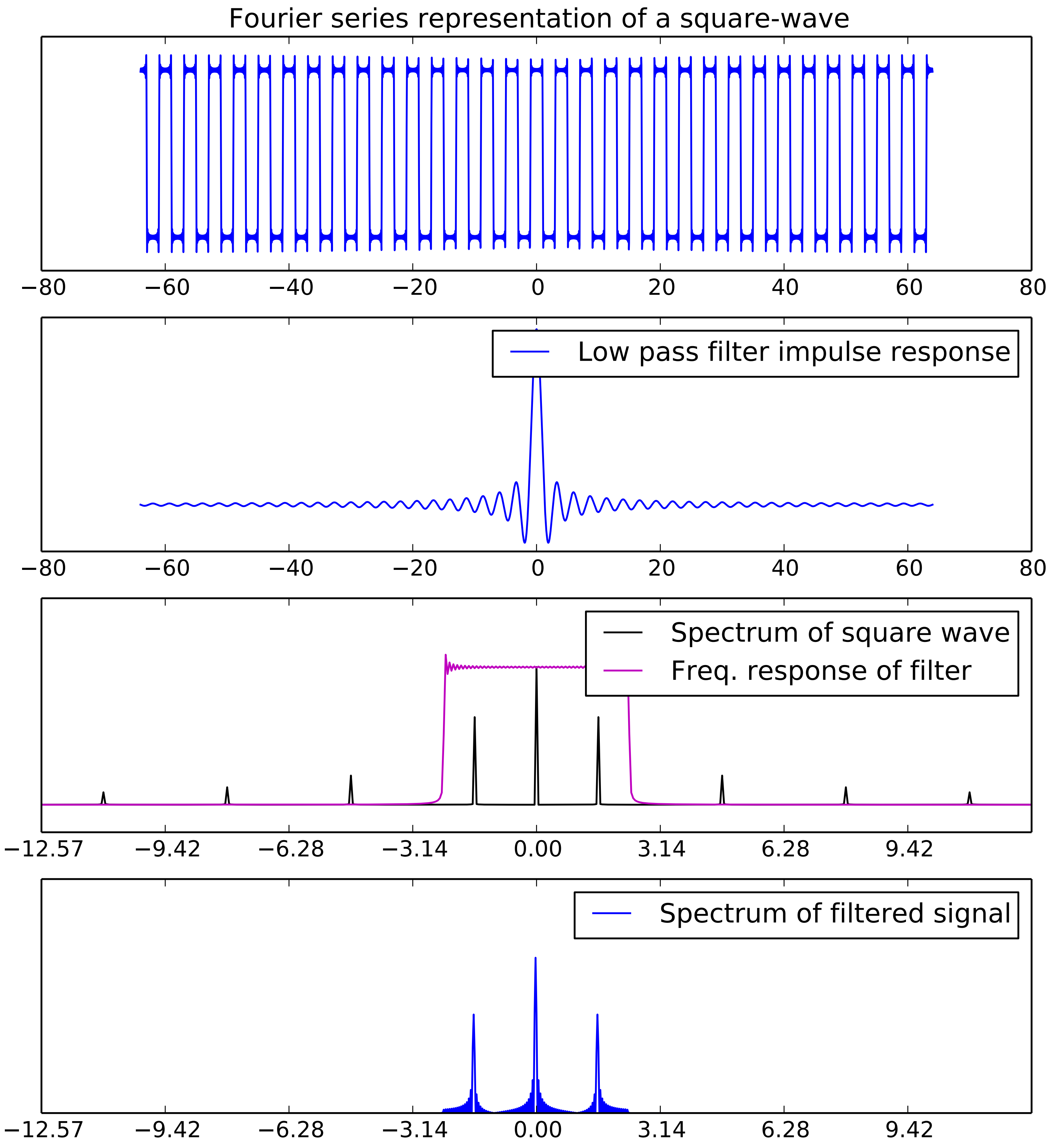
$F(\omega) = \pi\,\delta(\omega - \omega_c) + \pi\,\delta(\omega + \omega_c)$

$F(\omega)$

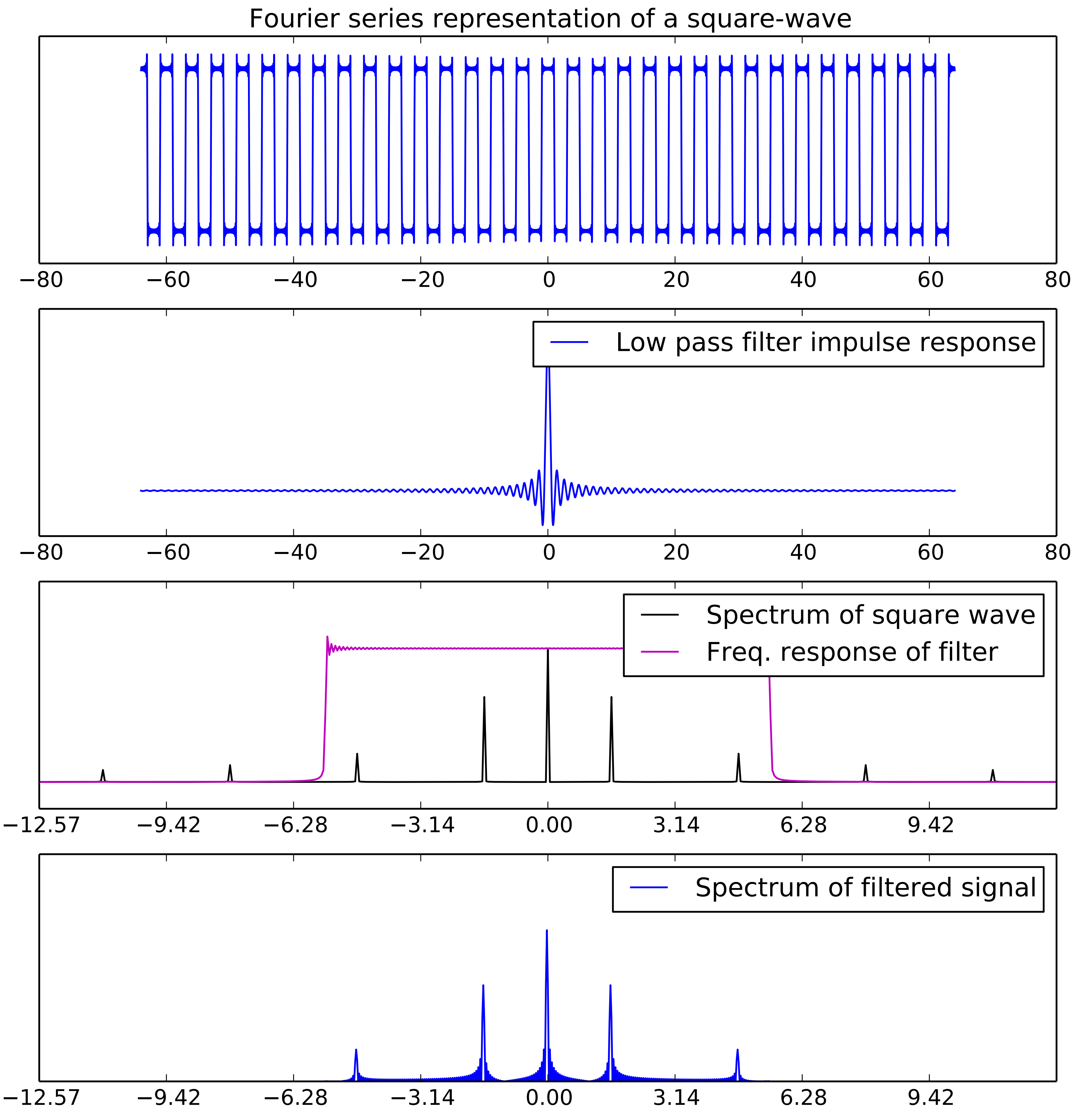




* Convolution shifts the center to center of impulse



Fourier series representation of a square-wave
−80
−60
−40
−20
0
20
40
60
80
Low pass filter impulse response
Spectrum of square wave
Freq. response of filter
−12.57
−9.42
−6.28
−3.14
0.00
3.14
6.28
9.42
Spectrum of filtered signal



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Low pass filter impulse response
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Spectrum of square wave
Freq. response of filter
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Spectrum of filtered signal
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