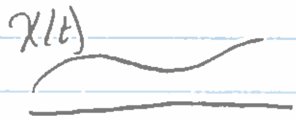
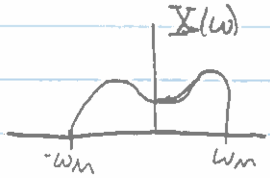
Problem Set 8

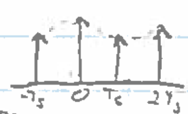
1) Consider a signal $x(t)$ which is band limited to ω_m. $x(t)$ and $X(\omega)$ are shown below



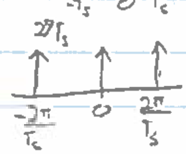


Additionally, let $p(t)$ be an impulse train with impulses seperated by T_s. Also $x_p(t) = x(t)(p(t))$

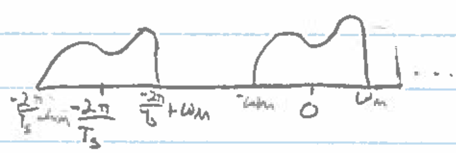
a. Sketch $x_p(t)$



b. Sketch $P(\omega)$



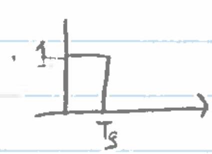
c. Sketch $X_p(\omega)$

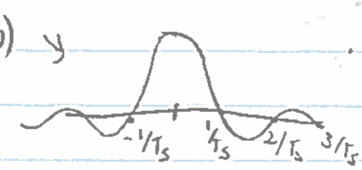


d. In order to fully regain the original signal $\frac{2\pi}{T_s}$ must be larger than $2 \times \omega_m$, otherwise the tiled signal will overlap and it will be impossible to get the original back

e. In order to recover the original signal, simply create a band pass filter around the signal so you only keep one copy. In the time domain this becomes a sinc function

f. Consider $z(t)$

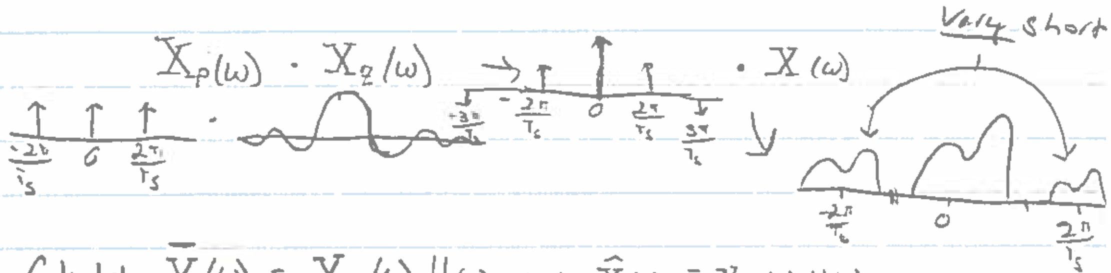




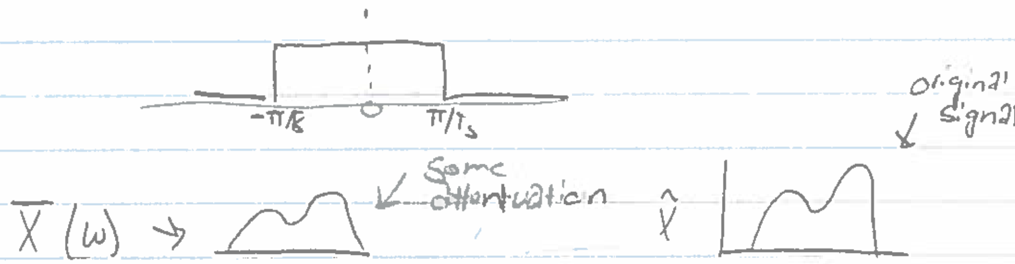
g. Sketch $x_z(t) = x_p * z(t)$ $\rightarrow$ $X_z(\omega) = X_p(\omega)\, Z(\omega)$

$x_z(t)$ is a zero-order hold reconstruction b/c we're convolving w/ a box w/ width T_s

h. Sketch $X_z(\omega)$



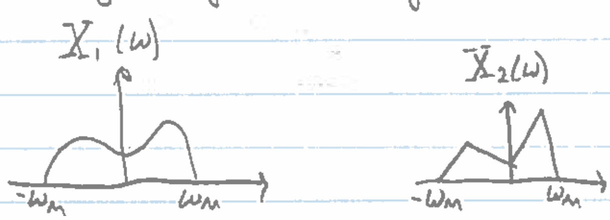
i. Sketch $\bar{X}(\omega) = X_z(\omega) H(\omega)$ and $\hat{X}(\omega) = X_p(\omega) H(\omega)$ where $H(\omega)$ is shown in



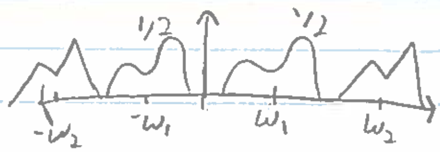
j. $\bar{X}(\omega)$ and $\hat{X}(\omega)$ are similar, but because of the convolution with the sinc, there seems to be some squishing down/ attenuation of $\bar{X}(\omega)$ not found in $\hat{X}(\omega)$.

k. $\bar{X}(\omega_m) / \hat{X}(\omega_m)$ should be $0/0$ when $\omega_m = \pi/T_s$ b/c both functions should be 0 there to prevent information loss

2) Consider a signal $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$. Suppose that $X_1(\omega) = 0$ and $X_2(\omega) = 0$ if $|\omega| > \omega_m$. Assume that $\omega_1 \gg \omega_m$ and $\omega_2 \gg \omega_m$ and $\omega_1 + 2\omega_m > \omega_2$. Suppose that $X_1(\omega)$ and $X_2(\omega)$ are given by the following.

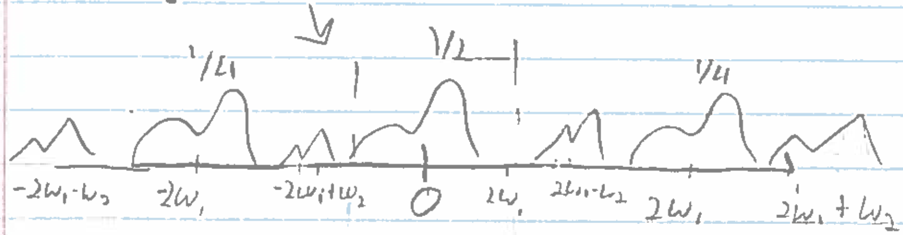


a. Please sketch $Y(\omega)$

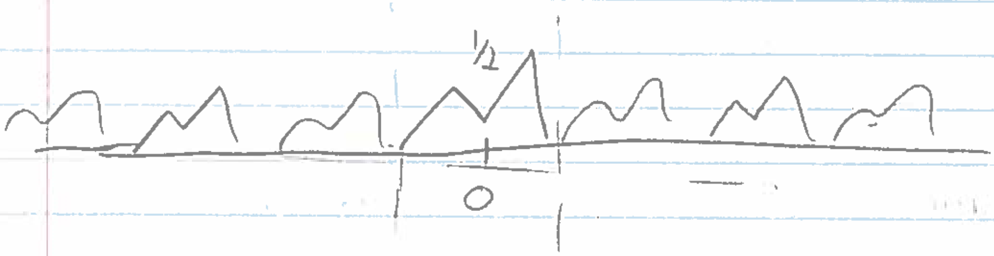


b. Please sketch the Fourier transforms of $y(t)\cos(\omega_1 t)$ and $y(t)\cos(\omega_2 t)$

$y(t)\cos(\omega_1 t)$



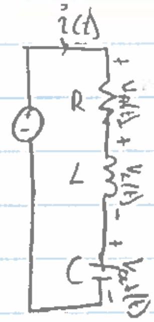
$y(t)\cos(\omega_2 t)$



c. How can you recover $x_2(t)$ and $x_1(t)$ from $y(t)$?
If we multiply $y(t)$ with a sin wave whose frequency matches the period of x_2 or x_1, we can get that signal back by taking a band pass around 0 and multiplying the result by 2 to recover the amplitude. (See dotted lines in part b) then doing the inverse Fourier transform to convert back into the time domain.

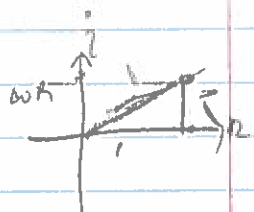
3) Consider the RLC circuit in figure 5. Recall the following:
$i(t) = C\frac{d}{dt}V_{out}(t) \quad V_L(t) = L\frac{d}{dt}i(t)$

figure 5



a. Write a differential equation relating $V_{out}(t)$ to $V_{in}(t)$.

$$V_{in} = V_R(t) + V_L(t) + V_{out}(t)$$



$$V_R(t) = R\,i(t) = R\,C\frac{d}{dt}V_{out}(t)$$

$$V_L(t) = L\frac{d}{dt}i(t) = L\frac{d}{dt}C\frac{d}{dt}V_{out}(t)$$

$$\boxed{V_{in} = RC\frac{d}{dt}V_{out}(t) + L\frac{d}{dt}C\frac{d}{dt}V_{out}(t) + V_{out}(t)}$$

b. $H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)}$ $\quad V_{in}(\omega) = j\omega RC\,V_{out}(\omega) - LC\omega^2 V_{out}(\omega) + V_{out}(\omega)$

$V_{out}(\omega) = V_{out}(\omega)$

$$\boxed{H(\omega) = \frac{1}{j\omega RC - LC\omega^2 + 1}}$$

c. $|H(\omega)| = \frac{1}{|j\omega RC - LC\omega^2 + 1|} = \frac{1}{\sqrt{(\omega RC)^2 + (LC\omega^2+1)^2}}$

d. $f(\omega) = \left((\omega RC)^2 + (LC\omega^2+1)^2\right)^{-1/2}$

to minimize, find when $f'(\omega) = 0$

$f'(\omega) = (2\omega R^2C^2 + 2(LC\omega^2+1)\cdot 2LC\omega)\cdot{-1/2}\,((\omega RC)^2 + (LC\omega^2+1)^2)^{2}$

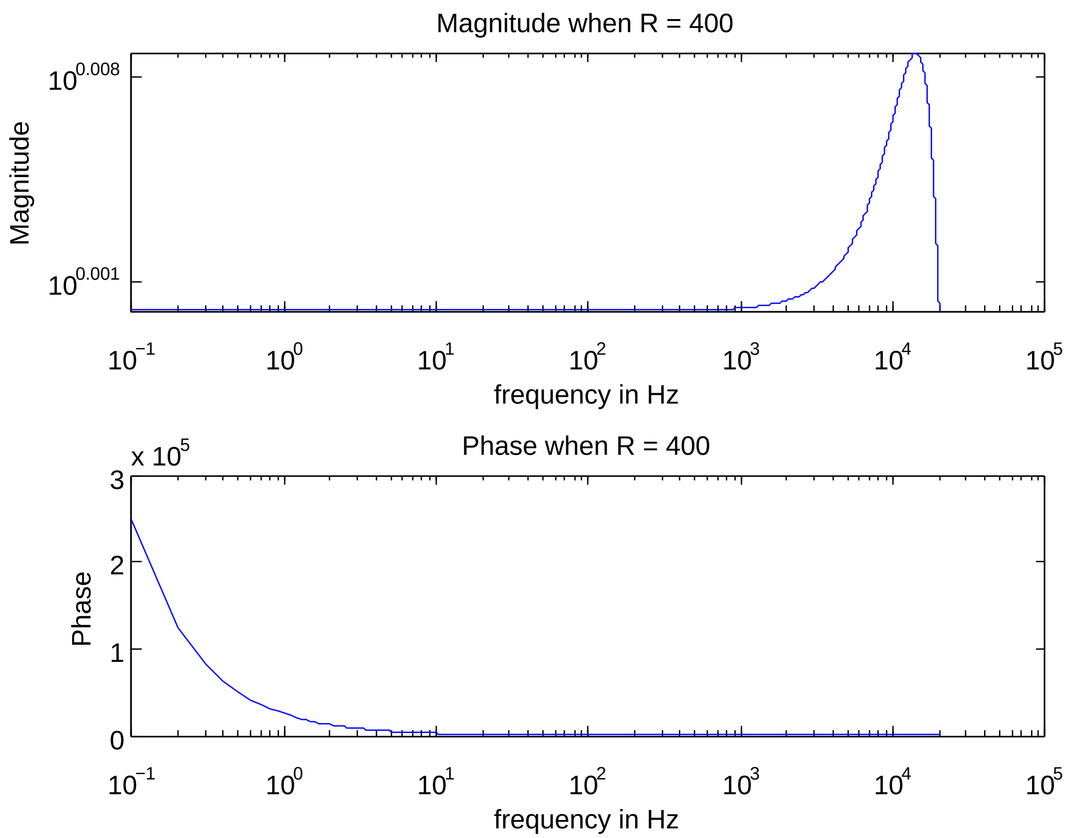
$= 0$

When $\omega = 0$ if $CL \neq 0$

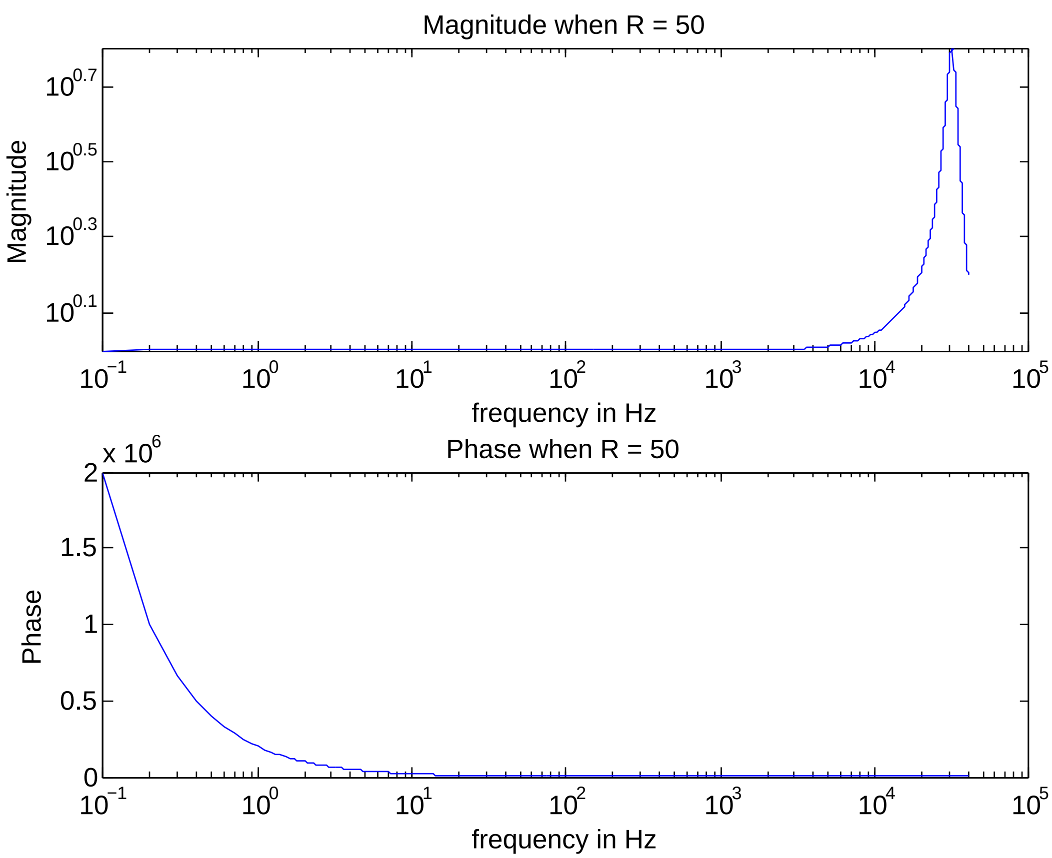
$$\boxed{\text{When } \omega = \pm\frac{\sqrt{2L} - R}{\sqrt{2}\sqrt{CL}}}$$ When $\sqrt{C}L \neq 0$, $4LLR + \frac{2}{L}C^2R^2 = 0$

e. See figures

Creates a band pass filter w/ behavior we derived in part d.



Magnitude when R = 400
Magnitude
$10^{0.008}$
$10^{0.001}$
10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
frequency in Hz
Phase when R = 400
x 10^5
Phase
3
2
1
0
10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
frequency in Hz



Magnitude when R = 50
Magnitude
$10^{0.7}$
$10^{0.5}$
$10^{0.3}$
$10^{0.1}$
10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
frequency in Hz
Phase when R = 50
$x\ 10^{6}$
Phase
2
1.5
1
0.5
0
10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
frequency in Hz