Intake_Rate_OGM

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1 Ontogenetic Growth Model - Intake Rate & Hyperallometric Fecundity

1.1 Derivation - Master Balance Equation (West *et al.*, 2001)

1.1.1 Growth from metabolic energy inflow

- B = incoming rate of energy flow, which is the average resting metabolic rate of the whole organism at time t
- B_c = the metabolic rate of a single cell
- E_c = the metabolic energy required to create a cell
- N_c = the total number of cells
- $\sum_c$ = over all types of tissue, assuming a typical cell as the fundamental unit
- tells us that dedication to growth **AFTER** metabolism is a fundamental axiom of energy/mass conservation equation
- growth is equal to surplus energy because assumed that energy is optimally allocated to

growth after miscellaneous costs

$$B = \sum_c \left[N_c B_c + E_c \frac{dN_c}{dt} \right]$$
$$B = N_c B_c + E_c \frac{dN_c}{dt}$$
$$\frac{dN_c}{dt} = \frac{B - N_c B_c}{E_c} = \frac{\text{Surplus energy}}{\text{Cost of creating single cell}}$$
$$= \text{multiply by mass of single cell}$$
$$\left(\frac{dN_c}{dt}\right) m_c = \left(\frac{B - N_c B_c}{E_c}\right) m_c$$
$$\frac{dm}{dt} = \left(\frac{B m_c}{E_c}\right) - \left(\frac{N_c m_c B_c}{E_c}\right)$$
$$\frac{dm}{dt} = \left(\frac{B m_c}{E_c}\right) - \left(\frac{m_{tot} B_c}{E_c}\right)$$
$$\frac{dm}{dt} = \left(\frac{m_c}{E_c}\right) B - \left(\frac{B_c}{E_c}\right) m_{tot}$$
$$\text{sub } B = B_0 m^{3/4}$$
$$\frac{dm}{dt} = \frac{m_c}{E_c} B_0 m^{3/4} - \frac{B_c}{E_c} m_{tot}$$
$$\text{sub } a = \frac{m_c}{E_c} B_0$$
$$b = \frac{B_c}{E_c}$$
$$\frac{dm}{dt} = a m^{3/4} - b m$$

1.1.2 Surplus is reduced by efficiency of use (ingestion versus energy flow into branching network)

- I = consumption rate $\left(\frac{mass}{time}\right)$
- t_I = "characteristic" time period for "sufficient" energy gain
- $t_I << t_{growth}$
- Need to tackle this time period, we know $t \propto \kappa m^{\gamma}$ i.e. bigger things can spend longer foraging, thus intake more (absolute not relative)
- But their digestion is then slower
- reasonable bounding of digestion times?
- Timescale at level of digestion is different to timescale of growth
- You may pass a threshold where you are digesting at maximum rate meaning that you are

limited to 3/4 exponent because limited by fractal dimension of capillary network

$$
\begin{aligned}
I_{tot} &= \int_{t_{start}}^{t_{end}} I dt \\
\frac{m_c}{E_c} B_0 m^{3/4} &= am^{3/4} \\
\frac{am^{3/4}}{m} &= am^{\frac{3}{4}-1} \\
\frac{am^{3/4}}{m} &= am^{-\frac{1}{4}} \\
I_{tot}\epsilon\left(am^{-\frac{1}{4}}\right)
\end{aligned}
$$

1.1.3 Can also introduce loss to reproduction

$$\frac{dm}{dt} = I_{tot}\epsilon\left(am^{-\frac{1}{4}}\right) - bm - cm^{\rho}$$

1.1.4 Objectives

- What data to we have?
  - intake rate
  - digestion rate
  - individual growth rate
- We want to plug into
  - intake rate data
- Fit across whole range of fish