Introductory Modeling of Carbon Dynamics: Project Outline

Luke Vaughan

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1 Introduction

Carbon levels are increasing throughout our planet. In response, massive research efforts have taken place in an attempt to gain insight into carbon dynamics. The problem (harmful greenhouse gases lingering in the atmosphere) must be thoroughly understood before suggestions and/or solutions (how best to help our Mother Earth) can be presented. Some key players in global carbon exchange include the soil, plant life, and root systems. Therefore, soil science is a rich area in which to apply models of carbon dynamics.

The model to be studied during this project is given by a two-component system of differential equations. The system, which is called the Introductory Carbon Balance Model (ICBM), is presented in source [1]. The ICBM can be used to address a few questions, including but not limited to the following. Is the system being studied retaining or losing soil carbon? What happens to soil carbon if temperatures increase? The goal for this project is to answer these questions and explore carbon dynamics by reproducing some plots from source [1] while consulting sources [2] - [3].

2 Methods

The ICBM is given by the system

$$\begin{aligned} \frac{dY}{dt} &= i - k_1 r Y, \\ \frac{dO}{dt} &= h k_1 r Y - k_2 r O. \end{aligned} \tag{1}$$

The following table contains explanations of the variables and parameters appearing in system (1).

Y	"Young" carbon pool: amount retained in soil (variable)
O	"Old" carbon pool: amount retained in soil (variable)
k_1	Affects how much carbon the young pool loses (parameter)
k_2	Affects how much carbon the old pool loses (parameter)
r	Climate & soil affecting decomposition rates for Y and O (parameter)
h	Fraction of carbon flowing yearly from Y to O (parameter)
i	Mean annual carbon input into Y (parameter)

Another goal for this project is to explain, in depth, the steps taken by the authors in order to find their solutions. System (1) can be solved explicitly, which is unsurprising, due to the fact that its equations are linear. Once we solve the first equation, we can substitute an expression for Y into the second equation. Then we can proceed to solve again. The authors' solutions are given as

$$\begin{aligned} Y &= \frac{i}{k_1 r} + \left(Y_0 - \frac{i}{k_1 r}\right) e^{-k_1 r t} \\ O &= h\frac{i}{k_2 r} + \left[O_0 - h\frac{i}{k_2 r} - h\frac{k_1 r Y_0 - i}{r(k_2 - k_1)}\right] e^{-k_2 r t} + \left[h\frac{k_1 r Y_0 - i}{r(k_2 - k_1)}\right] e^{-k_1 r t}. \end{aligned} \tag{2}$$

These explicit solutions allow us to model carbon dynamics with ease. For example, the authors use solutions (2) to extract "steady state" equations for Y and O:

$$\begin{aligned} Y_{ss} &= \frac{i}{r k_1}, \\ O_{ss} &= \frac{hi}{r k_2}. \end{aligned} \tag{3}$$

After discussing the steady state equations (3), the authors fit the ICBM to data taken from a 35-year experiment in Sweden. At this point, a shift from differential equations to statistics takes place. The authors use the statistical software SAS to create their plots. Therefore, re-familiarization with SAS may become necessary over the course of this project.

The authors make an important point in the introduction. "Selecting a simple model structure... made complex simulation techniques unnecessary - the model is analytically solved..." [1]. So numerical methods such as Forward Euler, Backward Euler, Trapezoidal Rule, etc. are not needed during the analysis of the ICBM. If this project needs more numerical focus, additional content can be included. Idea: use numerical methods to simulate differential equations appearing in source [2]. Simulations of related equations could enrich our study of carbon dynamics.

3 Results

The authors of source [1] create several plots to visualize carbon dynamics. One plot reveals that increased temperatures result in soil losing carbon to the atmosphere - see screenshot below [1]. However, the authors clarify that the introductory nature of the ICBM might result in some oversimplifications in model predictions. Nevertheless, the result shown here is interesting.

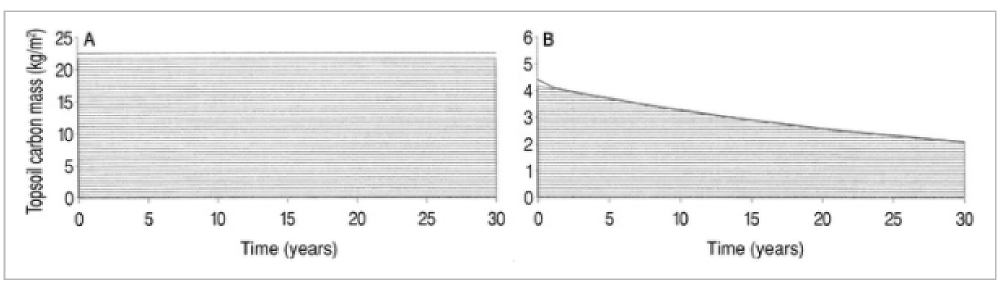

Figure 4 Open in figure viewer | PowerPoint

(A) Example of ICBM outputs when parameterized for an organic soil, Y_0 = 0.79, O_0 = 21.7, i = 0.63, h = 0.23; and (B) assuming a hot, humid climate, r = 5.36. All other parameters were set as in Fig. 3A.

4 Conclusion

Certain plots may not be able to be reproduced due to 1) having limited experience in SAS and 2) having no access to the authors' data. However, the goal of answering the questions presented in the introduction remains. We should expect to reach the same conclusion from source [1]: increased temperatures result in increased carbon levels in the atmosphere.

For a future direction, we could choose to study more sophisticated models. For example, we could study models with more than two carbon pools. We could also study models given by systems of nonlinear equations. Numerical methods would be invaluable during the analysis of these types of models.

5 Bibliography

1. Andrén, Olof, and Thomas Kätterer. "ICBM: The Introductory Carbon Balance Model for Exploration of Soil Carbon Balances." The Ecological Society of America, John Wiley & Sons, Ltd, 1 Nov. 1997, esajournals.onlinelibrary.wiley.com/doi/full/10.1890/1051-0761(1997)007[1226:ITICBM]2.0.CO;2?casa_token=y7Pa74mTw3cAAAAA

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3. Sala, Anna, et al. "Carbon Dynamics in Trees: Feast or Famine?" OUP Academic, Oxford University Press, 1 Feb. 2012, academic.oup.com/treephys/article/32/6/764/1663993.