Intermediate Disturbance Hypothesis (IDH)

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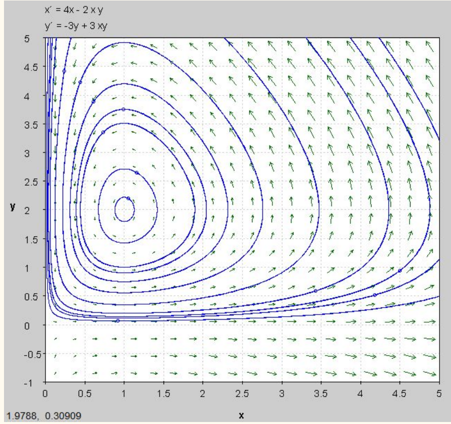
Topics in Mathematical Ecology

- **Botany**
- Zoology
- Differential equations
- Environmental science
- Other



4 Variables in IDH

- Disturbance
- Abundance
- Niche space
- Diversity

Disturbance

- Use the letter E
- Deforestation, forest fires, hurricanes, etc.
- E measures the number of plants killed in the given space
- Both niche space and abundance are functions of E

Abundance

- Use the letter A
- A measures the number of plants in the given space
- High abundance corresponds to large A value
- Linear relationship between A and E

$$A(E) = A_{\max} - aE \qquad (A_{\max} > 0 \text{ and } a > 0)$$

Niche Space

- Use the letter z
- Niche space z measures the fraction of given space available to new plants
- Value: $0 \leq z \leq 1$
- Linear relationship between z and E

$$z(E) = bE \quad (b > 0)$$

Diversity

- Use the letter S
- S measures the number of different plant species occupying the given space
- Nonlinear relationship: S is a function of A and z

$$S(A, z) = A^z$$

Intermediate Disturbance Hypothesis

- What is the connection between diversity and disturbance?
- IDH: number of different plant species **(diversity)** is **maximized** in areas with neither low nor high **disturbance**
- S_{max} occurs somewhere in between the two extremes: small E values and large E values

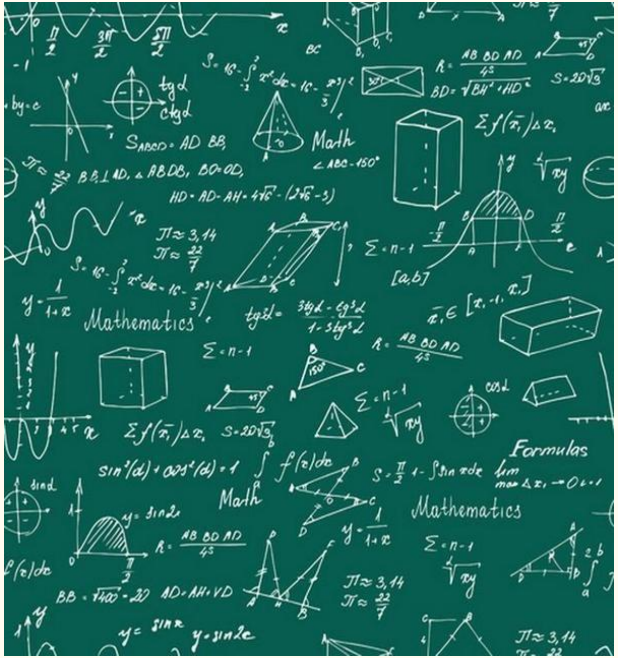
$$S = A^z$$

$$A = A_{\text{max}} - aE \qquad (A_{\text{max}} > 0 \text{ and } a > 0)$$

$$z = bE \qquad (b > 0)$$

Mathematical Implications

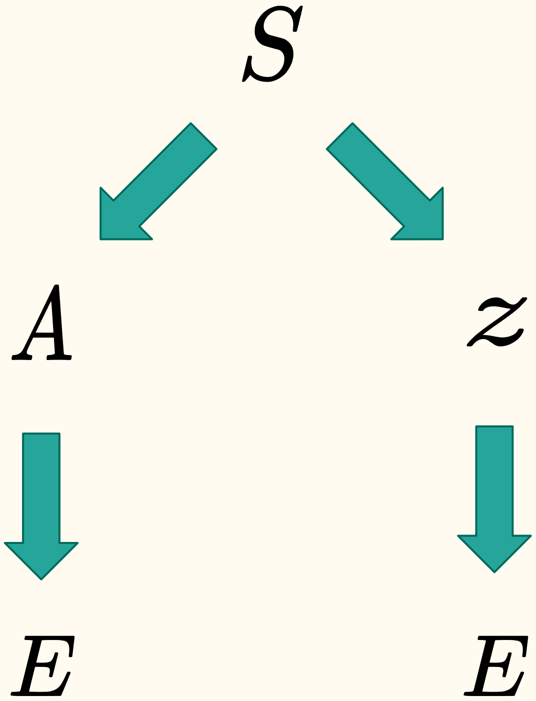
- Interested most of all in the relationship between diversity (S) and disturbance (E)
- First task: find an expression for the rate of change of diversity with respect to disturbance (find $\frac{dS}{dE}$)



Multivariable Calculus

- Flowchart visualization
- Multivariable chain rule:

$$\frac{dS}{dE} = \frac{\partial S}{\partial A}\frac{dA}{dE} + \frac{\partial S}{\partial z}\frac{dz}{dE}$$



Relevant Equations

$$\frac{dS}{dE} = \frac{\partial S}{\partial A}\frac{dA}{dE} + \frac{\partial S}{\partial z}\frac{dz}{dE}$$

$$A = A_{\text{max}} - aE$$

$$\frac{dA}{dE} = -a$$

$$S = A^z$$

$$\frac{\partial S}{\partial A} = zA^{z-1}$$

$$\frac{\partial S}{\partial z} = (\ln A)(A^z)$$

$$z = bE$$

$$\frac{dz}{dE} = b$$

$$\frac{dS}{dE} = (zA^{z-1})(-a) + (\ln A)(A^z)(b)$$

The Derivative Tells Us...

$$\frac{dS}{dE} = (zA^{z-1})(-a) + (\ln A)(A^z)(b)$$

Small E values (close to 0):

$A \approx A_{\text{max}}$

$z \approx 0$: $\frac{dS}{dE} = (\ln A)(b)$

$\frac{dS}{dE} > 0$

Large E values resulting in small A values (close to 0, fractional):

$z \approx 1$

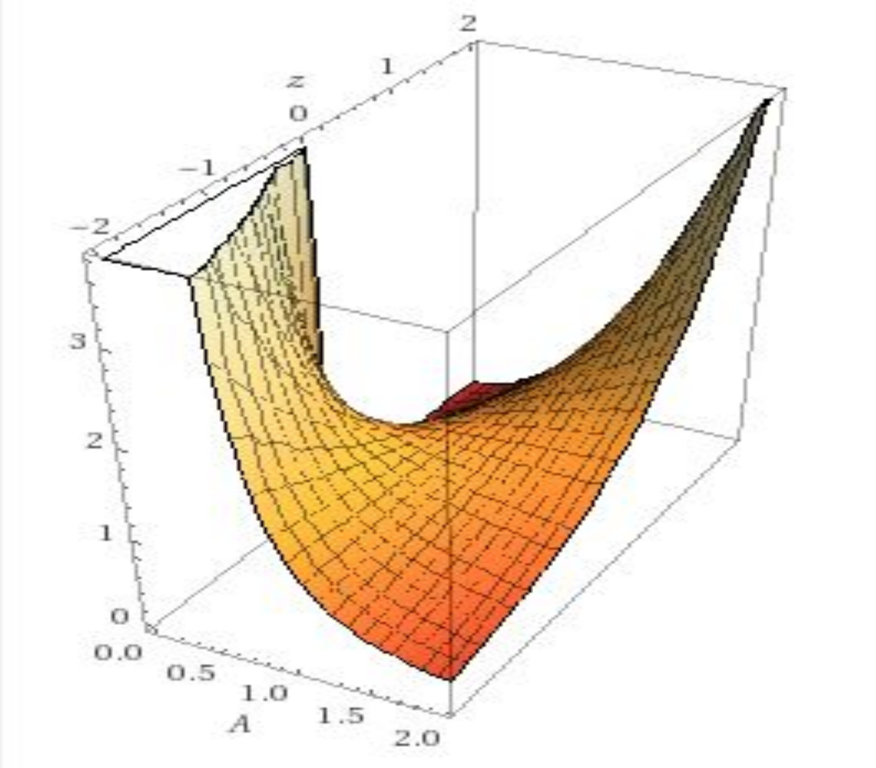
$\frac{dS}{dE} = -a + (\ln A)(A)(b)$

$\frac{dS}{dE} < 0$

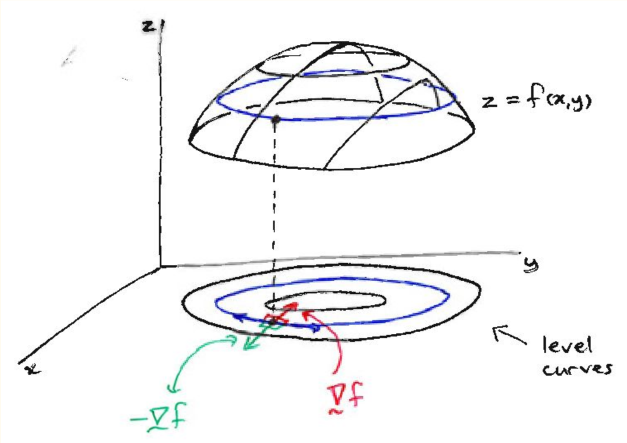
S maximum occurs between small and large E values.

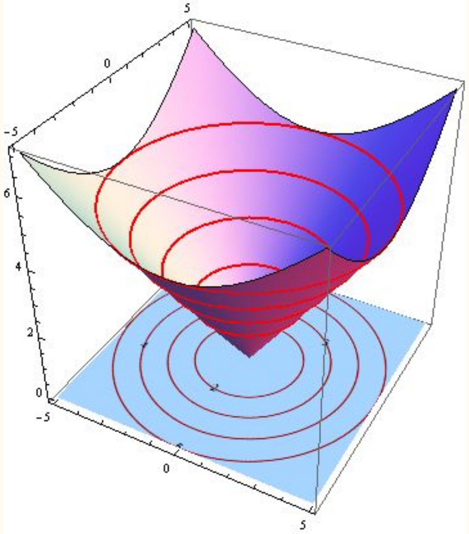
3D Graph: $S = A^z$

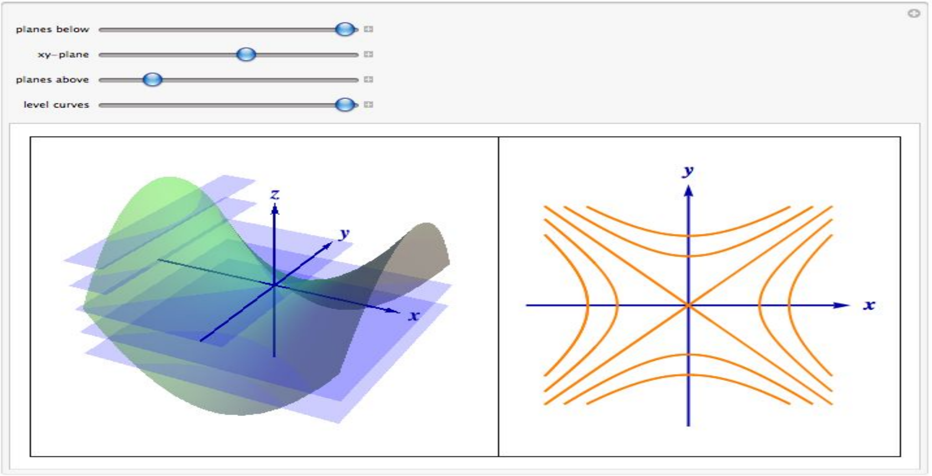
- Use Wolfram Alpha to plot surface S as a function of A and z
- Points on surface: (z, A, S)
- Values for diversity given by the "height" of each point



Level Curves







2D Graph: $S = A^z$

- Choose constant S values
- Level curves on zA-plane
- Algebra: get A in terms of z

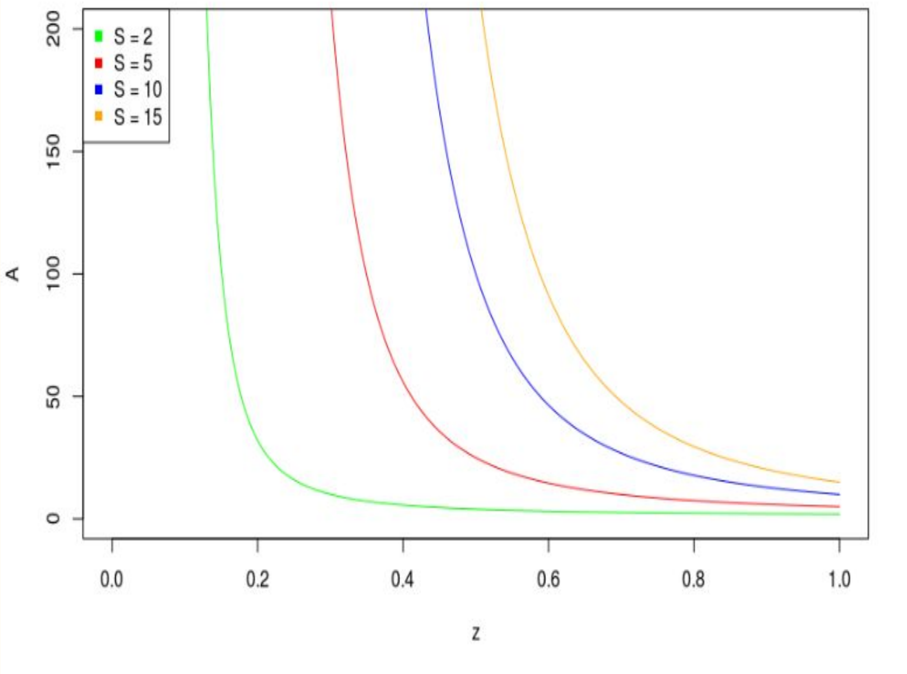
$$S = A^z$$

$$(S)^{1/z} = (A^z)^{1/z}$$

$$A = S^{1/z}$$

- Let $S = 2, 5, 10, 15$

Abundance (A) vs. Niche Space (z)

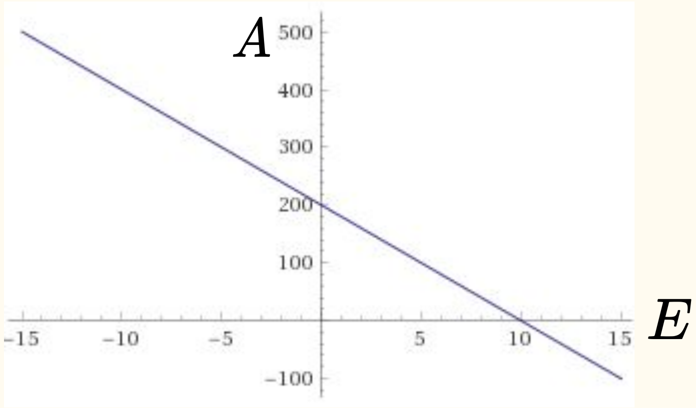


Abundance and Niche Space

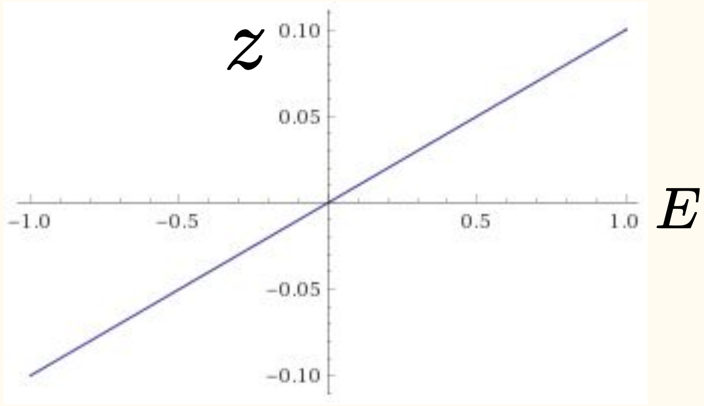
Choose a set of parameter values:

- $A_{\max} = 200$
- $a = 20$
- $b = 0.1$

$$A = 200 - 20E$$



$$z = 0.1E$$



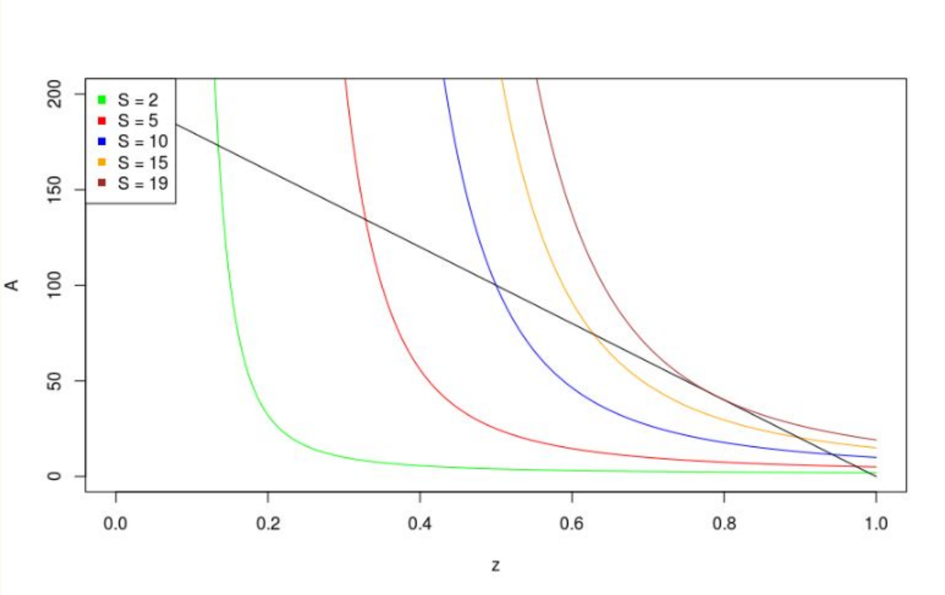
Level Curves Revisited

$A = 200 - 20E$

$z = 0.1E$

- Relationship between A and z desired: algebra yields $A = 200 - 200z$
- Superimpose line $A(z)$ over level curves

Abundance (A) vs. Niche Space (z)



Prediction: maximum of 19 species

Solution

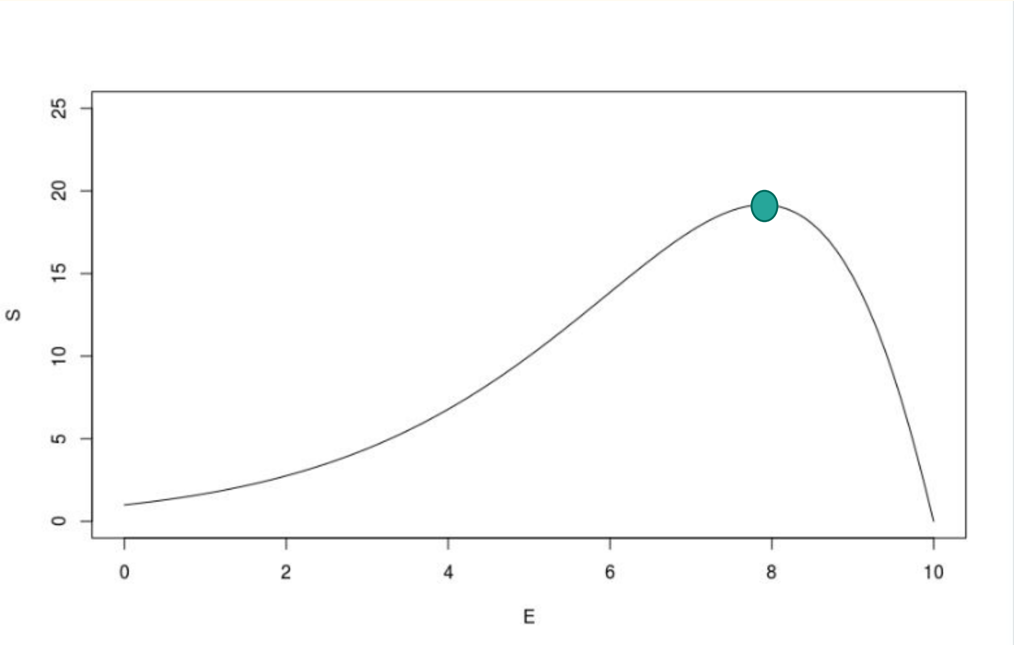
$$S = A^z$$

$$S = (200 - 20E)^{0.1E}$$

$$\boxed{\begin{aligned} S &= 19.15 \\ E &= 7.89 \end{aligned}}$$

$$A = 42.20 \quad z = 0.79$$

Diversity (S) vs. Disturbance (E)



Prediction confirmed: maximum occurs at (7.89, 19.15)

Other Curves

Sets of parameter values:

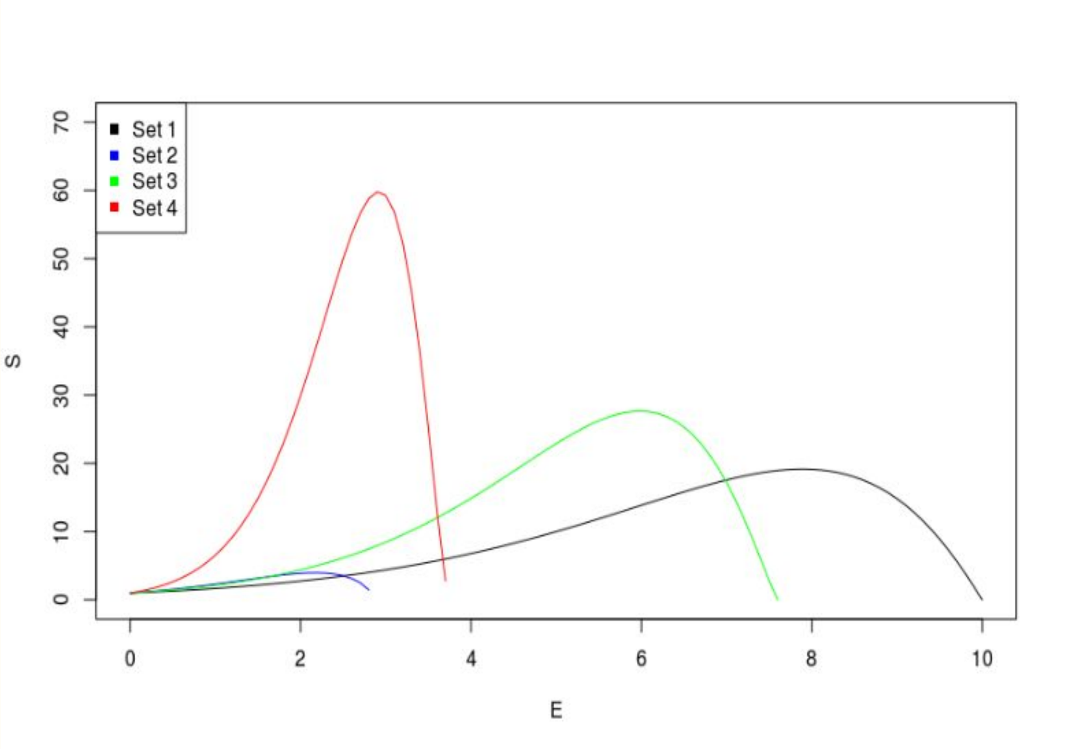
1) $A_{\max} = 200,\ a = 20,\ b = 0.1$

2) $A_{\max} = 100,\ a = 35,\ b = 0.2$

3) $A_{\max} = 190,\ a = 25,\ b = 0.15$

4) $A_{\max} = 150,\ a = 40,\ b = 0.4$

Diversity (S) vs. Disturbance (E)



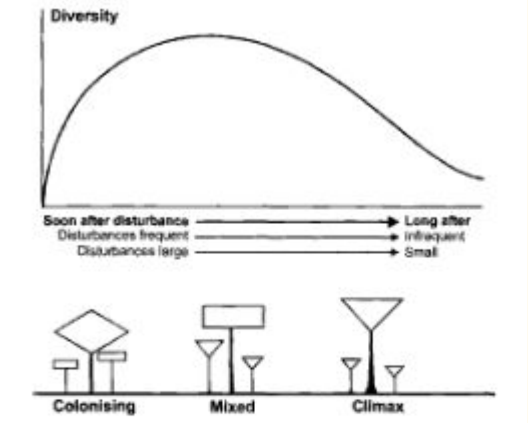
Further Investigation

What do ecologists have to say about the IDH? Check out some journal articles from studies done on three ecosystems:

- Budongo Forest (Uganda)
- Solomon Islands
- The Caribbean

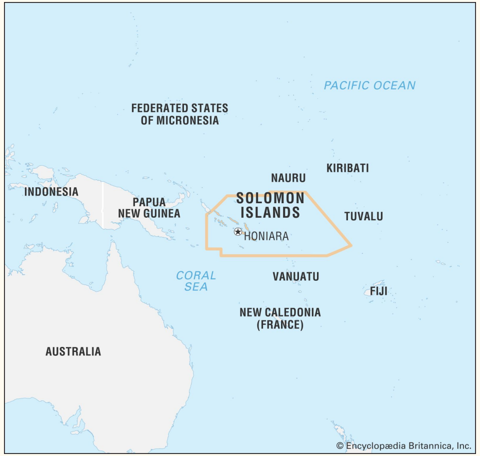
Budongo Forest (Uganda)

- Disturbance helps early-stage (disturbance-dependent) species in late-stage formations and can increase diversity
- Excessive disturbance is intolerable for some species, so diversity decreases
- “Best evidence” for concept (Connell)



Solomon Islands

- Tree species adaptation (regeneration)
- Kolombangara Island
- Undisturbed west coast: 122 species
- Cyclone-disturbed north coast: 129 species



The Caribbean

- Mangrove forests
- Dominant species: *Rhizophora mangle*, *Avicennia germinans*, *Laguncularia racemosa*
- Diversity depends on species dominance, which depends on frequency of destruction events (hurricanes & tropical storms)
- Most dominant species (*Laguncularia racemosa*): “pioneer-like”

Conclusions

- Function S and derivative $\frac{dS}{dE}$ verify the IDH
- Three journal articles provide some evidence in support of the IDH
- However, the IDH is merely a hypothesis and cannot be proven
- Real-world ecosystems are incredibly complex: the IDH does not apply in all cases
- Mathematical ecology is an extensive field: we have much more to explore

Sources

- Guided Project 67: Ecological Diversity. Roman Dial, Environmental Science Department, Alaska Pacific University, www.pearson.com.
- *Defining and Defending Connell's Intermediate Disturbance Hypothesis: A Response to Fox* (Douglas Sheil and David F.R.P. Burslem).
- *Tropical Forest Diversity, Environmental Change and Species Augmentation: After the Intermediate Disturbance Hypothesis* (Douglas Sheil).
- *Intermediate-Disturbance Hypothesis* (John F. Fox and J. H. Connell).
- *Testing the Intermediate Disturbance Hypothesis in Species-Poor Systems: A Simulation Experiment for Mangrove Forests* (Cyril Piou, Uta Berger, Hanno Hildenbrandt and Ilka C. Feller).