Homework3

October 28, 2020

#Problem 1

[58]:
```
import os
import pandas as pd
import numpy as np
import scipy
from scipy import stats
from scipy.stats import norm
from sklearn.linear_model import Ridge
from matplotlib import pyplot as plt
```

#Part a

[59]:
```
offset = 2

def toydata(n):
    class_1_size = n/2
    class_2_size = n - class_1_size
    classes = {}

    for example1,example2 in␣
 ↪zip(range(int(class_1_size)),range(int(class_2_size))):
        #draw random points from normal distribution
        point1 = np.random.normal(loc=[0,0], scale=[1,1])
        (x1,y1) = point1[0],point1[1]
        classes[(x1,y1)] = -1 #put into class1
        point2 = np.random.normal(loc=[offset,offset], scale=[1,1])
        (x2,y2) = point2[0],point2[1]
        classes[(x2,y2)] = 1 #put into class2

    return classes
```

#Part b

[70]:
```
def computeybar(dat):
    prob_x_1 = scipy.stats.multivariate_normal(mean=[0,0], cov=np.identity(2))␣
 ↪#class1
```

```
    prob_x_2 = scipy.stats.multivariate_normal(mean=[offset,offset], cov=np.
↪identity(2)) #class2
    ybar = []
    prob_y_1 = 0.5
    prob_y_2 = 0.5

    for x in dat:
        p_d_f = (-prob_x_1.pdf(x)*prob_y_1 + prob_x_2.pdf(x)*prob_y_2)/
↪(prob_x_1.pdf(x)*prob_y_1 + prob_x_2.pdf(x)*prob_y_2)
        ybar.append(p_d_f)

    #print(np.array(ybar))
    y_bar = np.array(ybar)
    #print(y_bar.shape)
    return y_bar
```

#Part c

```
[62]: def computehbar(lambda_,points,num_models=25): #points as calculated from␣
↪toydata
    #generate nmodel many models (Ridge regression)
    models = []
    for mod in range(num_models):
        ridge_mod = Ridge(alpha=10**lambda_)
        models.append(ridge_mod)


    #generate n-many training sets for these n-many models
    datasets = []
    for mod in range(num_models):
        dat = toydata(500)
        datasets.append(dat)


    #train the n-many models
    for i in range(num_models):
        X = np.array(list(datasets[i].keys()))
        y = np.array(list(datasets[i].values()))
        models[i].fit(X=X,y=y)

    classifications = []
    for mod in models:
        m_ = np.array(list(points.keys()))
        m_pred = mod.predict(m_)
        classifications.append(m_pred)
    #25 models trained over each of the 500 points
    #across rows, different predictions for the same point

    #mean of the classifications, for each of the 500 points
    #for each coordinate, in 500x1, it is the 25 model predictions for that X␣
↪(where the X is the point in question)
    hbar = np.mean(classifications, axis=0)
    return hbar,classifications
```

#Part d

[63]:
```
def computevariance(hbar,classifications):
    #25 Hds, applied to all 25 vectors

    sub_var = []
    for m in classifications:
        sub = m - hbar
        sub_var.append(sub**2)
    variance = np.mean(sub_var)
    return variance
```

#Part e

[64]:
```
def computeBias(hbar,ybar):
    bias = np.mean((hbar-ybar)**2)
    return np.mean(bias)
```

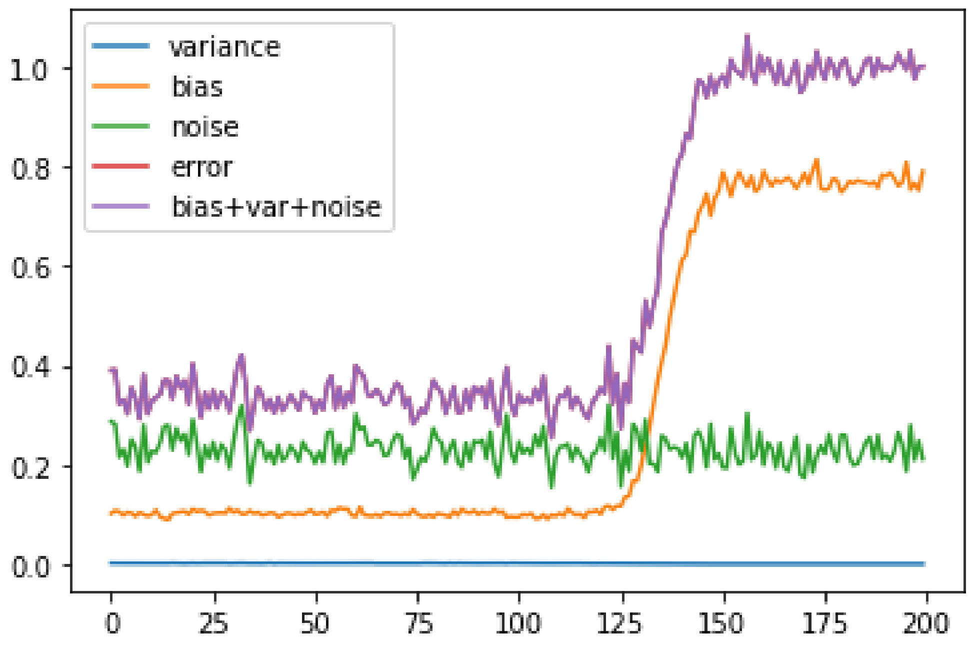
#Part f

[65]:
```
def computeNoise(ybar,Y):
    noise = np.mean((ybar-Y)**2)
    return noise
```

#Part g

[68]:
```
def biasvariancedemo():
    #plot: variance, bias, noise, test error, bias+variance+noise

    var_list = []
    bias_list = []
    noise_list = []
    error_list = []

    for lambda_ in np.arange(-10,10,0.1):
        dat = toydata(500)
        X = np.array(list(dat.keys()))
        Y = np.array(list(dat.values()))
        #get ybar
        ybar_ = computeybar(dat)
        #get hbar
        hbar_,classifications_ = computehbar(lambda_,dat)
        #get variance
        var = computevariance(hbar_,classifications_)
        var_list.append(var)
        #get bias
        bias_ = computeBias(hbar_,ybar_)
        bias_list.append(bias_)
        #get noise
        noise_ = computeNoise(ybar_,Y)
        noise_list.append(noise_)
        #get error
        error = var + bias_ + noise_
        error_list.append(error)
    return var_list,bias_list,noise_list,error_list
```

[71]:
```
v,b,n,e = biasvariancedemo()

b_v = np.add(b,v)
b_v_n = np.add(b_v,n)
plt.plot(v,label='variance')
plt.plot(b,label='bias')
plt.plot(n,label='noise')
plt.plot(e,label='error')
plt.plot(b_v_n,label='bias+var+noise')
plt.legend()
plt.show()
```



It’s hard to tell, but actually the variance IS decreasing.

#Part 2

[12]:
```
#imports

import pandas as pd
import numpy as np
import matplotlib
from sklearn.model_selection import train_test_split
from matplotlib import pyplot as plt
import math
```

[13]:
```
#create dataset of points
#f=open('/Users/benso/Desktop/Projects/MachineLearning/Assignment3/hw3_data2.
↪txt',"r")
f=open('/Users/lvbenson/Research_Projects/MachineLearning/Assignment3/hw3_data2.
↪txt',"r")

lines = f.readlines()
result=[]
for x in lines:
    result.append(x.split(',')[0])
f.close()

target = []
```

```
for sign in result:
    if sign == '+':
        target.append(1)
    else:
        target.append(-1)

#f=open('/Users/benso/Desktop/Projects/MachineLearning/Assignment3/hw3_data2.
 ↪txt',"r")
f=open('/Users/lvbenson/Research_Projects/MachineLearning/Assignment3/hw3_data2.
 ↪txt',"r")
lines1=f.readlines()
result2=[]
for x in lines1:
    result2.append(x.split(',')[1])
f.close()

x_coords = []
for coord in result2:
    x_coords.append(coord)

#f=open('/Users/benso/Desktop/Projects/MachineLearning/Assignment3/hw3_data2.
 ↪txt',"r")
f=open('/Users/lvbenson/Research_Projects/MachineLearning/Assignment3/hw3_data2.
 ↪txt',"r")
lines2=f.readlines()
result3=[]
for x in lines2:
    x_ = x.split(',')[2].rstrip('\n')
    result3.append(x_)
y_coords = []
for coord in result3:
    y_coords.append(coord)
data = []
for x,y in zip(x_coords,y_coords):
    data.append([x,y])
f.close()

X = np.array(data,dtype=float)
Y = np.array(target,dtype=float)
```

[4]:
```
#create test, train vectors
x_train,x_test,y_train,y_test = train_test_split(X,Y)
```

There are a ton of overlapping points, so it seems like using an rbf kernel, which is good at defining soft margins, so to avoid overfitting or underfitting, I'm choosing this kernel.

[14]:
```
def rbf_kernel(X,gamma):
    K = np.zeros((X.shape[0],X.shape[0]))
    for i in range(X.shape[0]):
        for j in range(X.shape[0]):
            K[i,j] = np.exp(-gamma*np.linalg.norm(X[i]-X[j])**2) # rbf kernel␣
 ↪alg, kinda like euclid distance. All this function does it calculate the␣
 ↪kernel, to be used later.
    return K
```

In reference to this document: http://www.robots.ox.ac.uk/~az/lectures/ml/lect3.pdf, and this document: https://towardsdatascience.com/optimization-loss-function-under-the-hood-part-iii-5dff33fa015d, as well as theoretical conversations with Daniel (where we discussed the sub-gradient approach to determining the change in hyperplane) the following is an implementation of accounting for the change in our w vector paired with our gradient descent. I call it 'wiggle' because the point of is to determine how much to change the hyperplane until it has finally correctly classified as much as possible.

[15]:
```
def Wiggle(w,K,X,Y,reg_term=1,C=1): #K is 2700 x 2700 matrix (for training␣
 ↪data), x is 2700x2, w is 2700x1 (training data)
    #Hinge loss determinism. This finds how much the w vector changes to get␣
 ↪closer and closer to complete classification.
    L_list = []
    size_y = range(len(y))
    for i in size_y:
        if -Y[i]*classification(i,w,Y,K)<1: #condition for needing to change␣
 ↪the w vector
            L_list.append(i)

    for i in size_y:
        for j in L_list:
            sub_grad_help = sum([Y[j]*K[j][i]]) #calculating the sub-gradient,␣
 ↪depending on our hinge loss list
        sub_grad_list = [(-Y[i]*sub_grad_help)]
        w_change = np.array(sub_grad_list) #an array of places and values for␣
 ↪which we need to change our w vector, to update the classifications

    return w_change
```

[16]:
```
#this is our overall classification function that we'll use to check to see if␣
 ↪the point is classified as positive or negative.
def classification(index_x,w,Y,K):
    size_y = range(len(Y))
    class_func = 0
    for i in size_y:
        k_calc = K[index_x][i]*w[i]
        class_func = class_func + Y[i]*k_calc
    return class_func
```

[17]:
```
def SVM(X,Y,K,epochs=30,learn_rate=1,reg_term=1,C=1,Gamma=1):
    #initialize random weights
    w = np.zeros(X.shape[0])
    for epoch in range(epochs):
        w_change = Wiggle(w,K,X,Y,reg_term,C) #calculates the wiggle
        w = np.subtract(w,w_change) #account for the wiggle in the w vector by␣
 ↪subtracting it from our prior w vector

    return w #gives us our new w vector, accounting for the required update in␣
 ↪values
```

[18]:
```
def solve_classifier(X,Y,epochs=30,learn_rate=1,reg_term=1,C=1,Gamma=1):
    K = rbf_kernel(X,Gamma) #create our kernel
    w_vector = SVM(X,Y,K,epochs,learn_rate) #this is what will return our w␣
 ↪vector, to then be wiggled and classified

    classifications = []
    for i in range(len(X)):
        class_ = classification(i,w_vector,Y,K) #get classifications for our␣
 ↪training data
        classifications.append(class_)

    return classifications,w_vector

classes,w = solve_classifier(x_train,y_train)
correct_classify = []
incorrect_classify = []
```

[19]:
```
#check out the training data accuracy

for i,y in zip(classes,y_train):
    if i > 0 and y == 1:
        correct_classify.append(1)
    elif i < 0 and y == -1:
        correct_classify.append(1)
    else:
        incorrect_classify.append(1)
total_correct = (len(correct_classify)) / (len(correct_classify) +␣
 ↪len(incorrect_classify))
print(total_correct,' :accuracy of training data')
```

```
0.952962962962963  :accuracy of training data
```

[20]:
```
#check out the test data accuracy. only calculate kernel and classes, use the w␣
 ↪that is already calculated from training.
K = rbf_kernel(x_test,1)
classifications_test = []
```

```
for i in range(len(x_test)):
    class_1 = classification(i,w,y_test,K)
    classifications_test.append(class_1)

correct_classify_test = []
incorrect_classify_test = []

#check out the training data accuracy

for i,y in zip(classifications_test,y_test):
    if i > 0 and y == 1:
        correct_classify_test.append(1)
    elif i < 0 and y == -1:
        correct_classify_test.append(1)
    else:
        incorrect_classify_test.append(1)
total_correct_test = (len(correct_classify_test)) / (len(correct_classify_test)␣
 ↪+ len(incorrect_classify_test))
print(total_correct_test,' :accuracy of test data')
```

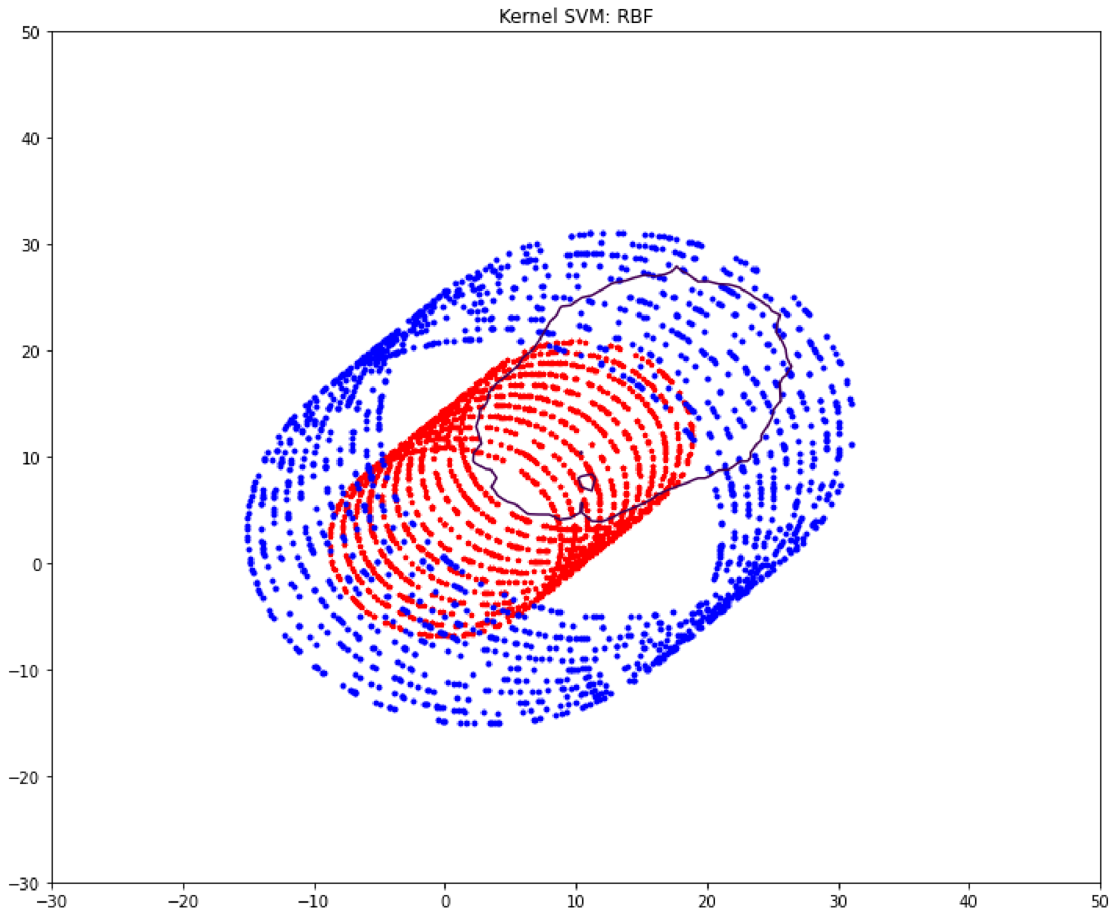
```
0.9655555555555555  :accuracy of test data
```

Our training and testing data perform almost identically, which is great! The classifier is probably working. Now let's plot it. Because I opted for a gaussian kernel, and the hyperplane is really long, I couldn't use normal plotting methods. I found this implementation online which I adapted for my plotting purposes: https://jakevdp.github.io/PythonDataScienceHandbook/05.07-support-vector-machines.html

It showed me how to use contourplot and a meshgrid to plot the hyperplane.

[32]:
```
#time to plot.

plot_x,plot_y = np.meshgrid(np.linspace(-30,50,100),np.linspace(-30,50,100))
```

[37]:
```
xy = np.vstack([xx.ravel(),yy.ravel()]).T

def plot_classification(pt,w=w,gamma=1,X=x_train,Y=y_train):
    point_func = 0
    for i in range(len(X)):
        new_k = np.exp(-gamma)
        new_k = np.exp(-gamma*np.linalg.norm(pt-X[i])**2)
        new_k *= w[i]*Y[i]
        point_func += new_k
    return point_func

z = []
for example in xy:
    z_p = plot_classification(pt=example)
    z.append(z_p)
new_z = np.array(z)

plt.figure(figsize=(12,10))
for d, sample in enumerate(X):
    if Y[d] == -1:
        plt.scatter(sample[0], sample[1], s=4, marker='o',␣
 ↪c='blue',linewidths=2)
    else:
        plt.scatter(sample[0], sample[1], s=4, marker='*', c='red',linewidths=2)
plt.contour(plot_x,plot_y,(new_z).reshape(plot_x.
 ↪shape),levels=[0],color='black')
plt.title('Kernel SVM: RBF')
plt.show()
```



It’s kind of hard to see, but the hyperplane is “on top” of the red ring.