In this section, you will:
- View vectors geometrically.
- Find magnitude and direction.
- Perform vector addition and scalar multiplication.
- Find the component form of a vector.
- Find the unit vector in the direction of *v*.
- Perform operations with vectors in terms of *i* and *j*.
- Find the dot product of two vectors.

8.8 VECTORS

An airplane is flying at an airspeed of 200 miles per hour headed on a SE bearing of 140°. A north wind (from north to south) is blowing at 16.2 miles per hour, as shown in **Figure 1**. What are the ground speed and actual bearing of the plane?
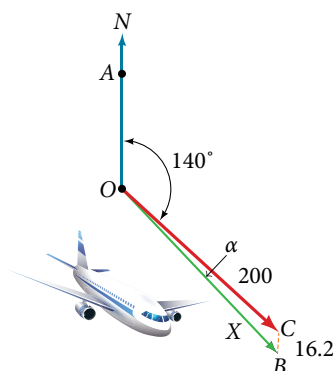

Figure 1

Ground speed refers to the speed of a plane relative to the ground. Airspeed refers to the speed a plane can travel relative to its surrounding air mass. These two quantities are not the same because of the effect of wind. In an earlier section, we used triangles to solve a similar problem involving the movement of boats. Later in this section, we will find the airplane's groundspeed and bearing, while investigating another approach to problems of this type. First, however, let's examine the basics of vectors.

A Geometric View of Vectors

A **vector** is a specific quantity drawn as a line segment with an arrowhead at one end. It has an **initial point**, where it begins, and a **terminal point**, where it ends. A vector is defined by its **magnitude**, or the length of the line, and its direction, indicated by an arrowhead at the terminal point. Thus, a vector is a directed line segment. There are various symbols that distinguish vectors from other quantities:

- Lower case, boldfaced type, with or without an arrow on top such as v, u, w, $\vec{v}$, $\vec{u}$, $\vec{w}$.
- Given initial point P and terminal point Q, a vector can be represented as $\overrightarrow{PQ}$. The arrowhead on top is what indicates that it is not just a line, but a directed line segment.
- Given an initial point of $(0, 0)$ and terminal point (a, b), a vector may be represented as $\langle a, b \rangle$.

This last symbol $\langle a, b \rangle$ has special significance. It is called the **standard position**. The position vector has an initial point $(0, 0)$ and a terminal point $\langle a, b \rangle$. To change any vector into the position vector, we think about the change in the x-coordinates and the change in the y-coordinates. Thus, if the initial point of a vector $\overrightarrow{CD}$ is $C(x_1, y_1)$ and the terminal point is $D(x_2, y_2)$, then the position vector is found by calculating

$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1 \rangle$$
$$= \langle a, b \rangle$$

In **Figure 2**, we see the original vector $\overrightarrow{CD}$ and the position vector $\overrightarrow{AB}$.
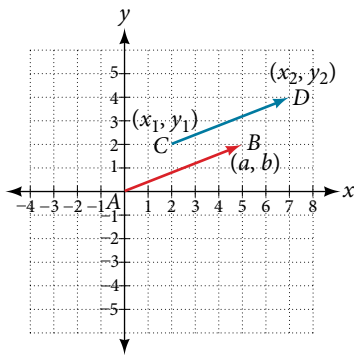

Figure 2

---

properties of vectors

A vector is a directed line segment with an initial point and a terminal point. Vectors are identified by magnitude, or the length of the line, and direction, represented by the arrowhead pointing toward the terminal point. The position vector has an initial point at $(0, 0)$ and is identified by its terminal point $\langle a, b \rangle$.

Example 1   **Find the Position Vector**

Consider the vector whose initial point is $P(2, 3)$ and terminal point is $Q(6, 4)$. Find the position vector.

Solution The position vector is found by subtracting one x-coordinate from the other x-coordinate, and one y-coordinate from the other y-coordinate. Thus

$$v = \langle 6 - 2, 4 - 3 \rangle$$
$$= \langle 4, 1 \rangle$$

The position vector begins at $(0, 0)$ and terminates at $(4, 1)$. The graphs of both vectors are shown in **Figure 3**.

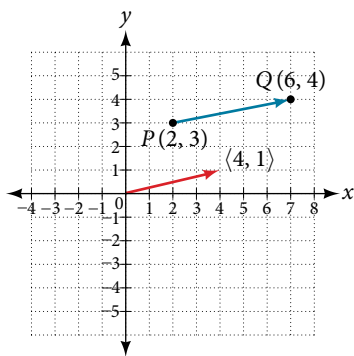
Figure 3

We see that the position vector is $\langle 4, 1 \rangle$.

Example 2   **Drawing a Vector with the Given Criteria and Its Equivalent Position Vector**

Find the position vector given that vector v has an initial point at $(-3, 2)$ and a terminal point at $(4, 5)$, then graph both vectors in the same plane.

Solution The position vector is found using the following calculation:

$$v = \langle 4 - (- 3), 5 - 2 \rangle$$
$$= \langle 7, 3 \rangle$$

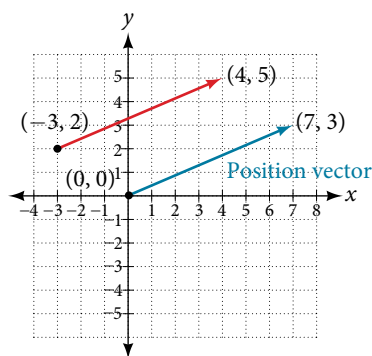Thus, the position vector begins at $(0, 0)$ and terminates at $(7, 3)$. See **Figure 4**.

Figure 4

Draw a vector v that connects from the origin to the point $(3, 5)$.

Finding Magnitude and Direction

To work with a vector, we need to be able to find its magnitude and its direction. We find its magnitude using the Pythagorean Theorem or the distance formula, and we find its direction using the inverse tangent function.

> ***magnitude and direction of a vector***
>
> Given a position vector $v = \langle a, b \rangle$, the magnitude is found by $|v| = \sqrt{a^2 + b^2}$. The direction is equal to the angle formed with the x-axis, or with the y-axis, depending on the application. For a position vector, the direction is found by $\tan \theta = \left(\frac{b}{a}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$, as illustrated in **Figure 5.**
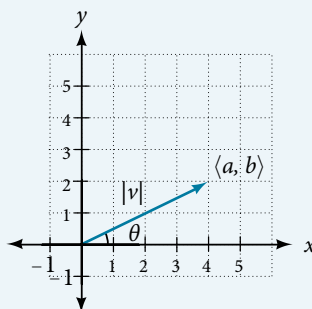>
>
>
> **Figure 5**
>
> Two vectors v and u are considered equal if they have the same magnitude and the same direction. Additionally, if both vectors have the same position vector, they are equal.

Example 3 **Finding the Magnitude and Direction of a Vector**

Find the magnitude and direction of the vector with initial point $P(-8, 1)$ and terminal point $Q(-2, -5)$. Draw the vector.

Solution First, find the position vector.

$$u = \langle -2, -(-8), -5-1 \rangle$$
$$= \langle 6, -6 \rangle$$

We use the Pythagorean Theorem to find the magnitude.

$$|u| = \sqrt{(6)^2 + (-6)^2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

The direction is given as

$$\tan\theta = \frac{-6}{6} = -1 \Rightarrow \theta = \tan^{-1}(-1)$$
$$= -45°$$

However, the angle terminates in the fourth quadrant, so we add 360° to obtain a positive angle. Thus, $-45° + 360° = 315°$. See **Figure 6**.
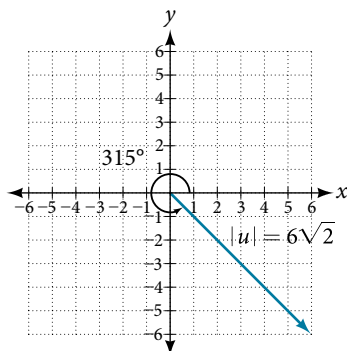

Figure 6

Example 4   Showing That Two Vectors Are Equal

Show that vector v with initial point at $(5, -3)$ and terminal point at $(-1, 2)$ is equal to vector u with initial point at $(-1, -3)$ and terminal point at $(-7, 2)$. Draw the position vector on the same grid as v and u. Next, find the magnitude and direction of each vector.

Solution   As shown in **Figure 7**, draw the vector v starting at initial $(5, -3)$ and terminal point $(-1, 2)$. Draw the vector u with initial point $(-1, -3)$ and terminal point $(-7, 2)$. Find the standard position for each.

Next, find and sketch the position vector for v and u. We have

$$v = \langle -1 - 5, 2 - (-3) \rangle$$
$$= \langle -6, 5 \rangle$$
$$u = \langle -7 - (-1), 2 - (-3) \rangle$$
$$= \langle -6, 5 \rangle$$

Since the position vectors are the same, v and u are the same.

An alternative way to check for vector equality is to show that the magnitude and direction are the same for both vectors. To show that the magnitudes are equal, use the Pythagorean Theorem.

$$|v| = \sqrt{(-1 - 5)^2 + (2 - (-3))^2}$$
$$= \sqrt{(-6)^2 + (5)^2}$$
$$= \sqrt{36 + 25}$$
$$= \sqrt{61}$$
$$|u| = \sqrt{(-7 - (-1))^2 + (2 - (-3))^2}$$
$$= \sqrt{(-6)^2 + (5)^2}$$
$$= \sqrt{36 + 25}$$
$$= \sqrt{61}$$

As the magnitudes are equal, we now need to verify the direction. Using the tangent function with the position vector gives

$$\tan\theta = -\frac{5}{6} \Rightarrow \theta = \tan^{-1}\left(-\frac{5}{6}\right)$$
$$= -39.8°$$

However, we can see that the position vector terminates in the second quadrant, so we add 180°. Thus, the direction is $-39.8° + 180° = 140.2°$
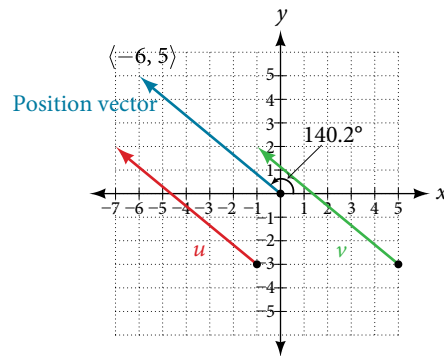
Figure 7

Performing Vector Addition and Scalar Multiplication

Now that we understand the properties of vectors, we can perform operations involving them. While it is convenient to think of the vector $u = \langle x, y \rangle$ as an arrow or directed line segment from the origin to the point (x, y), vectors can be situated anywhere in the plane. The sum of two vectors u and v, or **vector addition**, produces a third vector $u + v$, the **resultant** vector.

To find $u + v$, we first draw the vector u, and from the terminal end of u, we drawn the vector v. In other words, we have the initial point of v meet the terminal end of u. This position corresponds to the notion that we move along the first vector and then, from its terminal point, we move along the second vector. The sum $u + v$ is the resultant vector because it results from addition or subtraction of two vectors. The resultant vector travels directly from the beginning of u to the end of v in a straight path, as shown in **Figure 8**.
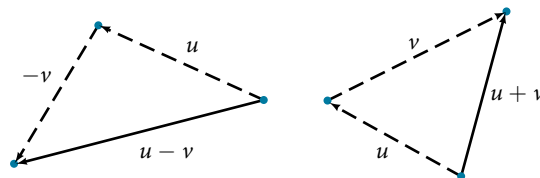

Figure 8

Vector subtraction is similar to vector addition. To find $u - v$, view it as $u + (-v)$. Adding $-v$ is reversing direction of v and adding it to the end of u. The new vector begins at the start of u and stops at the end point of $-v$. See **Figure 9** for a visual that compares vector addition and vector subtraction using parallelograms.

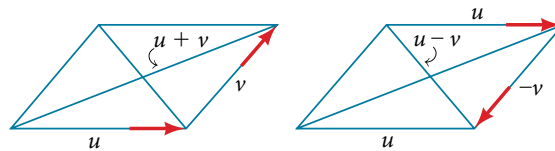Figure 9

Example 5 Adding and Subtracting Vectors

Given $u = \langle 3, -2 \rangle$ and $v = \langle -1, 4 \rangle$, find two new vectors $u + v$, and $u - v$.

Solution   To find the sum of two vectors, we add the components. Thus,

$$u + v = \langle 3, -2 \rangle + \langle -1, 4 \rangle$$
$$= \langle 3 + (-1), -2 + 4 \rangle$$
$$= \langle 2, 2 \rangle$$

See **Figure 10(a)**.

To find the difference of two vectors, add the negative components of v to u. Thus,

$$u + (-v) = \langle 3, -2 \rangle + \langle 1, -4 \rangle$$
$$= \langle 3 + 1, -2 + (-4) \rangle$$
$$= \langle 4, -6 \rangle$$
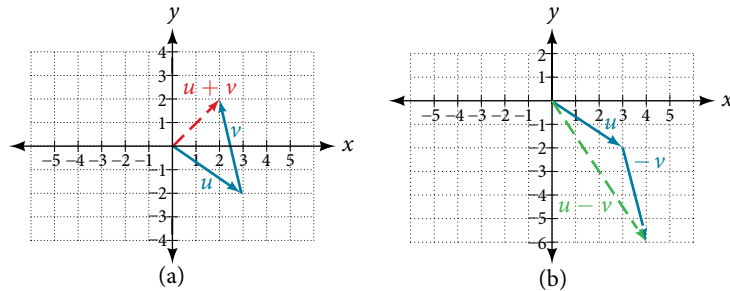
See **Figure 10(b)**.

Figure 10 (a) Sum of two vectors (b) Difference of two vectors

Multiplying By a Scalar

While adding and subtracting vectors gives us a new vector with a different magnitude and direction, the process of multiplying a vector by a **scalar**, a constant, changes only the magnitude of the vector or the length of the line. Scalar multiplication has no effect on the direction unless the scalar is negative, in which case the direction of the resulting vector is opposite the direction of the original vector.

> *scalar multiplication*
> **Scalar multiplication** involves the product of a vector and a scalar. Each component of the vector is multiplied by the scalar. Thus, to multiply $v = \langle a, b \rangle$ by k, we have
> $$kv = \langle ka, kb \rangle$$
> Only the magnitude changes, unless k is negative, and then the vector reverses direction.

Example 6   Performing Scalar Multiplication

Given vector $v = \langle 3, 1 \rangle$, find $3v$, $\frac{1}{2}v$, and $-v$.

Solution   See **Figure 11** for a geometric interpretation. If $v = \langle 3, 1 \rangle$, then

$$3v = \langle 3 \cdot 3, 3 \cdot 1 \rangle$$
$$= \langle 9, 3 \rangle$$

$$\frac{1}{2}v = \left\langle \frac{1}{2} \cdot 3, \frac{1}{2} \cdot 1 \right\rangle$$

$$= \left\langle \frac{3}{2}, \frac{1}{2} \right\rangle$$
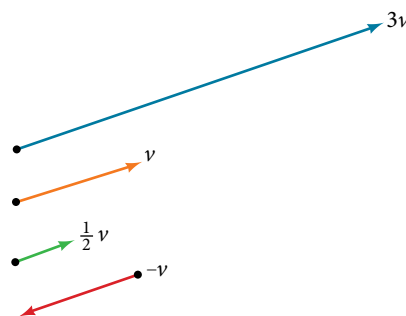
$$-v = \langle -3, -1 \rangle$$

Figure 11

Analysis   Notice that the vector $3v$ is three times the length of v, $\frac{1}{2}v$ is half the length of v, and $-v$ is the same length of v, but in the opposite direction.

Find the scalar multiple $3u$ given $u = \langle 5, 4 \rangle$.

Example 7   Using Vector Addition and Scalar Multiplication to Find a New Vector

Given $u = \langle 3, -2 \rangle$ and $v = \langle -1, 4 \rangle$, find a new vector $w = 3u + 2v$.

Solution   First, we must multiply each vector by the scalar.

$$3u = 3 \langle 3, -2 \rangle$$
$$= \langle 9, -6 \rangle$$
$$2v = 2 \langle -1, 4 \rangle$$
$$= \langle -2, 8 \rangle$$

Then, add the two together.

$$w = 3u + 2v$$
$$= \langle 9, -6 \rangle + \langle -2, 8 \rangle$$
$$= \langle 9 - 2, -6 + 8 \rangle$$
$$= \langle 7, 2 \rangle$$

So, $w = \langle 7, 2 \rangle$.

Finding Component Form

In some applications involving vectors, it is helpful for us to be able to break a vector down into its components. Vectors are comprised of two components: the horizontal component is the x direction, and the vertical component is the y direction. For example, we can see in the graph in **Figure 12** that the position vector $\langle 2, 3 \rangle$ comes from adding the vectors v_1 and v_2. We have v_1 with initial point $(0, 0)$ and terminal point $(2, 0)$.

$$v_1 = \langle 2 - 0, 0 - 0 \rangle$$
$$= \langle 2, 0 \rangle$$

We also have v_2 with initial point $(0, 0)$ and terminal point $(0, 3)$.

$$v_2 = \langle 0 - 0, 3 - 0 \rangle$$
$$= \langle 0, 3 \rangle$$

Therefore, the position vector is

$$v = \langle 2 + 0, 3 + 0 \rangle$$
$$= \langle 2, 3 \rangle$$

Using the Pythagorean Theorem, the magnitude of v_1 is 2, and the magnitude of v_2 is 3. To find the magnitude of v, use the formula with the position vector.

$$|v| = \sqrt{|v_1|^2 + |v_2|^2}$$
$$= \sqrt{2^2 + 3^2}$$
$$= \sqrt{13}$$

The magnitude of v is $\sqrt{13}$. To find the direction, we use the tangent function $\tan \theta = \dfrac{y}{x}$.

$$\tan \theta = \frac{v_2}{v_1}$$
$$\tan \theta = \frac{3}{2}$$
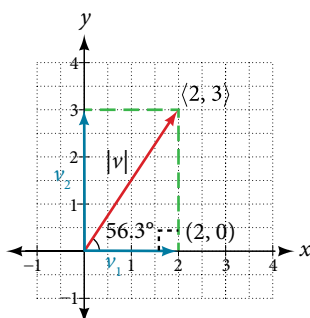$$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56.3°$$

Figure 12

Thus, the magnitude of v is $\sqrt{13}$ and the direction is 56.3° off the horizontal.

Example 8   Finding the Components of the Vector

Find the components of the vector v with initial point $(3, 2)$ and terminal point $(7, 4)$.

Solution   First find the standard position.

$$v = \langle 7 - 3, 4 - 2 \rangle$$
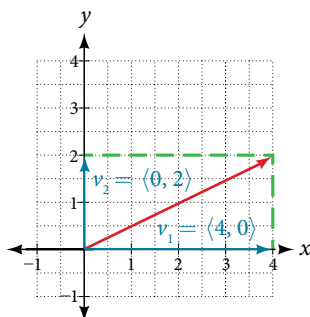$$= \langle 4, 2 \rangle$$

See the illustration in **Figure 13**.

Figure 13

The horizontal component is $v_1 = \langle 4, 0 \rangle$ and the vertical component is $v_2 = \langle 0, 2 \rangle$.

Finding the Unit Vector in the Direction of v

In addition to finding a vector's components, it is also useful in solving problems to find a vector in the same direction as the given vector, but of magnitude 1. We call a vector with a magnitude of 1 a **unit vector**. We can then preserve the direction of the original vector while simplifying calculations.

Unit vectors are defined in terms of components. The horizontal unit vector is written as $i = \langle 1, 0 \rangle$ and is directed along the positive horizontal axis. The vertical unit vector is written as $j = \langle 0, 1 \rangle$ and is directed along the positive vertical axis. See **Figure 14**.
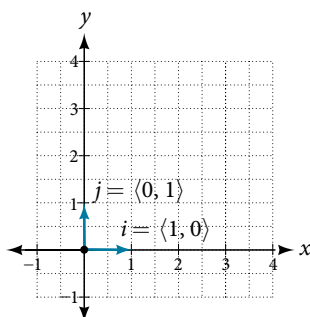

Figure 14

Example 9   Finding the Unit Vector in the Direction of *v*

Find a unit vector in the same direction as $v = \langle -5, 12 \rangle$.

Solution   First, we will find the magnitude.

$$
\begin{aligned}
|v| &= \sqrt{(-5)^2 + (12)^2} \\
&= \sqrt{25 + 144} \\
&= \sqrt{169} \\
&= 13
\end{aligned}
$$

Then we divide each component by $|v|$, which gives a unit vector in the same direction as v:

$$
\frac{v}{|v|} = -\frac{5}{13}i + \frac{12}{13}j
$$

or, in component form

$$
\frac{v}{|v|} = \left\langle -\frac{5}{13}, \frac{12}{13} \right\rangle
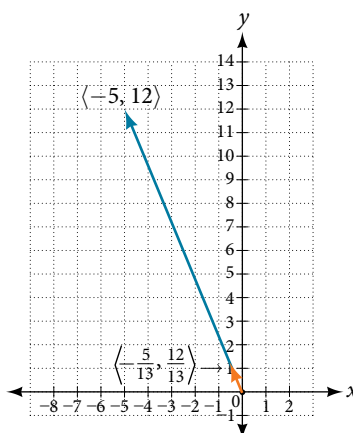$$

See **Figure 15**.

Figure 15

Verify that the magnitude of the unit vector equals 1. The magnitude of $-\dfrac{5}{13}i + \dfrac{12}{13}j$ is given as

$$
\sqrt{\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = \sqrt{\frac{25}{169} + \frac{144}{169}}
$$

$$
= \sqrt{\frac{169}{169}}
$$

$$
= 1
$$

The vector $u = \dfrac{5}{13}i + \dfrac{12}{13}j$ is the unit vector in the same direction as $v = \langle -5, 12 \rangle$.

Performing Operations with Vectors in Terms of *i* and *j*

So far, we have investigated the basics of vectors: magnitude and direction, vector addition and subtraction, scalar multiplication, the components of vectors, and the representation of vectors geometrically. Now that we are familiar with the general strategies used in working with vectors, we will represent vectors in rectangular coordinates in terms of *i* and *j*.

> **vectors in the rectangular plane**
> Given a vector v with initial point $P = (x_1, y_1)$ and terminal point $Q = (x_2, y_2)$, v is written as
> $$v = (x_2 - x_1)i + (y_1 - y_2)j$$
> The position vector from $(0, 0)$ to (a, b), where $(x_2 - x_1) = a$ and $(y_2 - y_1) = b$, is written as $v = ai + bj$. This vector sum is called a linear combination of the vectors i and j.
>
> The magnitude of $v = ai + bj$ is given as $|v| = \sqrt{a^2 + b^2}$. See **Figure 16**.
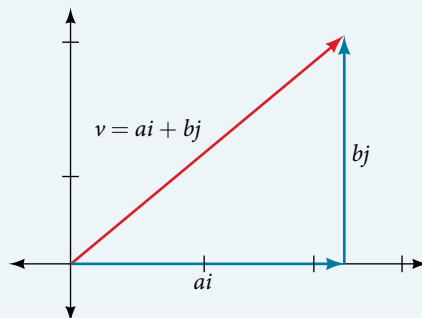>
>
>
> Figure 16

Example 10   Writing a Vector in Terms of *i* and *j*

Given a vector v with initial point $P = (2, -6)$ and terminal point $Q = (-6, 6)$, write the vector in terms of i and j.

Solution   Begin by writing the general form of the vector. Then replace the coordinates with the given values.

$$v = (x_2 - x_1)i + (y_2 - y_1)j$$
$$= (-6 - 2)i + (6 - (-6))j$$
$$= -8i + 12j$$

Example 11   Writing a Vector in Terms of *i* and *j* Using Initial and Terminal Points

Given initial point $P_1 = (-1, 3)$ and terminal point $P_2 = (2, 7)$, write the vector v in terms of i and j.

Solution   Begin by writing the general form of the vector. Then replace the coordinates with the given values.

$$v = (x_2 - x_1)i + (y_2 - y_1)j$$
$$v = (2 - (-1))i + (7 - 3)j$$
$$= 3i + 4j$$

Try It #3

Write the vector u with initial point $P = (-1, 6)$ and terminal point $Q = (7, -5)$ in terms of i and j.

Performing Operations on Vectors in Terms of *i* and *j*

When vectors are written in terms of i and j, we can carry out addition, subtraction, and scalar multiplication by performing operations on corresponding components.

> **adding and subtracting vectors in rectangular coordinates**
> Given $v = ai + bj$ and $u = ci + dj$, then
> $$v + u = (a + c)i + (b + d)j$$
> $$v - u = (a - c)i + (b - d)j$$

<u>Example 12</u> **Finding the Sum of the Vectors**

Find the sum of $v_1 = 2i - 3j$ and $v_2 = 4i + 5j$.

Solution   According to the formula, we have

$$v_1 + v_2 = (2+4)i + (-3+5)j$$
$$= 6i + 2j$$

Calculating the Component Form of a Vector: Direction

We have seen how to draw vectors according to their initial and terminal points and how to find the position vector. We have also examined notation for vectors drawn specifically in the Cartesian coordinate plane using i and j. For any of these vectors, we can calculate the magnitude. Now, we want to combine the key points, and look further at the ideas of magnitude and direction.

Calculating direction follows the same straightforward process we used for polar coordinates. We find the direction of the vector by finding the angle to the horizontal. We do this by using the basic trigonometric identities, but with $|v|$ replacing r.

---

vector components in terms of magnitude and direction
Given a position vector $v = \langle x, y \rangle$ and a direction angle θ,

$$\cos \theta = \frac{x}{|v|} \quad \text{and} \quad \sin \theta = \frac{y}{|v|}$$

$$x = |v| \cos \theta \qquad y = |v| \sin \theta$$

Thus, $v = xi + yj = |v|\cos \theta i + |v|\sin \theta j$, and magnitude is expressed as $|v| = \sqrt{x^2 + y^2}$.

<u>Example 13</u> **Writing a Vector in Terms of Magnitude and Direction**

Write a vector with length 7 at an angle of 135° to the positive x-axis in terms of magnitude and direction.

Solution   Using the conversion formulas $x = |v| \cos \theta i$ and $y = |v| \sin \theta j$, we find that

$$x = 7\cos(135°)i$$
$$= -\frac{7\sqrt{2}}{2}$$
$$y = 7\sin(135°)j$$
$$= \frac{7\sqrt{2}}{2}$$

This vector can be written as $v = 7\cos(135°)i + 7\sin(135°)j$ or simplified as

$$v = -\frac{7\sqrt{2}}{2}i + \frac{7\sqrt{2}}{2}j$$

Try It #4

A vector travels from the origin to the point (3, 5). Write the vector in terms of magnitude and direction.

Finding the Dot Product of Two Vectors

As we discussed earlier in the section, scalar multiplication involves multiplying a vector by a scalar, and the result is a vector. As we have seen, multiplying a vector by a number is called scalar multiplication. If we multiply a vector by a vector, there are two possibilities: the *dot product* and the *cross product*. We will only examine the dot product here; you may encounter the cross product in more advanced mathematics courses.

The dot product of two vectors involves multiplying two vectors together, and the result is a scalar.

> **_dot product_**
> The **dot product** of two vectors $v = \langle a, b \rangle$ and $u = \langle c, d \rangle$ is the sum of the product of the horizontal components and the product of the vertical components.
>
> $$v \cdot u = ac + bd$$
>
> To find the angle between the two vectors, use the formula below.
>
> $$\cos \theta = \frac{v}{|v|} \cdot \frac{u}{|u|}$$

Example 14 Finding the Dot Product of Two Vectors

Find the dot product of $v = \langle 5, 12 \rangle$ and $u = \langle -3, 4 \rangle$.

Solution   Using the formula, we have

$$
\begin{aligned}
v \cdot u &= \langle 5, 12 \rangle \cdot \langle -3, 4 \rangle \\
&= 5 \cdot (-3) + 12 \cdot 4 \\
&= -15 + 48 \\
&= 33
\end{aligned}
$$

Example 15 Finding the Dot Product of Two Vectors and the Angle between Them

Find the dot product of $v_1 = 5i + 2j$ and $v_2 = 3i + 7j$. Then, find the angle between the two vectors.

Solution   Finding the dot product, we multiply corresponding components.

$$
\begin{aligned}
v_1 \cdot v_2 &= \langle 5, 2 \rangle \cdot \langle 3, 7 \rangle \\
&= 5 \cdot 3 + 2 \cdot 7 \\
&= 15 + 14 \\
&= 29
\end{aligned}
$$

To find the angle between them, we use the formula $\cos \theta = \dfrac{v}{|v|} \cdot \dfrac{u}{|u|}$

$$
\frac{v}{|v|} \cdot \frac{u}{|u|} = \left\langle \frac{5}{\sqrt{29}} + \frac{2}{\sqrt{29}} \right\rangle \cdot \left\langle \frac{3}{\sqrt{58}} + \frac{7}{\sqrt{58}} \right\rangle
$$

$$
= \frac{5}{\sqrt{29}} \cdot \frac{3}{\sqrt{58}} + \frac{2}{\sqrt{29}} \cdot \frac{7}{\sqrt{58}}
$$

$$
= \frac{15}{\sqrt{1682}} + \frac{14}{\sqrt{1682}} = \frac{29}{\sqrt{1682}}
$$

$$
= 0.707107
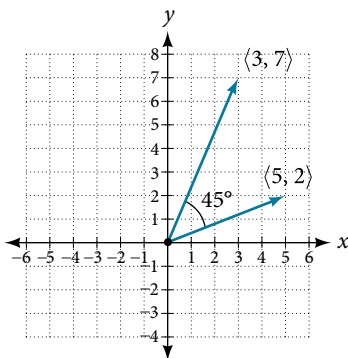$$

$$
\cos^{-1}(0.707107) = 45°
$$

See **Figure 17**.

Figure 17

Example 16 Finding the Angle between Two Vectors

Find the angle between $u = \langle -3, 4 \rangle$ and $v = \langle 5, 12 \rangle$.

Solution Using the formula, we have

$$\theta = \cos^{-1}\left(\frac{u}{|u|} \cdot \frac{v}{|v|} \right)$$

$$\left(\frac{u}{|u|} \cdot \frac{v}{|v|} \right) = \frac{-3i + 4j}{5} \cdot \frac{5i + 12j}{13}$$

$$= \left(-\frac{3}{5} \cdot \frac{5}{13} \right) + \left(\frac{4}{5} \cdot \frac{12}{13} \right)$$

$$= -\frac{15}{65} + \frac{48}{65}$$

$$= \frac{33}{65}$$

$$\theta = \cos^{-1}\left(\frac{33}{65} \right)$$
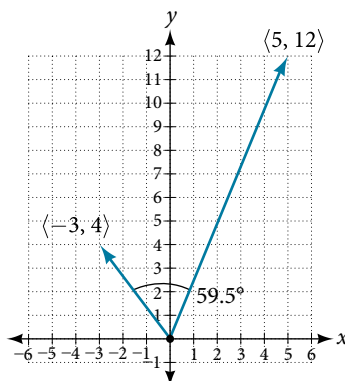
$$= 59.5°$$

See **Figure 18**.

Figure 18

Example 17 Finding Ground Speed and Bearing Using Vectors

We now have the tools to solve the problem we introduced in the opening of the section.

An airplane is flying at an airspeed of 200 miles per hour headed on a SE bearing of 140°. A north wind (from north to south) is blowing at 16.2 miles per hour. What are the ground speed and actual bearing of the plane? See **Figure 19**.
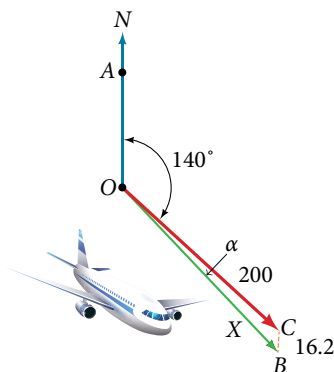

Figure 19

Solution The ground speed is represented by x in the diagram, and we need to find the angle α in order to calculate the adjusted bearing, which will be $140° + \alpha$.

Notice in **Figure 19**, that angle BCO must be equal to angle AOC by the rule of alternating interior angles, so angle BCO is 140°. We can find x by the Law of Cosines:

$$x^2 = (16.2)^2 + (200)^2 - 2(16.2)(200)\cos(140°)$$

$$x^2 = 45{,}226.41$$

$$x = \sqrt{45{,}226.41}$$

$$x = 212.7$$

The ground speed is approximately 213 miles per hour. Now we can calculate the bearing using the Law of Sines.

$$\frac{\sin\alpha}{16.2} = \frac{\sin(140°)}{212.7}$$

$$\sin\alpha = \frac{16.2\sin(140°)}{212.7}$$

$$= 0.04896$$

$$\sin^{-1}(0.04896) = 2.8°$$

Therefore, the plane has a SE bearing of $140° + 2.8° = 142.8°$. The ground speed is 212.7 miles per hour.

Access these online resources for additional instruction and practice with vectors.

- Introduction to Vectors (http://openstaxcollege.org/l/introvectors)
- Vector Operations (http://openstaxcollege.org/l/vectoroperation)
- The Unit Vector (http://openstaxcollege.org/l/unitvector)