CHAPTER 6 REVIEW

Key Terms

amplitude the vertical height of a function; the constant A appearing in the definition of a sinusoidal function

arccosine another name for the inverse cosine; $\arccos x = \cos^{-1} x$

arcsine another name for the inverse sine; $\arcsin x = \sin^{-1} x$

arctangent another name for the inverse tangent; $\arctan x = \tan^{-1} x$

inverse cosine function the function $\cos^{-1} x$, which is the inverse of the cosine function and the angle that has a cosine equal to a given number

inverse sine function the function $\sin^{-1} x$, which is the inverse of the sine function and the angle that has a sine equal to a given number

inverse tangent function the function $\tan^{-1} x$, which is the inverse of the tangent function and the angle that has a tangent equal to a given number

midline the horizontal line $y = D$, where D appears in the general form of a sinusoidal function

periodic function a function $f(x)$ that satisfies $f(x+P) = f(x)$ for a specific constant P and any value of x

phase shift the horizontal displacement of the basic sine or cosine function; the constant $\frac{C}{B}$

sinusoidal function any function that can be expressed in the form $f(x) = A\sin(Bx - C) + D$ or $f(x) = A\cos(Bx - C) + D$

Key Equations

Sinusoidal functions	$f(x) = A\sin(Bx - C) + D$
	$f(x) = A\cos(Bx - C) + D$
Shifted, compressed, and/or stretched tangent function	$y = A\tan(Bx - C) + D$
Shifted, compressed, and/or stretched secant function	$y = A\sec(Bx - C) + D$
Shifted, compressed, and/or stretched cosecant function	$y = A\csc(Bx - C) + D$
Shifted, compressed, and/or stretched cotangent function	$y = A\cot(Bx - C) + D$

Key Concepts

6.1 Graphs of the Sine and Cosine Functions

- Periodic functions repeat after a given value. The smallest such value is the period. The basic sine and cosine functions have a period of 2π.
- The function $\sin x$ is odd, so its graph is symmetric about the origin. The function $\cos x$ is even, so its graph is symmetric about the y-axis.
- The graph of a sinusoidal function has the same general shape as a sine or cosine function.
- In the general formula for a sinusoidal function, the period is $P = \frac{2\pi}{|B|}$ See **Example 1**.
- In the general formula for a sinusoidal function, $|A|$ represents amplitude. If $|A| > 1$, the function is stretched, whereas if $|A| < 1$, the function is compressed. See **Example 2**.
- The value $\frac{C}{B}$ in the general formula for a sinusoidal function indicates the phase shift. See **Example 3**.
- The value D in the general formula for a sinusoidal function indicates the vertical shift from the midline. See **Example 4**.
- Combinations of variations of sinusoidal functions can be detected from an equation. See **Example 5**.
- The equation for a sinusoidal function can be determined from a graph. See **Example 6** and **Example 7**.
- A function can be graphed by identifying its amplitude and period. See **Example 8** and **Example 9**.
- A function can also be graphed by identifying its amplitude, period, phase shift, and horizontal shift. See **Example 10**.
- Sinusoidal functions can be used to solve real-world problems. See **Example 11**, **Example 12**, and **Example 13**.

6.2 Graphs of the Other Trigonometric Functions

- The tangent function has period π.
- $f(x) = A\tan(Bx - C) + D$ is a tangent with vertical and/or horizontal stretch/compression and shift. See **Example 1**, **Example 2**, and **Example 3**.
- The secant and cosecant are both periodic functions with a period of 2π. $f(x) = A\sec(Bx - C) + D$ gives a shifted, compressed, and/or stretched secant function graph. See **Example 4** and **Example 5**.
- $f(x) = A\csc(Bx - C) + D$ gives a shifted, compressed, and/or stretched cosecant function graph. See **Example 6** and **Example 7**.
- The cotangent function has period π and vertical asymptotes at $0, \pm\pi, \pm 2\pi, \ldots$
- The range of cotangent is $(-\infty, \infty)$, and the function is decreasing at each point in its range.
- The cotangent is zero at $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$
- $f(x) = A\cot(Bx - C) + D$ is a cotangent with vertical and/or horizontal stretch/compression and shift. See **Example 8** and **Example 9**.
- Real-world scenarios can be solved using graphs of trigonometric functions. See **Example 10**.

6.3 Inverse Trigonometric Functions

- An inverse function is one that "undoes" another function. The domain of an inverse function is the range of the original function and the range of an inverse function is the domain of the original function.
- Because the trigonometric functions are not one-to-one on their natural domains, inverse trigonometric functions are defined for restricted domains.
- For any trigonometric function $f(x)$, if $x = f^{-1}(y)$, then $f(x) = y$. However, $f(x) = y$ only implies $x = f^{-1}(y)$ if x is in the restricted domain of f. See **Example 1.**
- Special angles are the outputs of inverse trigonometric functions for special input values; for example, $\frac{\pi}{4} = \tan^{-1}(1)$ and $\frac{\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right)$. See **Example 2**.
- A calculator will return an angle within the restricted domain of the original trigonometric function. See **Example 3**.
- Inverse functions allow us to find an angle when given two sides of a right triangle. See **Example 4**.
- In function composition, if the inside function is an inverse trigonometric function, then there are exact expressions; for example, $\sin(\cos^{-1}(x)) = \sqrt{1 - x^2}$. See **Example 5**.
- If the inside function is a trigonometric function, then the only possible combinations are $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ if $0 \le x \le \pi$ and $\cos^{-1}(\sin x) = \frac{\pi}{2} - x$ if $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. See **Example 6** and **Example 7**.
- When evaluating the composition of a trigonometric function with an inverse trigonometric function, draw a reference triangle to assist in determining the ratio of sides that represents the output of the trigonometric function. See **Example 8**.
- When evaluating the composition of a trigonometric function with an inverse trigonometric function, you may use trig identities to assist in determining the ratio of sides. See **Example 9**.