CHAPTER 9 REVIEW

Key Terms

addition method an algebraic technique used to solve systems of linear equations in which the equations are added in a way that eliminates one variable, allowing the resulting equation to be solved for the remaining variable; substitution is then used to solve for the first variable

augmented matrix a coefficient matrix adjoined with the constant column separated by a vertical line within the matrix brackets

break-even point the point at which a cost function intersects a revenue function; where profit is zero

coefficient matrix a matrix that contains only the coefficients from a system of equations

column a set of numbers aligned vertically in a matrix

consistent system a system for which there is a single solution to all equations in the system and it is an independent system, or if there are an infinite number of solutions and it is a dependent system

cost function the function used to calculate the costs of doing business; it usually has two parts, fixed costs and variable costs

Cramer's Rule a method for solving systems of equations that have the same number of equations as variables using determinants

dependent system a system of linear equations in which the two equations represent the same line; there are an infinite number of solutions to a dependent system

determinant a number calculated using the entries of a square matrix that determines such information as whether there is a solution to a system of equations

entry an element, coefficient, or constant in a matrix

feasible region the solution to a system of nonlinear inequalities that is the region of the graph where the shaded regions of each inequality intersect

Gaussian elimination using elementary row operations to obtain a matrix in row-echelon form

identity matrix a square matrix containing ones down the main diagonal and zeros everywhere else; it acts as a 1 in matrix algebra

inconsistent system a system of linear equations with no common solution because they represent parallel lines, which have no point or line in common

independent system a system of linear equations with exactly one solution pair (x, y)

main diagonal entries from the upper left corner diagonally to the lower right corner of a square matrix

matrix a rectangular array of numbers

multiplicative inverse of a matrix a matrix that, when multiplied by the original, equals the identity matrix

nonlinear inequality an inequality containing a nonlinear expression

partial fraction decomposition the process of returning a simplified rational expression to its original form, a sum or difference of simpler rational expressions

partial fractions the individual fractions that make up the sum or difference of a rational expression before combining them into a simplified rational expression

profit function the profit function is written as $P(x) = R(x) - C(x)$, revenue minus cost

revenue function the function that is used to calculate revenue, simply written as $R = xp$, where $x =$ quantity and $p =$ price

row a set of numbers aligned horizontally in a matrix

row operations adding one row to another row, multiplying a row by a constant, interchanging rows, and so on, with the goal of achieving row-echelon form

row-echelon form after performing row operations, the matrix form that contains ones down the main diagonal and zeros at every space below the diagonal

row-equivalent two matrices A and B are row-equivalent if one can be obtained from the other by performing basic row operations

scalar multiple an entry of a matrix that has been multiplied by a scalar

solution set the set of all ordered pairs or triples that satisfy all equations in a system of equations

substitution method an algebraic technique used to solve systems of linear equations in which one of the two equations is

system of linear equations a set of two or more equations in two or more variables that must be considered simultaneously.

system of nonlinear equations a system of equations containing at least one equation that is of degree larger than one

system of nonlinear inequalities a system of two or more inequalities in two or more variables containing at least one inequality that is not linear

Key Equations

Identity matrix for a 2×2 matrix	$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Identity matrix for a 3×3 matrix	$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Multiplicative inverse of a 2×2 matrix	$A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$

Key Concepts

9.1 Systems of Linear Equations: Two Variables

- A system of linear equations consists of two or more equations made up of two or more variables such that all equations in the system are considered simultaneously.
- The solution to a system of linear equations in two variables is any ordered pair that satisfies each equation independently. See **Example 1**.
- Systems of equations are classified as independent with one solution, dependent with an infinite number of solutions, or inconsistent with no solution.
- One method of solving a system of linear equations in two variables is by graphing. In this method, we graph the equations on the same set of axes. See **Example 2**.
- Another method of solving a system of linear equations is by substitution. In this method, we solve for one variable in one equation and substitute the result into the second equation. See **Example 3**.
- A third method of solving a system of linear equations is by addition, in which we can eliminate a variable by adding opposite coefficients of corresponding variables. See **Example 4**.
- It is often necessary to multiply one or both equations by a constant to facilitate elimination of a variable when adding the two equations together. See **Example 5**, **Example 6**, and **Example 7**.
- Either method of solving a system of equations results in a false statement for inconsistent systems because they are made up of parallel lines that never intersect. See **Example 8**.
- The solution to a system of dependent equations will always be true because both equations describe the same line. See **Example 9**.
- Systems of equations can be used to solve real-world problems that involve more than one variable, such as those relating to revenue, cost, and profit. See **Example 10** and **Example 11**.

9.2 Systems of Linear Equations: Three Variables

- A solution set is an ordered triple $\{(x, y, z)\}$ that represents the intersection of three planes in space. See **Example 1**.
- A system of three equations in three variables can be solved by using a series of steps that forces a variable to be eliminated. The steps include interchanging the order of equations, multiplying both sides of an equation by a nonzero constant, and adding a nonzero multiple of one equation to another equation. See **Example 2**.
- Systems of three equations in three variables are useful for solving many different types of real-world problems. See **Example 3**.
- A system of equations in three variables is inconsistent if no solution exists. After performing elimination operations, the result is a contradiction. See **Example 4**.
- Systems of equations in three variables that are inconsistent could result from three parallel planes, two parallel planes and one intersecting plane, or three planes that intersect the other two but not at the same location.

- A system of equations in three variables is dependent if it has an infinite number of solutions. After performing elimination operations, the result is an identity. See **Example 5**.
- Systems of equations in three variables that are dependent could result from three identical planes, three planes intersecting at a line, or two identical planes that intersect the third on a line.

9.3 Systems of Nonlinear Equations and Inequalities: Two Variables

- There are three possible types of solutions to a system of equations representing a line and a parabola: (1) no solution, the line does not intersect the parabola; (2) one solution, the line is tangent to the parabola; and (3) two solutions, the line intersects the parabola in two points. See **Example 1**.
- There are three possible types of solutions to a system of equations representing a circle and a line: (1) no solution, the line does not intersect the circle; (2) one solution, the line is tangent to the parabola; (3) two solutions, the line intersects the circle in two points. See **Example 2**.
- There are five possible types of solutions to the system of nonlinear equations representing an ellipse and a circle: (1) no solution, the circle and the ellipse do not intersect; (2) one solution, the circle and the ellipse are tangent to each other; (3) two solutions, the circle and the ellipse intersect in two points; (4) three solutions, the circle and ellipse intersect in three places; (5) four solutions, the circle and the ellipse intersect in four points. See **Example 3**.
- An inequality is graphed in much the same way as an equation, except for $>$ or $<$, we draw a dashed line and shade the region containing the solution set. See **Example 4**.
- Inequalities are solved the same way as equalities, but solutions to systems of inequalities must satisfy both inequalities. See **Example 5**.

9.4 Partial Fractions

- Decompose $\frac{P(x)}{Q(x)}$ by writing the partial fractions as $\frac{A}{a_1x + b_1} + \frac{B}{a_2x + b_2}$. Solve by clearing the fractions, expanding the right side, collecting like terms, and setting corresponding coefficients equal to each other, then setting up and solving a system of equations. See **Example 1**.
- The decomposition of $\frac{P(x)}{Q(x)}$ with repeated linear factors must account for the factors of the denominator in increasing powers. See **Example 2**.
- The decomposition of $\frac{P(x)}{Q(x)}$ with a nonrepeated irreducible quadratic factor needs a linear numerator over the quadratic factor, as in $\frac{A}{x} + \frac{Bx + C}{(ax^2 + bx + c)}$. See **Example 3**.
- In the decomposition of $\frac{P(x)}{Q(x)}$, where $Q(x)$ has a repeated irreducible quadratic factor, when the irreducible quadratic factors are repeated, powers of the denominator factors must be represented in increasing powers as $\frac{Ax + B}{(ax^2 + bx + c)} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \ldots + \frac{A_nx + B_n}{(ax^2 + bx + c)^n}$. See **Example 4**.

9.5 Matrices and Matrix Operations

- A matrix is a rectangular array of numbers. Entries are arranged in rows and columns.
- The dimensions of a matrix refer to the number of rows and the number of columns. A 3×2 matrix has three rows and two columns. See **Example 1**.
- We add and subtract matrices of equal dimensions by adding and subtracting corresponding entries of each matrix. See **Example 2**, **Example 3**, **Example 4**, and **Example 5**.
- Scalar multiplication involves multiplying each entry in a matrix by a constant. See **Example 6**.
- Scalar multiplication is often required before addition or subtraction can occur. See **Example 7**.
- Multiplying matrices is possible when inner dimensions are the same—the number of columns in the first matrix must match the number of rows in the second.
- The product of two matrices, A and B, is obtained by multiplying each entry in row 1 of A by each entry in column 1 of B; then multiply each entry of row 1 of A by each entry in columns 2 of B, and so on. See **Example 8** and **Example 9**.

- Many real-world problems can often be solved using matrices. See **Example 10**.
- We can use a calculator to perform matrix operations after saving each matrix as a matrix variable. See **Example 11**.

9.6 Solving Systems with Gaussian Elimination

- An augmented matrix is one that contains the coefficients and constants of a system of equations. See **Example 1**.
- A matrix augmented with the constant column can be represented as the original system of equations. See **Example 2**.
- Row operations include multiplying a row by a constant, adding one row to another row, and interchanging rows.
- We can use Gaussian elimination to solve a system of equations. See **Example 3**, **Example 4**, and **Example 5**.
- Row operations are performed on matrices to obtain row-echelon form. See **Example 6**.
- To solve a system of equations, write it in augmented matrix form. Perform row operations to obtain row-echelon form. Back-substitute to find the solutions. See **Example 7** and **Example 8**.
- A calculator can be used to solve systems of equations using matrices. See **Example 9**.
- Many real-world problems can be solved using augmented matrices. See **Example 10** and **Example 11**.

9.7 Solving Systems with Inverses

- An identity matrix has the property $AI = IA = A$. See **Example 1**.
- An invertible matrix has the property $AA^{-1} = A^{-1}A = I$. See **Example 2**.
- Use matrix multiplication and the identity to find the inverse of a 2×2 matrix. See **Example 3**.
- The multiplicative inverse can be found using a formula. See **Example 4**.
- Another method of finding the inverse is by augmenting with the identity. See **Example 5**.
- We can augment a 3×3 matrix with the identity on the right and use row operations to turn the original matrix into the identity, and the matrix on the right becomes the inverse. See **Example 6**.
- Write the system of equations as $AX = B$, and multiply both sides by the inverse of A: $A^{-1}AX = A^{-1}B$. See **Example 7** and **Example 8**.
- We can also use a calculator to solve a system of equations with matrix inverses. See **Example 9**.

9.8 Solving Systems with Cramer's Rule

- The determinant for $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $ad - bc$. See **Example 1**.
- Cramer's Rule replaces a variable column with the constant column. Solutions are $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$. See **Example 2**.
- To find the determinant of a 3×3 matrix, augment with the first two columns. Add the three diagonal entries (upper left to lower right) and subtract the three diagonal entries (lower left to upper right). See **Example 3**.
- To solve a system of three equations in three variables using Cramer's Rule, replace a variable column with the constant column for each desired solution: $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$. See **Example 4**.
- Cramer's Rule is also useful for finding the solution of a system of equations with no solution or infinite solutions. See **Example 5** and **Example 6**.
- Certain properties of determinants are useful for solving problems. For example:
  - If the matrix is in upper triangular form, the determinant equals the product of entries down the main diagonal.
  - When two rows are interchanged, the determinant changes sign.
  - If either two rows or two columns are identical, the determinant equals zero.
  - If a matrix contains either a row of zeros or a column of zeros, the determinant equals zero.
  - The determinant of an inverse matrix A^{-1} is the reciprocal of the determinant of the matrix A.
  - If any row or column is multiplied by a constant, the determinant is multiplied by the same factor. See **Example 7** and **Example 8**.