CHAPTER 11 REVIEW

Key Terms

Addition Principle if one event can occur in m ways and a second event with no common outcomes can occur in n ways, then the first or second event can occur in $m + n$ ways

annuity an investment in which the purchaser makes a sequence of periodic, equal payments

arithmetic sequence a sequence in which the difference between any two consecutive terms is a constant

arithmetic series the sum of the terms in an arithmetic sequence

binomial coefficient the number of ways to choose r objects from n objects where order does not matter; equivalent to $C(n, r)$, denoted $\binom{n}{r}$

binomial expansion the result of expanding $(x + y)^n$ by multiplying

Binomial Theorem a formula that can be used to expand any binomial

combination a selection of objects in which order does not matter

common difference the difference between any two consecutive terms in an arithmetic sequence

common ratio the ratio between any two consecutive terms in a geometric sequence

complement of an event the set of outcomes in the sample space that are not in the event E

diverge a series is said to diverge if the sum is not a real number

event any subset of a sample space

experiment an activity with an observable result

explicit formula a formula that defines each term of a sequence in terms of its position in the sequence

finite sequence a function whose domain consists of a finite subset of the positive integers $\{1, 2, \ldots n\}$ for some positive integer n

Fundamental Counting Principle if one event can occur in m ways and a second event can occur in n ways after the first event has occurred, then the two events can occur in $m \times n$ ways; also known as the Multiplication Principle

geometric sequence a sequence in which the ratio of a term to a previous term is a constant

geometric series the sum of the terms in a geometric sequence

index of summation in summation notation, the variable used in the explicit formula for the terms of a series and written below the sigma with the lower limit of summation

infinite sequence a function whose domain is the set of positive integers

infinite series the sum of the terms in an infinite sequence

lower limit of summation the number used in the explicit formula to find the first term in a series

Multiplication Principle if one event can occur in m ways and a second event can occur in n ways after the first event has occurred, then the two events can occur in $m \times n$ ways; also known as the Fundamental Counting Principle

mutually exclusive events events that have no outcomes in common

***n* factorial** the product of all the positive integers from 1 to n

***n*th partial sum** the sum of the first n terms of a sequence

***n*th term of a sequence** a formula for the general term of a sequence

outcomes the possible results of an experiment

permutation a selection of objects in which order matters

probability a number from 0 to 1 indicating the likelihood of an event

probability model a mathematical description of an experiment listing all possible outcomes and their associated

recursive formula a formula that defines each term of a sequence using previous term(s)

sample space the set of all possible outcomes of an experiment

sequence a function whose domain is a subset of the positive integers

series the sum of the terms in a sequence

summation notation a notation for series using the Greek letter sigma; it includes an explicit formula and specifies the first and last terms in the series

term a number in a sequence

union of two events the event that occurs if either or both events occur

upper limit of summation the number used in the explicit formula to find the last term in a series

Key Equations

Formula for a factorial	$0! = 1$ $1! = 1$ $n! = n(n-1)(n-2)\cdots(2)(1)$, for $n \geq 2$
recursive formula for nth term of an arithmetic sequence	$a_n = a_{n-1} + d;\ n \geq 2$
explicit formula for nth term of an arithmetic sequence	$a_n = a_1 + d(n-1)$
recursive formula for nth term of a geometric sequence	$a_n = ra_{n-1},\ n \geq 2$
explicit formula for nth term of a geometric sequence	$a_n = a_1 r^{n-1}$
sum of the first n terms of an arithmetic series	$S_n = \dfrac{n(a_1 + a_n)}{2}$
sum of the first n terms of a geometric series	$S_n = \dfrac{a_1(1-r^n)}{1-r},\ r \neq 1$
sum of an infinite geometric series with $-1 < r < 1$	$S_n = \dfrac{a_1}{1-r},\ r \neq 1$
number of permutations of n distinct objects taken r at a time	$P(n, r) = \dfrac{n!}{(n-r)!}$
number of combinations of n distinct objects taken r at a time	$C(n, r) = \dfrac{n!}{r!(n-r)!}$
number of permutations of n non-distinct objects	$\dfrac{n!}{r_1!r_2! \ldots r_k!}$
Binomial Theorem	$(x+y)^n = \sum\limits_{k-0}^{n} \binom{n}{k} x^{n-k} y^k$
$(r+1)^{\text{th}}$ term of a binomial expansion	$\binom{n}{r} x^{n-r} y^r$
probability of an event with equally likely outcomes	$P(E) = \dfrac{n(E)}{n(S)}$
probability of the union of two events	$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
probability of the union of mutually exclusive events	$P(E \cup F) = P(E) + P(F)$
probability of the complement of an event	$P(E') = 1 - P(E)$

Key Concepts

11.1 Sequences and Their Notations

- A sequence is a list of numbers, called terms, written in a specific order.
- Explicit formulas define each term of a sequence using the position of the term. See **Example 1**, **Example 2**, and **Example 3**.
- An explicit formula for the nth term of a sequence can be written by analyzing the pattern of several terms. See **Example 4**.
- Recursive formulas define each term of a sequence using previous terms.
- Recursive formulas must state the initial term, or terms, of a sequence.
- A set of terms can be written by using a recursive formula. See **Example 5** and **Example 6**.
- A factorial is a mathematical operation that can be defined recursively.
- The factorial of n is the product of all integers from 1 to n See **Example 7**.

11.2 Arithmetic Sequences

- An arithmetic sequence is a sequence where the difference between any two consecutive terms is a constant.
- The constant between two consecutive terms is called the common difference.
- The common difference is the number added to any one term of an arithmetic sequence that generates the subsequent term. See **Example 1**.
- The terms of an arithmetic sequence can be found by beginning with the initial term and adding the common difference repeatedly. See **Example 2** and **Example 3**.
- A recursive formula for an arithmetic sequence with common difference d is given by $a_n = a_{n-1} + d, n \geq 2$. See **Example 4**.
- As with any recursive formula, the initial term of the sequence must be given.
- An explicit formula for an arithmetic sequence with common difference d is given by $a_n = a_1 + d(n-1)$. See **Example 5**.
- An explicit formula can be used to find the number of terms in a sequence. See **Example 6**.
- In application problems, we sometimes alter the explicit formula slightly to $a_n = a_0 + dn$. See **Example 7**.

11.3 Geometric Sequences

- A geometric sequence is a sequence in which the ratio between any two consecutive terms is a constant.
- The constant ratio between two consecutive terms is called the common ratio.
- The common ratio can be found by dividing any term in the sequence by the previous term. See **Example 1**.
- The terms of a geometric sequence can be found by beginning with the first term and multiplying by the common ratio repeatedly. See **Example 2** and **Example 4**.
- A recursive formula for a geometric sequence with common ratio r is given by $a_n = ra_{n-1}$ for $n \geq 2$.
- As with any recursive formula, the initial term of the sequence must be given. See **Example 3**.
- An explicit formula for a geometric sequence with common ratio r is given by $a_n = a_1 r^{n-1}$. See **Example 5**.
- In application problems, we sometimes alter the explicit formula slightly to $a_n = a_0 r^n$. See **Example 6**.

11.4 Series and Their Notations

- The sum of the terms in a sequence is called a series.
- A common notation for series is called summation notation, which uses the Greek letter sigma to represent the sum. See **Example 1**.
- The sum of the terms in an arithmetic sequence is called an arithmetic series.
- The sum of the first n terms of an arithmetic series can be found using a formula. See **Example 2** and **Example 3**.
- The sum of the terms in a geometric sequence is called a geometric series.
- The sum of the first n terms of a geometric series can be found using a formula. See **Example 4** and **Example 5**.
- The sum of an infinite series exists if the series is geometric with $-1 < r < 1$.

- If the sum of an infinite series exists, it can be found using a formula. See **Example 6**, **Example 7**, and **Example 8**.
- An annuity is an account into which the investor makes a series of regularly scheduled payments. The value of an annuity can be found using geometric series. See **Example 9**.

11.5 Counting Principles

- If one event can occur in m ways and a second event with no common outcomes can occur in n ways, then the first or second event can occur in $m + n$ ways. See **Example 1**.
- If one event can occur in m ways and a second event can occur in n ways after the first event has occurred, then the two events can occur in $m \times n$ ways. See **Example 2**.
- A permutation is an ordering of n objects.
- If we have a set of n objects and we want to choose r objects from the set in order, we write $P(n, r)$.
- Permutation problems can be solved using the Multiplication Principle or the formula for $P(n, r)$. See **Example 3** and **Example 4**.
- A selection of objects where the order does not matter is a combination.
- Given n distinct objects, the number of ways to select r objects from the set is $C(n, r)$ and can be found using a formula. See **Example 5.**
- A set containing n distinct objects has 2^n subsets. See **Example 6**.
- For counting problems involving non-distinct objects, we need to divide to avoid counting duplicate permutations. See **Example 7**.

11.6 Binomial Theorem

- $\binom{n}{r}$ is called a binomial coefficient and is equal to $C(n, r)$. See **Example 1**.
- The Binomial Theorem allows us to expand binomials without multiplying. See **Example 2**.
- We can find a given term of a binomial expansion without fully expanding the binomial. See **Example 3**.

11.7 Probability

- Probability is always a number between 0 and 1, where 0 means an event is impossible and 1 means an event is certain.
- The probabilities in a probability model must sum to 1. See **Example 1**.
- When the outcomes of an experiment are all equally likely, we can find the probability of an event by dividing the number of outcomes in the event by the total number of outcomes in the sample space for the experiment. See **Example 2**.
- To find the probability of the union of two events, we add the probabilities of the two events and subtract the probability that both events occur simultaneously. See **Example 3**.
- To find the probability of the union of two mutually exclusive events, we add the probabilities of each of the events. See **Example 4**.
- The probability of the complement of an event is the difference between 1 and the probability that the event occurs. See **Example 5**.
- In some probability problems, we need to use permutations and combinations to find the number of elements in events and sample spaces. See **Example 6**.