CHAPTER 5 REVIEW

Key Terms

adjacent side in a right triangle, the side between a given angle and the right angle

angle the union of two rays having a common endpoint

angle of depression the angle between the horizontal and the line from the object to the observer's eye, assuming the object is positioned lower than the observer

angle of elevation the angle between the horizontal and the line from the object to the observer's eye, assuming the object is positioned higher than the observer

angular speed the angle through which a rotating object travels in a unit of time

arc length the length of the curve formed by an arc

area of a sector area of a portion of a circle bordered by two radii and the intercepted arc; the fraction $\frac{\theta}{2\pi}$ multiplied by the area of the entire circle

cosecant the reciprocal of the sine function: on the unit circle, $\csc t = \frac{1}{y}, y \neq 0$

cosine function the x-value of the point on a unit circle corresponding to a given angle

cotangent the reciprocal of the tangent function: on the unit circle, $\cot t = \frac{x}{y}, y \neq 0$

coterminal angles description of positive and negative angles in standard position sharing the same terminal side

degree a unit of measure describing the size of an angle as one-360th of a full revolution of a circle

hypotenuse the side of a right triangle opposite the right angle

identities statements that are true for all values of the input on which they are defined

initial side the side of an angle from which rotation begins

linear speed the distance along a straight path a rotating object travels in a unit of time; determined by the arc length

measure of an angle the amount of rotation from the initial side to the terminal side

negative angle description of an angle measured clockwise from the positive x-axis

opposite side in a right triangle, the side most distant from a given angle

period the smallest interval P of a repeating function f such that $f(x + P) = f(x)$

positive angle description of an angle measured counterclockwise from the positive x-axis

Pythagorean Identity a corollary of the Pythagorean Theorem stating that the square of the cosine of a given angle plus the square of the sine of that angle equals 1

quadrantal angle an angle whose terminal side lies on an axis

radian the measure of a central angle of a circle that intercepts an arc equal in length to the radius of that circle

radian measure the ratio of the arc length formed by an angle divided by the radius of the circle

ray one point on a line and all points extending in one direction from that point; one side of an angle

reference angle the measure of the acute angle formed by the terminal side of the angle and the horizontal axis

secant the reciprocal of the cosine function: on the unit circle, $\sec t = \frac{1}{x}, x \neq 0$

sine function the y-value of the point on a unit circle corresponding to a given angle

standard position the position of an angle having the vertex at the origin and the initial side along the positive x-axis

tangent the quotient of the sine and cosine: on the unit circle, $\tan t = \frac{y}{x}, x \neq 0$

terminal side the side of an angle at which rotation ends

unit circle a circle with a center at (0, 0) and radius 1.

vertex the common endpoint of two rays that form an angle

Key Equations

arc length	$s = r\theta$
area of a sector	$A = \frac{1}{2}\theta r^2$
angular speed	$\omega = \frac{\theta}{t}$
linear speed	$v = \frac{s}{t}$
linear speed related to angular speed	$v = r\omega$
cosine	$\cos t = x$
sine	$\sin t = y$
Pythagorean Identity	$\cos^2 t + \sin^2 t = 1$
tangent function	$\tan t = \frac{\sin t}{\cos t}$
secant function	$\sec t = \frac{1}{\cos t}$
cosecant function	$\csc t = \frac{1}{\sin t}$
cotangent function	$\cot t = \frac{1}{\tan t} = \frac{\cos t}{\sin t}$
cofunction identities	$\cos t = \sin\left(\frac{\pi}{2} - t\right)$
	$\sin t = \cos\left(\frac{\pi}{2} - t\right)$
	$\tan t = \cot\left(\frac{\pi}{2} - t\right)$
	$\cot t = \tan\left(\frac{\pi}{2} - t\right)$
	$\sec t = \csc\left(\frac{\pi}{2} - t\right)$
	$\csc t = \sec\left(\frac{\pi}{2} - t\right)$

Key Concepts

5.1 Angles

- An angle is formed from the union of two rays, by keeping the initial side fixed and rotating the terminal side. The amount of rotation determines the measure of the angle.
- An angle is in standard position if its vertex is at the origin and its initial side lies along the positive x-axis. A positive angle is measured counterclockwise from the initial side and a negative angle is measured clockwise.
- To draw an angle in standard position, draw the initial side along the positive x-axis and then place the terminal side according to the fraction of a full rotation the angle represents. See **Example 1**.
- In addition to degrees, the measure of an angle can be described in radians. See **Example 2**.
- To convert between degrees and radians, use the proportion $\frac{\theta}{180} = \frac{\theta^R}{\pi}$. See **Example 3** and **Example 4**.
- Two angles that have the same terminal side are called coterminal angles.
- We can find coterminal angles by adding or subtracting 360° or 2π. See **Example 5** and **Example 6**.
- Coterminal angles can be found using radians just as they are for degrees. See **Example 7**.
- The length of a circular arc is a fraction of the circumference of the entire circle. See **Example 8**.

- The area of sector is a fraction of the area of the entire circle. See **Example 9**.
- An object moving in a circular path has both linear and angular speed.
- The angular speed of an object traveling in a circular path is the measure of the angle through which it turns in a unit of time. See **Example 10**.
- The linear speed of an object traveling along a circular path is the distance it travels in a unit of time. See **Example 11**.

5.2 Unit Circle: Sine and Cosine Functions

- Finding the function values for the sine and cosine begins with drawing a unit circle, which is centered at the origin and has a radius of 1 unit.
- Using the unit circle, the sine of an angle t equals the y-value of the endpoint on the unit circle of an arc of length t whereas the cosine of an angle t equals the x-value of the endpoint. See **Example 1**.
- The sine and cosine values are most directly determined when the corresponding point on the unit circle falls on an axis. See **Example 2**.
- When the sine or cosine is known, we can use the Pythagorean Identity to find the other. The Pythagorean Identity is also useful for determining the sines and cosines of special angles. See **Example 3**.
- Calculators and graphing software are helpful for finding sines and cosines if the proper procedure for entering information is known. See **Example 4**.
- The domain of the sine and cosine functions is all real numbers.
- The range of both the sine and cosine functions is $[-1, 1]$.
- The sine and cosine of an angle have the same absolute value as the sine and cosine of its reference angle.
- The signs of the sine and cosine are determined from the x- and y-values in the quadrant of the original angle.
- An angle's reference angle is the size angle, t, formed by the terminal side of the angle t and the horizontal axis. See **Example 5**.
- Reference angles can be used to find the sine and cosine of the original angle. See **Example 6**.
- Reference angles can also be used to find the coordinates of a point on a circle. See **Example 7**.

5.3 The Other Trigonometric Functions

- The tangent of an angle is the ratio of the y-value to the x-value of the corresponding point on the unit circle.
- The secant, cotangent, and cosecant are all reciprocals of other functions. The secant is the reciprocal of the cosine function, the cotangent is the reciprocal of the tangent function, and the cosecant is the reciprocal of the sine function.
- The six trigonometric functions can be found from a point on the unit circle. See **Example 1**.
- Trigonometric functions can also be found from an angle. See **Example 2**.
- Trigonometric functions of angles outside the first quadrant can be determined using reference angles. See **Example 3**.
- A function is said to be even if $f(-x) = f(x)$ and odd if $f(-x) = -f(x)$.
- Cosine and secant are even; sine, tangent, cosecant, and cotangent are odd.
- Even and odd properties can be used to evaluate trigonometric functions. See **Example 4**.
- The Pythagorean Identity makes it possible to find a cosine from a sine or a sine from a cosine.
- Identities can be used to evaluate trigonometric functions. See **Example 5** and **Example 6**.
- Fundamental identities such as the Pythagorean Identity can be manipulated algebraically to produce new identities. See **Example 7**.
- The trigonometric functions repeat at regular intervals.
- The period P of a repeating function f is the smallest interval such that $f(x + P) = f(x)$ for any value of x.
- The values of trigonometric functions of special angles can be found by mathematical analysis.
- To evaluate trigonometric functions of other angles, we can use a calculator or computer software. See **Example 8**.

5.4 Right Triangle Trigonometry

- We can define trigonometric functions as ratios of the side lengths of a right triangle. See **Example 1**.
- The same side lengths can be used to evaluate the trigonometric functions of either acute angle in a right triangle. See **Example 2**.
- We can evaluate the trigonometric functions of special angles, knowing the side lengths of the triangles in which they occur. See **Example 3**.
- Any two complementary angles could be the two acute angles of a right triangle.
- If two angles are complementary, the cofunction identities state that the sine of one equals the cosine of the other and vice versa. See **Example 4**.
- We can use trigonometric functions of an angle to find unknown side lengths.
- Select the trigonometric function representing the ratio of the unknown side to the known side. See **Example 5**.
- Right-triangle trigonometry permits the measurement of inaccessible heights and distances.
- The unknown height or distance can be found by creating a right triangle in which the unknown height or distance is one of the sides, and another side and angle are known. See **Example 6**.