CHAPTER 1 REVIEW

Key Terms

absolute maximum the greatest value of a function over an interval

absolute minimum the lowest value of a function over an interval

absolute value equation an equation of the form $|A| = B$, with $B \geq 0$; it will have solutions when $A = B$ or $A = -B$

absolute value inequality a relationship in the form $|A| < B$, $|A| \leq B$, $|A| > B$, or $|A| \geq B$

average rate of change the difference in the output values of a function found for two values of the input divided by the difference between the inputs

composite function the new function formed by function composition, when the output of one function is used as the input of another

decreasing function a function is decreasing in some open interval if $f(b) < f(a)$ for any two input values a and b in the given interval where $b > a$

dependent variable an output variable

domain the set of all possible input values for a relation

even function a function whose graph is unchanged by horizontal reflection, $f(x) = f(-x)$, and is symmetric about the y-axis

function a relation in which each input value yields a unique output value

horizontal compression a transformation that compresses a function's graph horizontally, by multiplying the input by a constant $b > 1$

horizontal line test a method of testing whether a function is one-to-one by determining whether any horizontal line intersects the graph more than once

horizontal reflection a transformation that reflects a function's graph across the y-axis by multiplying the input by -1

horizontal shift a transformation that shifts a function's graph left or right by adding a positive or negative constant to the input

horizontal stretch a transformation that stretches a function's graph horizontally by multiplying the input by a constant $0 < b < 1$

increasing function a function is increasing in some open interval if $f(b) > f(a)$ for any two input values a and b in the given interval where $b > a$

independent variable an input variable

input each object or value in a domain that relates to another object or value by a relationship known as a function

interval notation a method of describing a set that includes all numbers between a lower limit and an upper limit; the lower and upper values are listed between brackets or parentheses, a square bracket indicating inclusion in the set, and a parenthesis indicating exclusion

inverse function for any one-to-one function $f(x)$, the inverse is a function $f^{-1}(x)$ such that $f^{-1}(f(x)) = x$ for all x in the domain of f; this also implies that $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}

local extrema collectively, all of a function's local maxima and minima

local maximum a value of the input where a function changes from increasing to decreasing as the input value increases.

local minimum a value of the input where a function changes from decreasing to increasing as the input value increases.

odd function a function whose graph is unchanged by combined horizontal and vertical reflection, $f(x) = -f(-x)$, and is symmetric about the origin

one-to-one function a function for which each value of the output is associated with a unique input value

output each object or value in the range that is produced when an input value is entered into a function

piecewise function a function in which more than one formula is used to define the output

range the set of output values that result from the input values in a relation

rate of change the change of an output quantity relative to the change of the input quantity

relation a set of ordered pairs

set-builder notation a method of describing a set by a rule that all of its members obey; it takes the form $\{x \mid \text{statement about } x\}$

vertical compression a function transformation that compresses the function's graph vertically by multiplying the output by a constant $0 < a < 1$

vertical line test a method of testing whether a graph represents a function by determining whether a vertical line intersects the graph no more than once

vertical reflection a transformation that reflects a function's graph across the x-axis by multiplying the output by -1

vertical shift a transformation that shifts a function's graph up or down by adding a positive or negative constant to the output

vertical stretch a transformation that stretches a function's graph vertically by multiplying the output by a constant $a > 1$

Key Equations

Constant function	$f(x) = c$, where c is a constant
Identity function	$f(x) = x$
Absolute value function	$f(x) = \lvert x \rvert$
Quadratic function	$f(x) = x^2$
Cubic function	$f(x) = x^3$
Reciprocal function	$f(x) = \frac{1}{x}$
Reciprocal squared function	$f(x) = \frac{1}{x^2}$
Square root function	$f(x) = \sqrt{x}$
Cube root function	$f(x) = \sqrt[3]{x}$
Average rate of change	$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$
Composite function	$(f \circ g)(x) = f(g(x))$
Vertical shift	$g(x) = f(x) + k$ (up for $k > 0$)
Horizontal shift	$g(x) = f(x - h)$ (right for $h > 0$)
Vertical reflection	$g(x) = -f(x)$
Horizontal reflection	$g(x) = f(-x)$
Vertical stretch	$g(x) = af(x)$ ($a > 0$)
Vertical compression	$g(x) = af(x)$ ($0 < a < 1$)
Horizontal stretch	$g(x) = f(bx)$ ($0 < b < 1$)
Horizontal compression	$g(x) = f(bx)$ ($b > 1$)

Key Concepts

1.1 Functions and Function Notation

- A relation is a set of ordered pairs. A function is a specific type of relation in which each domain value, or input, leads to exactly one range value, or output. See **Example 1** and **Example 2**.
- Function notation is a shorthand method for relating the input to the output in the form $y = f(x)$. See **Example 3** and **Example 4**.
- In tabular form, a function can be represented by rows or columns that relate to input and output values. See **Example 5**.
- To evaluate a function, we determine an output value for a corresponding input value. Algebraic forms of a function can be evaluated by replacing the input variable with a given value. See **Example 6** and **Example 7**.
- To solve for a specific function value, we determine the input values that yield the specific output value. See **Example 8**.
- An algebraic form of a function can be written from an equation. See **Example 9** and **Example 10**.
- Input and output values of a function can be identified from a table. See **Example 11**.
- Relating input values to output values on a graph is another way to evaluate a function. See **Example 12**.
- A function is one-to-one if each output value corresponds to only one input value. See **Example 13**.
- A graph represents a function if any vertical line drawn on the graph intersects the graph at no more than one point. See **Example 14**.
- The graph of a one-to-one function passes the horizontal line test. See **Example 15**.

1.2 Domain and Range

- The domain of a function includes all real input values that would not cause us to attempt an undefined mathematical operation, such as dividing by zero or taking the square root of a negative number.
- The domain of a function can be determined by listing the input values of a set of ordered pairs. See **Example 1.**
- The domain of a function can also be determined by identifying the input values of a function written as an equation. See **Example 2**, **Example 3**, and **Example 4**.
- Interval values represented on a number line can be described using inequality notation, set-builder notation, and interval notation. See **Example 5**.
- For many functions, the domain and range can be determined from a graph. See **Example 6** and **Example 7**.
- An understanding of toolkit functions can be used to find the domain and range of related functions. See **Example 8**, **Example 9**, and **Example 10**.
- A piecewise function is described by more than one formula. See **Example 11** and **Example 12**.
- A piecewise function can be graphed using each algebraic formula on its assigned subdomain. See **Example 13**.

1.3 Rates of Change and Behavior of Graphs

- A rate of change relates a change in an output quantity to a change in an input quantity. The average rate of change is determined using only the beginning and ending data. See **Example 1**.
- Identifying points that mark the interval on a graph can be used to find the average rate of change. See **Example 2**.
- Comparing pairs of input and output values in a table can also be used to find the average rate of change. See **Example 3**.
- An average rate of change can also be computed by determining the function values at the endpoints of an interval described by a formula. See **Example 4** and **Example 5**.
- The average rate of change can sometimes be determined as an expression. See **Example 6**.
- A function is increasing where its rate of change is positive and decreasing where its rate of change is negative. See **Example 7**.
- A local maximum is where a function changes from increasing to decreasing and has an output value larger (more positive or less negative) than output values at neighboring input values.

- A local minimum is where the function changes from decreasing to increasing (as the input increases) and has an output value smaller (more negative or less positive) than output values at neighboring input values.
- Minima and maxima are also called extrema.
- We can find local extrema from a graph. See **Example 8** and **Example 9**.
- The highest and lowest points on a graph indicate the maxima and minima. See **Example 10**.

1.4 Composition of Functions

- We can perform algebraic operations on functions. See **Example 1**.
- When functions are combined, the output of the first (inner) function becomes the input of the second (outer) function.
- The function produced by combining two functions is a composite function. See **Example 2** and **Example 3**.
- The order of function composition must be considered when interpreting the meaning of composite functions. See **Example 4**.
- A composite function can be evaluated by evaluating the inner function using the given input value and then evaluating the outer function taking as its input the output of the inner function.
- A composite function can be evaluated from a table. See **Example 5**.
- A composite function can be evaluated from a graph. See **Example 6**.
- A composite function can be evaluated from a formula. See **Example 7**.
- The domain of a composite function consists of those inputs in the domain of the inner function that correspond to outputs of the inner function that are in the domain of the outer function. See **Example 8** and **Example 9**.
- Just as functions can be combined to form a composite function, composite functions can be decomposed into simpler functions.
- Functions can often be decomposed in more than one way. See **Example 10**.

1.5 Transformation of Functions

- A function can be shifted vertically by adding a constant to the output. See **Example 1** and **Example 2**.
- A function can be shifted horizontally by adding a constant to the input. See **Example 3**, **Example 4**, and **Example 5**.
- Relating the shift to the context of a problem makes it possible to compare and interpret vertical and horizontal shifts. See **Example 6**.
- Vertical and horizontal shifts are often combined. See **Example 7** and **Example 8**.
- A vertical reflection reflects a graph about the x-axis. A graph can be reflected vertically by multiplying the output by -1.
- A horizontal reflection reflects a graph about the y-axis. A graph can be reflected horizontally by multiplying the input by -1.
- A graph can be reflected both vertically and horizontally. The order in which the reflections are applied does not affect the final graph. See **Example 9**.
- A function presented in tabular form can also be reflected by multiplying the values in the input and output rows or columns accordingly. See **Example 10**.
- A function presented as an equation can be reflected by applying transformations one at a time. See **Example 11**.
- Even functions are symmetric about the y-axis, whereas odd functions are symmetric about the origin.
- Even functions satisfy the condition $f(x) = f(-x)$.
- Odd functions satisfy the condition $f(x) = -f(-x)$.
- A function can be odd, even, or neither. See **Example 12**.
- A function can be compressed or stretched vertically by multiplying the output by a constant. See **Example 13**, **Example 14**, and **Example 15**.
- A function can be compressed or stretched horizontally by multiplying the input by a constant. See **Example 16**, **Example 17**, and **Example 18**.

- The order in which different transformations are applied does affect the final function. Both vertical and horizontal transformations must be applied in the order given. However, a vertical transformation may be combined with a horizontal transformation in any order. See **Example 19** and **Example 20**.

1.6 Absolute Value Functions

- The absolute value function is commonly used to measure distances between points. See **Example 1**.
- Applied problems, such as ranges of possible values, can also be solved using the absolute value function. See **Example 2**.
- The graph of the absolute value function resembles a letter V. It has a corner point at which the graph changes direction. See **Example 3**.
- In an absolute value equation, an unknown variable is the input of an absolute value function.
- If the absolute value of an expression is set equal to a positive number, expect two solutions for the unknown variable. See **Example 4**.
- An absolute value equation may have one solution, two solutions, or no solutions. See **Example 5**.
- An absolute value inequality is similar to an absolute value equation but takes the form $|A| < B$, $|A| \leq B$, $|A| > B$, or $|A| \geq B$. It can be solved by determining the boundaries of the solution set and then testing which segments are in the set. See **Example 6**.
- Absolute value inequalities can also be solved graphically. See **Example 7**.

1.7 Inverse Functions

- If $g(x)$ is the inverse of $f(x)$, then $g(f(x)) = f(g(x)) = x$. See **Example 1**, **Example 2**, and **Example 3**.
- Each of the toolkit functions has an inverse. See **Example 4**.
- For a function to have an inverse, it must be one-to-one (pass the horizontal line test).
- A function that is not one-to-one over its entire domain may be one-to-one on part of its domain.
- For a tabular function, exchange the input and output rows to obtain the inverse. See **Example 5**.
- The inverse of a function can be determined at specific points on its graph. See **Example 6**.
- To find the inverse of a formula, solve the equation $y = f(x)$ for x as a function of y. Then exchange the labels x and y. See **Example 7**, **Example 8**, and **Example 9**.
- The graph of an inverse function is the reflection of the graph of the original function across the line $y = x$. See **Example 10**.