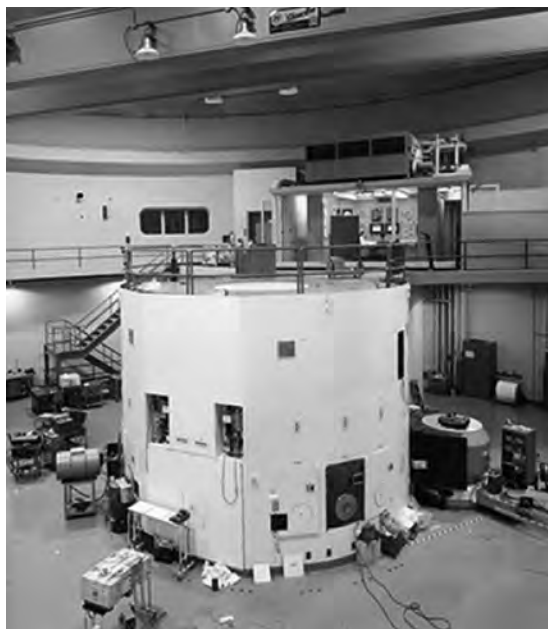In this section, you will:

- Model exponential growth and decay.
- Use Newton's Law of Cooling.
- Use logistic-growth models.
- Choose an appropriate model for data.
- Express an exponential model in base e.

4.7   EXPONENTIAL AND LOGARITHMIC MODELS

Figure 1 A nuclear research reactor inside the Neely Nuclear Research Center on the Georgia Institute of Technology campus. (credit: Georgia Tech Research Institute)

We have already explored some basic applications of exponential and logarithmic functions. In this section, we explore some important applications in more depth, including radioactive isotopes and Newton's Law of Cooling.

Modeling Exponential Growth and Decay

In real-world applications, we need to model the behavior of a function. In mathematical modeling, we choose a familiar general function with properties that suggest that it will model the real-world phenomenon we wish to analyze. In the case of rapid growth, we may choose the exponential growth function:

$$y = A_0 e^{kt}$$

where A_0 is equal to the value at time zero, e is Euler's constant, and k is a positive constant that determines the rate (percentage) of growth. We may use the exponential growth function in applications involving **doubling time**, the time it takes for a quantity to double. Such phenomena as wildlife populations, financial investments, biological samples, and natural resources may exhibit growth based on a doubling time. In some applications, however, as we will see when we discuss the logistic equation, the logistic model sometimes fits the data better than the exponential model.

On the other hand, if a quantity is falling rapidly toward zero, without ever reaching zero, then we should probably choose the exponential decay model. Again, we have the form $y = A_0 e^{kt}$ where A_0 is the starting value, and e is Euler's constant. Now k is a negative constant that determines the rate of decay. We may use the exponential decay model when we are calculating **half-life**, or the time it takes for a substance to exponentially decay to half of its original quantity. We use half-life in applications involving radioactive isotopes.

In our choice of a function to serve as a mathematical model, we often use data points gathered by careful observation and measurement to construct points on a graph and hope we can recognize the shape of the graph. Exponential growth and decay graphs have a distinctive shape, as we can see in **Figure 2** and **Figure 3**. It is important to remember that, although parts of each of the two graphs seem to lie on the x-axis, they are really a tiny distance above the x-axis.
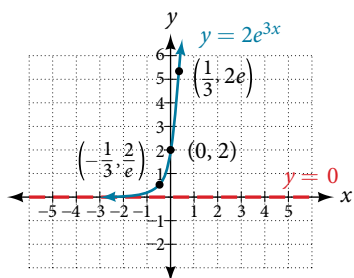

Figure 2 A graph showing exponential growth. The equation is $y = 2e^{3x}$.
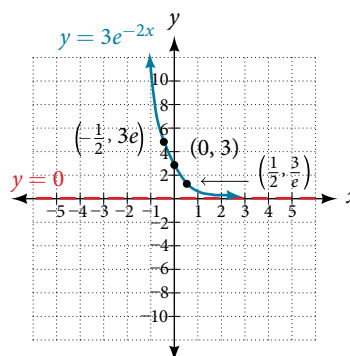

Figure 3 A graph showing exponential decay. The equation is $y = 3e^{-2x}$.

Exponential growth and decay often involve very large or very small numbers. To describe these numbers, we often use orders of magnitude. The **order of magnitude** is the power of ten, when the number is expressed in scientific notation, with one digit to the left of the decimal. For example, the distance to the nearest star, Proxima Centauri, measured in kilometers, is 40,113,497,200,000 kilometers. Expressed in scientific notation, this is $4.01134972 \times 10^{13}$. So, we could describe this number as having order of magnitude 10^{13}.

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characteristics of the exponential function, $y = A_0 e^{kt}$

An exponential function with the form $y = A_0 e^{kt}$ has the following characteristics:
- one-to-one function
- horizontal asymptote: $y = 0$
- domain: $(-\infty, \infty)$
- range: $(0, \infty)$
- x-intercept: none
- y-intercept: $(0, A_0)$
- increasing if $k > 0$ (see **Figure 4**)
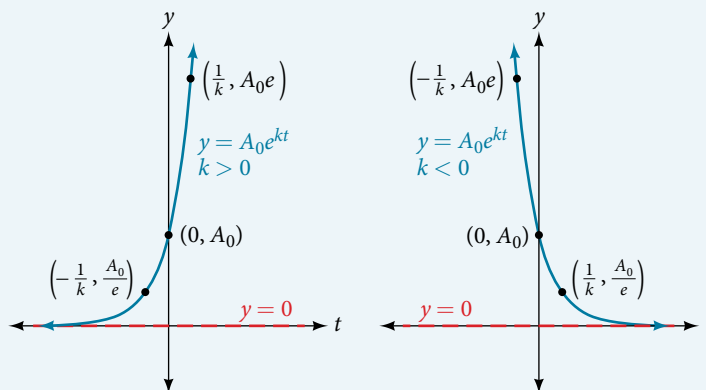- decreasing if $k < 0$ (see **Figure 4**)

Figure 4 An exponential function models exponential growth when $k > 0$ and exponential decay when $k < 0$.

Example 1   Graphing Exponential Growth

A population of bacteria doubles every hour. If the culture started with 10 bacteria, graph the population as a function of time.

Solution   When an amount grows at a fixed percent per unit time, the growth is exponential. To find A_0 we use the fact that A_0 is the amount at time zero, so $A_0 = 10$. To find k, use the fact that after one hour ($t = 1$) the population doubles from 10 to 20. The formula is derived as follows

$$20 = 10e^{k \cdot 1}$$

$$2 = e^k \qquad \text{Divide by 10}$$

$$\ln 2 = k \qquad \text{Take the natural logarithm}$$

so $k = \ln(2)$. Thus the equation we want to graph is $y = 10e^{(\ln 2)t} = 10(e^{\ln 2})^t = 10 \cdot 2^t$. The graph is shown in **Figure 5**.
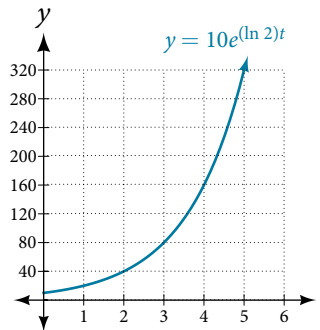
Figure 5 The graph of $y = 10e^{(\ln 2)t}$.

Analysis The population of bacteria after ten hours is 10,240. We could describe this amount is being of the order of magnitude 10^4. The population of bacteria after twenty hours is 10,485,760 which is of the order of magnitude 10^7, so we could say that the population has increased by three orders of magnitude in ten hours.

Half-Life

We now turn to exponential decay. One of the common terms associated with exponential decay, as stated above, is **half-life**, the length of time it takes an exponentially decaying quantity to decrease to half its original amount. Every radioactive isotope has a half-life, and the process describing the exponential decay of an isotope is called radioactive decay.

To find the half-life of a function describing exponential decay, solve the following equation:

$$\frac{1}{2} A_0 = A_0 e^{kt}$$

We find that the half-life depends only on the constant k and not on the starting quantity A_0.

The formula is derived as follows

$$\frac{1}{2} A_0 = A_0 e^{kt}$$

$$\frac{1}{2} = e^{kt} \qquad \text{Divide by } A_0.$$

$$\ln\left(\frac{1}{2}\right) = kt \qquad \text{Take the natural log.}$$

$$-\ln(2) = kt \qquad \text{Apply laws of logarithms.}$$

$$-\frac{\ln(2)}{k} = t \qquad \text{Divide by } k.$$

Since t, the time, is positive, k must, as expected, be negative. This gives us the half-life formula

$$t = -\frac{\ln(2)}{k}$$

How To…
Given the half-life, find the decay rate.

1. Write $A = A_0 e^{kt}$.

2. Replace A by $\frac{1}{2} A_0$ and replace t by the given half-life.

3. Solve to find k. Express k as an exact value (do not round).

Note: It is also possible to find the decay rate using $k = -\frac{\ln(2)}{t}$.

Example 2   Finding the Function that Describes Radioactive Decay

The half-life of carbon-14 is 5,730 years. Express the amount of carbon-14 remaining as a function of time, t.

Solution   This formula is derived as follows.

$$A = A_0 e^{kt}$$ The continuous growth formula.

$$0.5A_0 = A_0 e^{k \cdot 5730}$$ Substitute the half-life for t and $0.5A_0$ for $f(t)$.

$$0.5 = e^{5730k}$$ Divide by A_0.

$$\ln(0.5) = 5730k$$ Take the natural log of both sides.

$$k = \frac{\ln(0.5)}{5730}$$ Divide by the coefficient of k.

$$A = A_0 e^{\left(\frac{\ln(0.5)}{5730}\right)t}$$ Substitute for k in the continuous growth formula.

The function that describes this continuous decay is $f(t) = A_0 e^{\left(\frac{\ln(0.5)}{5730}\right)t}$. We observe that the coefficient of t, $\frac{\ln(0.5)}{5730} \approx -1.2097$ is negative, as expected in the case of exponential decay.

Try It #14

The half-life of plutonium-244 is 80,000,000 years. Find function gives the amount of carbon-14 remaining as a function of time, measured in years.

Radiocarbon Dating

The formula for radioactive decay is important in radiocarbon dating, which is used to calculate the approximate date a plant or animal died. Radiocarbon dating was discovered in 1949 by Willard Libby, who won a Nobel Prize for his discovery. It compares the difference between the ratio of two isotopes of carbon in an organic artifact or fossil to the ratio of those two isotopes in the air. It is believed to be accurate to within about 1% error for plants or animals that died within the last 60,000 years.

Carbon-14 is a radioactive isotope of carbon that has a half-life of 5,730 years. It occurs in small quantities in the carbon dioxide in the air we breathe. Most of the carbon on earth is carbon-12, which has an atomic weight of 12 and is not radioactive. Scientists have determined the ratio of carbon-14 to carbon-12 in the air for the last 60,000 years, using tree rings and other organic samples of known dates—although the ratio has changed slightly over the centuries.

As long as a plant or animal is alive, the ratio of the two isotopes of carbon in its body is close to the ratio in the atmosphere. When it dies, the carbon-14 in its body decays and is not replaced. By comparing the ratio of carbon-14 to carbon-12 in a decaying sample to the known ratio in the atmosphere, the date the plant or animal died can be approximated.

Since the half-life of carbon-14 is 5,730 years, the formula for the amount of carbon-14 remaining after t years is

$$A \approx A_0 e^{\left(\frac{\ln(0.5)}{5730}\right)t}$$

where

- A is the amount of carbon-14 remaining
- A_0 is the amount of carbon-14 when the plant or animal began decaying.

This formula is derived as follows:

$$A = A_0 e^{kt}$$ The continuous growth formula.

$$0.5A_0 = A_0 e^{k \cdot 5730}$$ Substitute the half-life for t and $0.5A_0$ for $f(t)$.

$$0.5 = e^{5730k}$$ Divide by A_0.

$$\ln(0.5) = 5730k$$ Take the natural log of both sides.

$$k = \frac{\ln(0.5)}{5730}$$ Divide by the coefficient of k.

$$A = A_0 e^{\left(\frac{\ln(0.5)}{5730}\right)t}$$ Substitute for r in the continuous growth formula.

To find the age of an object, we solve this equation for t:

$$t = \frac{\ln\left(\frac{A}{A_0}\right)}{}$$

Out of necessity, we neglect here the many details that a scientist takes into consideration when doing carbon-14 dating, and we only look at the basic formula. The ratio of carbon-14 to carbon-12 in the atmosphere is approximately 0.0000000001%. Let r be the ratio of carbon-14 to carbon-12 in the organic artifact or fossil to be dated, determined by a method called liquid scintillation. From the equation $A \approx A_0 e^{-0.000121t}$ we know the ratio of the percentage of carbon-14 in the object we are dating to the percentage of carbon-14 in the atmosphere is $r = \dfrac{A}{A_0} \approx e^{-0.000121t}$. We solve this equation for t, to get

$$t = \frac{\ln(r)}{-0.000121}$$

How To...

Given the percentage of carbon-14 in an object, determine its age.

1. Express the given percentage of carbon-14 as an equivalent decimal, k.

2. Substitute for k in the equation $t = \dfrac{\ln(r)}{-0.000121}$ and solve for the age, t.

Example 3   Finding the Age of a Bone

A bone fragment is found that contains 20% of its original carbon-14. To the nearest year, how old is the bone?

Solution   We substitute 20% = 0.20 for k in the equation and solve for t :

$$t = \frac{\ln(r)}{-0.000121} \qquad \text{Use the general form of the equation.}$$

$$= \frac{\ln(0.20)}{-0.000121} \qquad \text{Substitute for } r.$$

$$\approx 13301 \qquad \text{Round to the nearest year.}$$

The bone fragment is about 13,301 years old.

Analysis   The instruments that measure the percentage of carbon-14 are extremely sensitive and, as we mention above, a scientist will need to do much more work than we did in order to be satisfied. Even so, carbon dating is only accurate to about 1%, so this age should be given as 13,301 years $\pm$ 1% or 13,301 years $\pm$ 133 years.

Try It #15

Cesium-137 has a half-life of about 30 years. If we begin with 200 mg of cesium-137, will it take more or less than 230 years until only 1 milligram remains?

Calculating Doubling Time

For decaying quantities, we determined how long it took for half of a substance to decay. For growing quantities, we might want to find out how long it takes for a quantity to double. As we mentioned above, the time it takes for a quantity to double is called the **doubling time**.

Given the basic exponential growth equation $A = A_0 e^{kt}$, doubling time can be found by solving for when the original quantity has doubled, that is, by solving $2A_0 = A_0 e^{kt}$.

The formula is derived as follows:

$$2A_0 = A_0 e^{kt}$$

$$2 = e^{kt} \qquad \text{Divide by } A_0.$$

$$\ln(2) = kt \qquad \text{Take the natural logarithm.}$$

$$t = \frac{\ln(2)}{k} \qquad \text{Divide by the coefficient of } t.$$

Thus the doubling time is

$$t = \frac{\ln(2)}{k}$$

Example 4   **Finding a Function That Describes Exponential Growth**

According to Moore's Law, the doubling time for the number of transistors that can be put on a computer chip is approximately two years. Give a function that describes this behavior.

Solution   The formula is derived as follows:

$$t = \frac{\ln(2)}{k}$$ The doubling time formula.

$$2 = \frac{\ln(2)}{k}$$ Use a doubling time of two years.

$$k = \frac{\ln(2)}{2}$$ Multiply by k and divide by 2.

$$A = A_0 e^{\frac{\ln(2)}{2}t}$$ Substitute k into the continuous growth formula.

The function is $A_0 e^{\frac{\ln(2)}{2}t}$.

Try It #16

Recent data suggests that, as of 2013, the rate of growth predicted by Moore's Law no longer holds. Growth has slowed to a doubling time of approximately three years. Find the new function that takes that longer doubling time into account.

Using Newton's Law of Cooling

Exponential decay can also be applied to temperature. When a hot object is left in surrounding air that is at a lower temperature, the object's temperature will decrease exponentially, leveling off as it approaches the surrounding air temperature. On a graph of the temperature function, the leveling off will correspond to a horizontal asymptote at the temperature of the surrounding air. Unless the room temperature is zero, this will correspond to a vertical shift of the generic exponential decay function. This translation leads to **Newton's Law of Cooling**, the scientific formula for temperature as a function of time as an object's temperature is equalized with the ambient temperature

$$T(t) = Ae^{kt} + T_s$$

This formula is derived as follows:

$$T(t) = Ab^{ct} + T_s$$

$$T(t) = Ae^{\ln(b^{ct})} + T_s$$ Laws of logarithms.

$$T(t) = Ae^{ct\ln(b)} + T_s$$ Laws of logarithms.

$$T(t) = Ae^{kt} + T_s$$ Rename the constant $c\ln(b)$, calling it k.

Newton's law of cooling

The temperature of an object, T, in surrounding air with temperature T_s will behave according to the formula

$$T(t) = Ae^{kt} + T_s$$

where

- t is time
- A is the difference between the initial temperature of the object and the surroundings
- k is a constant, the continuous rate of cooling of the object

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How To…

Given a set of conditions, apply Newton's Law of Cooling.

1. Set T_s equal to the y-coordinate of the horizontal asymptote (usually the ambient temperature).
2. Substitute the given values into the continuous growth formula $T(t) = Ae^{kt} + T_s$ to find the parameters A and k.
3. Substitute in the desired time to find the temperature or the desired temperature to find the time.

<u>Example 5</u>   **Using Newton's Law of Cooling**

A cheesecake is taken out of the oven with an ideal internal temperature of 165°F, and is placed into a 35°F refrigerator. After 10 minutes, the cheesecake has cooled to 150°F. If we must wait until the cheesecake has cooled to 70°F before we eat it, how long will we have to wait?

Solution   Because the surrounding air temperature in the refrigerator is 35 degrees, the cheesecake's temperature will decay exponentially toward 35, following the equation

$$T(t) = Ae^{kt} + 35$$

We know the initial temperature was 165, so $T(0) = 165$.

$$165 = Ae^{k0} + 35 \qquad\qquad \text{Substitute } (0, 165).$$
$$A = 130 \qquad\qquad \text{Solve for } A.$$

We were given another data point, $T(10) = 150$, which we can use to solve for k.

$$150 = 130e^{k10} + 35 \qquad\qquad \text{Substitute } (10, 150).$$
$$115 = 130e^{k10} \qquad\qquad \text{Subtract 35.}$$
$$\frac{115}{130} = e^{10k} \qquad\qquad \text{Divide by 130.}$$
$$\ln\left(\frac{115}{130}\right) = 10k \qquad\qquad \text{Take the natural log of both sides.}$$
$$k = \frac{\ln\left(\frac{115}{130}\right)}{10} \approx -0.0123 \quad \text{Divide by the coefficient of } k.$$

This gives us the equation for the cooling of the cheesecake: $T(t) = 130e^{-0.0123t} + 35$.

Now we can solve for the time it will take for the temperature to cool to 70 degrees.

$$70 = 130e^{-0.0123t} + 35 \qquad\qquad \text{Substitute in 70 for } T(t).$$
$$35 = 130e^{-0.0123t} \qquad\qquad \text{Subtract 35.}$$
$$\frac{35}{130} = e^{-0.0123t} \qquad\qquad \text{Divide by 130.}$$
$$\ln\left(\frac{35}{130}\right) = -0.0123t \qquad\qquad \text{Take the natural log of both sides}$$
$$t = \frac{\ln\left(\frac{35}{130}\right)}{-0.0123} \approx 106.68 \quad \text{Divide by the coefficient of } t.$$

It will take about 107 minutes, or one hour and 47 minutes, for the cheesecake to cool to 70°F.

Try It #17

A pitcher of water at 40 degrees Fahrenheit is placed into a 70 degree room. One hour later, the temperature has risen to 45 degrees. How long will it take for the temperature to rise to 60 degrees?

Using Logistic Growth Models

Exponential growth cannot continue forever. Exponential models, while they may be useful in the short term, tend to fall apart the longer they continue. Consider an aspiring writer who writes a single line on day one and plans to double the number of lines she writes each day for a month. By the end of the month, she must write over 17 billion lines, or one-half-billion pages. It is impractical, if not impossible, for anyone to write that much in such a short period of time. Eventually, an exponential model must begin to approach some limiting value, and then the growth is forced to slow. For this reason, it is often better to use a model with an upper bound instead of an exponential growth model, though the exponential growth model is still useful over a short term, before approaching the limiting value.

The **logistic growth model** is approximately exponential at first, but it has a reduced rate of growth as the output approaches the model's upper bound, called the **carrying capacity**. For constants a, b, and c, the logistic growth of a population over time x is represented by the model

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

The graph in **Figure 6** shows how the growth rate changes over time. The graph increases from left to right, but the growth rate only increases until it reaches its point of maximum growth rate, at which point the rate of increase decreases.
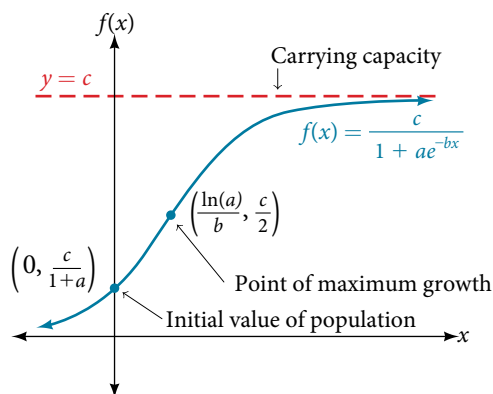

Figure 6

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logistic growth

The logistic growth model is

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

where

- $\dfrac{c}{1 + a}$ is the initial value
- c is the *carrying capacity, or limiting value*
- b is a constant determined by the rate of growth.

Example 6   Using the Logistic-Growth Model

An influenza epidemic spreads through a population rapidly, at a rate that depends on two factors: The more people who have the flu, the more rapidly it spreads, and also the more uninfected people there are, the more rapidly it spreads. These two factors make the logistic model a good one to study the spread of communicable diseases. And, clearly, there is a maximum value for the number of people infected: the entire population.

For example, at time $t = 0$ there is one person in a community of 1,000 people who has the flu. So, in that community, at most 1,000 people can have the flu. Researchers find that for this particular strain of the flu, the logistic growth constant is $b = 0.6030$. Estimate the number of people in this community who will have had this flu after ten days. Predict how many people in this community will have had this flu after a long period of time has passed.

Solution   We substitute the given data into the logistic growth model

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

Because at most 1,000 people, the entire population of the community, can get the flu, we know the limiting value is $c = 1000$. To find a, we use the formula that the number of cases at time $t = 0$ is $\dfrac{c}{1 + a} = 1$, from which it follows that $a = 999$. This model predicts that, after ten days, the number of people who have had the flu is $f(x) = \dfrac{1000}{1 + 999e^{-0.6030x}} \approx 293.8$. Because the actual number must be a whole number (a person has either had the flu or not) we round to 294. In the long term, the number of people who will contract the flu is the limiting value, $c = 1000$.

Analysis   *Remember that, because we are dealing with a virus, we cannot predict with certainty the number of people infected. The model only approximates the number of people infected and will not give us exact or actual values. The graph in* **Figure 7** *gives a good picture of how this model fits the data.*
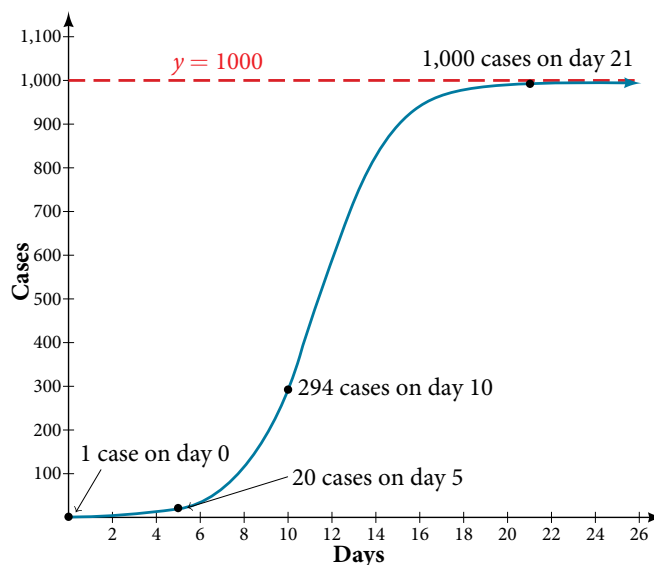
Figure 7 The graph of $f(x) = \dfrac{1000}{1 + 999e^{-0.6030x}}$

Try It #18

Using the model in **Example 6**, estimate the number of cases of flu on day 15.

Choosing an Appropriate Model for Data

Now that we have discussed various mathematical models, we need to learn how to choose the appropriate model for the raw data we have. Many factors influence the choice of a mathematical model, among which are experience, scientific laws, and patterns in the data itself. Not all data can be described by elementary functions. Sometimes, a function is chosen that approximates the data over a given interval. For instance, suppose data were gathered on the number of homes bought in the United States from the years 1960 to 2013. After plotting these data in a scatter plot, we notice that the shape of the data from the years 2000 to 2013 follow a logarithmic curve. We could restrict the interval from 2000 to 2010, apply regression analysis using a logarithmic model, and use it to predict the number of home buyers for the year 2015.

Three kinds of functions that are often useful in mathematical models are linear functions, exponential functions, and logarithmic functions. If the data lies on a straight line, or seems to lie approximately along a straight line, a linear model may be best. If the data is non-linear, we often consider an exponential or logarithmic model, though other models, such as quadratic models, may also be considered.

In choosing between an exponential model and a logarithmic model, we look at the way the data curves. This is called the concavity. If we draw a line between two data points, and all (or most) of the data between those two points lies above that line, we say the curve is concave down. We can think of it as a bowl that bends downward and therefore cannot hold water. If all (or most) of the data between those two points lies below the line, we say the curve is concave up. In this case, we can think of a bowl that bends upward and can therefore hold water. An exponential curve, whether rising or falling, whether representing growth or decay, is always concave up away from its horizontal asymptote. A logarithmic curve is always concave away from its vertical asymptote. In the case of positive data, which is the most common case, an exponential curve is always concave up, and a logarithmic curve always concave down.

A logistic curve changes concavity. It starts out concave up and then changes to concave down beyond a certain point, called a point of inflection.

After using the graph to help us choose a type of function to use as a model, we substitute points, and solve to find the parameters. We reduce round-off error by choosing points as far apart as possible.

Example 7   Choosing a Mathematical Model

Does a linear, exponential, logarithmic, or logistic model best fit the values listed in **Table 1**? Find the model, and use a graph to check your choice.

x	1	2	3	4	5	6	7	8	9
y	0	1.386	2.197	2.773	3.219	3.584	3.892	4.159	4.394

Table 1

Solution   First, plot the data on a graph as in **Figure 8**. For the purpose of graphing, round the data to two significant digits.
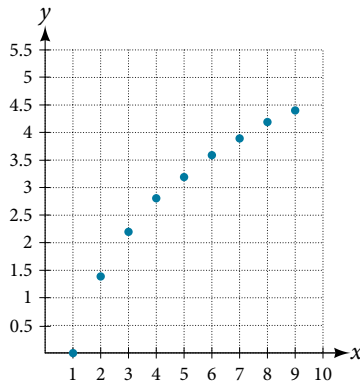

Figure 8

Clearly, the points do not lie on a straight line, so we reject a linear model. If we draw a line between any two of the points, most or all of the points between those two points lie above the line, so the graph is concave down, suggesting a logarithmic model. We can try $y = a\ln(bx)$. Plugging in the first point, (1,0), gives $0 = a\ln b$.

We reject the case that $a = 0$ (if it were, all outputs would be 0), so we know $\ln(b) = 0$. Thus $b = 1$ and $y = a\ln(x)$. Next we can use the point (9,4.394) to solve for a:

$$y = a\ln(x)$$
$$4.394 = a\ln(9)$$
$$a = \frac{4.394}{\ln(9)}$$

Because $a = \dfrac{4.394}{\ln(9)} \approx 2$, an appropriate model for the data is $y = 2\ln(x)$.

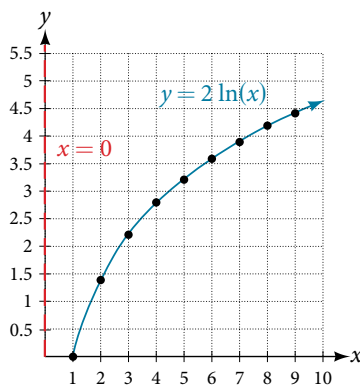To check the accuracy of the model, we graph the function together with the given points as in **Figure 9**.

Figure 9   The graph of $y = 2\ln x$.

We can conclude that the model is a good fit to the data.

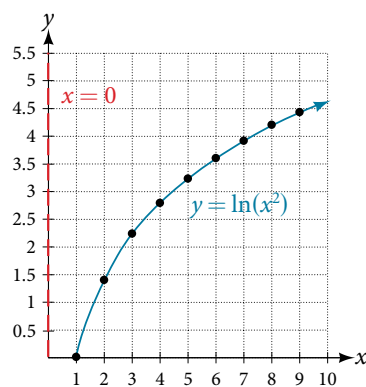Compare **Figure 9** to the graph of $y = \ln(x^2)$ shown in **Figure 10**.

Figure 10 The graph of $y = \ln(x^2)$

The graphs appear to be identical when $x > 0$. A quick check confirms this conclusion: $y = \ln(x^2) = 2\ln(x)$ for $x > 0$.

However, if $x < 0$, the graph of $y = \ln(x^2)$ includes a "extra" branch, as shown in **Figure 11**. This occurs because, while $y = 2\ln(x)$ cannot have negative values in the domain (as such values would force the argument to be negative), the function $y = \ln(x^2)$ can have negative domain values.
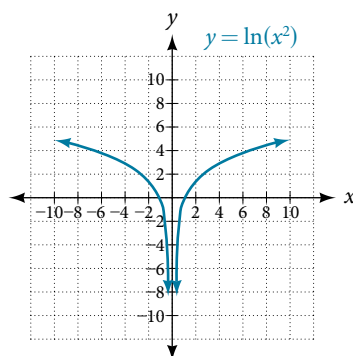

Figure 11

Try It #19

Does a linear, exponential, or logarithmic model best fit the data in **Table 2**? Find the model.

x	1	2	3	4	5	6	7	8	9
y	3.297	5.437	8.963	14.778	24.365	40.172	66.231	109.196	180.034

Table 2

Expressing an Exponential Model in Base e

While powers and logarithms of any base can be used in modeling, the two most common bases are 10 and e. In science and mathematics, the base e is often preferred. We can use laws of exponents and laws of logarithms to change any base to base e.

How To...

Given a model with the form $y = ab^x$, change it to the form $y = A_0e^{kx}$.

1. Rewrite $y = ab^x$ as $y = ae^{\ln(b^x)}$.

2. Use the power rule of logarithms to rewrite y as $y = ae^{x\ln(b)} = ae^{\ln(b)x}$.

3. Note that $a = A_0$ and $k = \ln(b)$ in the equation $y = A_0e^{kx}$.

<u>Example 8</u>   **Changing to base e**

Change the function $y = 2.5(3.1)^x$ so that this same function is written in the form $y = A_0 e^{kx}$.

Solution

The formula is derived as follows

$$y = 2.5(3.1)^x$$
$$= 2.5e^{\ln(3.1^x)} \quad \text{Insert exponential and its inverse.}$$
$$= 2.5e^{x\ln 3.1} \quad \text{Laws of logs.}$$
$$= 2.5e^{(\ln 3.1)x} \quad \text{Commutative law of multiplication}$$

Try It #20

Change the function $y = 3(0.5)^x$ to one having e as the base.

Access these online resources for additional instruction and practice with exponential and logarithmic models.

- Logarithm Application – pH (http://openstaxcollege.org/l/logph)
- Exponential Model – Age Using Half-Life (http://openstaxcollege.org/l/expmodelhalf)
- Newton's Law of Cooling (http://openstaxcollege.org/l/newtoncooling)
- Exponential Growth Given Doubling Time (http://openstaxcollege.org/l/expgrowthdbl)
- Exponential Growth – Find Initial Amount Given Doubling Time (http://openstaxcollege.org/l/initialdouble)