CHAPTER 10 REVIEW

Key Terms

angle of rotation an acute angle formed by a set of axes rotated from the Cartesian plane where, if $\cot(2\theta) > 0$, then θ is between $(0°, 45°)$; if $\cot(2\theta) < 0$, then θ is between $(45°, 90°)$; and if $\cot(2\theta) = 0$, then $\theta = 45°$

center of a hyperbola the midpoint of both the transverse and conjugate axes of a hyperbola

center of an ellipse the midpoint of both the major and minor axes

conic section any shape resulting from the intersection of a right circular cone with a plane

conjugate axis the axis of a hyperbola that is perpendicular to the transverse axis and has the co-vertices as its endpoints

degenerate conic sections any of the possible shapes formed when a plane intersects a double cone through the apex. Types of degenerate conic sections include a point, a line, and intersecting lines.

directrix a line perpendicular to the axis of symmetry of a parabola; a line such that the ratio of the distance between the points on the conic and the focus to the distance to the directrix is constant

eccentricity the ratio of the distances from a point P on the graph to the focus F and to the directrix D represented by $e = \frac{PF}{PD}$, where e is a positive real number

ellipse the set of all points (x, y) in a plane such that the sum of their distances from two fixed points is a constant

foci plural of focus

focus (of a parabola) a fixed point in the interior of a parabola that lies on the axis of symmetry

focus (of an ellipse) one of the two fixed points on the major axis of an ellipse such that the sum of the distances from these points to any point (x, y) on the ellipse is a constant

hyperbola the set of all points (x, y) in a plane such that the difference of the distances between (x, y) and the foci is a positive constant

latus rectum the line segment that passes through the focus of a parabola parallel to the directrix, with endpoints on the parabola

major axis the longer of the two axes of an ellipse

minor axis the shorter of the two axes of an ellipse

nondegenerate conic section a shape formed by the intersection of a plane with a double right cone such that the plane does not pass through the apex; nondegenerate conics include circles, ellipses, hyperbolas, and parabolas

parabola the set of all points (x, y) in a plane that are the same distance from a fixed line, called the directrix, and a fixed point (the focus) not on the directrix

polar equation an equation of a curve in polar coordinates r and θ

transverse axis the axis of a hyperbola that includes the foci and has the vertices as its endpoints

Key Equations

Horizontal ellipse, center at origin	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b$
Vertical ellipse, center at origin	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1,\ a > b$
Horizontal ellipse, center (h, k)	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,\ a > b$
Vertical ellipse, center (h, k)	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1,\ a > b$
Hyperbola, center at origin, transverse axis on x-axis	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Hyperbola, center at origin, transverse axis on y-axis	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Hyperbola, center at (h, k), transverse axis parallel to x-axis	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$
Hyperbola, center at (h, k), transverse axis parallel to y-axis	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Parabola, vertex at origin, axis of symmetry on x-axis	$y^2=4px$
Parabola, vertex at origin, axis of symmetry on y-axis	$x^2=4py$
Parabola, vertex at (h, k), axis of symmetry on x-axis	$(y-k)^2=4p(x-h)$
Parabola, vertex at (h, k), axis of symmetry on y-axis	$(x-h)^2=4p(y-k)$
General Form equation of a conic section	$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$
Rotation of a conic section	$x=x'\cos\theta-y'\sin\theta$ $y=x'\sin\theta+y'\cos\theta$
Angle of rotation	θ, where $\cot(2\theta)=\frac{A-C}{B}$

Key Concepts

10.1 The Ellipse

- An ellipse is the set of all points (x, y) in a plane such that the sum of their distances from two fixed points is a constant. Each fixed point is called a focus (plural: foci).
- When given the coordinates of the foci and vertices of an ellipse, we can write the equation of the ellipse in standard form. See **Example 1** and **Example 2**.
- When given an equation for an ellipse centered at the origin in standard form, we can identify its vertices, co-vertices, foci, and the lengths and positions of the major and minor axes in order to graph the ellipse. See **Example 3** and **Example 4**.
- When given the equation for an ellipse centered at some point other than the origin, we can identify its key features and graph the ellipse. See **Example 5** and **Example 6**.
- Real-world situations can be modeled using the standard equations of ellipses and then evaluated to find key features, such as lengths of axes and distance between foci. See **Example 7**.

10.2 The Hyperbola

- A hyperbola is the set of all points (x, y) in a plane such that the difference of the distances between (x, y) and the foci is a positive constant.
- The standard form of a hyperbola can be used to locate its vertices and foci. See **Example 1**.
- When given the coordinates of the foci and vertices of a hyperbola, we can write the equation of the hyperbola in standard form. See **Example 2** and **Example 3**.
- When given an equation for a hyperbola, we can identify its vertices, co-vertices, foci, asymptotes, and lengths and positions of the transverse and conjugate axes in order to graph the hyperbola. See **Example 4** and **Example 5**.
- Real-world situations can be modeled using the standard equations of hyperbolas. For instance, given the dimensions of a natural draft cooling tower, we can find a hyperbolic equation that models its sides. See **Example 6**.

10.3 The Parabola

- A parabola is the set of all points (x, y) in a plane that are the same distance from a fixed line, called the directrix, and a fixed point (the focus) not on the directrix.
- The standard form of a parabola with vertex $(0, 0)$ and the x-axis as its axis of symmetry can be used to graph the parabola. If $p > 0$, the parabola opens right. If $p < 0$, the parabola opens left. See **Example 1**.
- The standard form of a parabola with vertex $(0, 0)$ and the y-axis as its axis of symmetry can be used to graph the parabola. If $p > 0$, the parabola opens up. If $p < 0$, the parabola opens down. See **Example 2**.
- When given the focus and directrix of a parabola, we can write its equation in standard form. See **Example 3**.
- The standard form of a parabola with vertex (h, k) and axis of symmetry parallel to the x-axis can be used to graph the parabola. If $p > 0$, the parabola opens right. If $p < 0$, the parabola opens left. See **Example 4**.
- The standard form of a parabola with vertex (h, k) and axis of symmetry parallel to the y-axis can be used to graph the parabola. If $p > 0$, the parabola opens up. If $p < 0$, the parabola opens down. See **Example 5**.
- Real-world situations can be modeled using the standard equations of parabolas. For instance, given the diameter and focus of a cross-section of a parabolic reflector, we can find an equation that models its sides. See **Example 6**.

10.4 Rotation of Axes

- Four basic shapes can result from the intersection of a plane with a pair of right circular cones connected tail to tail. They include an ellipse, a circle, a hyperbola, and a parabola.
- A nondegenerate conic section has the general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ where A, B and C are not all zero. The values of A, B, and C determine the type of conic. See **Example 1**.
- Equations of conic sections with an xy term have been rotated about the origin. See **Example 2**.
- The general form can be transformed into an equation in the x' and y' coordinate system without the $x'y'$ term. See **Example 3** and **Example 4**.
- An expression is described as invariant if it remains unchanged after rotating. Because the discriminant is invariant, observing it enables us to identify the conic section. See **Example 5**.

10.5 Conic Sections in Polar Coordinates

- Any conic may be determined by a single focus, the corresponding eccentricity, and the directrix. We can also define a conic in terms of a fixed point, the focus $P(r, \theta)$ at the pole, and a line, the directrix, which is perpendicular to the polar axis.
- A conic is the set of all points $e = \frac{PF}{PD}$, where eccentricity e is a positive real number. Each conic may be written in terms of its polar equation. See **Example 1**.
- The polar equations of conics can be graphed. See **Example 2**, **Example 3**, and **Example 4**.
- Conics can be defined in terms of a focus, a directrix, and eccentricity. See **Example 5** and **Example 6**.
- We can use the identities $r = \sqrt{x^2 + y^2}$, $x = r\cos\theta$, and $y = r\sin\theta$ to convert the equation for a conic from polar to rectangular form. See **Example 7**.