CHAPTER 12 REVIEW

Key Terms

average rate of change the slope of the line connecting the two points $(a, f(a))$ and $(a+h, f(a+h))$ on the curve of $f(x)$; it is given by $\text{AROC} = \frac{f(a+h) - f(a)}{h}$.

continuous function a function that has no holes or breaks in its graph

derivative the slope of a function at a given point; denoted $f'(a)$, at a point $x = a$ it is $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, providing the limit exists.

differentiable a function $f(x)$ for which the derivative exists at $x = a$. In other words, if $f'(a)$ exists.

discontinuous function a function that is not continuous at $x = a$

instantaneous rate of change the slope of a function at a given point; at $x = a$ it is given by $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

instantaneous velocity the change in speed or direction at a given instant; a function $s(t)$ represents the position of an object at time t, and the instantaneous velocity or velocity of the object at time $t = a$ is given by $s'(a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$.

jump discontinuity a point of discontinuity in a function $f(x)$ at $x = a$ where both the left and right-hand limits exist, but $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$

left-hand limit the limit of values of $f(x)$ as x approaches a from the left, denoted $\lim_{x \to a^-} f(x) = L$. The values of $f(x)$ can get as close to the limit L as we like by taking values of x sufficiently close to a such that $x < a$ and $x \neq a$. Both a and L are real numbers.

limit when it exists, the value, L, that the output of a function $f(x)$ approaches as the input x gets closer and closer to a but does not equal a. The value of the output, $f(x)$, can get as close to L as we choose to make it by using input values of x sufficiently near to $x = a$, but not necessarily at $x = a$. Both a and L are real numbers, and L is denoted $\lim_{x \to a} f(x) = L$.

properties of limits a collection of theorems for finding limits of functions by performing mathematical operations on the limits

removable discontinuity a point of discontinuity in a function $f(x)$ where the function is discontinuous, but can be redefined to make it continuous

right-hand limit the limit of values of $f(x)$ as x approaches a from the right, denoted $\lim_{x \to a^+} f(x) = L$. The values of $f(x)$ can get as close to the limit L as we like by taking values of x sufficiently close to a where $x > a$, and $x \neq a$. Both a and L are real numbers.

secant line a line that intersects two points on a curve

tangent line a line that intersects a curve at a single point

two-sided limit the limit of a function $f(x)$, as x approaches a, is equal to L, that is, $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

Key Equations

average rate of change	$\text{AROC} = \frac{f(a+h) - f(a)}{h}$
derivative of a function	$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Key Concepts

12.1 Finding Limits: Numerical and Graphical Approaches

- A function has a limit if the output values approach some value L as the input values approach some quantity a. See **Example 1**.
- A shorthand notation is used to describe the limit of a function according to the form $\lim_{x \to a} f(x) = L$, which indicates that as x approaches a, both from the left of $x = a$ and the right of $x = a$, the output value gets close to L.
- A function has a left-hand limit if $f(x)$ approaches L as x approaches a where $x < a$. A function has a right-hand limit if $f(x)$ approaches L as x approaches a where $x > a$.
- A two-sided limit exists if the left-hand limit and the right-hand limit of a function are the same. A function is said to have a limit if it has a two-sided limit.
- A graph provides a visual method of determining the limit of a function.
- If the function has a limit as x approaches a, the branches of the graph will approach the same y- coordinate near $x = a$ from the left and the right. See **Example 2**.
- A table can be used to determine if a function has a limit. The table should show input values that approach a from both directions so that the resulting output values can be evaluated. If the output values approach some number, the function has a limit. See **Example 3**.
- A graphing utility can also be used to find a limit. See **Example 4**.

12.2 Finding Limits: Properties of Limits

- The properties of limits can be used to perform operations on the limits of functions rather than the functions themselves. See **Example 1**.
- The limit of a polynomial function can be found by finding the sum of the limits of the individual terms. See **Example 2** and **Example 3**.
- The limit of a function that has been raised to a power equals the same power of the limit of the function. Another method is direct substitution. See **Example 4**.
- The limit of the root of a function equals the corresponding root of the limit of the function.
- One way to find the limit of a function expressed as a quotient is to write the quotient in factored form and simplify. See **Example 5**.
- Another method of finding the limit of a complex fraction is to find the LCD. See **Example 6**.
- A limit containing a function containing a root may be evaluated using a conjugate. See **Example 7**.
- The limits of some functions expressed as quotients can be found by factoring. See **Example 8**.
- One way to evaluate the limit of a quotient containing absolute values is by using numeric evidence. Setting it up piecewise can also be useful. See **Example 9**.

12.3 Continuity

- A continuous function can be represented by a graph without holes or breaks.
- A function whose graph has holes is a discontinuous function.
- A function is continuous at a particular number if three conditions are met:
  - Condition 1: $f(a)$ exists.
  - Condition 2: $\lim_{x \to a} f(x)$ exists at $x = a$.
  - Condition 3: $\lim_{x \to a} f(x) = f(a)$.
- A function has a jump discontinuity if the left- and right-hand limits are different, causing the graph to “jump.”
- A function has a removable discontinuity if it can be redefined at its discontinuous point to make it continuous. See **Example 1**.

- Some functions, such as polynomial functions, are continuous everywhere. Other functions, such as logarithmic functions, are continuous on their domain. See **Example 2** and **Example 3**.
- For a piecewise function to be continuous each piece must be continuous on its part of the domain and the function as a whole must be continuous at the boundaries. See **Example 4** and **Example 5**.

12.4 Derivatives

- The slope of the secant line connecting two points is the average rate of change of the function between those points. See **Example 1**.
- The derivative, or instantaneous rate of change, is a measure of the slope of the curve of a function at a given point, or the slope of the line tangent to the curve at that point. See **Example 2**, **Example 3**, and **Example 4**.
- The difference quotient is the quotient in the formula for the instantaneous rate of change: $\dfrac{f(a+h)-f(a)}{h}$
- Instantaneous rates of change can be used to find solutions to many real-world problems. See **Example 5**.
- The instantaneous rate of change can be found by observing the slope of a function at a point on a graph by drawing a line tangent to the function at that point. See **Example 6**.
- Instantaneous rates of change can be interpreted to describe real-world situations. See **Example 7** and **Example 8**.
- Some functions are not differentiable at a point or points. See **Example 9**.
- The point-slope form of a line can be used to find the equation of a line tangent to the curve of a function. See **Example 10**.
- Velocity is a change in position relative to time. Instantaneous velocity describes the velocity of an object at a given instant. Average velocity describes the velocity maintained over an interval of time.
- Using the derivative makes it possible to calculate instantaneous velocity even though there is no elapsed time. See **Example 11**.