CHAPTER 8 REVIEW

Key Terms

altitude a perpendicular line from one vertex of a triangle to the opposite side, or in the case of an obtuse triangle, to the line containing the opposite side, forming two right triangles

ambiguous case a scenario in which more than one triangle is a valid solution for a given oblique SSA triangle

Archimedes' spiral a polar curve given by $r = \theta$. When multiplied by a constant, the equation appears as $r = a\theta$. As $r = \theta$, the curve continues to widen in a spiral path over the domain.

argument the angle associated with a complex number; the angle between the line from the origin to the point and the positive real axis

cardioid a member of the limaçon family of curves, named for its resemblance to a heart; its equation is given as $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$, where $\frac{a}{b} = 1$

convex limaçon a type of one-loop limaçon represented by $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$ such that $\frac{a}{b} \geq 2$

De Moivre's Theorem formula used to find the nth power or nth roots of a complex number; states that, for a positive integer n, z^n is found by raising the modulus to the nth power and multiplying the angles by n

dimpled limaçon a type of one-loop limaçon represented by $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$ such that $1 < \frac{a}{b} < 2$

dot product given two vectors, the sum of the product of the horizontal components and the product of the vertical components

Generalized Pythagorean Theorem an extension of the Law of Cosines; relates the sides of an oblique triangle and is used for SAS and SSS triangles

initial point the origin of a vector

inner-loop limaçon a polar curve similar to the cardioid, but with an inner loop; passes through the pole twice; represented by $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$ where $a < b$

Law of Cosines states that the square of any side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of the other two sides and the cosine of the included angle

Law of Sines states that the ratio of the measurement of one angle of a triangle to the length of its opposite side is equal to the remaining two ratios of angle measure to opposite side; any pair of proportions may be used to solve for a missing angle or side

lemniscate a polar curve resembling a figure 8 and given by the equation $r^2 = a^2 \cos 2\theta$ and $r^2 = a^2 \sin 2\theta$, $a \neq 0$

magnitude the length of a vector; may represent a quantity such as speed, and is calculated using the Pythagorean Theorem

modulus the absolute value of a complex number, or the distance from the origin to the point (x, y); also called the amplitude

oblique triangle any triangle that is not a right triangle

one-loop limaçon a polar curve represented by $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$ such that $a > 0$, $b > 0$, and $\frac{a}{b} > 1$; may be dimpled or convex; does not pass through the pole

parameter a variable, often representing time, upon which x and y are both dependent

polar axis on the polar grid, the equivalent of the positive x-axis on the rectangular grid

polar coordinates on the polar grid, the coordinates of a point labeled (r, θ), where θ indicates the angle of rotation from the polar axis and r represents the radius, or the distance of the point from the pole in the direction of θ

polar equation an equation describing a curve on the polar grid

polar form of a complex number a complex number expressed in terms of an angle θ and its distance from the origin r; can be found by using conversion formulas $x = r\cos\theta$, $y = r\sin\theta$, and $r = \sqrt{x^2 + y^2}$

pole the origin of the polar grid

resultant a vector that results from addition or subtraction of two vectors, or from scalar multiplication

rose curve a polar equation resembling a flower, given by the equations $r = a\cos n\theta$ and $r = a\sin n\theta$; when n is even there are $2n$ petals, and the curve is highly symmetrical; when n is odd there are n petals.

scalar a quantity associated with magnitude but not direction; a constant

scalar multiplication the product of a constant and each component of a vector

standard position the placement of a vector with the initial point at (0, 0) and the terminal point (a, b), represented by the change in the x-coordinates and the change in the y-coordinates of the original vector

terminal point the end point of a vector, usually represented by an arrow indicating its direction

unit vector a vector that begins at the origin and has magnitude of 1; the horizontal unit vector runs along the x-axis and is defined as $v_1 = \langle 1, 0 \rangle$ the vertical unit vector runs along the y-axis and is defined as $v_2 = \langle 0, 1 \rangle$.

vector a quantity associated with both magnitude and direction, represented as a directed line segment with a starting point (initial point) and an end point (terminal point)

vector addition the sum of two vectors, found by adding corresponding components

Key Equations

Law of Sines	$\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c}$ $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$
Area for oblique triangles	$\text{Area} = \dfrac{1}{2}bc\sin\alpha$ $= \dfrac{1}{2}ac\sin\beta$ $= \dfrac{1}{2}ab\sin\gamma$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = a^2 + c^2 - 2ac\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$
Heron's formula	$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{(a+b+c)}{2}$
Conversion formulas	$\cos\theta = \dfrac{x}{r} \rightarrow x = r\cos\theta$ $\sin\theta = \dfrac{y}{r} \rightarrow y = r\sin\theta$ $r^2 = x^2 + y^2$ $\tan\theta = \dfrac{y}{x}$

Key Concepts

8.1 Non-right Triangles: Law of Sines

- The Law of Sines can be used to solve oblique triangles, which are non-right triangles.
- According to the Law of Sines, the ratio of the measurement of one of the angles to the length of its opposite side equals the other two ratios of angle measure to opposite side.

- There are three possible cases: ASA, AAS, SSA. Depending on the information given, we can choose the appropriate equation to find the requested solution. See **Example 1**.
- The ambiguous case arises when an oblique triangle can have different outcomes.
- There are three possible cases that arise from SSA arrangement—a single solution, two possible solutions, and no solution. See **Example 2** and **Example 3**.
- The Law of Sines can be used to solve triangles with given criteria. See **Example 4**.
- The general area formula for triangles translates to oblique triangles by first finding the appropriate height value. See **Example 5**.
- There are many trigonometric applications. They can often be solved by first drawing a diagram of the given information and then using the appropriate equation. See **Example 6**.

8.2 Non-right Triangles: Law of Cosines

- The Law of Cosines defines the relationship among angle measurements and lengths of sides in oblique triangles.
- The Generalized Pythagorean Theorem is the Law of Cosines for two cases of oblique triangles: SAS and SSS. Dropping an imaginary perpendicular splits the oblique triangle into two right triangles or forms one right triangle, which allows sides to be related and measurements to be calculated. See **Example 1** and **Example 2**.
- The Law of Cosines is useful for many types of applied problems. The first step in solving such problems is generally to draw a sketch of the problem presented. If the information given fits one of the three models (the three equations), then apply the Law of Cosines to find a solution. See **Example 3** and **Example 4**.
- Heron's formula allows the calculation of area in oblique triangles. All three sides must be known to apply Heron's formula. See **Example 5** and See **Example 6**.

8.3 Polar Coordinates

- The polar grid is represented as a series of concentric circles radiating out from the pole, or origin.
- To plot a point in the form (r, θ), $\theta > 0$, move in a counterclockwise direction from the polar axis by an angle of θ, and then extend a directed line segment from the pole the length of r in the direction of θ. If θ is negative, move in a clockwise direction, and extend a directed line segment the length of r in the direction of θ. See **Example 1**.
- If r is negative, extend the directed line segment in the opposite direction of θ. See **Example 2**.
- To convert from polar coordinates to rectangular coordinates, use the formulas $x = r\cos\theta$ and $y = r\sin\theta$. See **Example 3** and **Example 4**.
- To convert from rectangular coordinates to polar coordinates, use one or more of the formulas: $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$, $\tan\theta = \frac{y}{x}$, and $r = \sqrt{x^2 + y^2}$. See **Example 5**.
- Transforming equations between polar and rectangular forms means making the appropriate substitutions based on the available formulas, together with algebraic manipulations. See **Example 6**, **Example 7**, and **Example 8**.
- Using the appropriate substitutions makes it possible to rewrite a polar equation as a rectangular equation, and then graph it in the rectangular plane. See **Example 9**, **Example 10**, and **Example 11**.

8.4 Polar Coordinates: Graphs

- It is easier to graph polar equations if we can test the equations for symmetry with respect to the line $\theta = \frac{\pi}{2}$, the polar axis, or the pole.
- There are three symmetry tests that indicate whether the graph of a polar equation will exhibit symmetry. If an equation fails a symmetry test, the graph may or may not exhibit symmetry. See **Example 1**.
- Polar equations may be graphed by making a table of values for θ and r.
- The maximum value of a polar equation is found by substituting the value θ that leads to the maximum value of the trigonometric expression.
- The zeros of a polar equation are found by setting $r = 0$ and solving for θ. See **Example 2**.
- Some formulas that produce the graph of a circle in polar coordinates are given by $r = a\cos\theta$ and $r = a\sin\theta$. See **Example 3**.
- The formulas that produce the graphs of a cardioid are given by $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$, for $a > 0$,

- The formulas that produce the graphs of a one-loop limaçon are given by $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$ for $1 < \frac{a}{b} < 2$. See **Example 5**.
- The formulas that produce the graphs of an inner-loop limaçon are given by $r = a \pm b\cos\theta$ and $r = a \pm b\sin\theta$ for $a > 0$, $b > 0$, and $a < \mathrm{b}$. See **Example 6**.
- The formulas that produce the graphs of a lemniscates are given by $r^2 = a^2 \cos 2\theta$ and $r^2 = a^2 \sin 2\theta$, where $a \neq 0$. See **Example 7**.
- The formulas that produce the graphs of rose curves are given by $r = a\cos n\theta$ and $r = a\sin n\theta$, where $a \neq 0$; if n is even, there are $2n$ petals, and if n is odd, there are n petals. See **Example 8** and **Example 9**.
- The formula that produces the graph of an Archimedes' spiral is given by $r = \theta$, $\theta \geq 0$. See **Example 10**.

8.5 Polar Form of Complex Numbers

- Complex numbers in the form $a + bi$ are plotted in the complex plane similar to the way rectangular coordinates are plotted in the rectangular plane. Label the x-axis as the real axis and the y-axis as the imaginary axis. See **Example 1**.
- The absolute value of a complex number is the same as its magnitude. It is the distance from the origin to the point: $|z| = \sqrt{a^2 + b^2}$. See **Example 2** and **Example 3**.
- To write complex numbers in polar form, we use the formulas $x = r\cos\theta$, $y = r\sin\theta$, and $r = \sqrt{x^2 + y^2}$. Then, $z = r(\cos\theta + i\sin\theta)$. See **Example 4** and **Example 5**.
- To convert from polar form to rectangular form, first evaluate the trigonometric functions. Then, multiply through by r. See **Example 6** and **Example 7**.
- To find the product of two complex numbers, multiply the two moduli and add the two angles. Evaluate the trigonometric functions, and multiply using the distributive property. See **Example 8**.
- To find the quotient of two complex numbers in polar form, find the quotient of the two moduli and the difference of the two angles. See **Example 9**.
- To find the power of a complex number z^n, raise r to the power n, and multiply θ by n. See **Example 10**.
- Finding the roots of a complex number is the same as raising a complex number to a power, but using a rational exponent. See **Example 11**.

8.6 Parametric Equations

- Parameterizing a curve involves translating a rectangular equation in two variables, x and y, into two equations in three variables, x, y, and t. Often, more information is obtained from a set of parametric equations. See **Example 1**, **Example 2**, and **Example 3**.
- Sometimes equations are simpler to graph when written in rectangular form. By eliminating t, an equation in x and y is the result.
- To eliminate t, solve one of the equations for t, and substitute the expression into the second equation. See **Example 4**, **Example 5**, **Example 6**, and **Example 7**.
- Finding the rectangular equation for a curve defined parametrically is basically the same as eliminating the parameter. Solve for t in one of the equations, and substitute the expression into the second equation. See **Example 8**.
- There are an infinite number of ways to choose a set of parametric equations for a curve defined as a rectangular equation.
- Find an expression for x such that the domain of the set of parametric equations remains the same as the original rectangular equation. See **Example 9**.

8.7 Parametric Equations: Graphs

- When there is a third variable, a third parameter on which x and y depend, parametric equations can be used.
- To graph parametric equations by plotting points, make a table with three columns labeled t, $x(t)$, and $y(t)$. Choose values for t in increasing order. Plot the last two columns for x and y. See **Example 1** and **Example 2**.
- When graphing a parametric curve by plotting points, note the associated t-values and show arrows on the graph indicating the orientation of the curve. See **Example 3** and **Example 4**.
- Parametric equations allow the direction or the orientation of the curve to be shown on the graph. Equations that are not functions can be graphed and used in many applications involving motion. See **Example 5**.
- Projectile motion depends on two parametric equations: $x = (v_0 \cos \theta)t$ and $y = -16t^2 + (v_0 \sin \theta)t + h$. Initial velocity is symbolized as v_0. θ represents the initial angle of the object when thrown, and h represents the height at which the object is propelled.

8.8 Vectors

- The position vector has its initial point at the origin. See **Example 1**.
- If the position vector is the same for two vectors, they are equal. See **Example 2**. Vectors are defined by their magnitude and direction. See **Example 3**.
- If two vectors have the same magnitude and direction, they are equal. See **Example 4**.
- Vector addition and subtraction result in a new vector found by adding or subtracting corresponding elements. See **Example 5**.
- Scalar multiplication is multiplying a vector by a constant. Only the magnitude changes; the direction stays the same. See **Example 6** and **Example 7**.
- Vectors are comprised of two components: the horizontal component along the positive x-axis, and the vertical component along the positive y-axis. See **Example 8**.
- The unit vector in the same direction of any nonzero vector is found by dividing the vector by its magnitude.
- The magnitude of a vector in the rectangular coordinate system is $|v| = \sqrt{a^2 + b^2}$. See **Example 9**.
- In the rectangular coordinate system, unit vectors may be represented in terms of i and j where i represents the horizontal component and j represents the vertical component. Then, $v = ai + bj$ is a scalar multiple of v by real numbers a and b. See **Example 10** and **Example 11**.
- Adding and subtracting vectors in terms of i and j consists of adding or subtracting corresponding coefficients of i and corresponding coefficients of j. See **Example 12**.
- A vector $v = ai + bj$ is written in terms of magnitude and direction as $v = |v|\cos \theta \mathrm{i} + |v|\sin \theta j$. See **Example 13**.
- The dot product of two vectors is the product of the i terms plus the product of the j terms. See **Example 14**.
- We can use the dot product to find the angle between two vectors. **Example 15** and **Example 16**.
- Dot products are useful for many types of physics applications. See **Example 17**.