CHAPTER 3 REVIEW

Key Terms

arrow notation a way to symbolically represent the local and end behavior of a function by using arrows to indicate that an input or output approaches a value

axis of symmetry a vertical line drawn through the vertex of a parabola around which the parabola is symmetric; it is defined by $x = -\frac{b}{2a}$.

coefficient a nonzero real number multiplied by a variable raised to an exponent

complex conjugate the complex number in which the sign of the imaginary part is changed and the real part of the number is left unchanged; when added to or multiplied by the original complex number, the result is a real number

complex number the sum of a real number and an imaginary number, written in the standard form $a + bi$, where a is the real part, and bi is the imaginary part

complex plane a coordinate system in which the horizontal axis is used to represent the real part of a complex number and the vertical axis is used to represent the imaginary part of a complex number

constant of variation the non-zero value k that helps define the relationship between variables in direct or inverse variation

continuous function a function whose graph can be drawn without lifting the pen from the paper because there are no breaks in the graph

degree the highest power of the variable that occurs in a polynomial

Descartes' Rule of Signs a rule that determines the maximum possible numbers of positive and negative real zeros based on the number of sign changes of $f(x)$ and $f(-x)$

direct variation the relationship between two variables that are a constant multiple of each other; as one quantity increases, so does the other

Division Algorithm given a polynomial dividend $f(x)$ and a non-zero polynomial divisor $d(x)$ where the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that $f(x) = d(x)q(x) + r(x)$ where $q(x)$ is the quotient and $r(x)$ is the remainder. The remainder is either equal to zero or has degree strictly less than $d(x)$.

end behavior the behavior of the graph of a function as the input decreases without bound and increases without bound

Factor Theorem k is a zero of polynomial function $f(x)$ if and only if $(x - k)$ is a factor of $f(x)$

Fundamental Theorem of Algebra a polynomial function with degree greater than 0 has at least one complex zero

general form of a quadratic function the function that describes a parabola, written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

global maximum highest turning point on a graph; $f(a)$ where $f(a) \geq f(x)$ for all x.

global minimum lowest turning point on a graph; $f(a)$ where $f(a) \leq f(x)$ for all x.

horizontal asymptote a horizontal line $y = b$ where the graph approaches the line as the inputs increase or decrease without bound.

Intermediate Value Theorem for two numbers a and b in the domain of f, if $a < b$ and $f(a) \neq f(b)$, then the function f takes on every value between $f(a)$ and $f(b)$; specifically, when a polynomial function changes from a negative value to a positive value, the function must cross the x-axis

inverse variation the relationship between two variables in which the product of the variables is a constant

inversely proportional a relationship where one quantity is a constant divided by the other quantity; as one quantity increases, the other decreases

invertible function any function that has an inverse function

imaginary number a number in the form bi where $i = \sqrt{-1}$

joint variation a relationship where a variable varies directly or inversely with multiple variables

leading coefficient the coefficient of the leading term

leading term the term containing the highest power of the variable

Linear Factorization Theorem allowing for multiplicities, a polynomial function will have the same number of factors as its degree, and each factor will be in the form $(x - c)$, where c is a complex number

multiplicity the number of times a given factor appears in the factored form of the equation of a polynomial; if a polynomial contains a factor of the form $(x - h)^p$, $x = h$ is a zero of multiplicity p.

polynomial function a function that consists of either zero or the sum of a finite number of non-zero terms, each of which is a product of a number, called the coefficient of the term, and a variable raised to a non-negative integer power.

power function a function that can be represented in the form $f(x) = kx^p$ where k is a constant, the base is a variable, and the exponent, p, is a constant

rational function a function that can be written as the ratio of two polynomials

Rational Zero Theorem the possible rational zeros of a polynomial function have the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the leading coefficient.

Remainder Theorem if a polynomial $f(x)$ is divided by $x - k$, then the remainder is equal to the value $f(k)$

removable discontinuity a single point at which a function is undefined that, if filled in, would make the function continuous; it appears as a hole on the graph of a function

smooth curve a graph with no sharp corners

standard form of a quadratic function the function that describes a parabola, written in the form $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex.

synthetic division a shortcut method that can be used to divide a polynomial by a binomial of the form $x - k$

term of a polynomial function any $a_i x^i$ of a polynomial function in the form $f(x) = a_n x^n + ... + a_2 x^2 + a_1 x + a_0$

turning point the location at which the graph of a function changes direction

varies directly a relationship where one quantity is a constant multiplied by the other quantity

varies inversely a relationship where one quantity is a constant divided by the other quantity

vertex the point at which a parabola changes direction, corresponding to the minimum or maximum value of the quadratic function

vertex form of a quadratic function another name for the standard form of a quadratic function

vertical asymptote a vertical line $x = a$ where the graph tends toward positive or negative infinity as the inputs approach a

zeros in a given function, the values of x at which $y = 0$, also called roots

Key Equations

general form of a quadratic function	$f(x) = ax^2 + bx + c$
the quadratic formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
standard form of a quadratic function	$f(x) = a(x - h)^2 + k$
general form of a polynomial function	$f(x) = a_n x^n + ... + a_2 x^2 + a_1 x + a_0$
Division Algorithm	$f(x) = d(x)q(x) + r(x)$ where $q(x) \neq 0$
Rational Function	$f(x) = \dfrac{P(x)}{Q(x)} = \dfrac{a_p x^p + a_{p-1} x^{p-1} + ... + a_1 x + a_0}{b_q x^q + b_{q-1} x^{q-1} + ... + b_1 x + b_0}, Q(x) \neq 0$
Direct variation	$y = kx^n$, k is a nonzero constant.
Inverse variation	$y = \dfrac{k}{x^n}$, k is a nonzero constant.

Key Concepts

3.1 Complex Numbers

- The square root of any negative number can be written as a multiple of i. See **Example 1**.
- To plot a complex number, we use two number lines, crossed to form the complex plane. The horizontal axis is the real axis, and the vertical axis is the imaginary axis. See **Example 2**.
- Complex numbers can be added and subtracted by combining the real parts and combining the imaginary parts. See **Example 3**.
- Complex numbers can be multiplied and divided.
- To multiply complex numbers, distribute just as with polynomials. See **Example 4**, **Example 5**, and **Example 8**.
- To divide complex numbers, multiply both the numerator and denominator by the complex conjugate of the denominator to eliminate the complex number from the denominator. See **Example 6**, **Example 7**, and **Example 9**.
- The powers of i are cyclic, repeating every fourth one. See **Example 10**.

3.2 Quadratic Functions

- A polynomial function of degree two is called a quadratic function.
- The graph of a quadratic function is a parabola. A parabola is a U-shaped curve that can open either up or down.
- The axis of symmetry is the vertical line passing through the vertex. The zeros, or x-intercepts, are the points at which the parabola crosses the x-axis. The y-intercept is the point at which the parabola crosses the y-axis. See **Example 1**, **Example 7**, and **Example 8**.
- Quadratic functions are often written in general form. Standard or vertex form is useful to easily identify the vertex of a parabola. Either form can be written from a graph. See **Example 2**.
- The vertex can be found from an equation representing a quadratic function. See **Example 3**.
- The domain of a quadratic function is all real numbers. The range varies with the function. See **Example 4**.
- A quadratic function's minimum or maximum value is given by the y-value of the vertex.
- The minimum or maximum value of a quadratic function can be used to determine the range of the function and to solve many kinds of real-world problems, including problems involving area and revenue. See **Example 5** and **Example 6**.
- Some quadratic equations must be solved by using the quadratic formula. See **Example 9**.
- The vertex and the intercepts can be identified and interpreted to solve real-world problems. See **Example 10**.

3.3 Power Functions and Polynomial Functions

- A power function is a variable base raised to a number power. See **Example 1**.
- The behavior of a graph as the input decreases beyond bound and increases beyond bound is called the end behavior.
- The end behavior depends on whether the power is even or odd. See **Example 2** and **Example 3**.
- A polynomial function is the sum of terms, each of which consists of a transformed power function with positive whole number power. See **Example 4**.
- The degree of a polynomial function is the highest power of the variable that occurs in a polynomial. The term containing the highest power of the variable is called the leading term. The coefficient of the leading term is called the leading coefficient. See **Example 5**.
- The end behavior of a polynomial function is the same as the end behavior of the power function represented by the leading term of the function. See **Example 6** and **Example 7**.
- A polynomial of degree n will have at most n x-intercepts and at most $n - 1$ turning points. See **Example 8**, **Example 9**, **Example 10**, **Example 11**, and **Example 12**.

3.4 Graphs of Polynomial Functions

- Polynomial functions of degree 2 or more are smooth, continuous functions. See **Example 1**.
- To find the zeros of a polynomial function, if it can be factored, factor the function and set each factor equal to zero. See **Example 2**, **Example 3**, and **Example 4**.
- Another way to find the x-intercepts of a polynomial function is to graph the function and identify the points at which the graph crosses the x-axis. See **Example 5**.
- The multiplicity of a zero determines how the graph behaves at the x-intercepts. See **Example 6**.
- The graph of a polynomial will cross the horizontal axis at a zero with odd multiplicity.
- The graph of a polynomial will touch the horizontal axis at a zero with even multiplicity.
- The end behavior of a polynomial function depends on the leading term.
- The graph of a polynomial function changes direction at its turning points.
- A polynomial function of degree n has at most $n - 1$ turning points. See **Example 7**.
- To graph polynomial functions, find the zeros and their multiplicities, determine the end behavior, and ensure that the final graph has at most $n - 1$ turning points. See **Example 8** and **Example 10**.
- Graphing a polynomial function helps to estimate local and global extremas. See **Example 11**.
- The Intermediate Value Theorem tells us that if $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value c between a and b for which $f(c) = 0$. See **Example 9**.

3.5 Dividing Polynomials

- Polynomial long division can be used to divide a polynomial by any polynomial with equal or lower degree. See **Example 1** and **Example 2**.
- The Division Algorithm tells us that a polynomial dividend can be written as the product of the divisor and the quotient added to the remainder.
- Synthetic division is a shortcut that can be used to divide a polynomial by a binomial in the form $x - k$. See **Example 3**, **Example 4**, and **Example 5**.
- Polynomial division can be used to solve application problems, including area and volume. See **Example 6**.

3.6 Zeros of Polynomial Functions

- To find $f(k)$, determine the remainder of the polynomial $f(x)$ when it is divided by $x - k$. See **Example 1**.
- k is a zero of $f(x)$ if and only if $(x - k)$ is a factor of $f(x)$. See **Example 2**.
- Each rational zero of a polynomial function with integer coefficients will be equal to a factor of the constant term divided by a factor of the leading coefficient. See **Example 3** and **Example 4**.
- When the leading coefficient is 1, the possible rational zeros are the factors of the constant term.
- Synthetic division can be used to find the zeros of a polynomial function. See **Example 5**.
- According to the Fundamental Theorem, every polynomial function has at least one complex zero. See **Example 6**.
- Every polynomial function with degree greater than 0 has at least one complex zero.
- Allowing for multiplicities, a polynomial function will have the same number of factors as its degree. Each factor will be in the form $(x - c)$, where c is a complex number. See **Example 7**.
- The number of positive real zeros of a polynomial function is either the number of sign changes of the function or less than the number of sign changes by an even integer.
- The number of negative real zeros of a polynomial function is either the number of sign changes of $f(-x)$ or less than the number of sign changes by an even integer. See **Example 8**.
- Polynomial equations model many real-world scenarios. Solving the equations is easiest done by synthetic division. See **Example 9**.

3.7 Rational Functions

- We can use arrow notation to describe local behavior and end behavior of the toolkit functions $f(x) = \frac{1}{x}$ and $f(x) = \frac{1}{x^2}$. See **Example 1**.
- A function that levels off at a horizontal value has a horizontal asymptote. A function can have more than one vertical asymptote. See **Example 2**.
- Application problems involving rates and concentrations often involve rational functions. See **Example 3**.
- The domain of a rational function includes all real numbers except those that cause the denominator to equal zero. See **Example 4**.
- The vertical asymptotes of a rational function will occur where the denominator of the function is equal to zero and the numerator is not zero. See **Example 5**.
- A removable discontinuity might occur in the graph of a rational function if an input causes both numerator and denominator to be zero. See **Example 6**.
- A rational function's end behavior will mirror that of the ratio of the leading terms of the numerator and denominator functions. See **Example 7**, **Example 8**, **Example 9**, and **Example 10**.
- Graph rational functions by finding the intercepts, behavior at the intercepts and asymptotes, and end behavior. See **Example 11**.
- If a rational function has x-intercepts at $x = x_1, x_2, \ldots, x_n$, vertical asymptotes at $x = v_1, v_2, \ldots, v_m$, and no $x_i =$ any v_j, then the function can be written in the form

$$f(x) = a\frac{(x - x_1)^{p_1}(x - x_2)^{p_2}\ldots(x - x_n)^{p_n}}{(x - v_1)^{q_1}(x - v_2)^{q_2}\ldots(x - v_m)^{q_n}}$$

  See **Example 12**.

3.8 Inverses and Radical Functions

- The inverse of a quadratic function is a square root function.
- If f^{-1} is the inverse of a function f, then f is the inverse of the function f^{-1}. See **Example 1**.
- While it is not possible to find an inverse of most polynomial functions, some basic polynomials are invertible. See **Example 2**.
- To find the inverse of certain functions, we must restrict the function to a domain on which it will be one-to-one. See **Example 3** and **Example 4**.
- When finding the inverse of a radical function, we need a restriction on the domain of the answer. See **Example 5** and **Example 7**.
- Inverse and radical and functions can be used to solve application problems. See **Example 6** and **Example 8**.

3.9 Modeling Using Variation

- A relationship where one quantity is a constant multiplied by another quantity is called direct variation. See **Example 1**.
- Two variables that are directly proportional to one another will have a constant ratio.
- A relationship where one quantity is a constant divided by another quantity is called inverse variation. See **Example 2**.
- Two variables that are inversely proportional to one another will have a constant multiple. See **Example 3**.
- In many problems, a variable varies directly or inversely with multiple variables. We call this type of relationship joint variation. See **Example 4**.