CHAPTER 7 REVIEW

Key Terms

damped harmonic motion oscillating motion that resembles periodic motion and simple harmonic motion, except that the graph is affected by a damping factor, an energy dissipating influence on the motion, such as friction

double-angle formulas identities derived from the sum formulas for sine, cosine, and tangent in which the angles are equal

even-odd identities set of equations involving trigonometric functions such that if $f(-x) = -f(x)$, the identity is odd, and if $f(-x) = f(x)$, the identity is even

half-angle formulas identities derived from the reduction formulas and used to determine half-angle values of trigonometric functions

product-to-sum formula a trigonometric identity that allows the writing of a product of trigonometric functions as a sum or difference of trigonometric functions

Pythagorean identities set of equations involving trigonometric functions based on the right triangle properties

quotient identities pair of identities based on the fact that tangent is the ratio of sine and cosine, and cotangent is the ratio of cosine and sine

reciprocal identities set of equations involving the reciprocals of basic trigonometric definitions

reduction formulas identities derived from the double-angle formulas and used to reduce the power of a trigonometric function

simple harmonic motion a repetitive motion that can be modeled by periodic sinusoidal oscillation

sum-to-product formula a trigonometric identity that allows, by using substitution, the writing of a sum of trigonometric functions as a product of trigonometric functions

Key Equations

Pythagorean identities	$\sin^2\theta + \cos^2\theta = 1$ $1 + \cot^2\theta = \csc^2\theta$ $1 + \tan^2\theta = \sec^2\theta$
Even-odd identities	$\tan(-\theta) = -\tan\theta$ $\cot(-\theta) = -\cot\theta$ $\sin(-\theta) = -\sin\theta$ $\csc(-\theta) = -\csc\theta$ $\cos(-\theta) = \cos\theta$ $\sec(-\theta) = \sec\theta$
Reciprocal identities	$\sin\theta = \dfrac{1}{\csc\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$ $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

Quotient identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Sum Formula for Cosine $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

Difference Formula for Cosine $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

Sum Formula for Sine $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

Difference Formula for Sine $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

Sum Formula for Tangent $\tan(\alpha + \beta) = \dfrac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

Difference Formula for Tangent $\tan(\alpha - \beta) = \dfrac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

Cofunction identities

$$\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan \theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\cot \theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\sec \theta = \csc\left(\frac{\pi}{2} - \theta\right)$$

$$\csc \theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

Double-angle formulas

$$\begin{aligned} \sin(2\theta) &= 2\sin \theta \cos \theta \\ \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2\sin^2 \theta \\ &= 2\cos^2 \theta - 1 \\ \tan(2\theta) &= \frac{2\tan \theta}{1 - \tan^2 \theta} \end{aligned}$$

Reduction formulas

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Half-angle formulas	$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$
	$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$
	$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$
Product-to-sum Formulas	$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$
	$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$
	$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$
	$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$
Sum-to-product Formulas	$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
	$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$
	$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$
	$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$
Standard form of sinusoidal equation	$y = A\sin(Bt-C)+D$ or $y = A\cos(Bt-C)+D$
Simple harmonic motion	$d = a\cos(\omega t)$ or $d = a\sin(\omega t)$
Damped harmonic motion	$f(t) = ae^{-ct}\sin(\omega t)$ or $f(t) = ae^{-ct}\cos(\omega t)$

Key Concepts

7.1 Solving Trigonometric Equations with Identities

- There are multiple ways to represent a trigonometric expression. Verifying the identities illustrates how expressions can be rewritten to simplify a problem.
- Graphing both sides of an identity will verify it. See **Example 1**.
- Simplifying one side of the equation to equal the other side is another method for verifying an identity. See **Example 2** and **Example 3**.
- The approach to verifying an identity depends on the nature of the identity. It is often useful to begin on the more complex side of the equation. See **Example 4**.
- We can create an identity by simplifying an expression and then verifying it. See **Example 5**.
- Verifying an identity may involve algebra with the fundamental identities. See **Example 6** and **Example 7**.

- Algebraic techniques can be used to simplify trigonometric expressions. We use algebraic techniques throughout this text, as they consist of the fundamental rules of mathematics. See **Example 8**, **Example 9**, and **Example 10**.

7.2 Sum and Difference Identities

- The sum formula for cosines states that the cosine of the sum of two angles equals the product of the cosines of the angles minus the product of the sines of the angles. The difference formula for cosines states that the cosine of the difference of two angles equals the product of the cosines of the angles plus the product of the sines of the angles.
- The sum and difference formulas can be used to find the exact values of the sine, cosine, or tangent of an angle. See **Example 1** and **Example 2**.
- The sum formula for sines states that the sine of the sum of two angles equals the product of the sine of the first angle and cosine of the second angle plus the product of the cosine of the first angle and the sine of the second angle. The difference formula for sines states that the sine of the difference of two angles equals the product of the sine of the first angle and cosine of the second angle minus the product of the cosine of the first angle and the sine of the second angle. See **Example 3**.
- The sum and difference formulas for sine and cosine can also be used for inverse trigonometric functions. See **Example 4**.
- The sum formula for tangent states that the tangent of the sum of two angles equals the sum of the tangents of the angles divided by 1 minus the product of the tangents of the angles. The difference formula for tangent states that the tangent of the difference of two angles equals the difference of the tangents of the angles divided by 1 plus the product of the tangents of the angles. See **Example 5**.
- The Pythagorean Theorem along with the sum and difference formulas can be used to find multiple sums and differences of angles. See **Example 6**.
- The cofunction identities apply to complementary angles and pairs of reciprocal functions. See **Example 7**.
- Sum and difference formulas are useful in verifying identities. See **Example 8** and **Example 9**.
- Application problems are often easier to solve by using sum and difference formulas. See **Example 10** and **Example 11**.

7.3 Double-Angle, Half-Angle, and Reduction Formulas

- Double-angle identities are derived from the sum formulas of the fundamental trigonometric functions: sine, cosine, and tangent. See **Example 1**, **Example 2**, **Example 3**, and **Example 4**.
- Reduction formulas are especially useful in calculus, as they allow us to reduce the power of the trigonometric term. See **Example 5** and **Example 6**.
- Half-angle formulas allow us to find the value of trigonometric functions involving half-angles, whether the original angle is known or not. See **Example 7**, **Example 8**, and **Example 9**.

7.4 Sum-to-Product and Product-to-Sum Formulas

- From the sum and difference identities, we can derive the product-to-sum formulas and the sum-to-product formulas for sine and cosine.
- We can use the product-to-sum formulas to rewrite products of sines, products of cosines, and products of sine and cosine as sums or differences of sines and cosines. See **Example 1**, **Example 2**, and **Example 3**.
- We can also derive the sum-to-product identities from the product-to-sum identities using substitution.
- We can use the sum-to-product formulas to rewrite sum or difference of sines, cosines, or products sine and cosine as products of sines and cosines. See **Example 4**.

- Trigonometric expressions are often simpler to evaluate using the formulas. See **Example 5**.
- The identities can be verified using other formulas or by converting the expressions to sines and cosines. To verify an identity, we choose the more complicated side of the equals sign and rewrite it until it is transformed into the other side. See **Example 6** and **Example 7**.

7.5 Solving Trigonometric Equations

- When solving linear trigonometric equations, we can use algebraic techniques just as we do solving algebraic equations. Look for patterns, like the difference of squares, quadratic form, or an expression that lends itself well to substitution. See **Example 1**, **Example 2**, and **Example 3**.
- Equations involving a single trigonometric function can be solved or verified using the unit circle. See **Example 4**, **Example 5**, and **Example 6**, and **Example 7**.
- We can also solve trigonometric equations using a graphing calculator. See **Example 8** and **Example 9**.
- Many equations appear quadratic in form. We can use substitution to make the equation appear simpler, and then use the same techniques we use solving an algebraic quadratic: factoring, the quadratic formula, etc. See **Example 10**, **Example 11**, **Example 12**, and **Example 13**.
- We can also use the identities to solve trigonometric equation. See **Example 14**, **Example 15**, and **Example 16**.
- We can use substitution to solve a multiple-angle trigonometric equation, which is a compression of a standard trigonometric function. We will need to take the compression into account and verify that we have found all solutions on the given interval. See **Example 17**.
- Real-world scenarios can be modeled and solved using the Pythagorean Theorem and trigonometric functions. See **Example 18**.

7.6 Modeling with Trigonometric Equations

- Sinusoidal functions are represented by the sine and cosine graphs. In standard form, we can find the amplitude, period, and horizontal and vertical shifts. See **Example 1** and **Example 2**.
- Use key points to graph a sinusoidal function. The five key points include the minimum and maximum values and the midline values. See **Example 3**.
- Periodic functions can model events that reoccur in set cycles, like the phases of the moon, the hands on a clock, and the seasons in a year. See **Example 4**, **Example 5**, **Example 6** and **Example 7**.
- Harmonic motion functions are modeled from given data. Similar to periodic motion applications, harmonic motion requires a restoring force. Examples include gravitational force and spring motion activated by weight. See **Example 8**.
- Damped harmonic motion is a form of periodic behavior affected by a damping factor. Energy dissipating factors, like friction, cause the displacement of the object to shrink. See **Example 9**, **Example 10**, **Example 11**, **Example 12**, and **Example 13**.
- Bounding curves delineate the graph of harmonic motion with variable maximum and minimum values. See **Example 14**.