

# Lecture 10: Recurrent Neural Networks

# Administrative

A1 grades will go out soon

A2 is due today (11:59pm)

Midterm is in-class on Tuesday!  
We will send out details on where to go soon

# Extra Credit: Train Game

More details on Piazza  
by early next week

The screenshot shows a web-based application titled "Train Game" running on a Mac OS X system. The interface includes a top navigation bar with tabs for "Train Game", "Test Game", and "About". The user is logged in as "syeyung" and can log out.

**Control Panel:** This section contains five sets of hyperparameter controls, each with a "Submit" button:

- Weight decay (0) with controls for "Same", "Up", and "Down".
- Network depth (6) with controls for "Same", "Up", and "Down".
- Dropout (0.4) with controls for "Same", "Up", and "Down".
- Learning rate (0.0001) with controls for "Same", "Up", and "Down".
- Network width (256) with controls for "Same", "Up", and "Down".

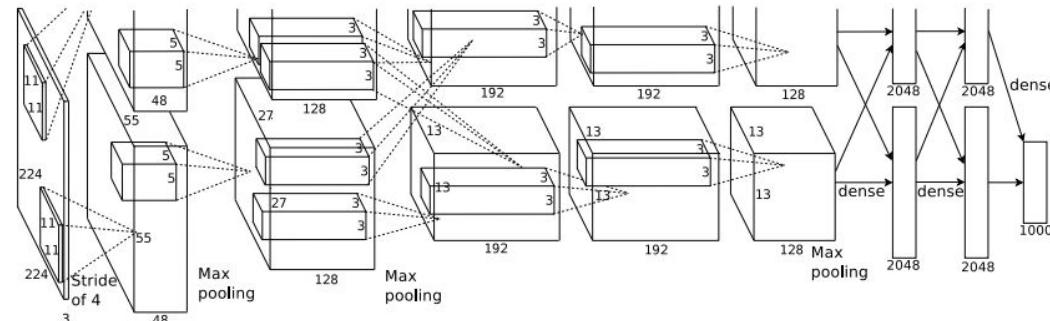
**Dataset Statistics:** This section provides information about the dataset splits:

- Classes: Classes: 3
- Input tensor size: Input tensor size: [3, 32, 32]
- Examples per split: Examples per split: Train (5000), Val (3000), Test(2000)

**Training Curves:** Two line graphs show the progression of training over 20 epochs. The left graph plots "Loss" (Y-axis, 0.4 to 1.1) against "Epoch" (X-axis, 0 to 20). The right graph plots "Accuracy" (Y-axis, 0.3 to 0.9) against "Epoch" (X-axis, 0 to 20). Both graphs compare "train" (blue line with square markers) and "val" (red line with square markers) data.

# Last Time: CNN Architectures

## AlexNet



## Revolution of Depth

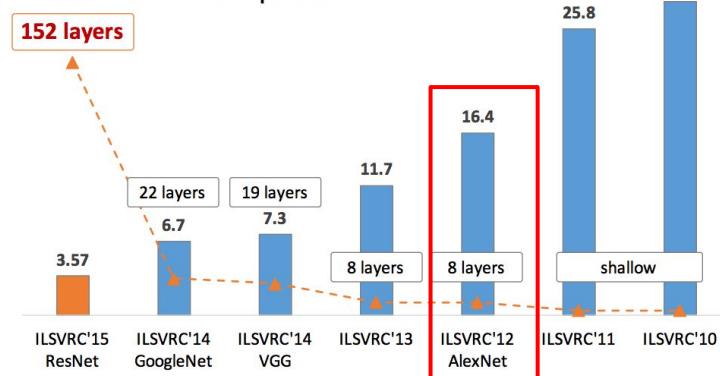
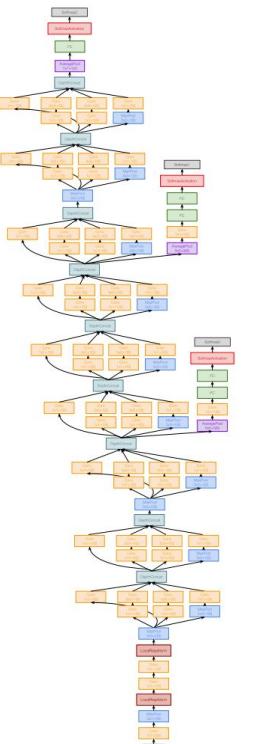
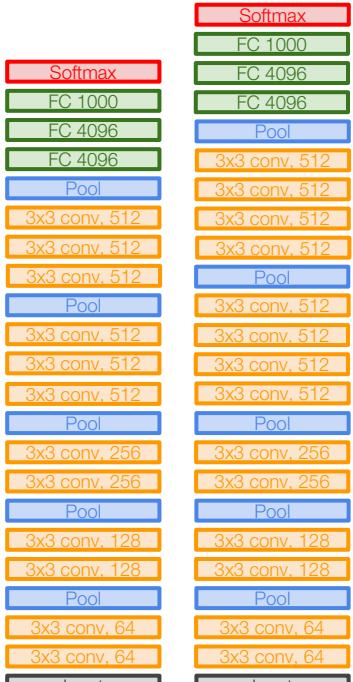


Figure copyright Kaiming He, 2016. Reproduced with permission.

# Last Time: CNN Architectures



GoogLeNet

VGG16

VGG19

## Revolution of Depth

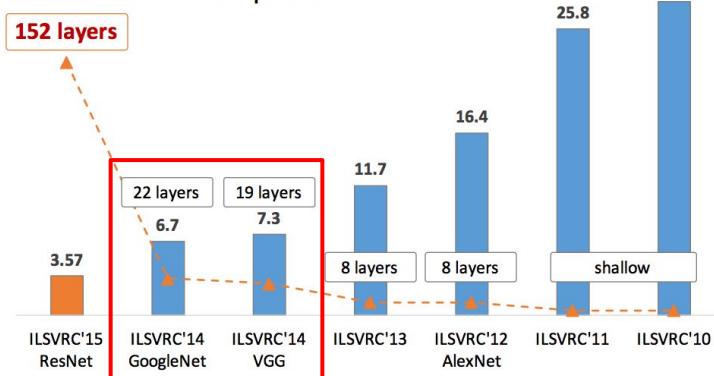
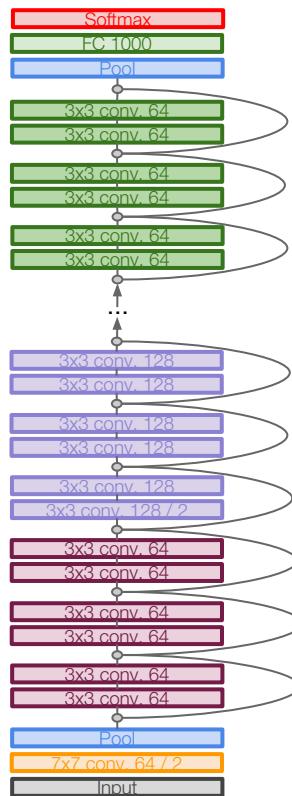
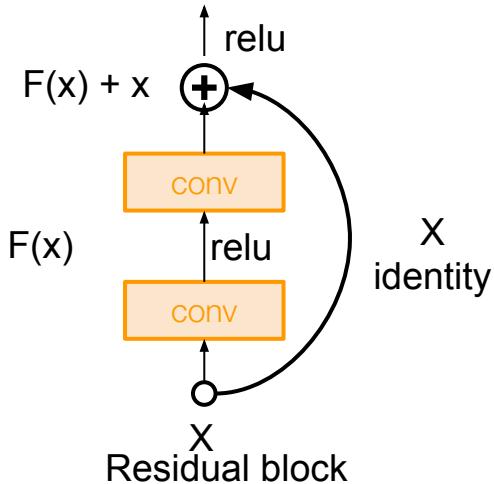


Figure copyright Kaiming He, 2016. Reproduced with permission.

# Last Time: CNN Architectures



Revolution of Depth

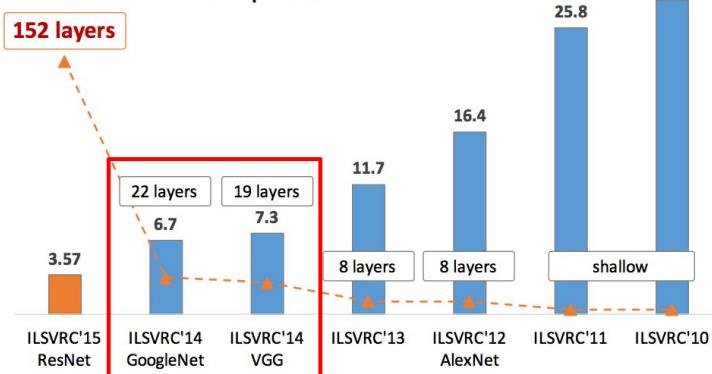
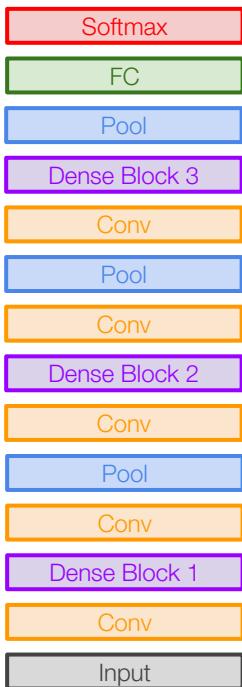
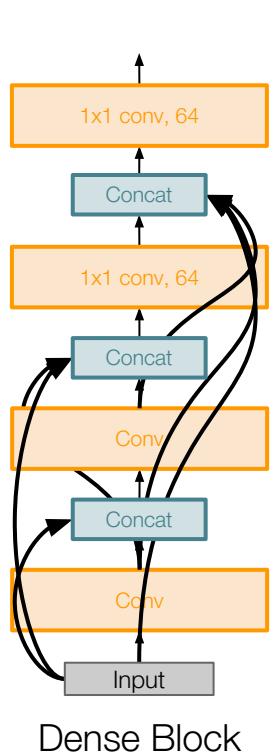


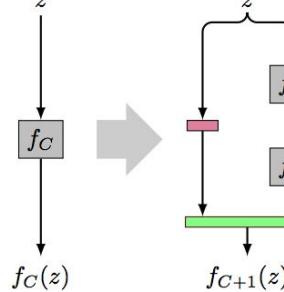
Figure copyright Kaiming He, 2016. Reproduced with permission.

# DenseNet



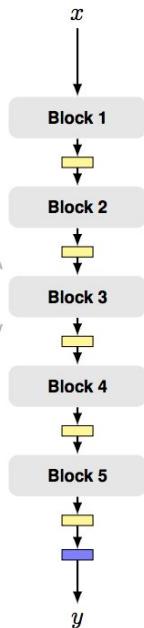
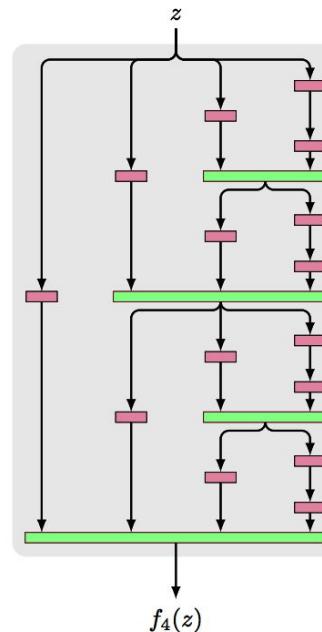
# FractalNet

**Fractal Expansion Rule**



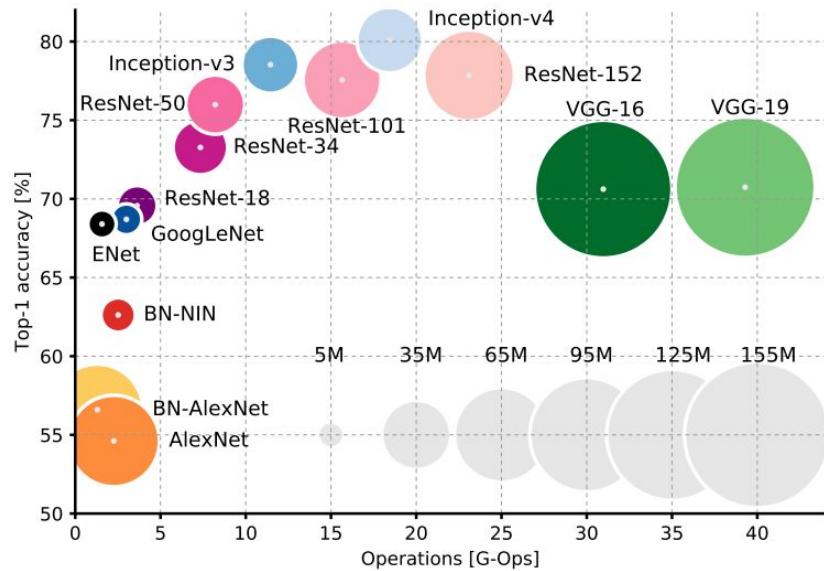
**Layer Key**

- Convolution (pink square)
- Join (green bar)
- Pool (yellow bar)
- Prediction (blue bar)

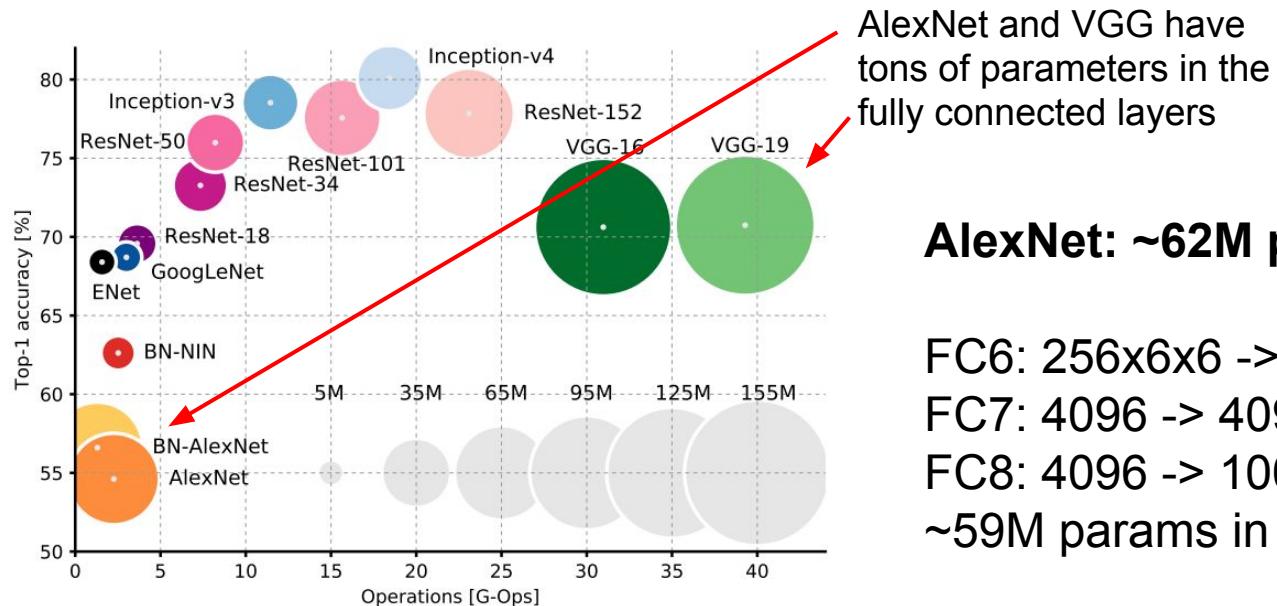


Figures copyright Larsson et al., 2017. Reproduced with permission.

# Last Time: CNN Architectures



# Last Time: CNN Architectures



AlexNet and VGG have tons of parameters in the fully connected layers

**AlexNet: ~62M parameters**

FC6: 256x6x6  $\rightarrow$  4096: 38M params

FC7: 4096  $\rightarrow$  4096: 17M params

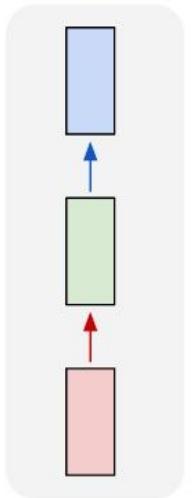
FC8: 4096  $\rightarrow$  1000: 4M params

~59M params in FC layers!

# Today: Recurrent Neural Networks

# “Vanilla” Neural Network

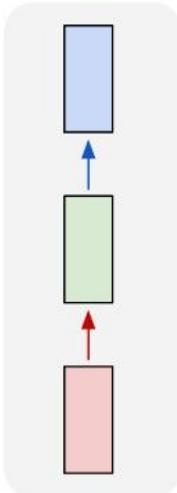
one to one



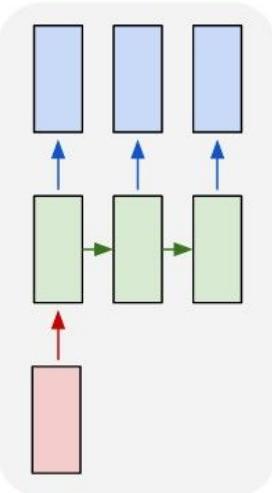
**Vanilla Neural Networks**

# Recurrent Neural Networks: Process Sequences

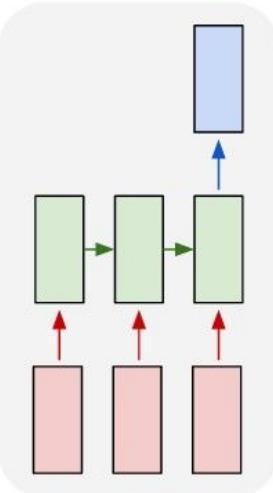
one to one



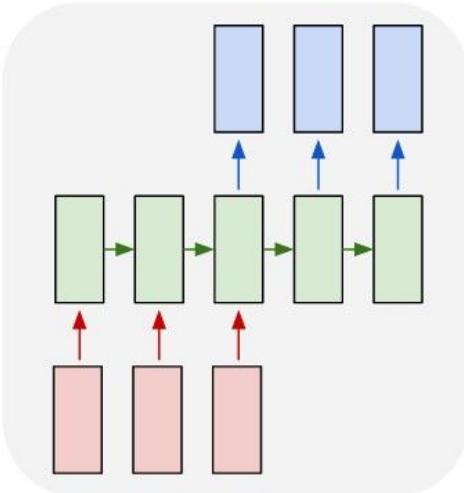
one to many



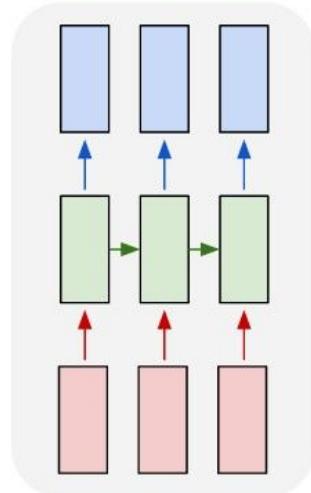
many to one



many to many



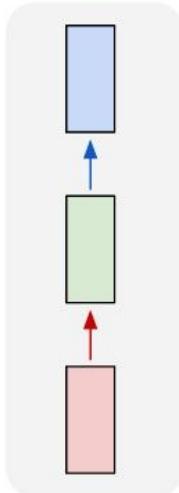
many to many



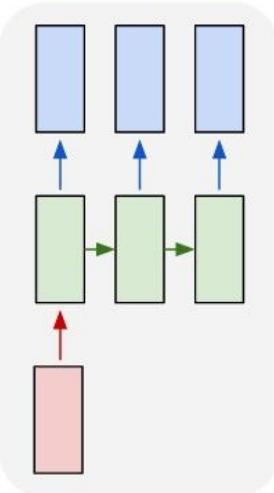
→  
e.g. **Image Captioning**  
image -> sequence of words

# Recurrent Neural Networks: Process Sequences

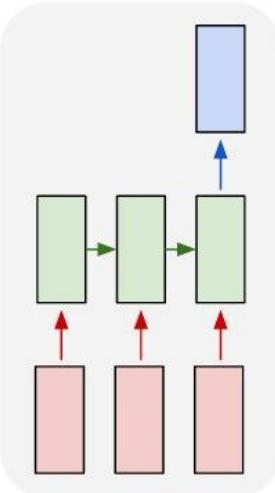
one to one



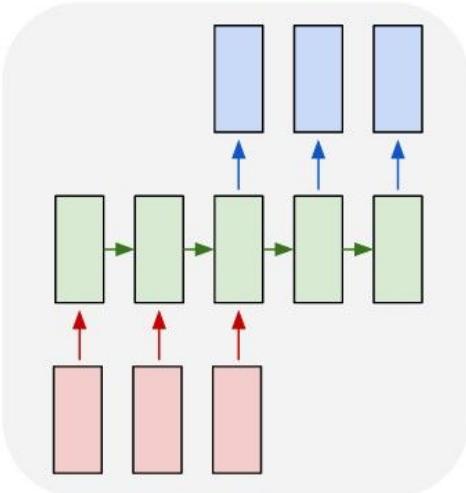
one to many



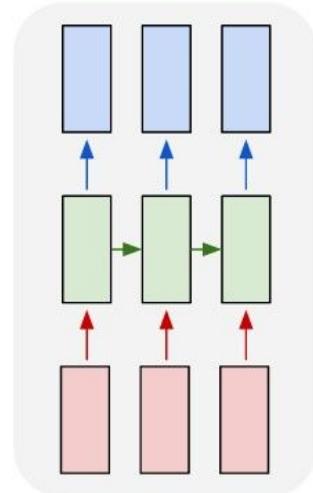
many to one



many to many



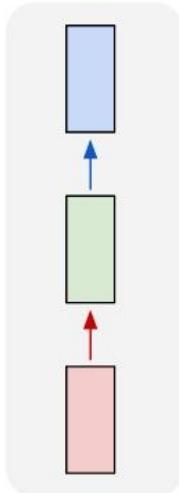
many to many



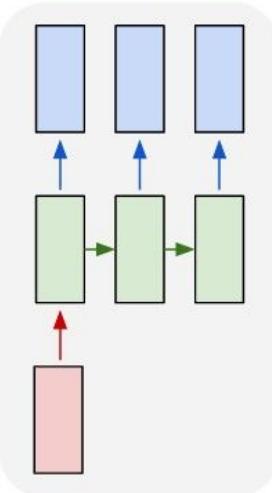
e.g. **Sentiment Classification**  
sequence of words -> sentiment

# Recurrent Neural Networks: Process Sequences

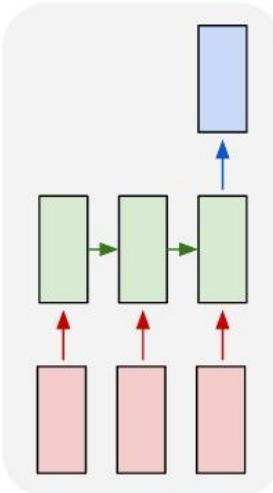
one to one



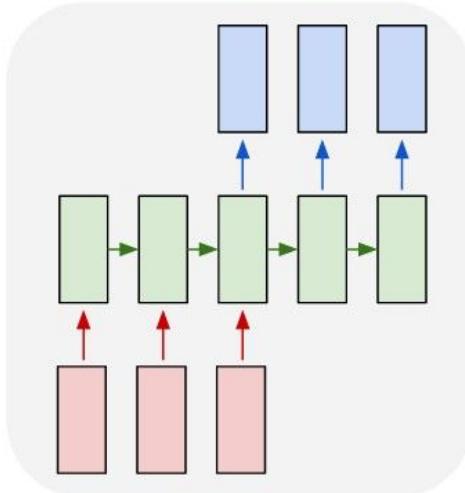
one to many



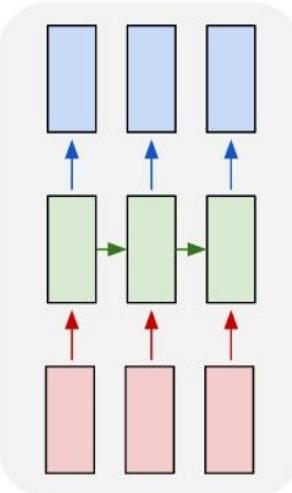
many to one



many to many



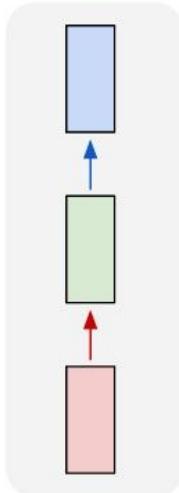
many to many



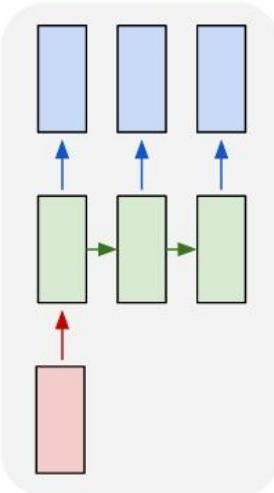
↑  
e.g. **Machine Translation**  
seq of words -> seq of words

# Recurrent Neural Networks: Process Sequences

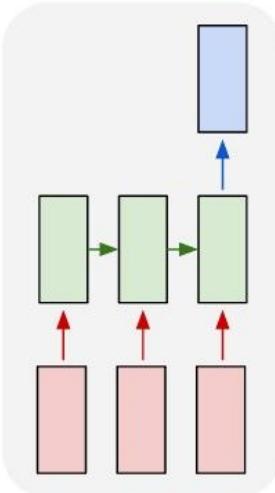
one to one



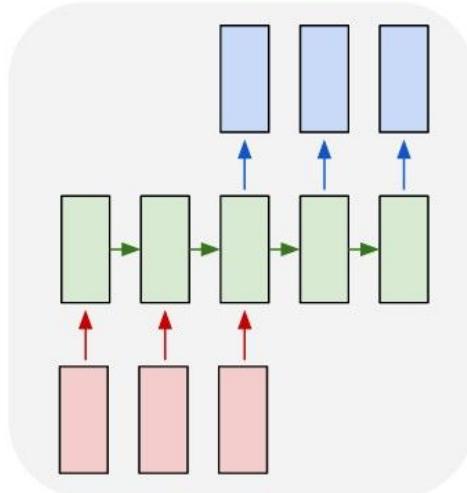
one to many



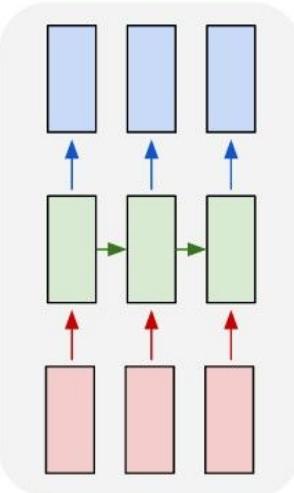
many to one



many to many



many to many



e.g. Video classification on frame level

# Sequential Processing of Non-Sequence Data

Classify images by taking a series of “glimpses”



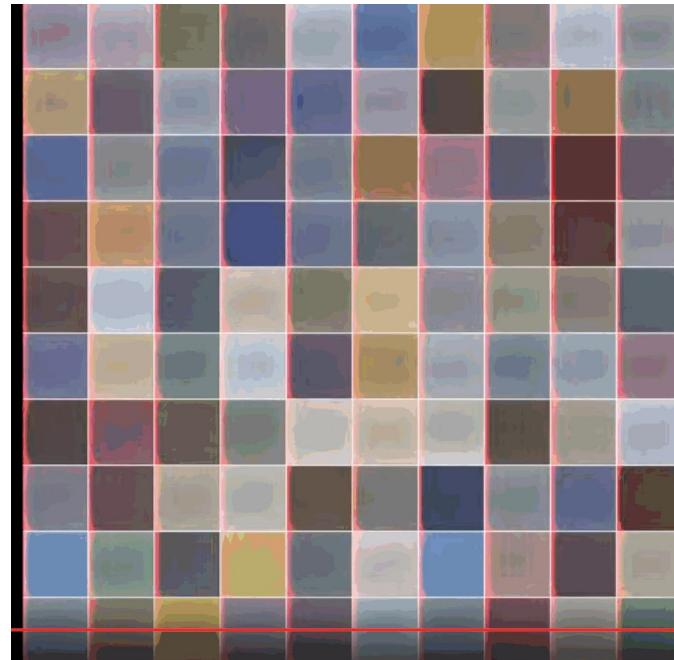
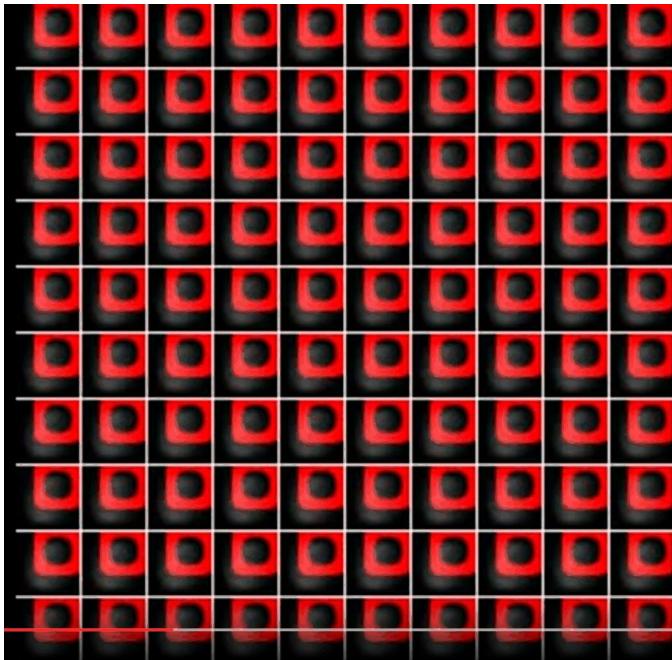
Ba, Mnih, and Kavukcuoglu, “Multiple Object Recognition with Visual Attention”, ICLR 2015.

Gregor et al, “DRAW: A Recurrent Neural Network For Image Generation”, ICML 2015

Figure copyright Karol Gregor, Ivo Danihelka, Alex Graves, Danilo Jimenez Rezende, and Daan Wierstra, 2015. Reproduced with permission.

# Sequential Processing of Non-Sequence Data

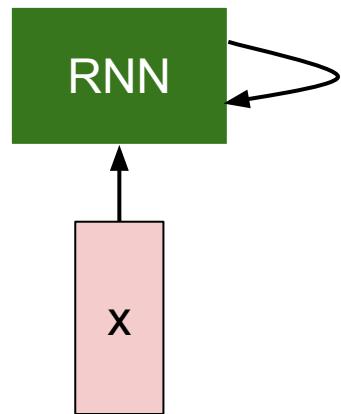
Generate images one piece at a time!



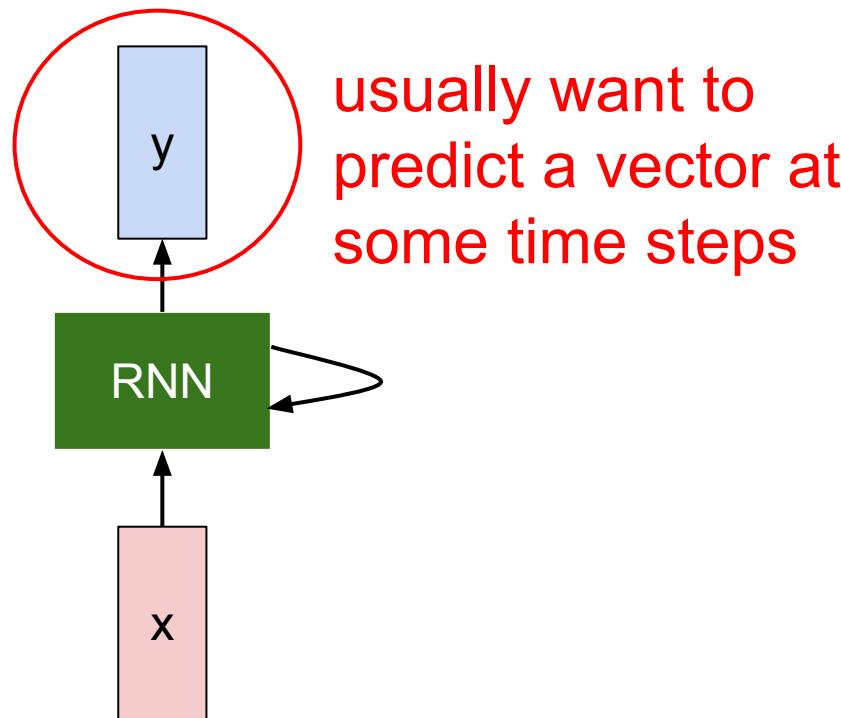
Gregor et al, "DRAW: A Recurrent Neural Network For Image Generation , ICML 2015

Figure copyright Karol Gregor, Ivo Danihelka, Alex Graves, Danilo Jimenez Rezende, and Daan Wierstra, 2015. Reproduced with permission.

# Recurrent Neural Network

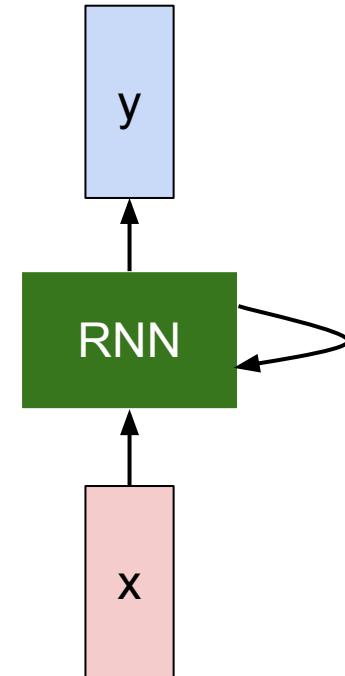


# Recurrent Neural Network



# Recurrent Neural Network

We can process a sequence of vectors  $\mathbf{x}$  by applying a **recurrence formula** at every time step:

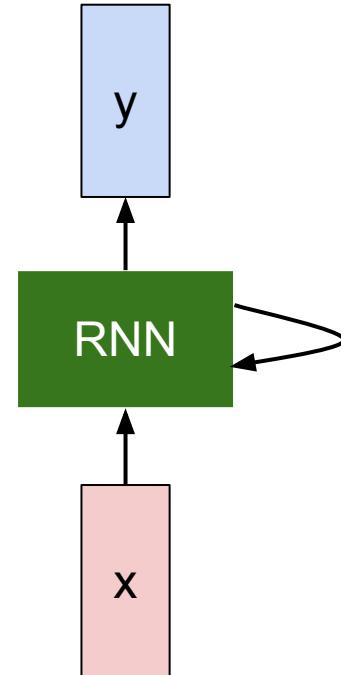


# Recurrent Neural Network

We can process a sequence of vectors  $\mathbf{x}$  by applying a **recurrence formula** at every time step:

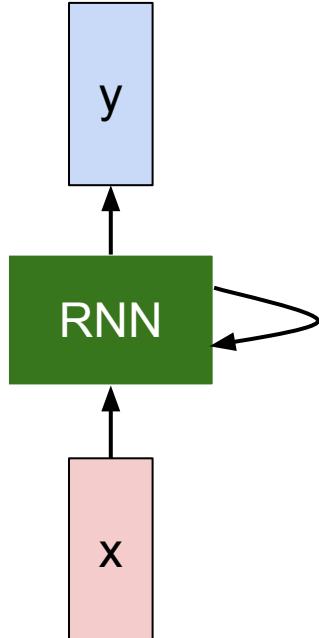
$$h_t = f_W(h_{t-1}, x_t)$$

Notice: the same function and the same set of parameters are used at every time step.



# (Vanilla) Recurrent Neural Network

The state consists of a single “*hidden*” vector  $\mathbf{h}$ :



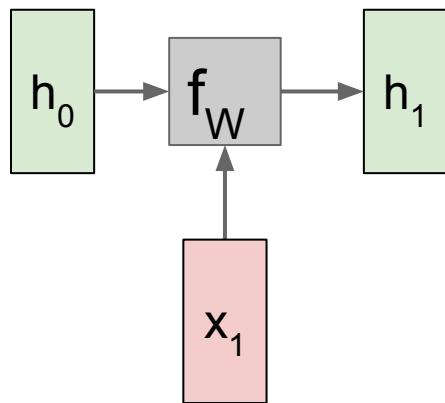
$$\mathbf{h}_t = f_W(\mathbf{h}_{t-1}, \mathbf{x}_t)$$



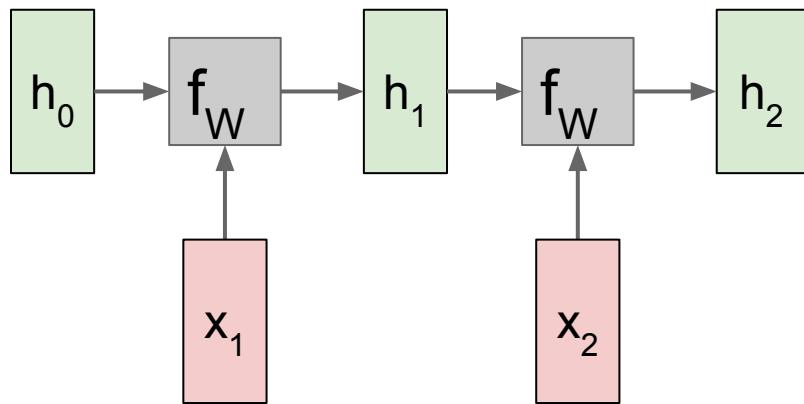
$$\mathbf{h}_t = \tanh(W_{hh}\mathbf{h}_{t-1} + W_{xh}\mathbf{x}_t)$$

$$y_t = W_{hy}\mathbf{h}_t$$

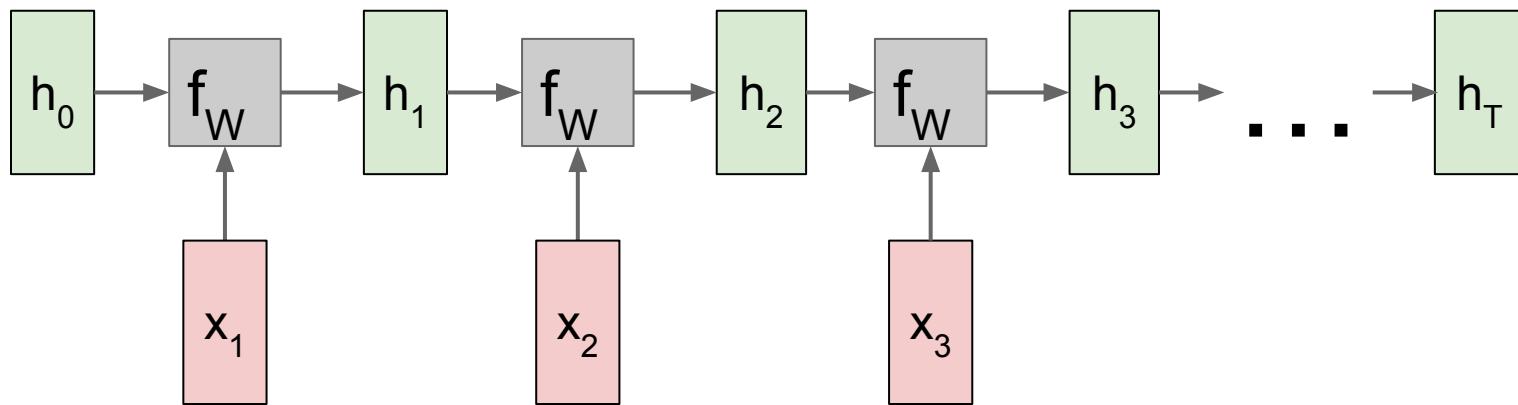
# RNN: Computational Graph



# RNN: Computational Graph

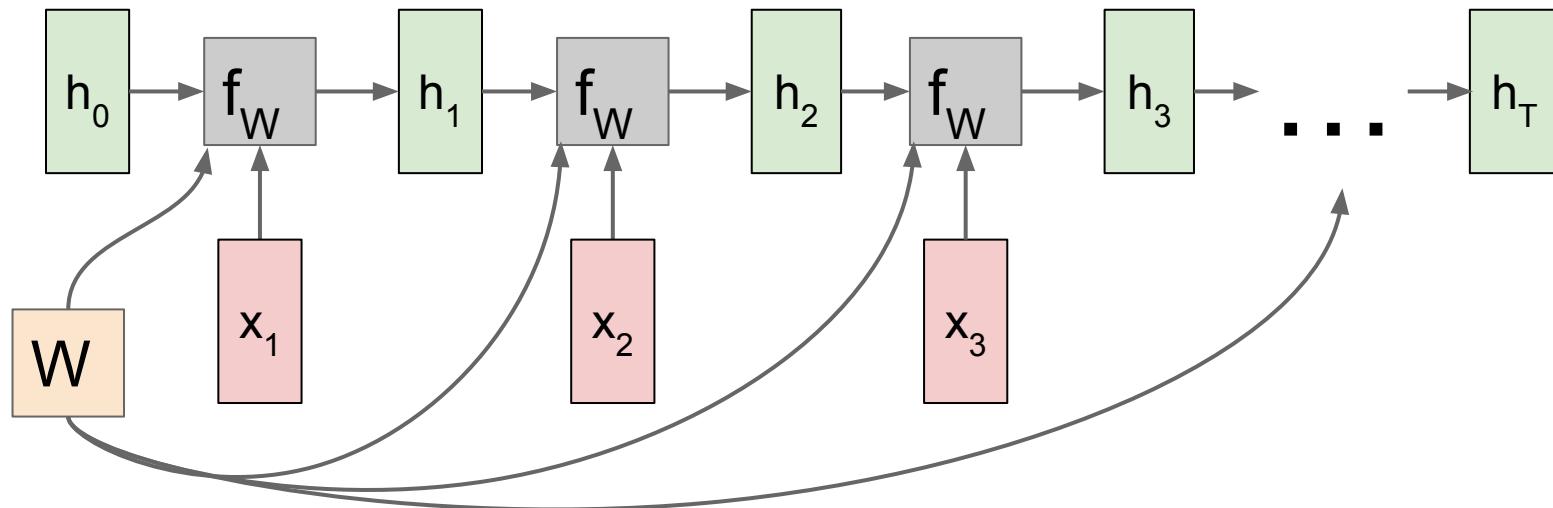


# RNN: Computational Graph

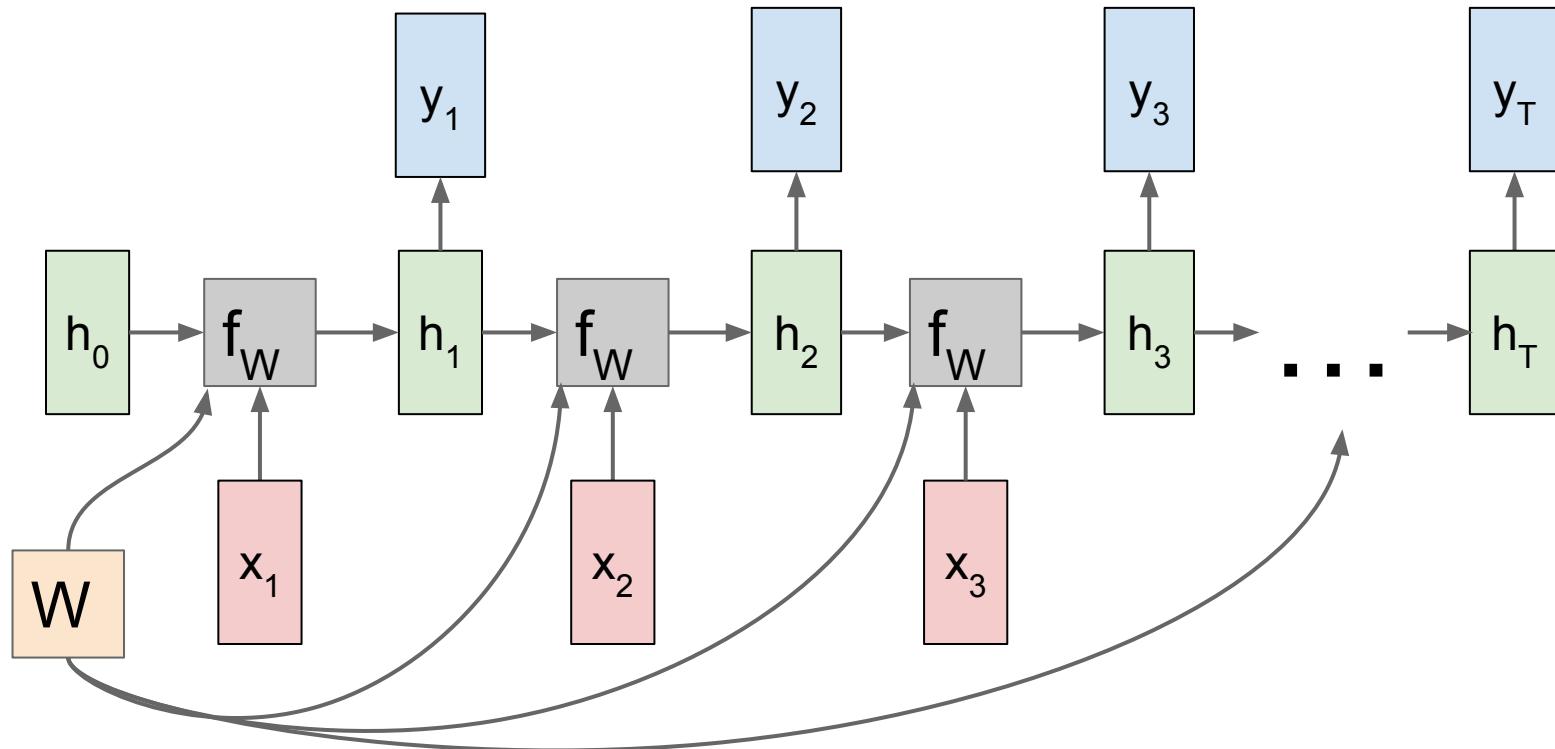


# RNN: Computational Graph

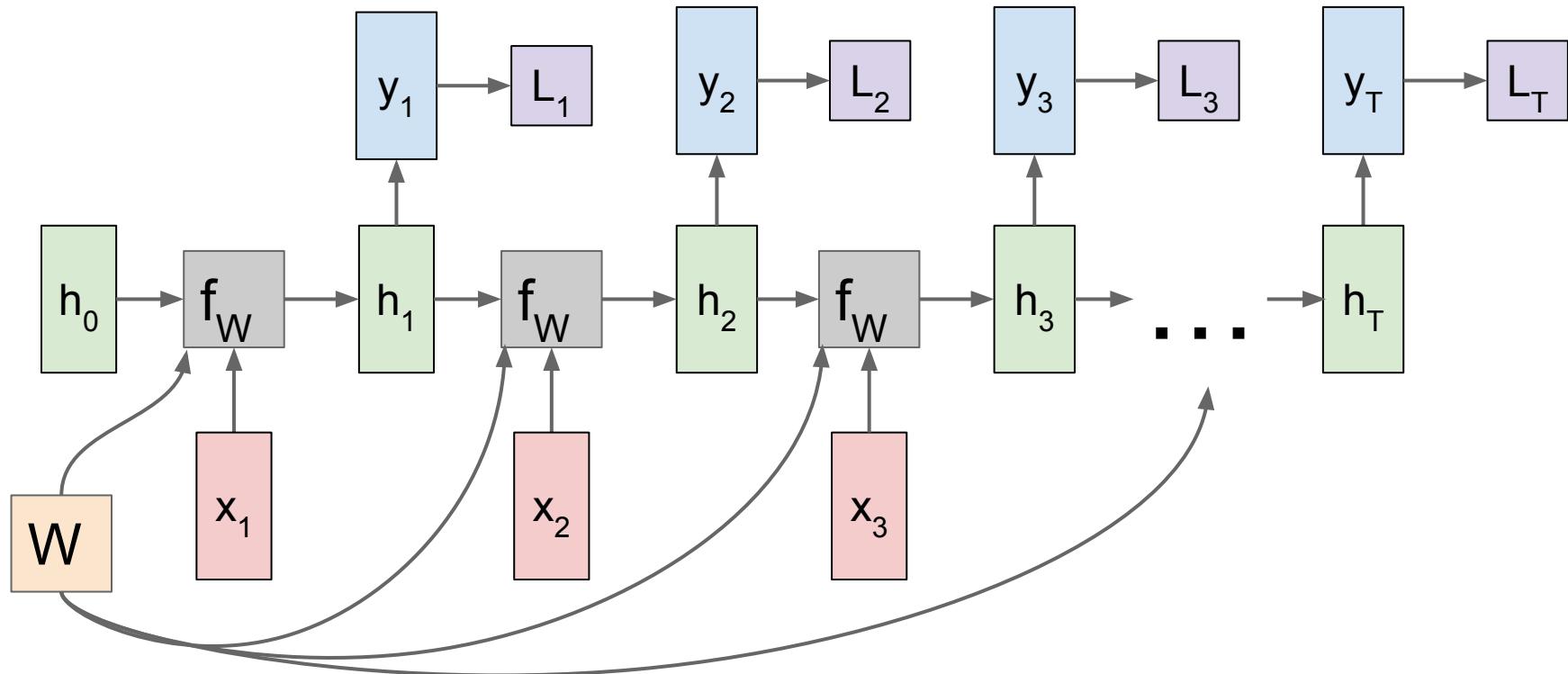
Re-use the same weight matrix at every time-step



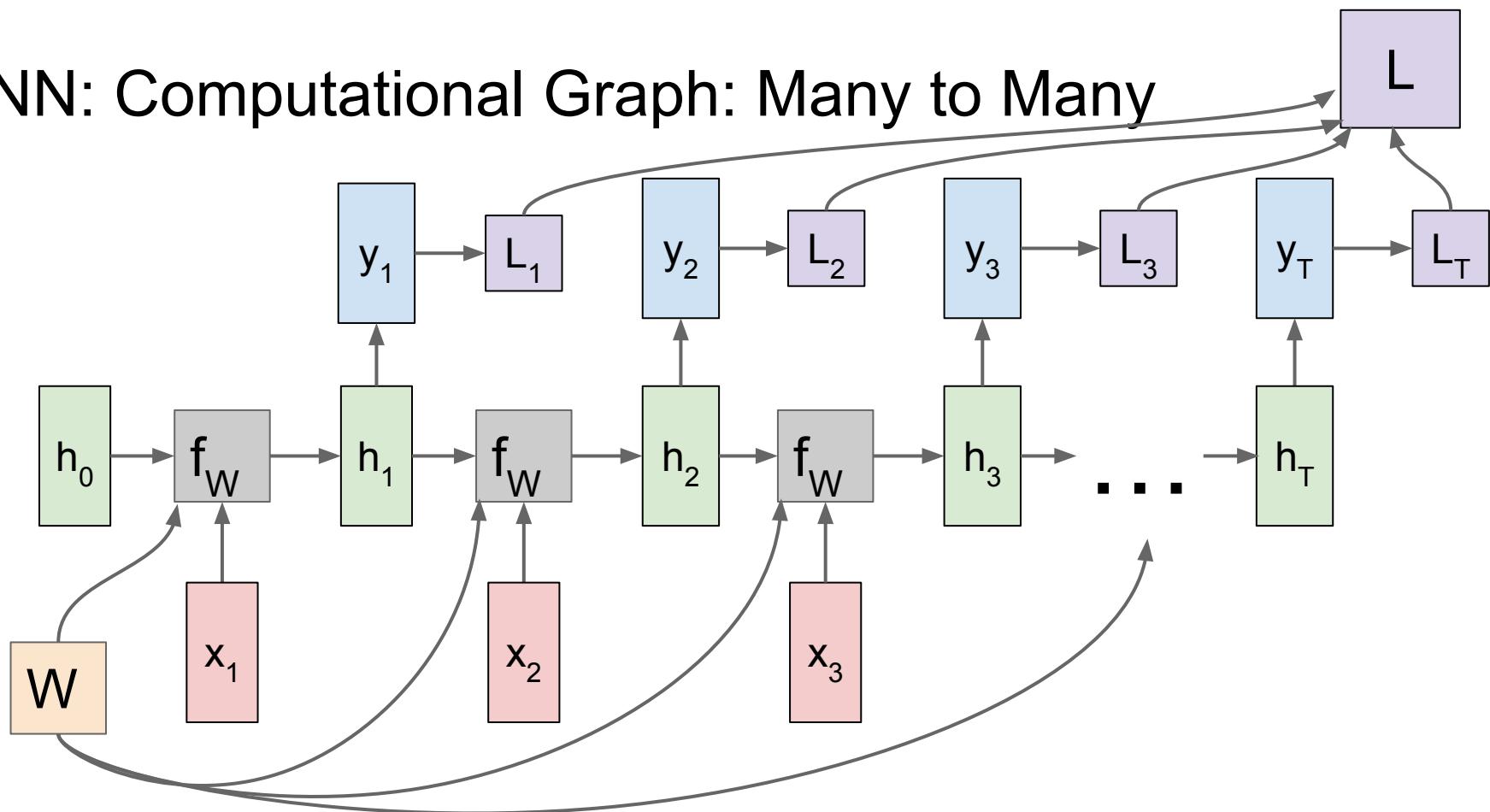
# RNN: Computational Graph: Many to Many



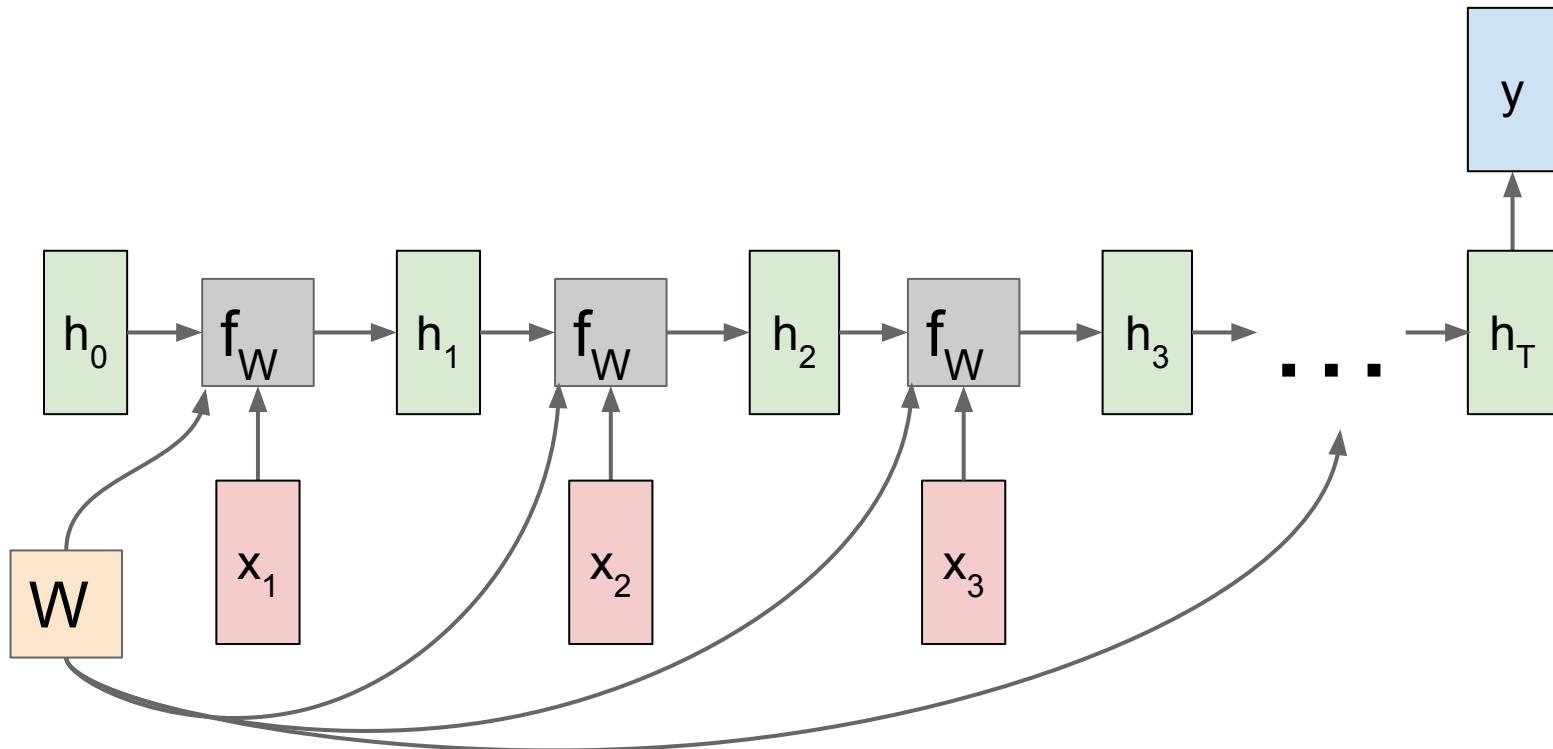
# RNN: Computational Graph: Many to Many



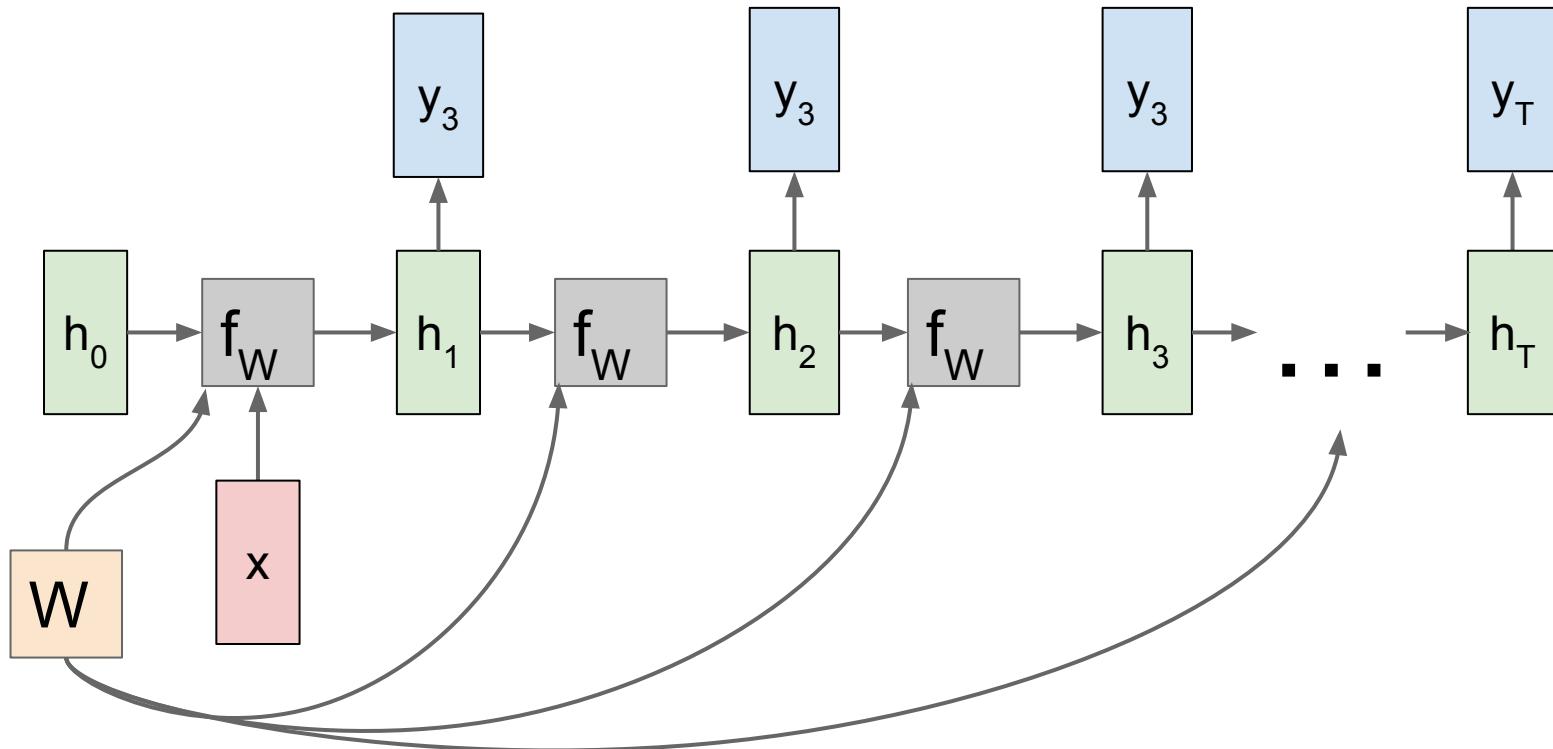
# RNN: Computational Graph: Many to Many



# RNN: Computational Graph: Many to One

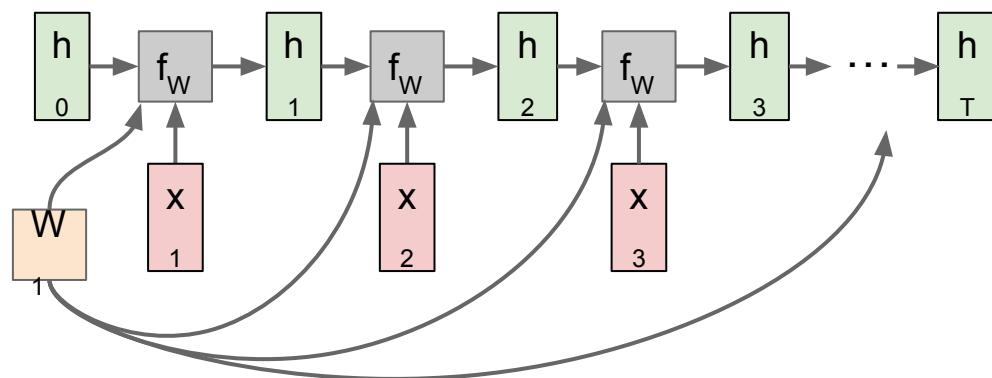


# RNN: Computational Graph: One to Many



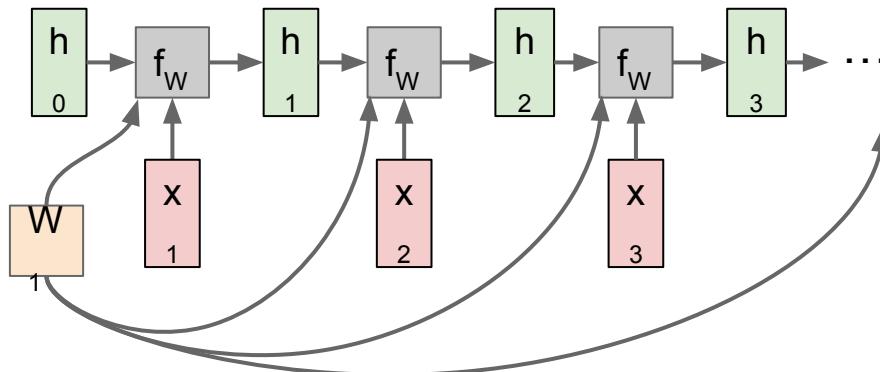
# Sequence to Sequence: Many-to-one + one-to-many

**Many to one:** Encode input sequence in a single vector

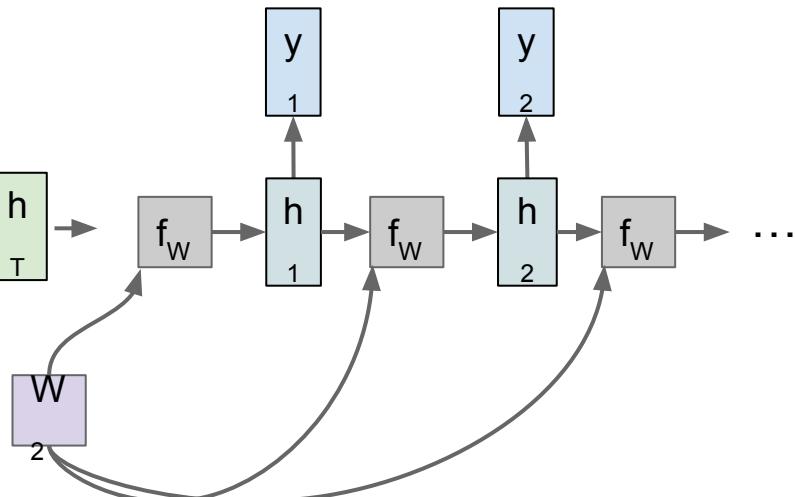


# Sequence to Sequence: Many-to-one + one-to-many

**Many to one:** Encode input sequence in a single vector



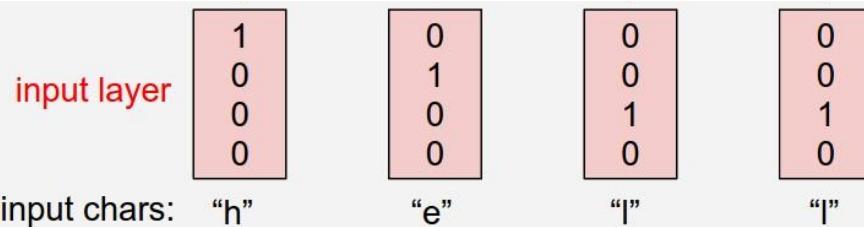
**One to many:** Produce output sequence from single input vector



# Example: Character-level Language Model

Vocabulary:  
[h,e,l,o]

Example training  
sequence:  
“hello”

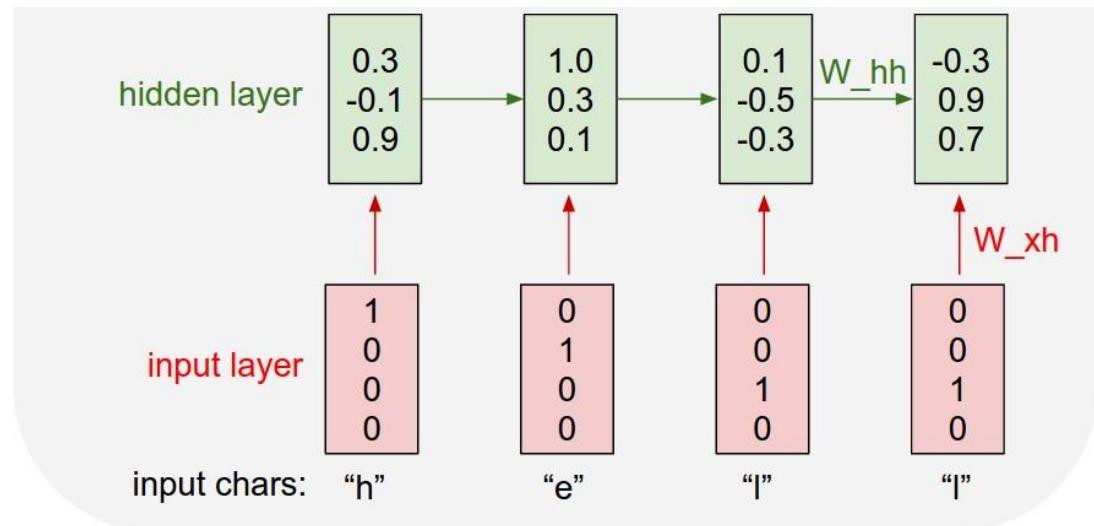


# Example: Character-level Language Model

Vocabulary:  
[h,e,l,o]

Example training  
sequence:  
“hello”

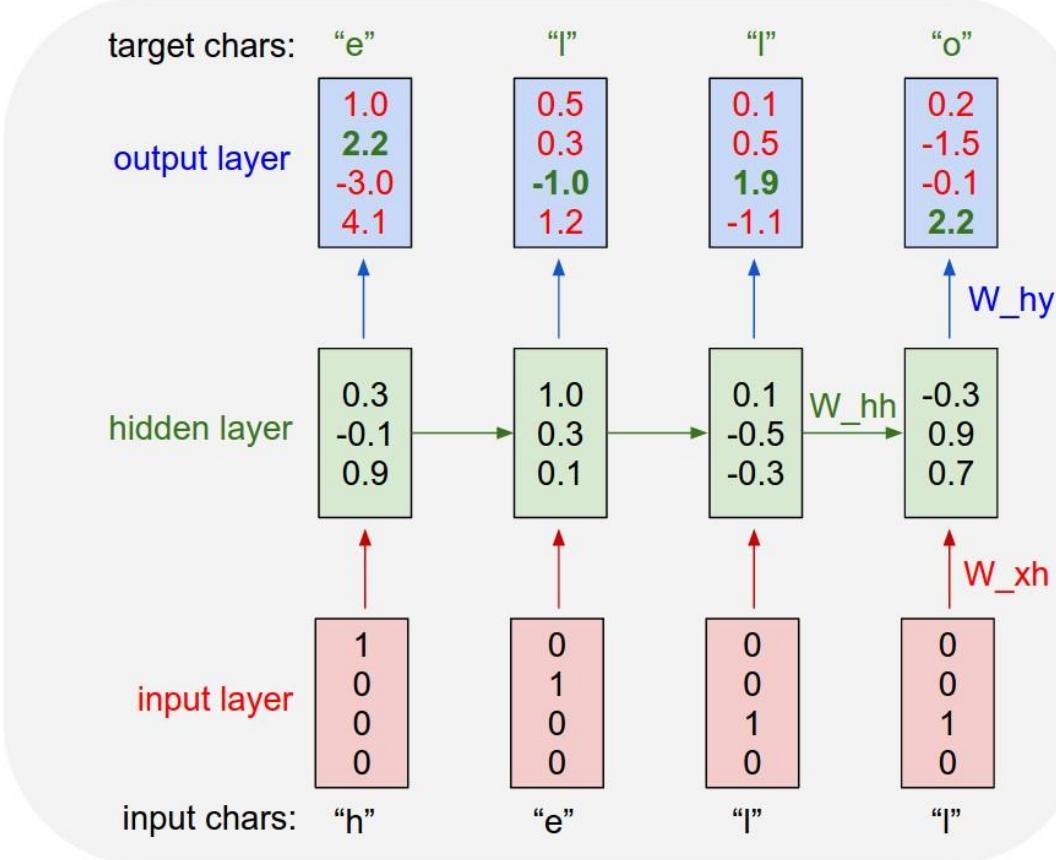
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$



# Example: Character-level Language Model

Vocabulary:  
[h,e,l,o]

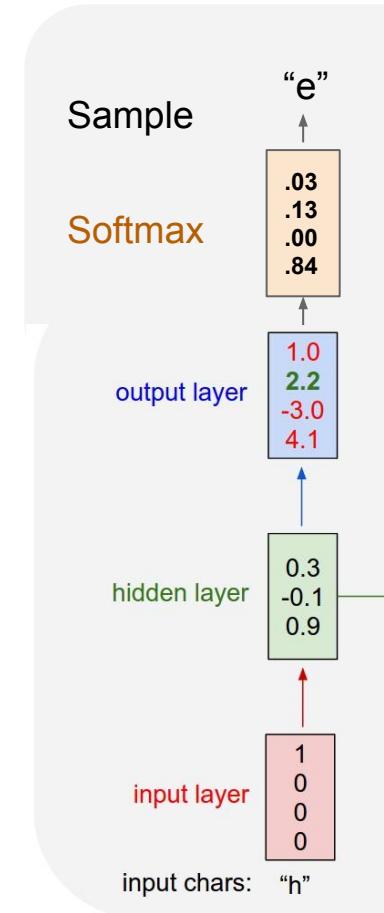
Example training  
sequence:  
“hello”



# Example: Character-level Language Model Sampling

Vocabulary:  
[h,e,l,o]

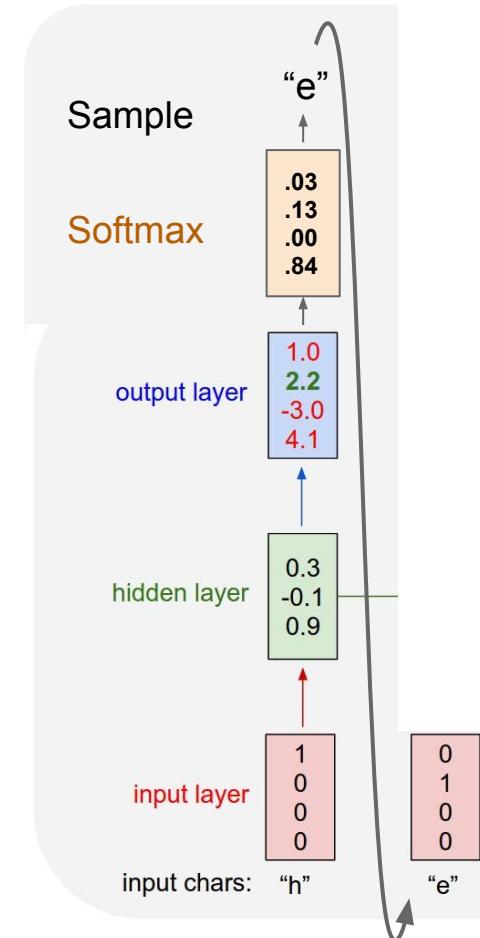
At test-time sample  
characters one at a time,  
feed back to model



# Example: Character-level Language Model Sampling

Vocabulary:  
[h,e,l,o]

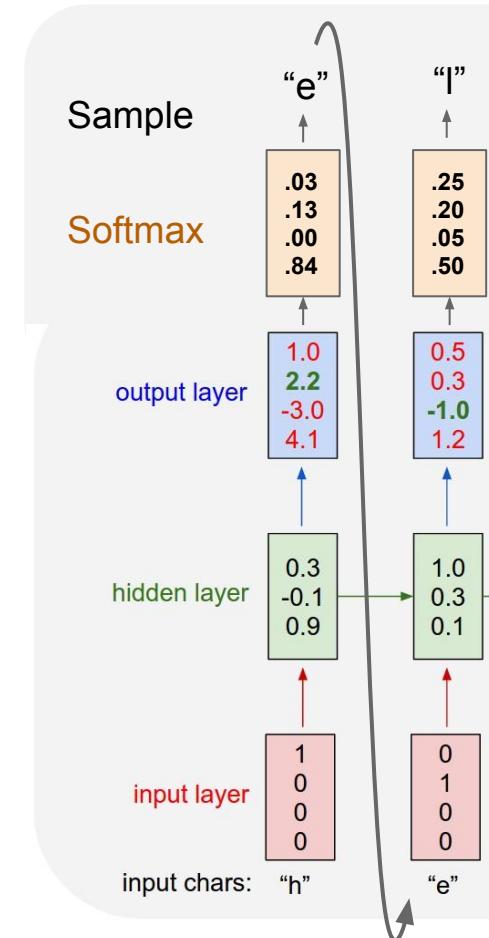
At test-time sample  
characters one at a time,  
feed back to model



# Example: Character-level Language Model Sampling

Vocabulary:  
[h,e,l,o]

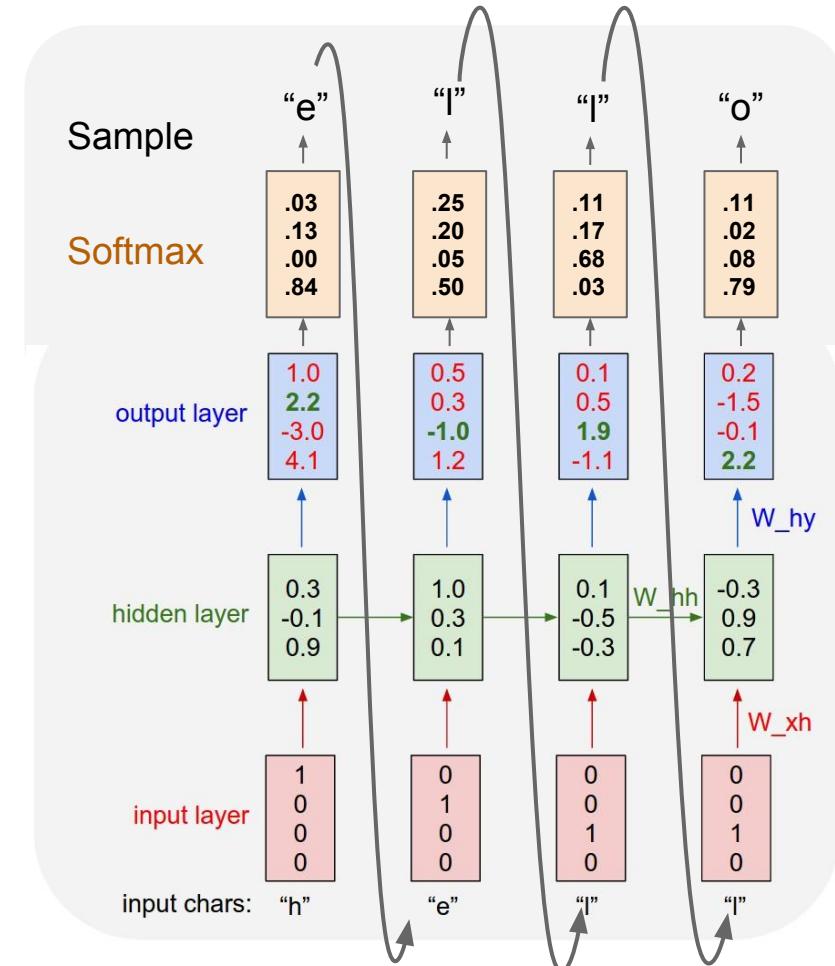
At test-time sample  
characters one at a time,  
feed back to model



# Example: Character-level Language Model Sampling

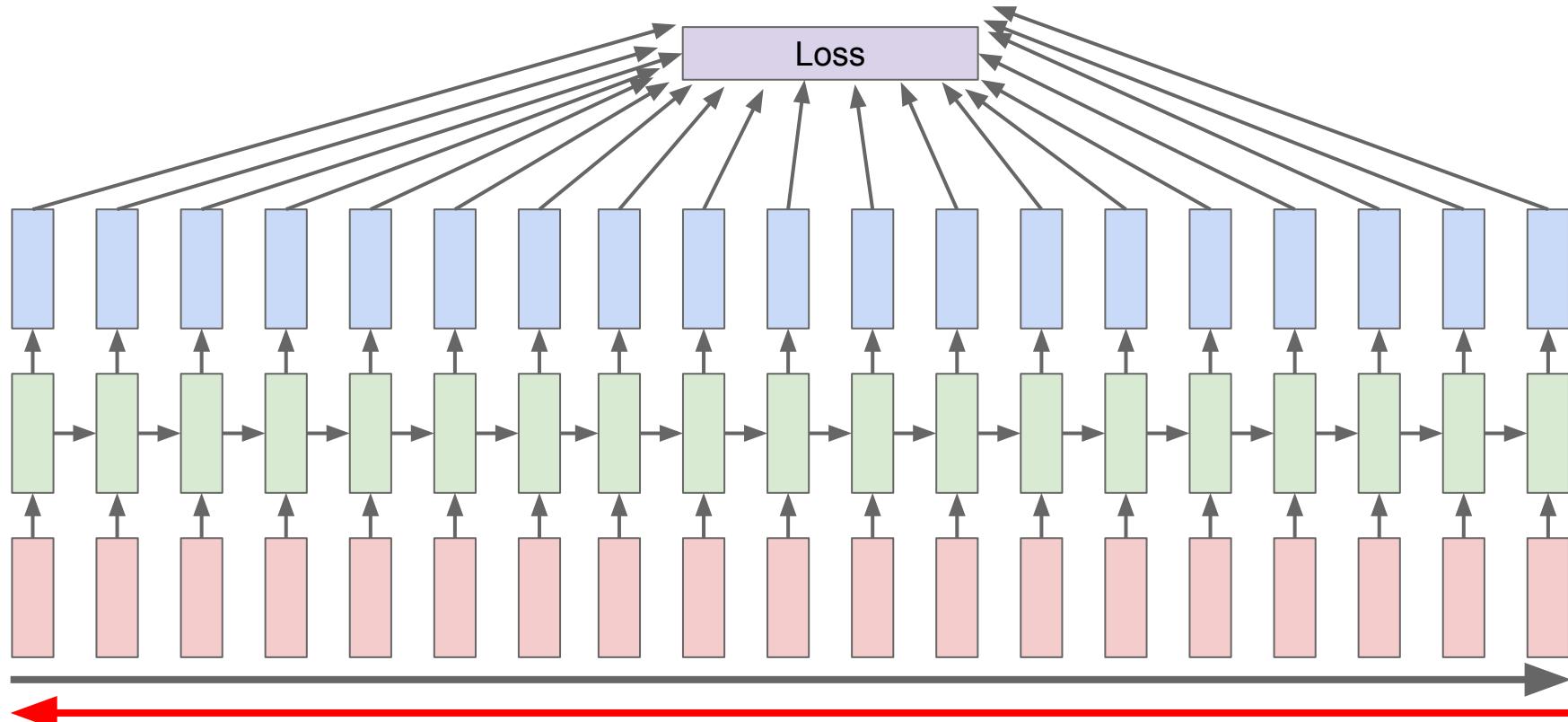
Vocabulary:  
[h,e,l,o]

At test-time sample  
characters one at a time,  
feed back to model

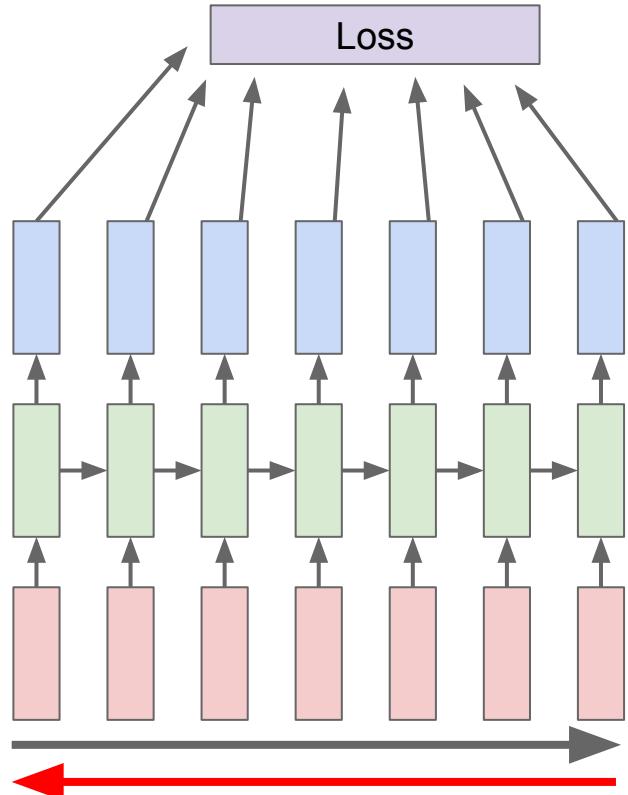


# Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

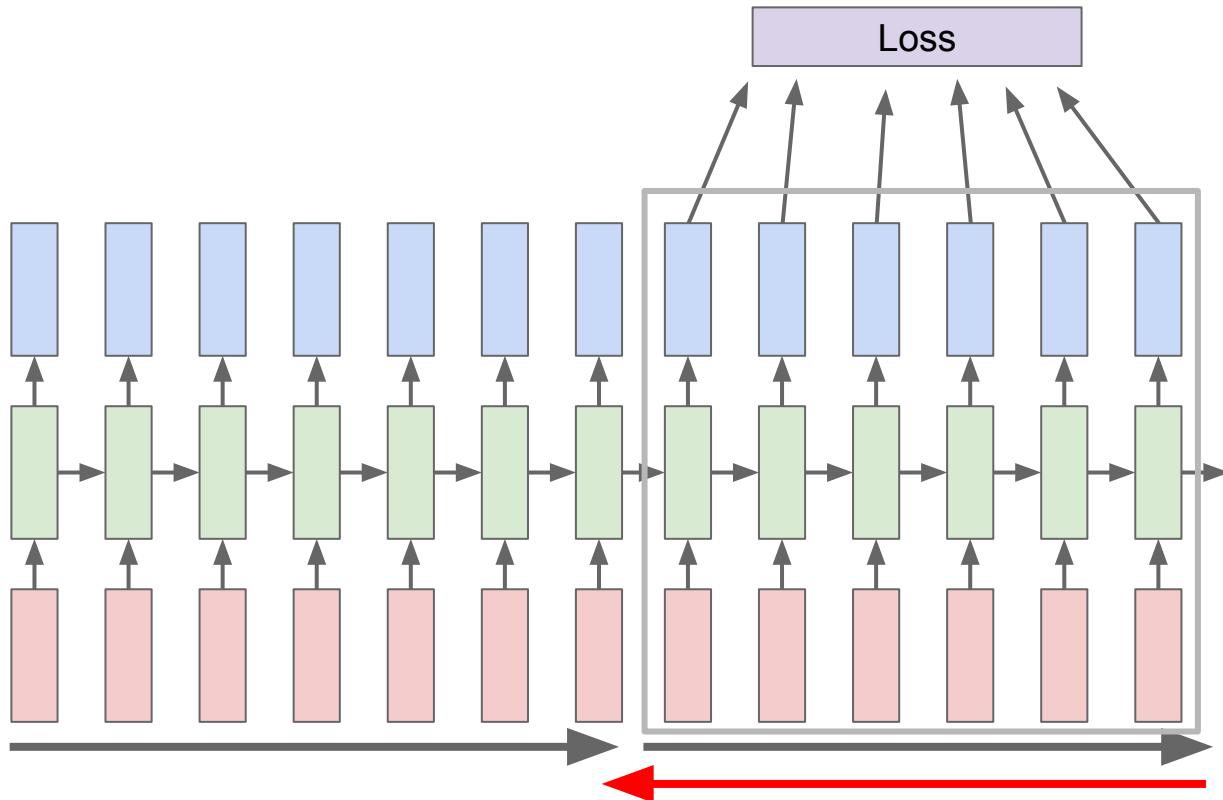


# Truncated Backpropagation through time



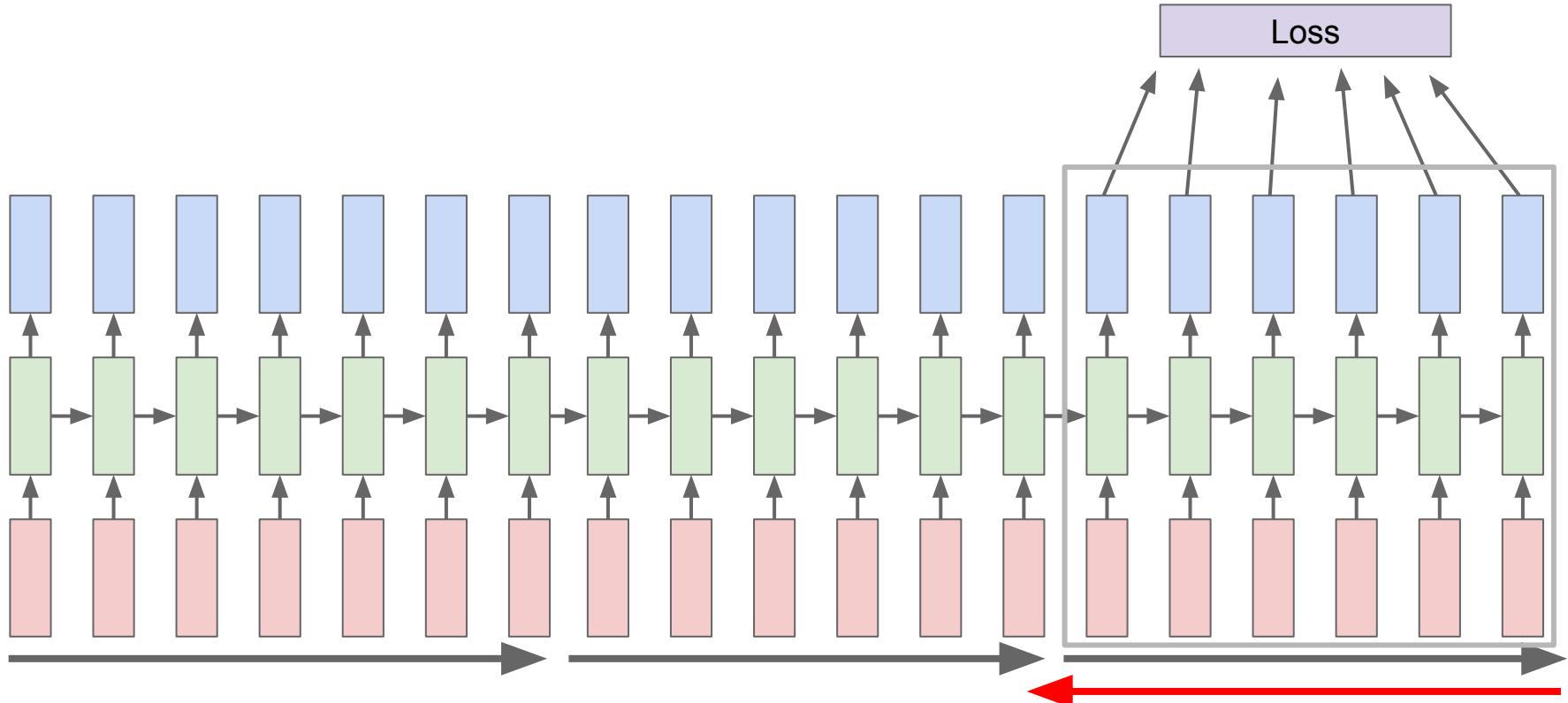
Run forward and backward  
through chunks of the  
sequence instead of whole  
sequence

# Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

# Truncated Backpropagation through time



# min-char-rnn.py gist: 112 lines of Python

```
1  """
2  Minimal character-level Vanilla RNN model. Written by Andrej Karpathy (@karpathy)
3  BSD License
4  """
5  import numpy as np
6
7  # data I/O
8  data = open('input.txt', 'r').read() # should be simple plain text file
9  chars = list(set(data))
10 data_size, vocab_size = len(data), len(chars)
11 print('data has %d characters, %d unique.' % (data_size, vocab_size))
12 char_to_ix = {ch:i for i,ch in enumerate(chars)}
13 ix_to_char = {i:ch for i,ch in enumerate(chars)}
14
15 # hyperparameters
16 hidden_size = 100 # size of hidden layer of neurons
17 seq_length = 25 # number of steps to unroll the RNN for
18 learning_rate = 1e-1
19
20 # model parameters
21 wkh = np.random.rand(hidden_size, vocab_size)*0.01 # input to hidden
22 whh = np.random.rand(hidden_size, hidden_size)*0.01 # hidden to hidden
23 why = np.random.rand(vocab_size, hidden_size)*0.01 # hidden to output
24 bh = np.zeros((hidden_size, 1)) # hidden bias
25 by = np.zeros((vocab_size, 1)) # output bias
26
27 def lossFun(inputs, targets, hprev):
28     """
29     inputs,targets are both list of integers.
30     hprev is Hx1 array of initial hidden state
31     returns the loss, gradients on model parameters, and last hidden state
32     """
33     hs, ys, ps = {}, {}, {}
34     hs[-1] = np.copy(hprev)
35     loss = 0
36     # forward pass
37     for t in xrange(len(inputs)):
38         xs[t] = np.zeros((vocab_size,1)) # encode in 1-of-k representation
39         xs[t][inputs[t]] = 1
40         hs[t] = np.tanh(np.dot(wkh, xs[t]) + np.dot(whh, hs[t-1]) + bh) # hidden state
41         ys[t] = np.dot(why, hs[t]) - by # unnormalized log probabilities for next chars
42         ps[t] = np.exp(ys[t]) / np.sum(np.exp(ys[t])) # probabilities for next chars
43         loss += -np.log(ps[t][targets[t],0]) # softmax (cross-entropy loss)
44
45         # backward pass: compute gradients going backwards
46         dwhh, dwhy = np.zeros_like(whh), np.zeros_like(why)
47         dbh, dby = np.zeros_like(bh), np.zeros_like(by)
48         dhnext = np.zeros_like(hs[0])
49         for t in reversed(xrange(len(inputs))):
50             dy = np.copy(ps[t])
51             dy[targets[t]] -= 1 # backprop into y
52             dyt = np.dot(dy, hs[t].T)
53             dh = np.dot(why.T, dy) + dhnext # backprop into h
54             ddraw = (i - hs[t].T) * dh # backprop through tanh nonlinearity
55             dwhh += np.dot(ddraw, xs[t].T)
56             dwhy += np.dot(ddraw, hs[t-1].T)
57             dbh += ddraw
58             dhnext = np.dot(why.T, ddraw)
59             for dparam in [dwhh, dwhy, dbh, dby]:
60                 np.clip(dparam, -5, 5, out=dparam) # clip to mitigate exploding gradients
61
62     return loss, dwhh, dwhy, dbh, dby, hs[len(inputs)-1]
```

```
63 def sample(h, seed_ix, n):
64     """
65     sample a sequence of integers from the model
66     h is memory state, seed_ix is seed letter for first time step
67     """
68     x = np.zeros((vocab_size, 1))
69     x[seed_ix] = 1
70     ixes = []
71     for t in xrange(n):
72         h = np.tanh(np.dot(wkh, x) + np.dot(whh, h) + bh)
73         y = np.dot(why, h) + by
74         p = np.exp(y) / np.sum(np.exp(y))
75         ix = np.random.choice(range(vocab_size), p=p.ravel())
76         x = np.zeros((vocab_size, 1))
77         x[ix] = 1
78         ixes.append(ix)
79
80     return ixes
81
82 n, p, b, 0, 0
83 mxwh, mwhh, mhhy = np.zeros_like(wkh), np.zeros_like(whh), np.zeros_like(why)
84 mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
85 smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
86 while True:
87     # prepare inputs (we're sweeping from left to right in steps seq_length long)
88     if p+seq_length >= len(data) or n == 0:
89         hprev = np.zeros((hidden_size,1)) # reset RNN memory
90         p = 0 # go from start of data
91         inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
92         targets = [char_to_ix[ch] for ch in data[p+seq_length+1:p+2*seq_length+1]]
93
94     # sample from the model now and then
95     if n % 100 == 0:
96         sample_ix = sample(hprev, inputs[0], 200)
97         txt = ''.join(ix_to_char[ix] for ix in sample_ix)
98         print('----\n%s\n----' % (txt, ))
99
100     # forward seq_length characters through the net and fetch gradient
101     loss, dwhh, dwhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
102     smooth_loss = smooth_loss * .999 + loss * .001
103     if n % 100 == 0: print('iter %d, loss: %f' % (n, smooth_loss)) # print progress
104
105     # perform parameter update with Adagrad
106     for param, dparam, mem in zip([wkh, whh, why, bh, by],
107                                   [dwhh, dwhy, dbh, dby],
108                                   [mxwh, mwhh, mhhy, mbh, mby]):
109         mem += dparam * dparam
110         param -= learning_rate * param / np.sqrt(mem + 1e-8) # adagrad update
111
112         p += seq_length # move data pointer
113         n += 1 # iteration counter
```

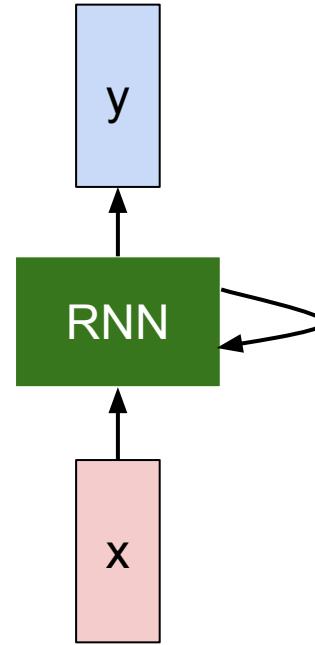
(<https://gist.github.com/karpathy/d4dee566867f8291f086>)

# THE SONNETS

by William Shakespeare

From fairest creatures we desire increase,  
That thereby beauty's rose might never die,  
But as the riper should by time decease,  
His tender heir might bear his memory:  
But thou, contracted to thine own bright eyes,  
Feed'st thy light's flame with self-substantial fuel,  
Making a famine where abundance lies,  
Thyself thy foe, to thy sweet self too cruel:  
Thou that art now the world's fresh ornament,  
And only herald to the gaudy spring,  
Within thine own bud buriest thy content,  
And tender churl mak'st waste in niggarding:  
    Pity the world, or else this glutton be,  
    To eat the world's due, by the grave and thee.

When forty winters shall besiege thy brow,  
And dig deep trenches in thy beauty's field,  
Thy youth's proud livery so gazed on now,  
Will be a tatter'd weed of small worth held:  
Then being asked, where all thy beauty lies,  
Where all the treasure of thy lusty days;  
To say, within thine own deep sunken eyes,  
Were an all-eating shame, and thriftless praise.  
How much more praise deserv'd thy beauty's use,  
If thou couldst answer 'This fair child of mine  
Shall sum my count, and make my old excuse,'  
Proving his beauty by succession thine!  
    This were to be new made when thou art old,  
    And see thy blood warm when thou feel'st it cold.



at first:

tyntd-iafhatawiaoihrdemot lytdws e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e  
plia tkldrgd t o idoe ns,smtt h ne etie h,hregtrs nigtike,aoaenns lng

↓ train more

"Tmont thithey" fomesscerliund  
Keushey. Thom here  
sheulke, anmerenith ol sivh I lalterthend Bleipile shuwyl fil on aseterlome  
coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."

↓ train more

Aftair fall unsuch that the hall for Prince Velzonski's that me of  
her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort  
how, and Gogition is so overelical and ofter.

↓ train more

"Why do what that day," replied Natasha, and wishing to himself the fact the  
princess, Princess Mary was easier, fed in had oftened him.  
Pierre aking his soul came to the packs and drove up his father-in-law women.

PANDARUS:

Alas, I think he shall be come approached and the day  
When little strain would be attain'd into being never fed,  
And who is but a chain and subjects of his death,  
I should not sleep.

Second Senator:

They are away this miseries, produced upon my soul,  
Breaking and strongly should be buried, when I perish  
The earth and thoughts of many states.

DUKE VINCENTIO:

Well, your wit is in the care of side and that.

Second Lord:

They would be ruled after this chamber, and  
my fair nues begun out of the fact, to be conveyed,  
Whose noble souls I'll have the heart of the wars.

Clown:

Come, sir, I will make did behold your worship.

VIOLA:

I'll drink it.

VIOLA:

Why, Salisbury must find his flesh and thought  
That which I am not aps, not a man and in fire,  
To show the reining of the raven and the wars  
To grace my hand reproach within, and not a fair are hand,  
That Caesar and my goodly father's world;  
When I was heaven of presence and our fleets,  
We spare with hours, but cut thy council I am great,  
Murdered and by thy master's ready there  
My power to give thee but so much as hell:  
Some service in the noble bondman here,  
Would show him to her wine.

KING LEAR:

O, if you were a feeble sight, the courtesy of your law,  
Your sight and several breath, will wear the gods  
With his heads, and my hands are wonder'd at the deeds,  
So drop upon your lordship's head, and your opinion  
Shall be against your honour.

# The Stacks Project: open source algebraic geometry textbook

The screenshot shows the homepage of the Stacks Project. At the top, there is a navigation bar with links: home, about, tags explained, tag lookup, browse, search, bibliography, recent comments, blog, and add slogans. Below the navigation bar, there is a section titled "Browse chapters". This section contains a table with two columns: "Part" and "Chapter". The "Part" column lists "Preliminaries", "Algebraic Spaces", "Deformation Theory", "Algebraic Stacks", and "Miscellany". The "Chapter" column lists numbered chapters from 1 to 10, each with three download links: "online", "TeX", and "pdf". To the right of the table, there is a sidebar with two sections: "Parts" and "Statistics". The "Parts" section lists the same five parts as the table. The "Statistics" section provides information about the project's size: 455910 lines of code, 14221 tags (56 inactive tags), and 2366 sections.

Part	Chapter	online	TeX	view pdf
Preliminaries	1. Introduction	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	2. Conventions	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	3. Set Theory	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	4. Categories	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	5. Topology	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	6. Sheaves on Spaces	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	7. Sites and Sheaves	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	8. Stacks	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	9. Fields	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>
	10. Commutative Algebra	<a href="#">online</a>	<a href="#">tex</a>	<a href="#">pdf</a>

Parts

- [Preliminaries](#)
- [Schemes](#)
- [Topics in Scheme Theory](#)
- [Algebraic Spaces](#)
- [Deformation Theory](#)
- [Algebraic Stacks](#)
- [Miscellany](#)

Statistics

The Stacks project now consists of

- 455910 lines of code
- 14221 tags (56 inactive tags)
- 2366 sections

Latex source

<http://stacks.math.columbia.edu/>  
The stacks project is licensed under the [GNU Free Documentation License](#)

For  $\bigoplus_{n=1,\dots,m} \mathcal{L}_{m,n} = 0$ , hence we can find a closed subset  $\mathcal{H}$  in  $\mathcal{H}$  and any sets  $\mathcal{F}$  on  $X$ ,  $U$  is a closed immersion of  $S$ , then  $U \rightarrow T$  is a separated algebraic space.

*Proof.* Proof of (1). It also start we get

$$S = \text{Spec}(R) = U \times_X U \times_X U$$

and the comparicoly in the fibre product covering we have to prove the lemma generated by  $\coprod Z \times_U U \rightarrow V$ . Consider the maps  $M$  along the set of points  $\text{Sch}_{fppf}$  and  $U \rightarrow U$  is the fibre category of  $S$  in  $U$  in Section, ?? and the fact that any  $U$  affine, see Morphisms, Lemma ???. Hence we obtain a scheme  $S$  and any open subset  $W \subset U$  in  $\text{Sh}(G)$  such that  $\text{Spec}(R') \rightarrow S$  is smooth or an

$$U = \bigcup U_i \times_{S_i} U_i$$

which has a nonzero morphism we may assume that  $f_i$  is of finite presentation over  $S$ . We claim that  $\mathcal{O}_{X,x}$  is a scheme where  $x, x', s'' \in S'$  such that  $\mathcal{O}_{X,x'} \rightarrow \mathcal{O}_{X',x'}$  is separated. By Algebra, Lemma ?? we can define a map of complexes  $\text{GL}_{S'}(x'/S'')$  and we win.  $\square$

To prove study we see that  $\mathcal{F}|_U$  is a covering of  $\mathcal{X}'$ , and  $\mathcal{T}_i$  is an object of  $\mathcal{F}_{X/S}$  for  $i > 0$  and  $\mathcal{F}_p$  exists and let  $\mathcal{F}_i$  be a presheaf of  $\mathcal{O}_X$ -modules on  $\mathcal{C}$  as a  $\mathcal{F}$ -module. In particular  $\mathcal{F} = U/\mathcal{F}$  we have to show that

$$\widetilde{M}^\bullet = \mathcal{I}^\bullet \otimes_{\text{Spec}(k)} \mathcal{O}_{S,s} - i_X^{-1} \mathcal{F}$$

is a unique morphism of algebraic stacks. Note that

$$\text{Arrows} = (\text{Sch}/S)_{fppf}^{\text{opp}}, (\text{Sch}/S)_{fppf}$$

and

$$V = \Gamma(S, \mathcal{O}) \rightarrow (U, \text{Spec}(A))$$

is an open subset of  $X$ . Thus  $U$  is affine. This is a continuous map of  $X$  is the inverse, the groupoid scheme  $S$ .

*Proof.* See discussion of sheaves of sets.  $\square$

The result for prove any open covering follows from the less of Example ???. It may replace  $S$  by  $X_{\text{spaces},\text{étale}}$  which gives an open subspace of  $X$  and  $T$  equal to  $S_{\text{Zar}}$ , see Descent, Lemma ???. Namely, by Lemma ?? we see that  $R$  is geometrically regular over  $S$ .

**Lemma 0.1.** Assume (3) and (3) by the construction in the description.

Suppose  $X = \lim |X|$  (by the formal open covering  $X$  and a single map  $\underline{\text{Proj}}_X(\mathcal{A}) = \text{Spec}(B)$  over  $U$  compatible with the complex

$$\text{Set}(\mathcal{A}) = \Gamma(X, \mathcal{O}_{X,\mathcal{O}_X}).$$

When in this case of to show that  $\mathcal{Q} \rightarrow \mathcal{C}_{Z/X}$  is stable under the following result in the second conditions of (1), and (3). This finishes the proof. By Definition ?? (without element is when the closed subschemes are catenary. If  $T$  is surjective we may assume that  $T$  is connected with residue fields of  $S$ . Moreover there exists a closed subspace  $Z \subset X$  of  $X$  where  $U$  in  $X'$  is proper (some defining as a closed subset of the uniqueness it suffices to check the fact that the following theorem

(1)  $f$  is locally of finite type. Since  $S = \text{Spec}(R)$  and  $Y = \text{Spec}(R)$ .

*Proof.* This is form all sheaves of sheaves on  $X$ . But given a scheme  $U$  and a surjective étale morphism  $U \rightarrow X$ . Let  $U \cap U = \coprod_{i=1,\dots,n} U_i$  be the scheme  $X$  over  $S$  at the schemes  $X_i \rightarrow X$  and  $U = \lim_i X_i$ .  $\square$

The following lemma surjective restrocomposes of this implies that  $\mathcal{F}_{x_0} = \mathcal{F}_{x_0} = \mathcal{F}_{x,\dots,x_0}$ .

**Lemma 0.2.** Let  $X$  be a locally Noetherian scheme over  $S$ ,  $E = \mathcal{F}_{X/S}$ . Set  $\mathcal{I} = \mathcal{J}_1 \subset \mathcal{I}'_n$ . Since  $\mathcal{I}^n \subset \mathcal{I}^n$  are nonzero over  $i_0 \leq p$  is a subset of  $\mathcal{J}_{n,0} \circ \mathcal{A}_2$  works.

**Lemma 0.3.** In Situation ???. Hence we may assume  $q' = 0$ .

*Proof.* We will use the property we see that  $p$  is the next functor (??). On the other hand, by Lemma ?? we see that

$$D(\mathcal{O}_{X'}) = \mathcal{O}_X(D)$$

where  $K$  is an  $F$ -algebra where  $\delta_{n+1}$  is a scheme over  $S$ .  $\square$

*Proof.* Omitted. □

**Lemma 0.1.** Let  $\mathcal{C}$  be a set of the construction.

Let  $\mathcal{C}$  be a gerber covering. Let  $\mathcal{F}$  be a quasi-coherent sheaves of  $\mathcal{O}$ -modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

*Proof.* This is an algebraic space with the composition of sheaves  $\mathcal{F}$  on  $X_{\text{étale}}$  we have

$$\mathcal{O}_X(\mathcal{F}) = \{\text{morph}_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where  $\mathcal{G}$  defines an isomorphism  $\mathcal{F} \rightarrow \mathcal{F}$  of  $\mathcal{O}$ -modules. □

**Lemma 0.2.** This is an integer  $\mathcal{Z}$  is injective.

*Proof.* See Spaces, Lemma ??.

**Lemma 0.3.** Let  $S$  be a scheme. Let  $X$  be a scheme and  $X$  is an affine open covering. Let  $\mathcal{U} \subset \mathcal{X}$  be a canonical and locally of finite type. Let  $X$  be a scheme. Let  $X$  be a scheme which is equal to the formal complex.

The following to the construction of the lemma follows.

Let  $X$  be a scheme. Let  $X$  be a scheme covering. Let

$$b : X \rightarrow Y' \rightarrow Y \rightarrow Y \rightarrow Y' \times_X Y \rightarrow X.$$

be a morphism of algebraic spaces over  $S$  and  $Y$ .

*Proof.* Let  $X$  be a nonzero scheme of  $X$ . Let  $X$  be an algebraic space. Let  $\mathcal{F}$  be a quasi-coherent sheaf of  $\mathcal{O}_X$ -modules. The following are equivalent

- (1)  $\mathcal{F}$  is an algebraic space over  $S$ .
- (2) If  $X$  is an affine open covering.

Consider a common structure on  $X$  and  $X$  the functor  $\mathcal{O}_X(U)$  which is locally of finite type. □

This since  $\mathcal{F} \in \mathcal{F}$  and  $x \in \mathcal{G}$  the diagram

$$\begin{array}{ccccc}
 S & \xrightarrow{\quad} & & & \\
 \downarrow & & & & \\
 \xi & \longrightarrow & \mathcal{O}_{X'} & \xrightarrow{\quad} & \\
 & \nearrow & \uparrow & \searrow & \\
 & & =\alpha' \longrightarrow & & \\
 & \uparrow & =\alpha' \longrightarrow & & \\
 \text{Spec}(K_\psi) & & \text{Mor}_{\text{Sets}} & & d(\mathcal{O}_{X/k}, \mathcal{G}) \\
 & & \downarrow & & \downarrow X \\
 & & & & \text{d}(\mathcal{O}_{X/k}, \mathcal{G})
 \end{array}$$

is a limit. Then  $\mathcal{G}$  is a finite type and assume  $S$  is a flat and  $\mathcal{F}$  and  $\mathcal{G}$  is a finite type  $f_*$ . This is of finite type diagrams, and

- the composition of  $\mathcal{G}$  is a regular sequence,
- $\mathcal{O}_{X'}$  is a sheaf of rings.

*Proof.* We have see that  $X = \text{Spec}(R)$  and  $\mathcal{F}$  is a finite type representable by algebraic space. The property  $\mathcal{F}$  is a finite morphism of algebraic stacks. Then the cohomology of  $X$  is an open neighbourhood of  $U$ . □

*Proof.* This is clear that  $\mathcal{G}$  is a finite presentation, see Lemmas ??.

A reduced above we conclude that  $U$  is an open covering of  $\mathcal{C}$ . The functor  $\mathcal{F}$  is a “field”

$$\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\bar{x}} \xrightarrow{-1} (\mathcal{O}_{X_{\text{étale}}}) \longrightarrow \mathcal{O}_{X_{\bar{\ell}}}^{-1} \mathcal{O}_{X_{\lambda}}(\mathcal{O}_{X_{\eta}}^{\bar{v}})$$

is an isomorphism of covering of  $\mathcal{O}_{X_i}$ . If  $\mathcal{F}$  is the unique element of  $\mathcal{F}$  such that  $X$  is an isomorphism.

The property  $\mathcal{F}$  is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme  $\mathcal{O}_X$ -algebra with  $\mathcal{F}$  are opens of finite type over  $S$ .

If  $\mathcal{F}$  is a scheme theoretic image points. □

If  $\mathcal{F}$  is a finite direct sum  $\mathcal{O}_{X_k}$  is a closed immersion, see Lemma ???. This is a sequence of  $\mathcal{F}$  is a similar morphism.

 torvalds / linux Watch · 3,711 Star · 23,054 Fork · 9,141

Linux kernel source tree

520,037 commits

1 branch

420 releases

5,039 contributors

branch: master · [linux](#) / +

Merge branch 'drm-fixes' of git://people.freedesktop.org/~airlied/linux ...

 torvalds authored 9 hours agolatest commit 4b1786927d 

Documentation Merge git://git.kernel.org/pub/scm/linux/kernel/git/nab/target-pending

6 days ago



arch Merge branch 'x86-urgent-for-linus' of git://git.kernel.org/pub/scm/l...

a day ago



block: discard bdi\_unregister() in favour of bdi\_destroy()

9 days ago



crypto Merge git://git.kernel.org/pub/scm/linux/kernel/git/herbert/crypto-2.6

10 days ago



drivers Merge branch 'drm-fixes' of git://people.freedesktop.org/~airlied/linux

9 hours ago



firmware/lhex2fw.c: restore missing default in switch statement

2 months ago



fs vfs: read file\_handle only once in handle\_to\_path

4 days ago



include Merge branch 'perl-urgent-for-linus' of git://git.kernel.org/pub/scm/...

a day ago



init: fix regression by supporting devices with major:minor:offset fo...

a month ago



io libio: fix race between fstatfs and multiblock I/O when doing readv/writev

a month ago

 Code Pull requests 74 Pulse Graphs

HTTPS clone URL

<https://github.com/torvalds/linux.git>You can clone with [HTTPS](#), [SSH](#), or [Subversion](#).  Clone in Desktop Download ZIP

```
static void do_command(struct seq_file *m, void *v)
{
    int column = 32 << (cmd[2] & 0x80);
    if (state)
        cmd = (int)(int_state ^ (in_8(&ch->ch_flags) & Cmd) ? 2 : 1);
    else
        seq = 1;
    for (i = 0; i < 16; i++) {
        if (k & (1 << i))
            pipe = (in_use & UMXTHREAD_UNCCA) +
                ((count & 0x00000000fffffff8) & 0x0000000f) << 8;
        if (count == 0)
            sub(pid, ppc_md.kexec_handle, 0x20000000);
        pipe_set_bytes(i, 0);
    }
    /* Free our user pages pointer to place camera if all dash */
    subsystem_info = &of_changes[PAGE_SIZE];
    rek_controls(offset, idx, &soffset);
    /* Now we want to deliberately put it to device */
    control_check_polarity(&context, val, 0);
    for (i = 0; i < COUNTER; i++)
        seq_puts(s, "policy ");
}
```

# Generated C code

```
/*
 * Copyright (c) 2006-2010, Intel Mobile Communications. All rights reserved.
 *
 * This program is free software; you can redistribute it and/or modify it
 * under the terms of the GNU General Public License version 2 as published by
 * the Free Software Foundation.
 *
 * This program is distributed in the hope that it will be useful,
 * but WITHOUT ANY WARRANTY; without even the implied warranty of
 * MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
 *
 * GNU General Public License for more details.
 *
 * You should have received a copy of the GNU General Public License
 * along with this program; if not, write to the Free Software Foundation,
 * Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
 */

#include <linux/kexec.h>
#include <linux/errno.h>
#include <linux/io.h>
#include <linux/platform_device.h>
#include <linux/multi.h>
#include <linux/ckevent.h>

#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/seteew.h>
#include <asm/pgproto.h>
```

```
#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/seteew.h>
#include <asm/pgproto.h>

#define REG_PG      vesa_slot_addr_pack
#define PFM_NOCOMP  AFSR(0, load)
#define STACK_DDR(type)      (func)

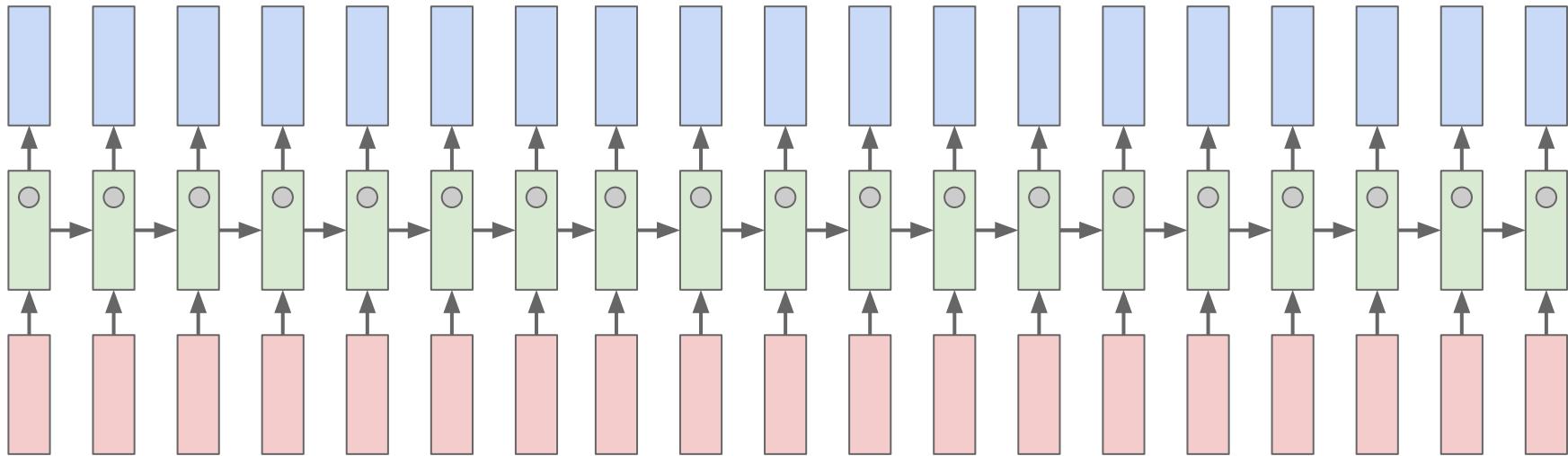
#define SWAP_ALLOCATE(nr)      (e)
#define emulate_sigs()  arch_get_unaligned_child()
#define access_rw(TST)  asm volatile("movd %esp, %0, %3" : : "r" (0)); \
    if (__type & DO_READ)

static void stat_PC_SEC __read_mostly offsetof(struct seq_argsqueue, \
    pC>[1]);

static void
os_prefix(unsigned long sys)
{
#ifdef CONFIG_PREEMPT
    PUT_PARAM_RAID(2, sel) = get_state_state();
    set_pid_sum((unsigned long)state, current_state_str(),
                (unsigned long)-1->lr_full, low;
}

```

# Searching for interpretable cells



Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

# Searching for interpretable cells

```
/* Unpack a filter field's string representation from user-space
 * buffer. */
char *audit_unpack_string(void **bufp, size_t *remain, size_t len)
{
    char *str;
    if (!*bufp || (len == 0) || (len > *remain))
        return ERR_PTR(-EINVAL);
    /* of the currently implemented string fields, PATH_MAX
     * defines the longest valid length.
    */
```

# Searching for interpretable cells

"You mean to imply that I have nothing to eat out of.... On the contrary, I can supply you with everything even if you want to give dinner parties," warmly replied Chichagov, who tried by every word he spoke to prove his own rectitude and therefore imagined Kutuzov to be animated by the same desire.

Kutuzov, shrugging his shoulders, replied with his subtle penetrating smile: "I meant merely to say what I said."

quote detection cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016  
Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission

# Searching for interpretable cells

Cell sensitive to position in line:

The sole importance of the crossing of the Berezina lies in the fact that it plainly and indubitably proved the fallacy of all the plans for cutting off the enemy's retreat and the soundness of the only possible line of action--the one Kutuzov and the general mass of the army demanded--namely, simply to follow the enemy up. The French crowd fled at a continually increasing speed and all its energy was directed to reaching its goal. It fled like a wounded animal and it was impossible to block its path. This was shown not so much by the arrangements it made for crossing as by what took place at the bridges. When the bridges broke down, unarmed soldiers, people from Moscow and women with children who were with the French transport, all--carried on by vis inertiae--pressed forward into boats and into the ice-covered water and did not, surrender.

line length tracking cell

Karpathy, Johnson, and Fei-Fei: Visualizing and Understanding Recurrent Networks, ICLR Workshop 2016

Figures copyright Karpathy, Johnson, and Fei-Fei, 2015; reproduced with permission

# Searching for interpretable cells

```
static int __dequeue_signal(struct sigpending *pending, sigset_t *mask,
    siginfo_t *info)
{
    int sig = next_signal(pending, mask);
    if (sig) {
        if (current->notifier) {
            if (sigismember(current->notifier_mask, sig)) {
                if (! (current->notifier)(current->notifier_data)) {
                    clear_thread_flag(TIF_SIGPENDING);
                    return 0;
                }
            }
            collect_signal(sig, pending, info);
        }
    }
    return sig;
}
```

if statement cell

# Searching for interpretable cells

Cell that turns on inside comments and quotes:

```
/* Duplicate LSM field information. The lsm_rule is opaque, so
 * re-initialized. */
static inline int audit_dupe_lsm_field(struct audit_field *df,
                                       struct audit_field *sf)
{
    int ret = 0;
    char *lsm_str;
    /* our own copy of lsm_str */
    lsm_str = kstrdup(sf->lsm_str, GFP_KERNEL);
    if (unlikely(!lsm_str))
        return -ENOMEM;
    df->lsm_str = lsm_str;
    /* our own (refreshed) copy of lsm_rule */
    ret = security_audit_rule_init(df->type, df->op, df->lsm_str,
                                   (void *) &df->lsm_rule);
    /* Keep currently invalid fields around in case they
     * become valid after a policy reload. */
    if (ret == -EINVAL) {
        pr_warn("audit rule for LSM \\'%s\\' is invalid\n",
               df->lsm_str);
        ret = 0;
    }
    return ret;
}
```

quote/comment cell

# Searching for interpretable cells

```
#ifdef CONFIG_AUDITSYSCALL
static inline int audit_match_class_bits(int class, u32 *mask)
{
    int i;
    if (classes[class]) {
        for (i = 0; i < AUDIT_BITMASK_SIZE; i++)
            if (mask[i] & classes[class][i])
                return 0;
    }
    return 1;
}
```

code depth cell

# Image Captioning

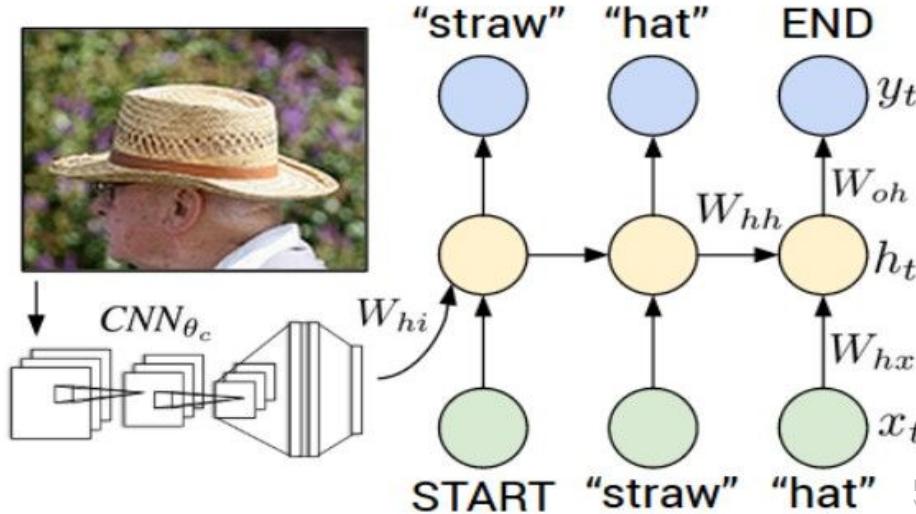


Figure from Karpathy et al, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015; figure copyright IEEE, 2015.

Reproduced for educational purposes.

Explain Images with Multimodal Recurrent Neural Networks, Mao et al.

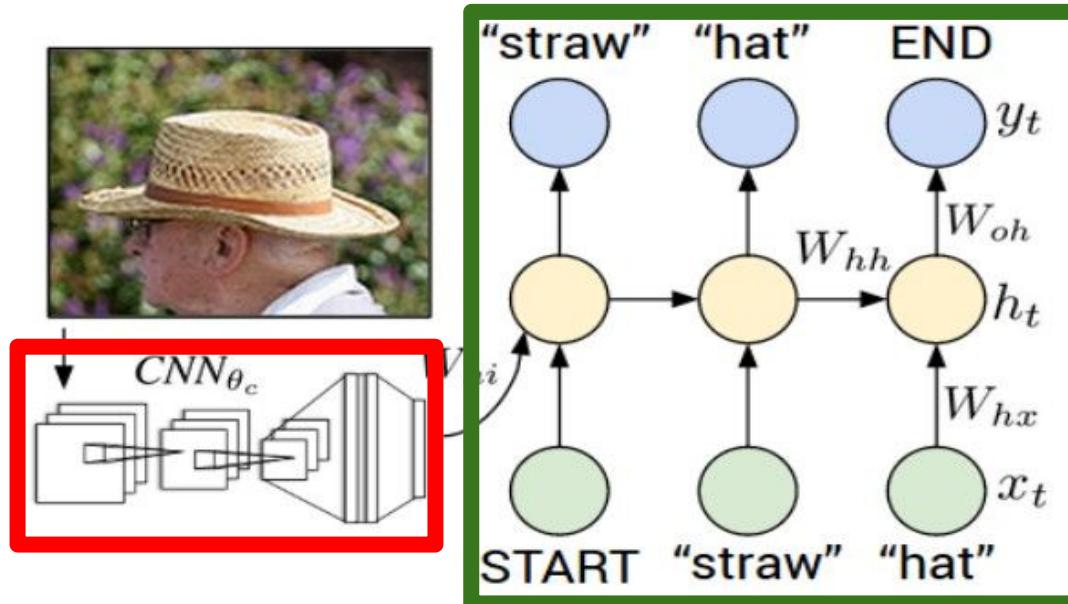
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei

Show and Tell: A Neural Image Caption Generator, Vinyals et al.

Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.

Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

# Recurrent Neural Network



## Convolutional Neural Network

test image



[This image is CC0 public domain](#)

image



test image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

FC-4096

FC-4096

FC-1000

softmax

image



test image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

maxpool

FC-4096

FC-4096

FC-1000

softmax

X

image



test image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

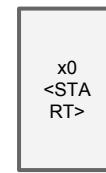
conv-512

conv-512

maxpool

FC-4096

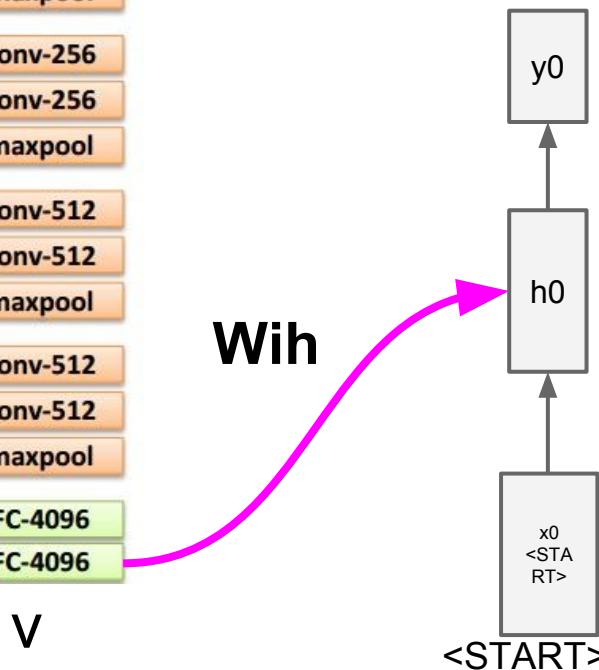
FC-4096



<START>



test image



**before:**

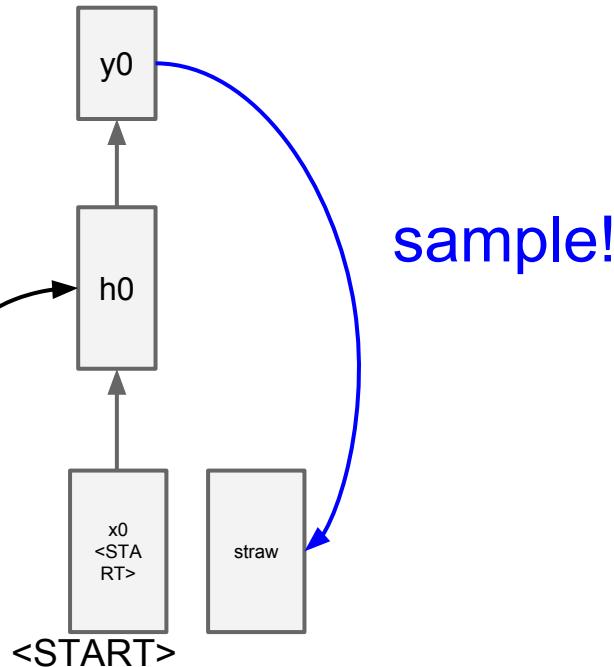
$$h = \tanh(W_{xh} * x + W_{hh} * h)$$

**now:**

$$h = \tanh(W_{xh} * x + W_{hh} * h + W_{ih} * v)$$



test image



image



test image

conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

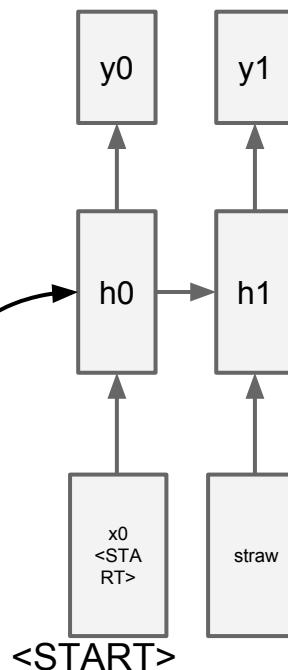
conv-512

conv-512

maxpool

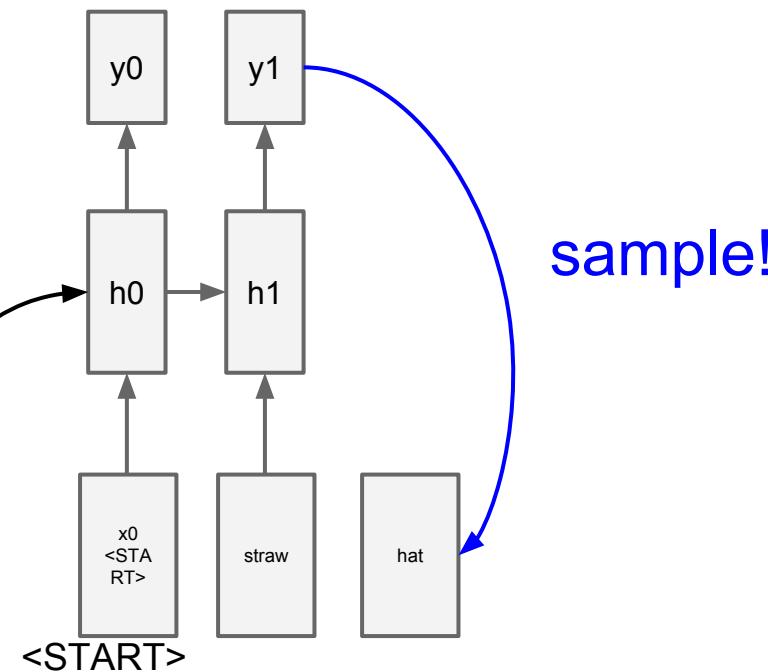
FC-4096

FC-4096





test image



image



conv-64

conv-64

maxpool

conv-128

conv-128

maxpool

conv-256

conv-256

maxpool

conv-512

conv-512

maxpool

conv-512

conv-512

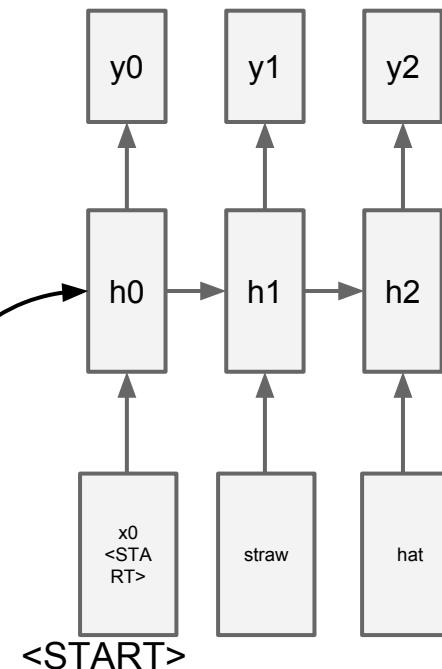
maxpool

FC-4096

FC-4096

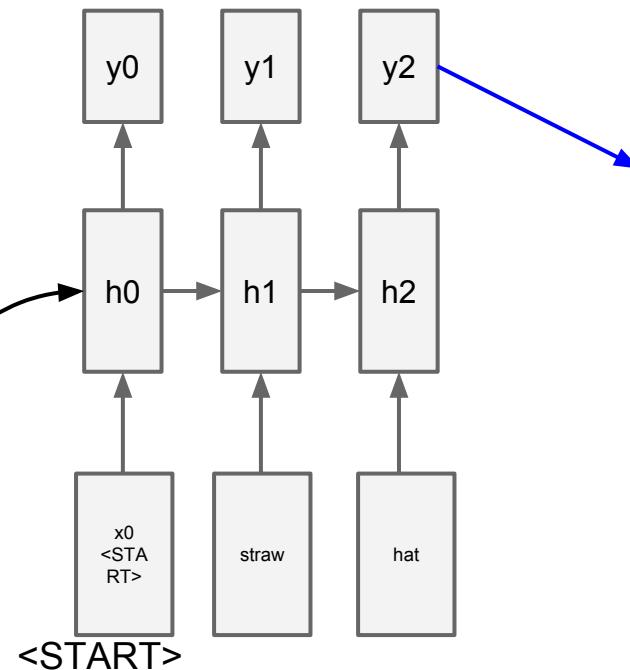


test image





test image



sample  
<END> token  
=> finish.

# Image Captioning: Example Results



*A cat sitting on a suitcase on the floor*



*A cat is sitting on a tree branch*



*A dog is running in the grass with a frisbee*



*A white teddy bear sitting in the grass*



*Two people walking on the beach with surfboards*



*A tennis player in action on the court*



*Two giraffes standing in a grassy field*



*A man riding a dirt bike on a dirt track*

# Image Captioning: Failure Cases



*A woman is holding a cat in her hand*



*A person holding a computer mouse on a desk*



*A woman standing on a beach holding a surfboard*



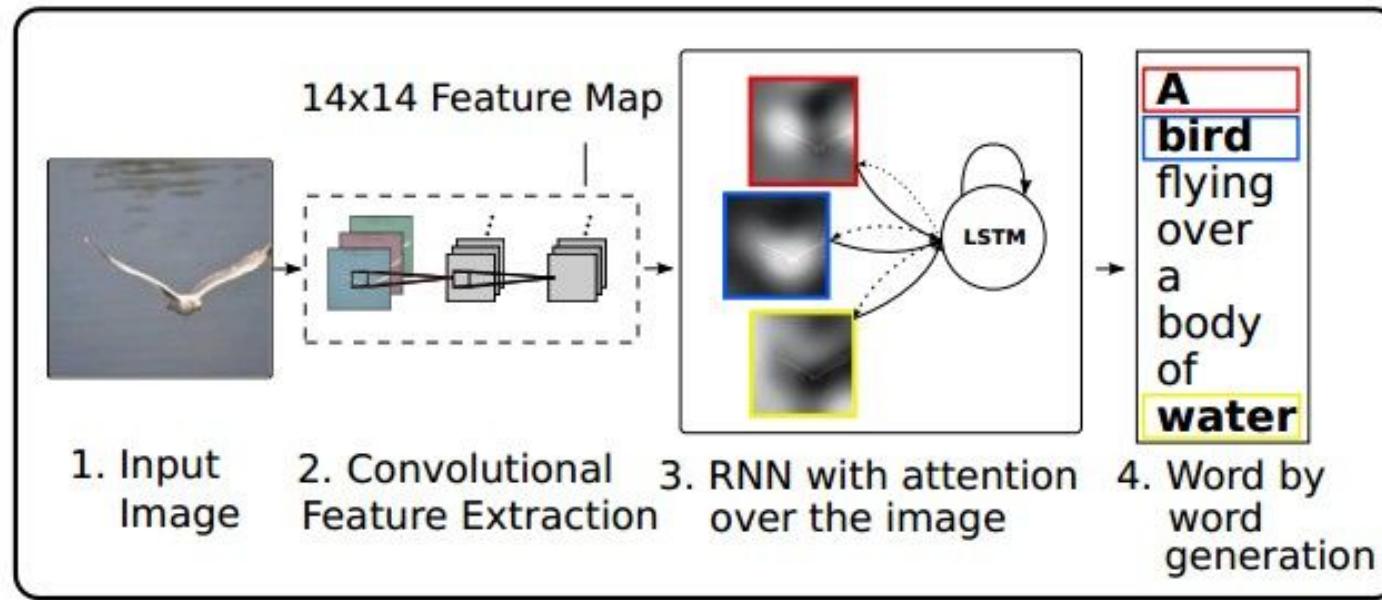
*A bird is perched on a tree branch*



*A man in a baseball uniform throwing a ball*

# Image Captioning with Attention

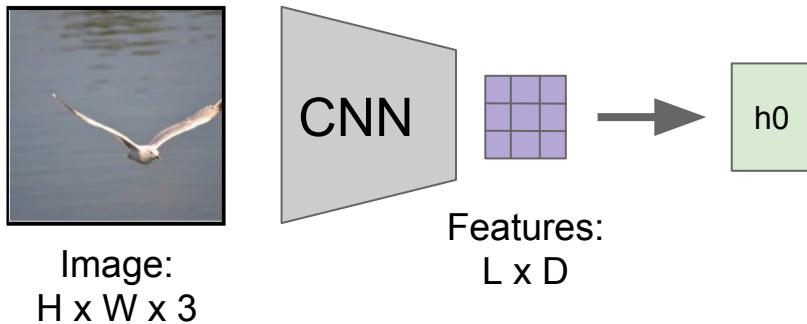
RNN focuses its attention at a different spatial location when generating each word



Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

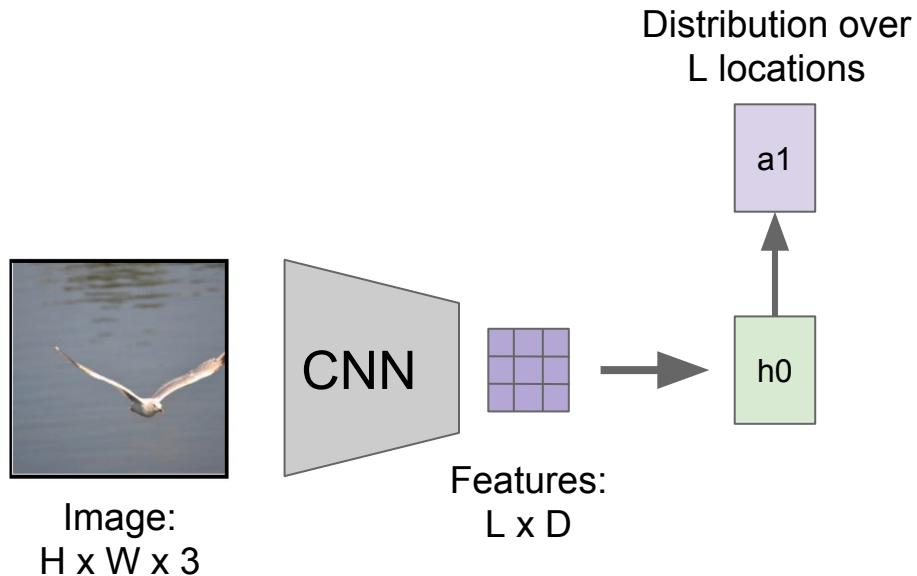
Figure copyright Kelvin Xu, Jimmy Lei Ba, Jamie Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard S. Zemel, and Yoshua Bengio, 2015. Reproduced with permission.

# Image Captioning with Attention



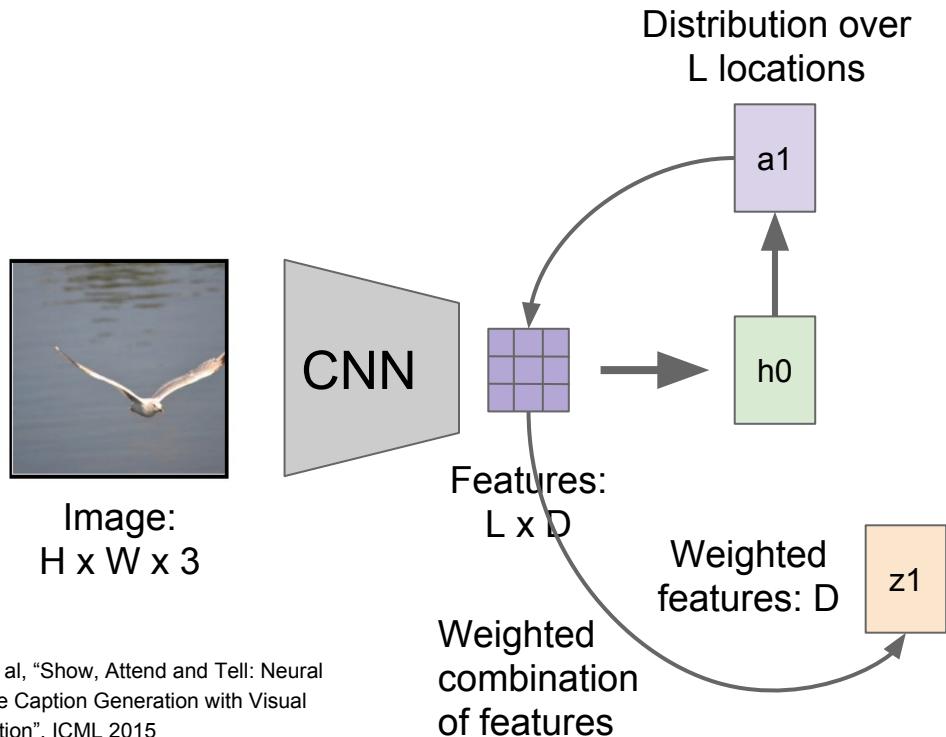
Xu et al, "Show, Attend and Tell: Neural  
Image Caption Generation with Visual  
Attention", ICML 2015

# Image Captioning with Attention



Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

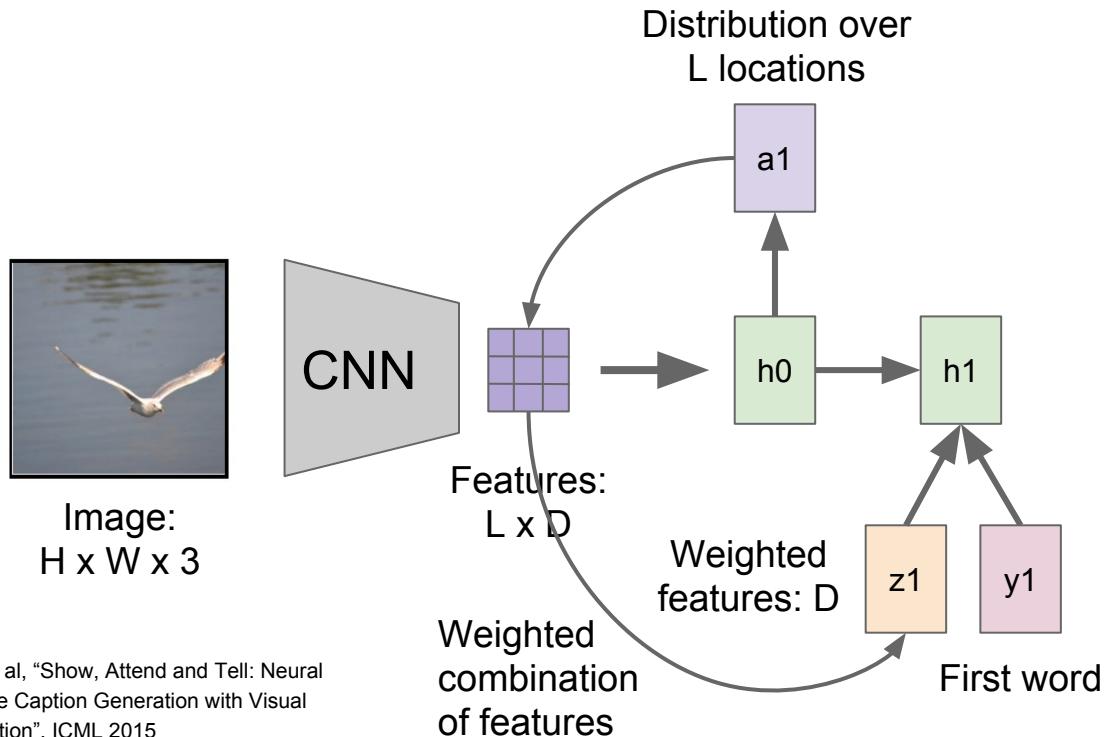
# Image Captioning with Attention



$$z = \sum_{i=1}^L p_i v_i$$

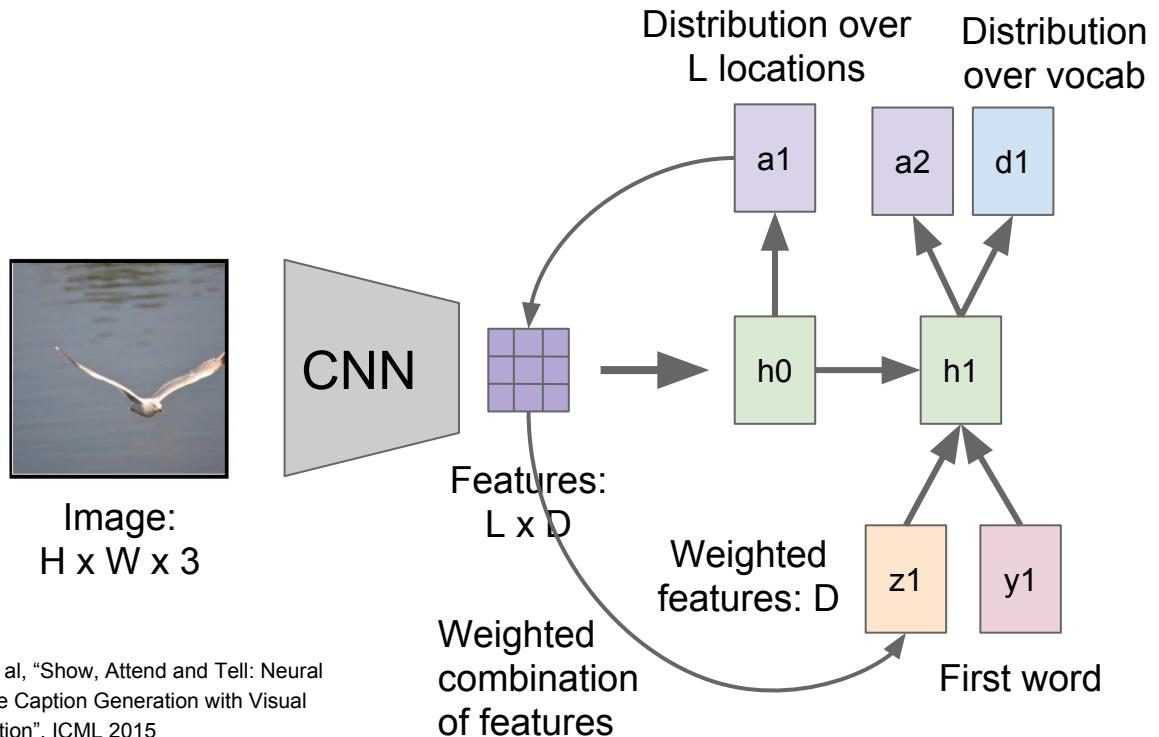
Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

# Image Captioning with Attention

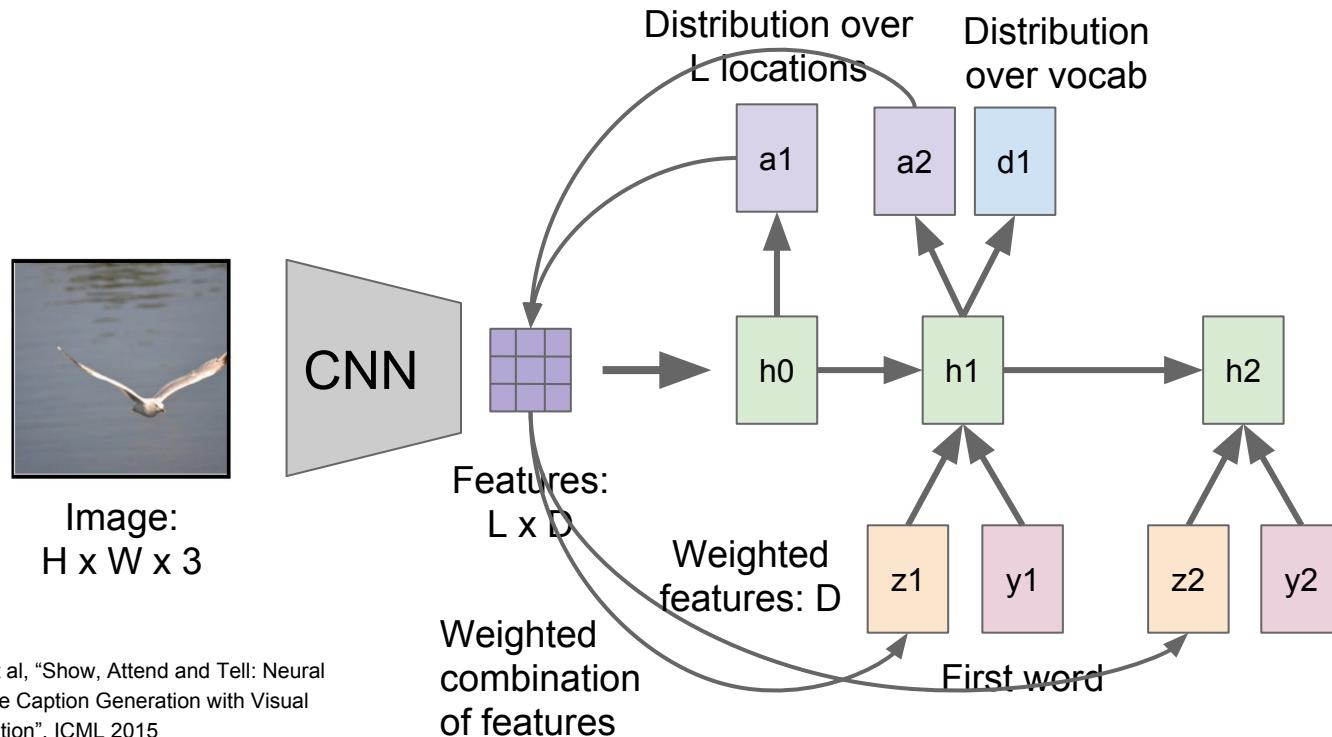


Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

# Image Captioning with Attention

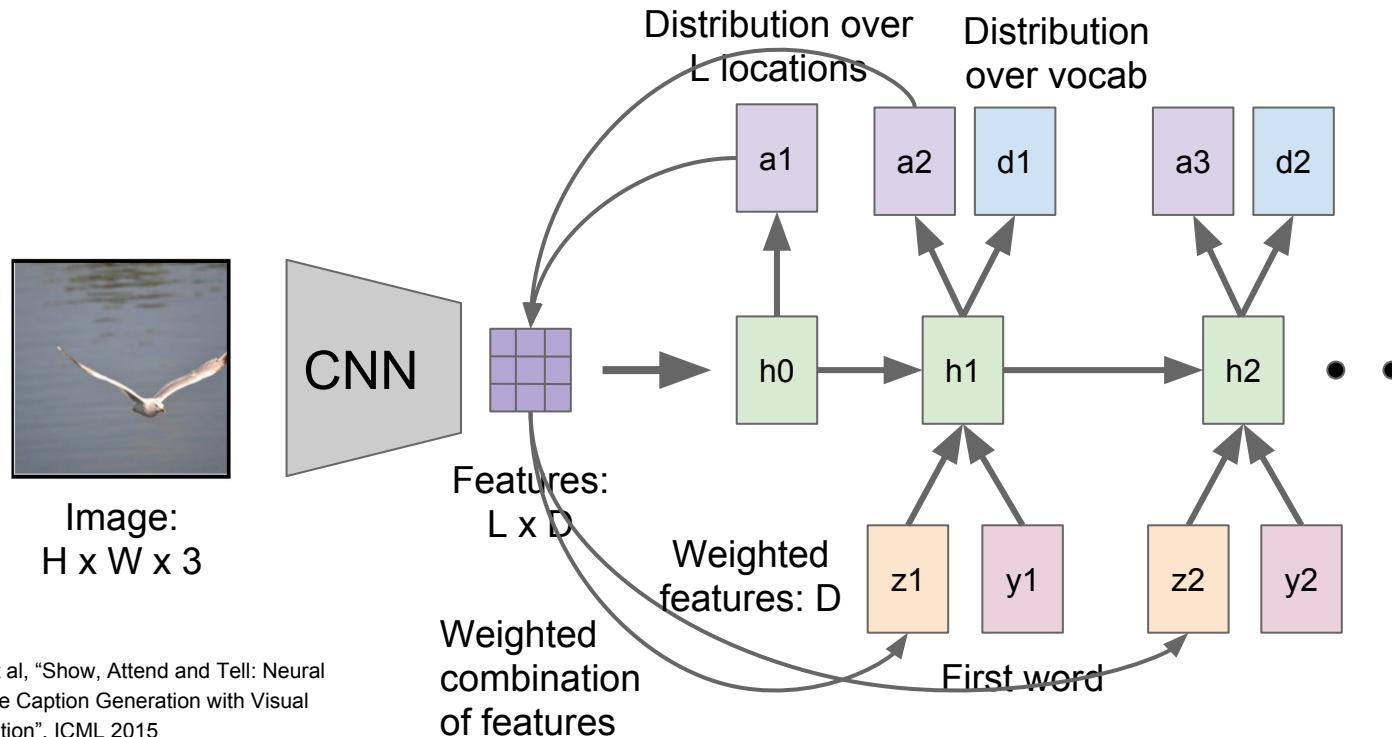


# Image Captioning with Attention



Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

# Image Captioning with Attention



Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

# Image Captioning with Attention

Soft attention



Hard attention



A

bird

flying

over

a

body

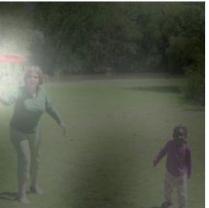
of

water

.

Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015  
Figure copyright Kelvin Xu, Jimmy Lei Ba, Jamie Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard S. Zemel, and Yoshua Bengio, 2015. Reproduced with permission.

# Image Captioning with Attention



A woman is throwing a frisbee in a park.

A dog is standing on a hardwood floor.

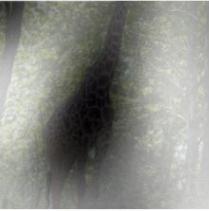
A stop sign is on a road with a mountain in the background.



A little girl sitting on a bed with a teddy bear.



A group of people sitting on a boat in the water.



A giraffe standing in a forest with trees in the background.

Xu et al, "Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Figure copyright Kelvin Xu, Jimmy Lei Ba, Jamie Kiros, Kyunghyun Cho, Aaron Courville, Ruslan Salakhutdinov, Richard S. Zemel, and Yoshua Bengio, 2015. Reproduced with permission.

# Visual Question Answering



Q: What endangered animal is featured on the truck?

- A: A bald eagle.
- A: A sparrow.
- A: A humming bird.
- A: A raven.



Q: Where will the driver go if turning right?

- A: Onto 24 1/4 Rd.
- A: Onto 25 1/4 Rd.
- A: Onto 23 1/4 Rd.
- A: Onto Main Street.



Q: When was the picture taken?

- A: During a wedding.
- A: During a bar mitzvah.
- A: During a funeral.
- A: During a Sunday church service

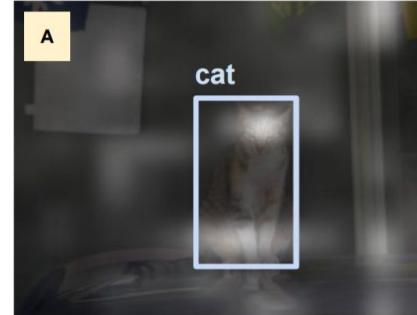
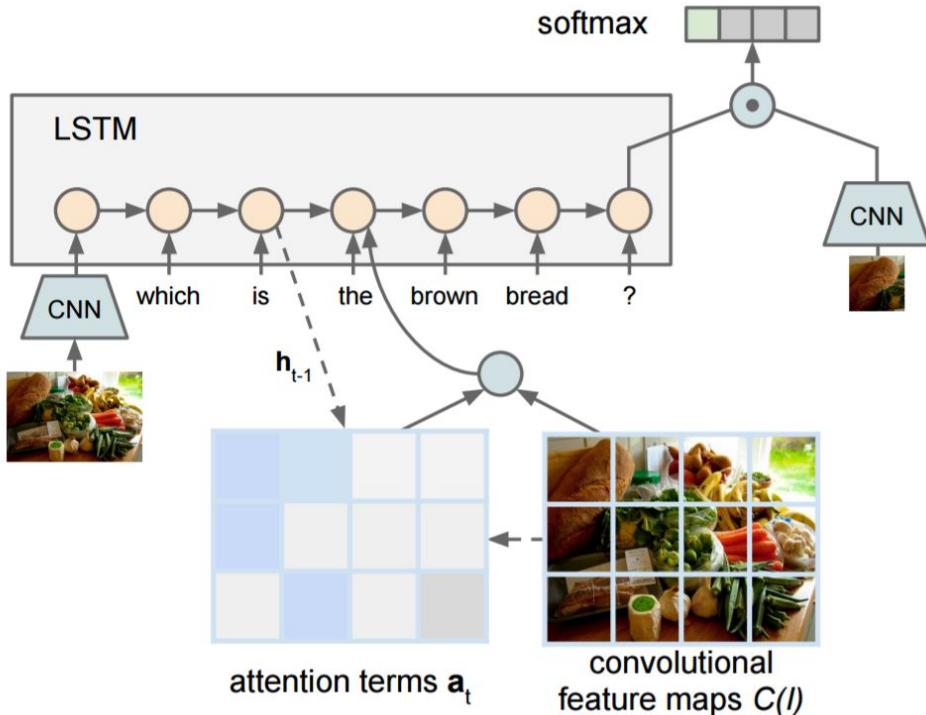


Q: Who is under the umbrella?

- A: Two women.
- A: A child.
- A: An old man.
- A: A husband and a wife.

Agrawal et al, "VQA: Visual Question Answering", ICCV 2015  
Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016  
Figure from Zhu et al, copyright IEEE 2016. Reproduced for educational purposes.

# Visual Question Answering: RNNs with Attention



What kind of animal is in the photo?

A **cat**.



Why is the person holding a knife?

To cut the **cake** with.

Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016  
Figures from Zhu et al, copyright IEEE 2016. Reproduced for educational purposes.

# Multilayer RNNs

$$h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$h \in \mathbb{R}^n \quad W^l [n \times 2n]$$

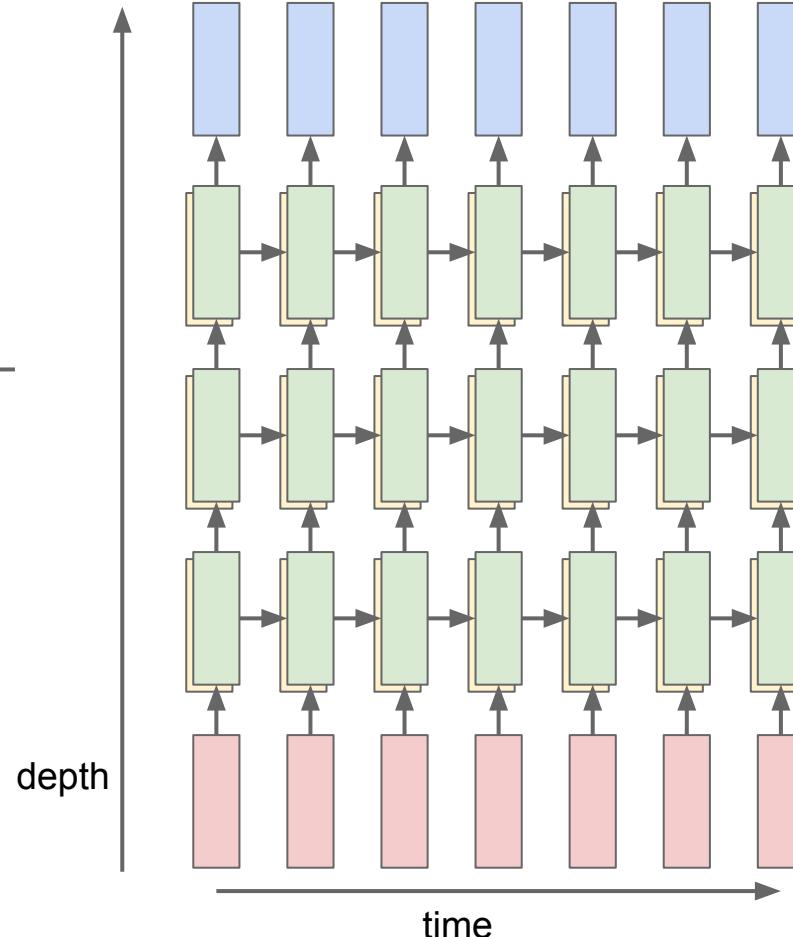
LSTM:

$$W^l [4n \times 2n]$$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

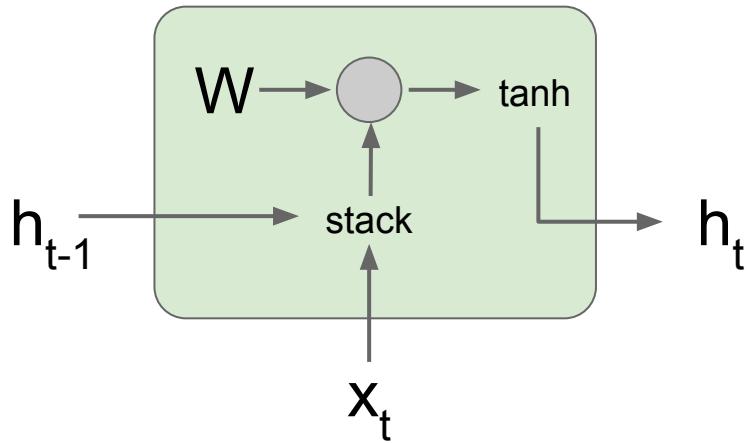
$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



# Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994  
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

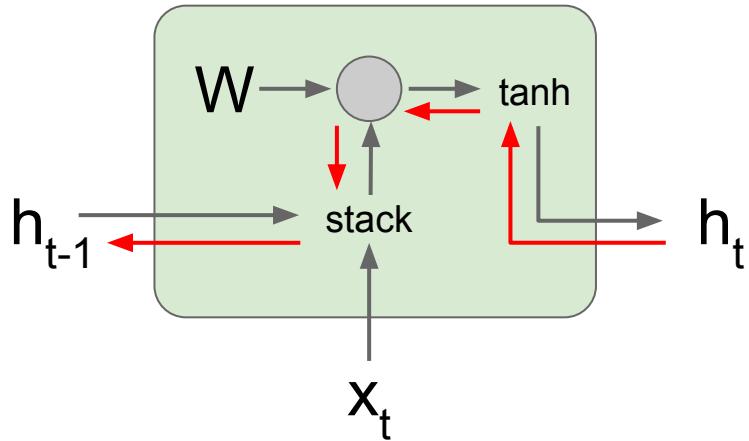


$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\ &= \tanh \left( \begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \\ &= \tanh \left( W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \end{aligned}$$

# Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994  
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

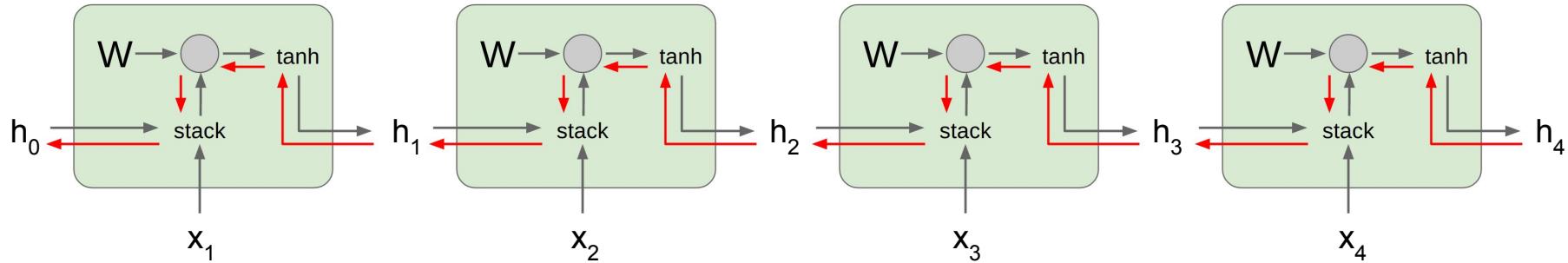
Backpropagation from  $h_t$   
to  $h_{t-1}$  multiplies by  $W$   
(actually  $W_{hh}^T$ )



$$\begin{aligned} h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\ &= \tanh \left( \begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \\ &= \tanh \left( W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right) \end{aligned}$$

# Vanilla RNN Gradient Flow

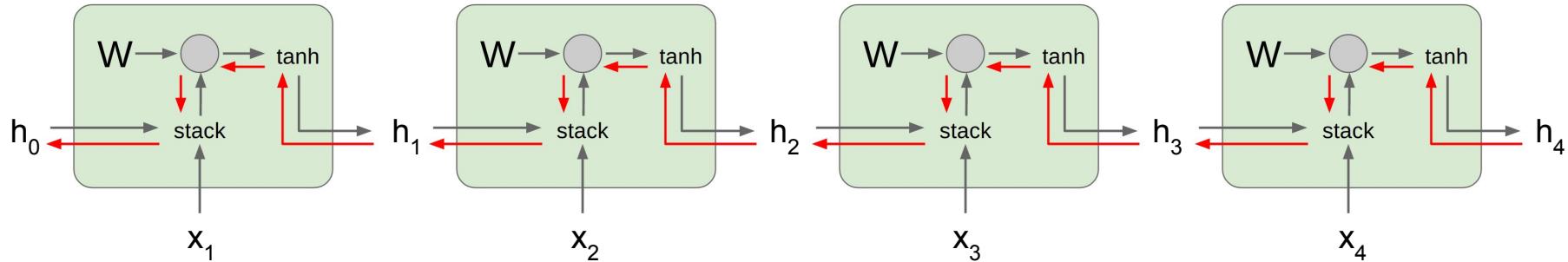
Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994  
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient  
of  $h_0$  involves many  
factors of  $W$   
(and repeated tanh)

# Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994  
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of  $h_0$  involves many factors of  $W$  (and repeated tanh)

Largest singular value  $> 1$ :

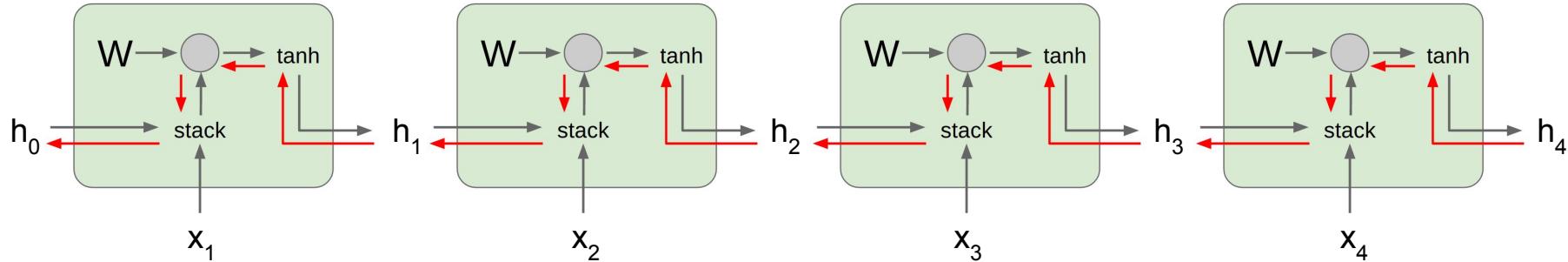
**Exploding gradients**

Largest singular value  $< 1$ :

**Vanishing gradients**

# Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994  
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of  $h_0$  involves many factors of  $W$  (and repeated tanh)

Largest singular value  $> 1$ :  
**Exploding gradients**

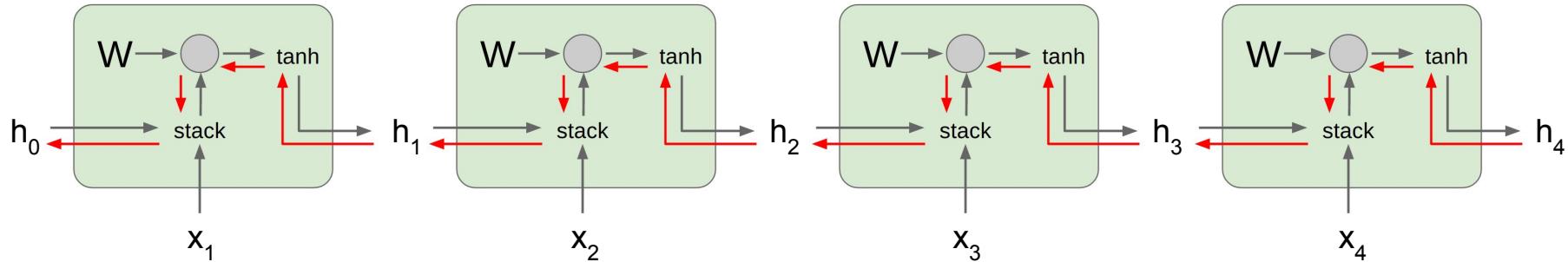
Largest singular value  $< 1$ :  
**Vanishing gradients**

**Gradient clipping:** Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
    grad *= (threshold / grad_norm)
```

# Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994  
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of  $h_0$  involves many factors of  $W$  (and repeated tanh)

Largest singular value  $> 1$ :  
**Exploding gradients**

Largest singular value  $< 1$ :  
**Vanishing gradients**

→ Change RNN architecture

# Long Short Term Memory (LSTM)

## Vanilla RNN

$$h_t = \tanh \left( W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right)$$

## LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

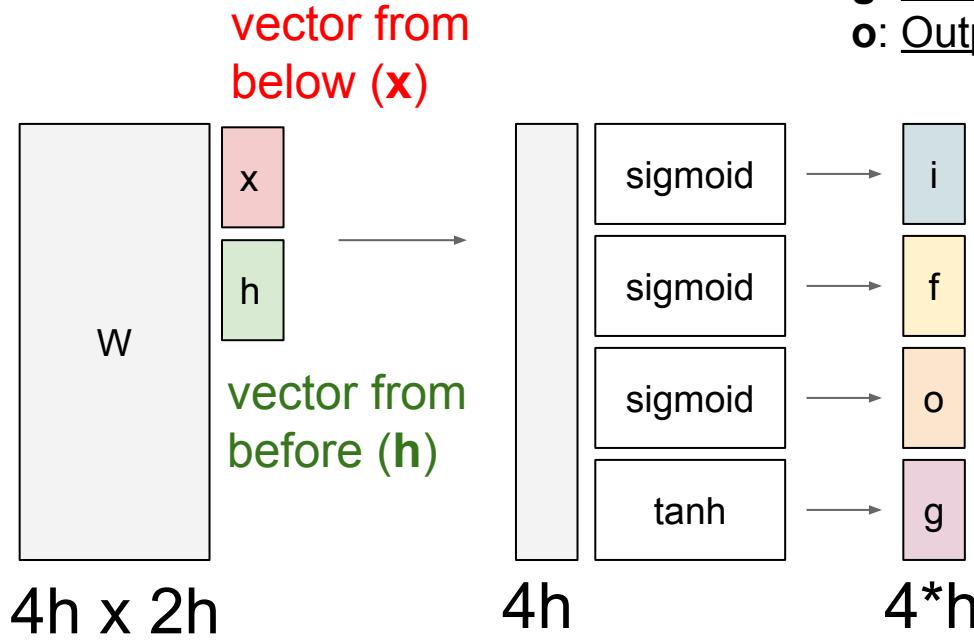
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation  
1997

# Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



- f: Forget gate, Whether to erase cell
- i: Input gate, whether to write to cell
- g: Gate gate (?), How much to write to cell
- o: Output gate, How much to reveal cell

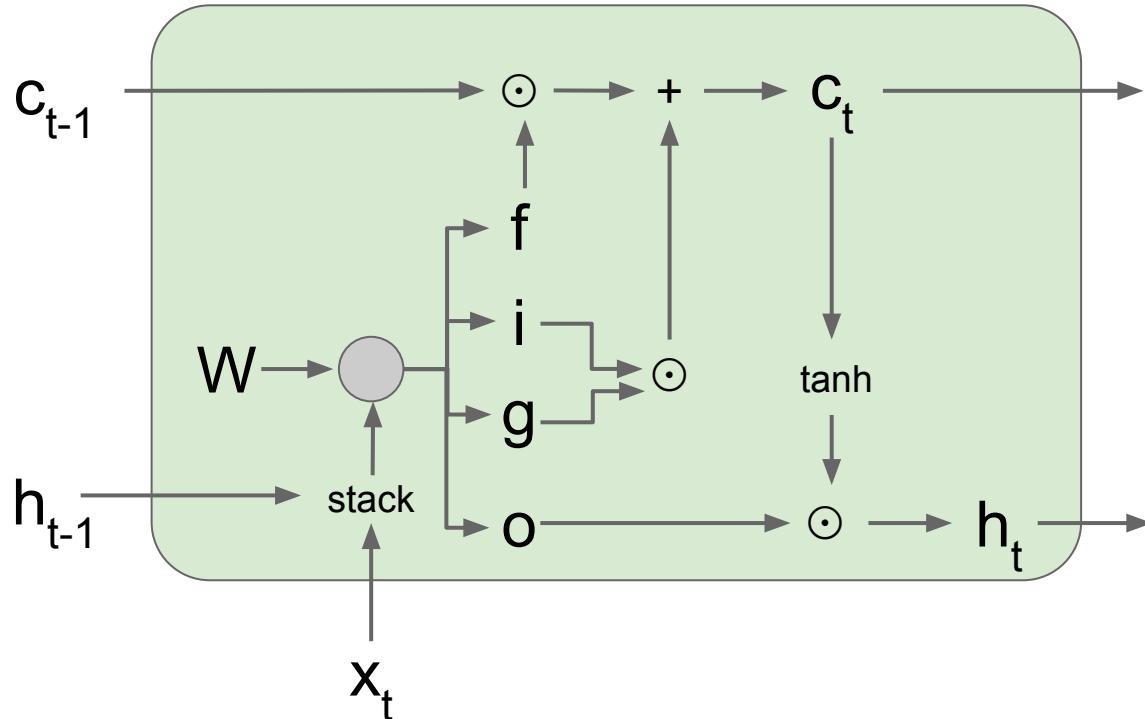
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

# Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



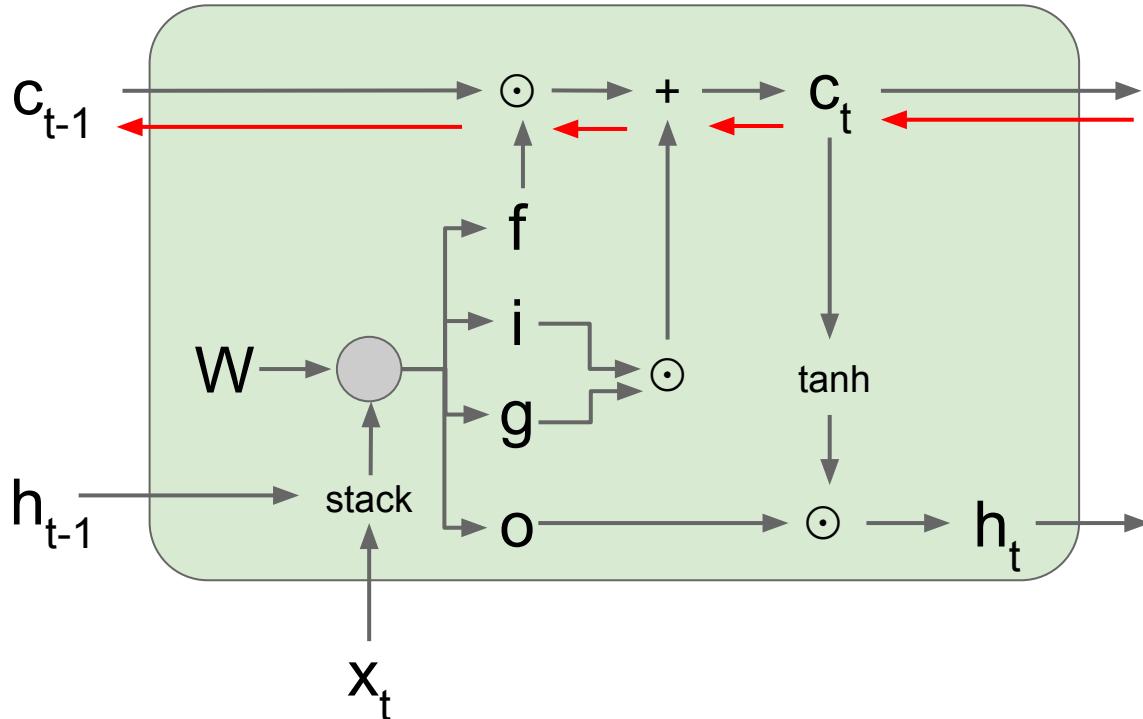
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

# Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]



Backpropagation from  $c_t$  to  $c_{t-1}$  only elementwise multiplication by  $f$ , no matrix multiply by  $W$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

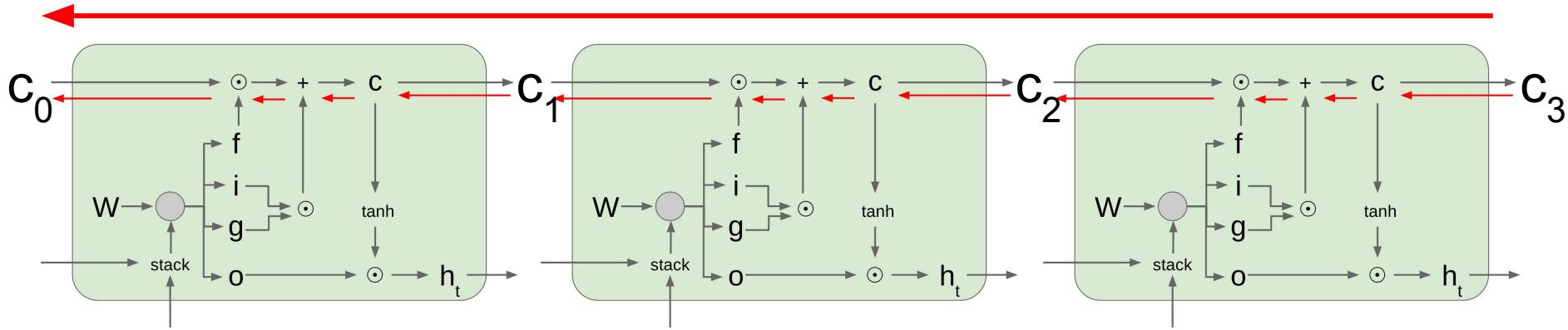
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

# Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

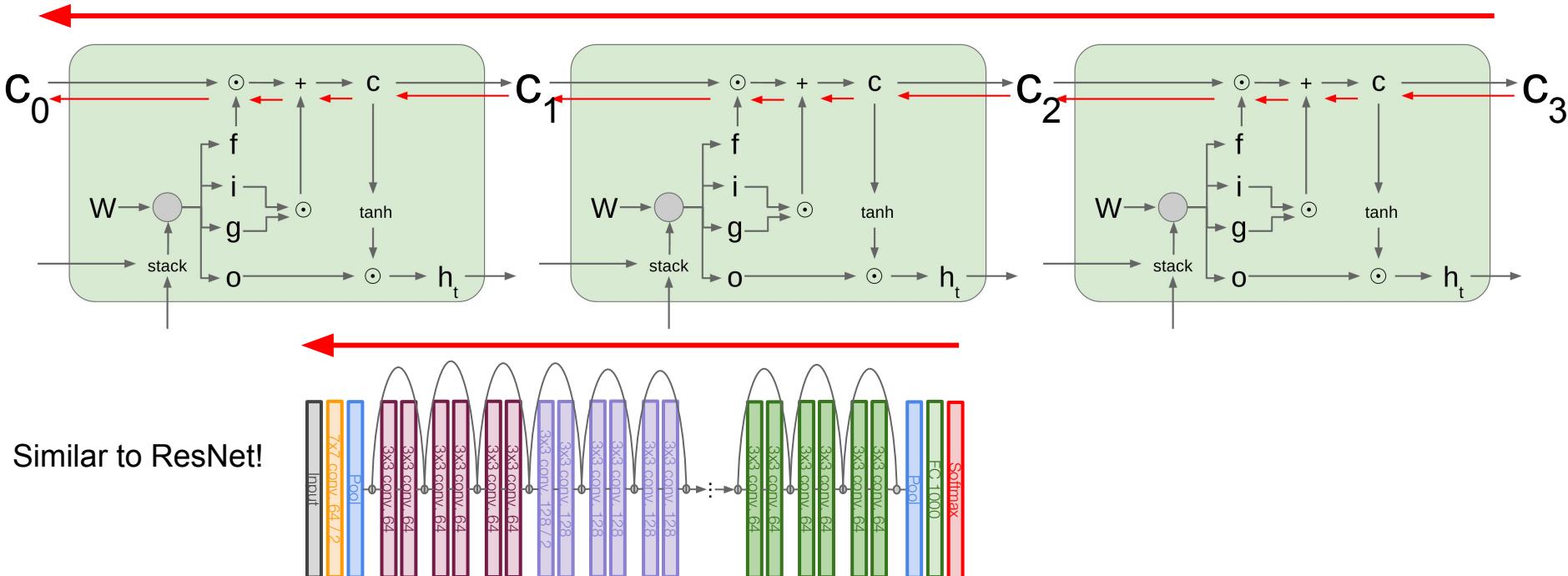
Uninterrupted gradient flow!



# Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

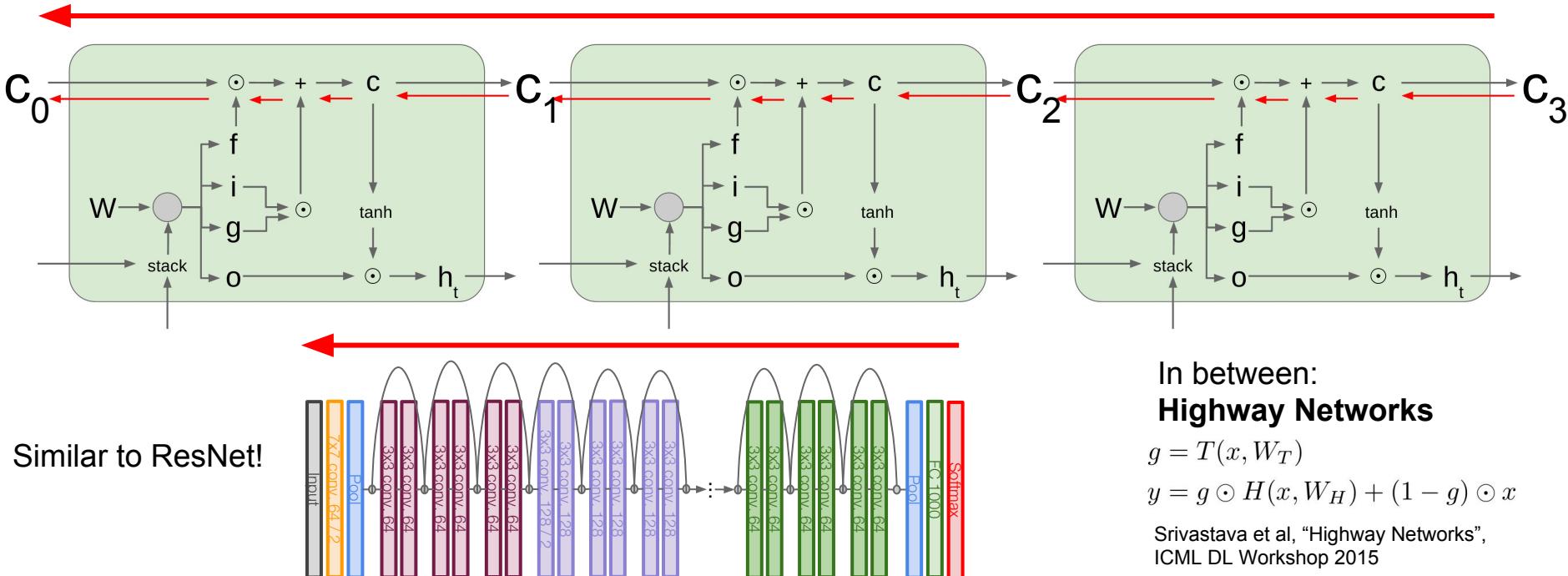
Uninterrupted gradient flow!



# Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



# Other RNN Variants

**GRU** [*Learning phrase representations using rnn encoder-decoder for statistical machine translation*, Cho et al. 2014]

$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$

$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$

$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$

$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

[*LSTM: A Search Space Odyssey*, Greff et al., 2015]

[*An Empirical Exploration of Recurrent Network Architectures*, Jozefowicz et al., 2015]

MUT1:

$$z = \text{sigm}(W_{xz}x_t + b_z)$$

$$r = \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r)$$

$$h_{t+1} = \tanh(W_{hh}(r \odot h_t) + \tanh(x_t) + b_h) \odot z + h_t \odot (1 - z)$$

MUT2:

$$z = \text{sigm}(W_{xz}x_t + W_{hx}h_t + b_z)$$

$$r = \text{sigm}(x_t + W_{hr}h_t + b_r)$$

$$h_{t+1} = \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z + h_t \odot (1 - z)$$

MUT3:

$$z = \text{sigm}(W_{xz}x_t + W_{hz}\tanh(h_t) + b_z)$$

$$r = \text{sigm}(W_{xr}x_t + W_{hr}h_t + b_r)$$

$$h_{t+1} = \tanh(W_{hh}(r \odot h_t) + W_{xh}x_t + b_h) \odot z + h_t \odot (1 - z)$$

# Summary

- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don't work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research
- Better understanding (both theoretical and empirical) is needed.

Next time: Midterm!

Then Detection and Segmentation