3D Graphics Systems | AI Graphics - Theory and Practice | IMPA 2023

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Course info: https://lvelho.impa.br/i3d23/

Lab Class #4 - Bundle Adjustment

Absolute camera orientation given set of relative camera pairs

The problem we deal with is defined as follows:

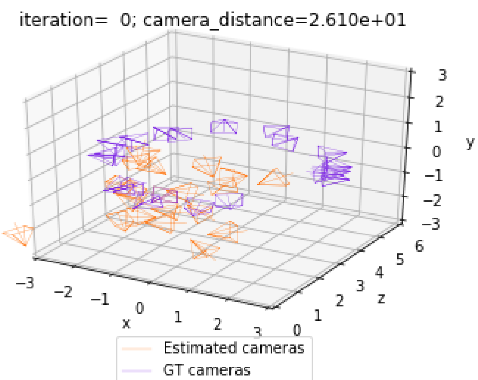
Given an optical system of N cameras with extrinsics $\{g_1, \ldots, g_N | g_i \in SE(3)\}$, and a set of relative camera positions $\{g_{ij} | g_{ij} \in SE(3)\}$ that map between coordinate frames of randomly selected pairs of cameras (i, j), we search for the absolute extrinsic parameters $\{g_1, \ldots, g_N\}$ that are consistent with the relative camera motions.

More formally:

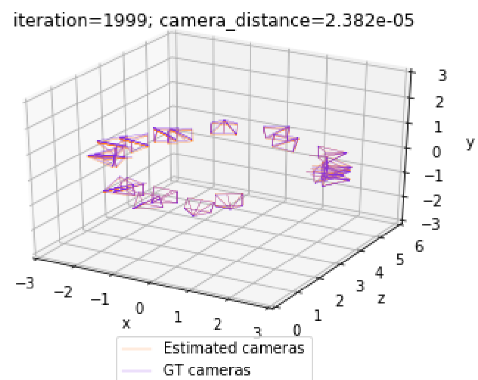
$$g_1, \ldots, g_N = \arg\min_{g_1, \ldots, g_N} \sum_{g_{ij}} d(g_{ij}, g_i^{-1} g_j),$$

, where $d(g_i, g_j)$ is a suitable metric that compares the extrinsics of cameras g_i and g_j.

Visually, the problem can be described as follows. The picture below depicts the situation at the beginning of our optimization. The ground truth cameras are plotted in purple while the randomly initialized estimated cameras are plotted in orange:



Our optimization seeks to align the estimated (orange) cameras with the ground truth (purple) cameras, by minimizing the discrepancies between pairs of relative cameras. Thus, the solution to the problem should look as follows:



In practice, the camera extrinsics g_{ij} and g_i are represented using objects from the `PerspectiveCameras` class initialized with the corresponding rotation and translation matrices `R_absolute` and `T_absolute` that define the extrinsic parameters $g = (R, T); R \in SO(3); T \in \mathbb{R}^3$. In order to ensure that `R_absolute` is a valid rotation matrix, we represent it using an exponential map (implemented with `so3_exponential_map`) of the axis-angle representation of the rotation `log_R_absolute`.

0. Install and Import Modules

Ensure `torch` and `torchvision` are installed. If `pytorch3d` is not installed, install it using the following cell:

```
import os
import sys
import torch
need_pytorch3d=False
try:
    import pytorch3d
except ModuleNotFoundError:
    need_pytorch3d=True
if need_pytorch3d:
    if torch.__version__.startswith(("2.1.")) and sys.platform.startswith("linux"):
        # We try to install PyTorch3D via a released wheel.
        pyt_version_str=torch.__version__.split("+")[0].replace(".", "")
        version_str="".join([
            f"py3{sys.version_info.minor}_cu",
            torch.version.cuda.replace(".",""),
            f"_pyt{pyt_version_str}"
        ])
        !pip install fvcore iopath
        !pip install --no-index --no-cache-dir pytorch3d -f https://dl.fbaipublicfiles.com/pytorch3d/packaging/wh
    else:
        # We try to install PyTorch3D from source.
        !pip install 'git+https://github.com/facebookresearch/pytorch3d.git@stable'
```

```
Collecting fvcore
  Downloading fvcore-0.1.5.post20221221.tar.gz (50 kB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 50.2/50.2 kB 1.7 MB/s eta 0:00:00
  Preparing metadata (setup.py) ... done
Collecting iopath
  Downloading iopath-0.1.10.tar.gz (42 kB)
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 42.2/42.2 kB 3.5 MB/s eta 0:00:00
  Preparing metadata (setup.py) ... done
Requirement already satisfied: numpy in /usr/local/lib/python3.10/dist-packages (from fvcore) (1.23.5)
Collecting yacs>=0.1.6 (from fvcore)
  Downloading yacs-0.1.8-py3-none-any.whl (14 kB)
Requirement already satisfied: pyyaml>=5.1 in /usr/local/lib/python3.10/dist-packages (from fvcore) (6.0.1)
Requirement already satisfied: tqdm in /usr/local/lib/python3.10/dist-packages (from fvcore) (4.66.1)
Requirement already satisfied: termcolor>=1.1 in /usr/local/lib/python3.10/dist-packages (from fvcore) (2.3.
Requirement already satisfied: Pillow in /usr/local/lib/python3.10/dist-packages (from fvcore) (9.4.0)
Requirement already satisfied: tabulate in /usr/local/lib/python3.10/dist-packages (from fvcore) (0.9.0)
Requirement already satisfied: typing_extensions in /usr/local/lib/python3.10/dist-packages (from iopath) (4
Collecting portalocker (from iopath)
  Downloading portalocker-2.8.2-py3-none-any.whl (17 kB)
Building wheels for collected packages: fvcore, iopath
  Building wheel for fvcore (setup.py) ... done
  Created wheel for fvcore: filename=fvcore-0.1.5.post20221221-py3-none-any.whl size=61400 sha256=ca9f94a8a4
  Stored in directory: /root/.cache/pip/wheels/01/c0/af/77c1cf53a1be9e42a52b48e5af2169d40ec2e89f7362489dd0
  Building wheel for iopath (setup.py) ... done
  Created wheel for iopath: filename=iopath-0.1.10-py3-none-any.whl size=31532 sha256=6ead971732f0140d4ff90a
  Stored in directory: /root/.cache/pip/wheels/9a/a3/b6/ac0fcd1b4ed5cfeb3db92e6a0e476cfd48ed0df92b91080c1d
Successfully built fvcore iopath
Installing collected packages: yacs, portalocker, iopath, fvcore
Successfully installed fvcore-0.1.5.post20221221 iopath-0.1.10 portalocker-2.8.2 yacs-0.1.8
Looking in links: https://dl.fbaipublicfiles.com/pytorch3d/packaging/wheels/py310_cu118_pyt210/download.html
Collecting pytorch3d
  Downloading https://dl.fbaipublicfiles.com/pytorch3d/packaging/wheels/py310_cu118_pyt210/pytorch3d-0.7.5-c
     ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 20.1/20.1 MB 194.9 MB/s eta 0:00:00
Requirement already satisfied: fvcore in /usr/local/lib/python3.10/dist-packages (from pytorch3d) (0.1.5.pos
Requirement already satisfied: iopath in /usr/local/lib/python3.10/dist-packages (from pytorch3d) (0.1.10)
Requirement already satisfied: numpy in /usr/local/lib/python3.10/dist-packages (from fvcore->pytorch3d) (1.
Requirement already satisfied: yacs>=0.1.6 in /usr/local/lib/python3.10/dist-packages (from fvcore->pytorch3
Requirement already satisfied: pyyaml>=5.1 in /usr/local/lib/python3.10/dist-packages (from fvcore->pytorch3
Requirement already satisfied: tqdm in /usr/local/lib/python3.10/dist-packages (from fvcore->pytorch3d) (4.6
Requirement already satisfied: termcolor>=1.1 in /usr/local/lib/python3.10/dist-packages (from fvcore->pytor
Requirement already satisfied: Pillow in /usr/local/lib/python3.10/dist-packages (from fvcore->pytorch3d) (9
```

```
Requirement already satisfied: tabulate in /usr/local/lib/python3.10/dist-packages (from fvcore->pytorch3d)
Requirement already satisfied: typing-extensions in /usr/local/lib/python3.10/dist-packages (from iopath->py
Requirement already satisfied: portalocker in /usr/local/lib/python3.10/dist-packages (from iopath->pytorch3
Installing collected packages: pytorch3d
Successfully installed pytorch3d-0.7.5
```

```
# imports
import matplotlib.pyplot as plt
import torch
from pytorch3d.transforms.so3 import (
    so3_exponential_map,
    so3_relative_angle,
)
from pytorch3d.renderer.cameras import (
    PerspectiveCameras,
)

from pytorch3d.io import load_objs_as_meshes
from pytorch3d.renderer import (
    RasterizationSettings, PointLights, MeshRenderer,
    MeshRasterizer, SoftPhongShader
)

# add path for demo utils
import sys
import os
sys.path.append(os.path.abspath(''))

# set for reproducibility
torch.manual_seed(42)
if torch.cuda.is_available():
    device = torch.device("cuda:0")
else:
    device = torch.device("cpu")
    print("WARNING: CPU only, this will be slow!")
```

If using **Google Colab**, fetch the utils file for plotting the camera scene, and the ground truth camera positions:

```
!wget https://raw.githubusercontent.com/hallpaz/3dsystems23/master/scripts/camera_visualization.py
!wget https://raw.githubusercontent.com/facebookresearch/pytorch3d/master/docs/tutorials/utils/plot_image_grid.py
from camera_visualization import plot_camera_scene
from plot_image_grid import image_grid

!mkdir data
!wget -P data https://raw.githubusercontent.com/facebookresearch/pytorch3d/master/docs/tutorials/data/camera_grapl
```

```
--2023-11-23 21:21:18--  https://raw.githubusercontent.com/hallpaz/3dsystems23/master/scripts/camera_visuali
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 185.199.108.133, 185.199.109.133, 185.199
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|185.199.108.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 2178 (2.1K) [text/plain]
Saving to: ‘camera_visualization.py’

camera_visualizatio 100%[===================>]   2.13K  --.-KB/s    in 0s

2023-11-23 21:21:19 (21.4 MB/s) - ‘camera_visualization.py’ saved [2178/2178]

--2023-11-23 21:21:19--  https://raw.githubusercontent.com/facebookresearch/pytorch3d/master/docs/tutorials/
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 185.199.108.133, 185.199.109.133, 185.199
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|185.199.108.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 1608 (1.6K) [text/plain]
Saving to: ‘plot_image_grid.py’

plot_image_grid.py  100%[===================>]   1.57K  --.-KB/s    in 0s

2023-11-23 21:21:19 (24.7 MB/s) - ‘plot_image_grid.py’ saved [1608/1608]

--2023-11-23 21:21:19--  https://raw.githubusercontent.com/facebookresearch/pytorch3d/master/docs/tutorials/
Resolving raw.githubusercontent.com (raw.githubusercontent.com)... 185.199.108.133, 185.199.109.133, 185.199
Connecting to raw.githubusercontent.com (raw.githubusercontent.com)|185.199.108.133|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 16896 (16K) [application/octet-stream]
Saving to: ‘data/camera_graph.pth’
```

```
camera_graph.pth    100%[===================>]  16.50K  --.-KB/s    in 0.001s

2023-11-23 21:21:20 (28.3 MB/s) - ‘data/camera_graph.pth’ saved [16896/16896]
```

OR if running **locally** uncomment and run the following cell:

```
# from utils import plot_camera_scene
# from utils import image_grid
```

1. Set up Cameras and load ground truth positions

```
# load the SE3 graph of relative/absolute camera positions
camera_graph_file = './data/camera_graph.pth'
(R_absolute_gt, T_absolute_gt), \
    (R_relative, T_relative), \
    relative_edges = \
        torch.load(camera_graph_file)

# create the relative cameras
cameras_relative = PerspectiveCameras(
    R = R_relative.to(device),
    T = T_relative.to(device),
    device = device,
)

# create the absolute ground truth cameras
cameras_absolute_gt = PerspectiveCameras(
    R = R_absolute_gt.to(device),
    T = T_absolute_gt.to(device),
    device = device,
)

# the number of absolute camera positions
N = R_absolute_gt.shape[0]
```

1.1 Check the ground truth values for rotation and translation of the first camera g_0. Do they look like measured values or arbitrary ones? Why do you think this decision was taken?

```
###############################################################################
# Code and explanation for 1.1
###############################################################################


# GT rotation for g0
R_absolute_gt[0]
```

```
tensor([[1., 0., 0.],
        [0., 1., 0.],
        [0., 0., 1.]])
```

```
# GT translation form g0
T_absolute_gt[0]
```

```
tensor([-0., -0., -0.])
```

2. Define optimization functions

Relative cameras and camera distance

We now define two functions crucial for the optimization.

`calc_camera_distance` compares a pair of cameras. This function is important as it defines the loss that we are minimizing. The method utilizes the `so3_relative_angle` function from the SO3 API.

`get_relative_camera` computes the parameters of a relative camera that maps between a pair of absolute cameras. Here we utilize the `compose` and `inverse` class methods from the PyTorch3D Transforms API.

```
def calc_camera_distance(cam_1, cam_2):
    """
    Calculates the divergence of a batch of pairs of cameras cam_1, cam_2.
    The distance is composed of the cosine of the relative angle between
    the rotation components of the camera extrinsics and the L2 distance
    between the translation vectors.
    """
    # rotation distance
    R_distance = (1.-so3_relative_angle(cam_1.R, cam_2.R, cos_angle=True)).mean()
    # translation distance
    T_distance = ((cam_1.T - cam_2.T)**2).sum(1).mean()
    # the final distance is the sum
    return R_distance + T_distance

def get_relative_camera(cams, edges):
    """
    For each pair of indices (i,j) in "edges" generate a camera
    that maps from the coordinates of the camera cams[i] to
    the coordinates of the camera cams[j]
    """

    # first generate the world-to-view Transform3d objects of each
    # camera pair (i, j) according to the edges argument
    trans_i, trans_j = [
        PerspectiveCameras(
            R = cams.R[edges[:, i]],
            T = cams.T[edges[:, i]],
            device = device,
        ).get_world_to_view_transform()
         for i in (0, 1)
    ]

    # compose the relative transformation as g_i^{-1} g_j
    trans_rel = trans_i.inverse().compose(trans_j)

    # generate a camera from the relative transform
    matrix_rel = trans_rel.get_matrix()
    cams_relative = PerspectiveCameras(
                        R = matrix_rel[:, :3, :3],
                        T = matrix_rel[:, 3, :3],
                        device = device,
                    )
    return cams_relative
```

2.1 In this task, we are parameterizing the 3D rotation group - $SO(3)$ - using rotation matrices. This choice has some drawbacks, as we need to ensure our matrices are valid rotation matrices. Which other choice(s) could we have used to parameterize rotations? Would it be a better choice?

```
###############################################################################
# Explanation for 2.1
###############################################################################
```

- PyTorch3D transforms code https://pytorch3d.readthedocs.io/en/latest/_modules/pytorch3d/transforms/so3.html
- Rodrigues Rotation formula: https://en.wikipedia.org/wiki/Rodrigues%27_rotation_formula
- Other representations of a rotation https://rotations.berkeley.edu/other-representations-of-a-rotation/
- Euler Angles and Gimbal Lock: http://webserver2.tecgraf.puc-rio.br/~mgattass/demo/EulerAnglesRotations-GimballLock/euler.html

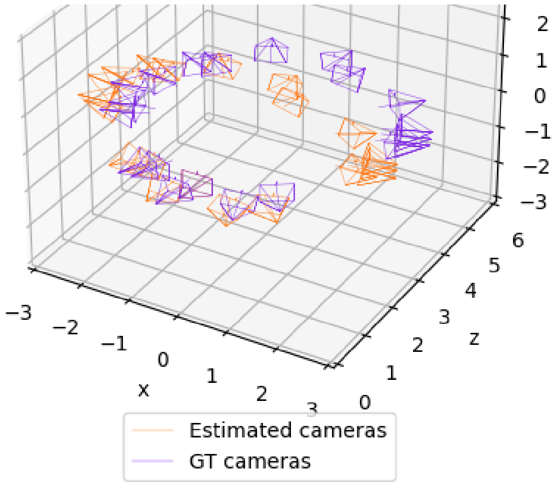
3. Optimization

Finally, we start the optimization of the absolute cameras.

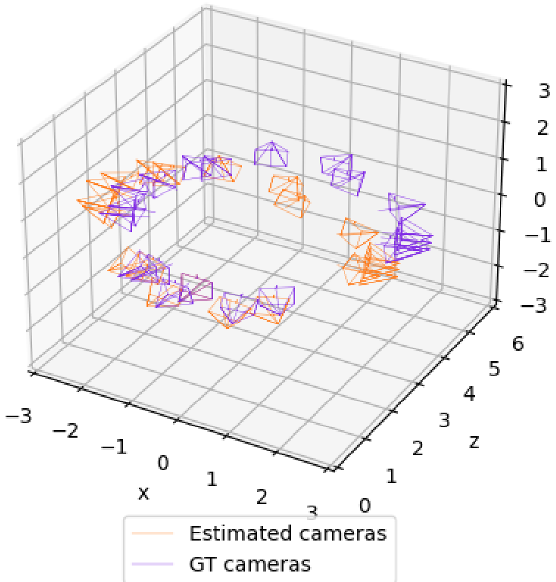
We use SGD with momentum and optimize over `log_R_absolute` and `T_absolute`.

As mentioned earlier, `log_R_absolute` is the axis angle representation of the rotation part of our absolute cameras. We can obtain the 3x3 rotation matrix `R_absolute` that corresponds to `log_R_absolute` with:

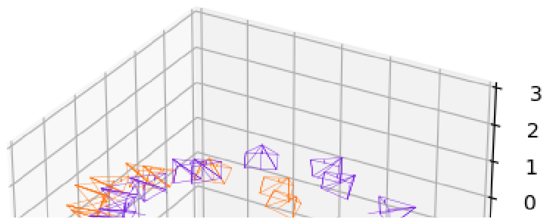
```
    R_absolute = so3_exponential_map(log_R_absolute)

# initialize the absolute log-rotations/translations with random entries
log_R_absolute_init = torch.randn(N, 3, dtype=torch.float32, device=device)
T_absolute_init = torch.randn(N, 3, dtype=torch.float32, device=device)

# furthermore, we know that the first camera is a trivial one
# (check exercise 1.1 above)
log_R_absolute_init[0, :] = 0.
T_absolute_init[0, :] = 0.

# instantiate a copy of the initialization of log_R / T
log_R_absolute = log_R_absolute_init.clone().detach()
log_R_absolute.requires_grad = True
T_absolute = T_absolute_init.clone().detach()
T_absolute.requires_grad = True

# the mask the specifies which cameras are going to be optimized
#     (since we know the first camera is already correct,
#      we only optimize over the 2nd-to-last cameras)
camera_mask = torch.ones(N, 1, dtype=torch.float32, device=device)
camera_mask[0] = 0.

# init the optimizer
optimizer = torch.optim.SGD([log_R_absolute, T_absolute], lr=.1, momentum=0.9)

losses = []
# run the optimization
n_iter = 2000  # fix the number of iterations
for it in range(n_iter):
    # re-init the optimizer gradients
    optimizer.zero_grad()

    # compute the absolute camera rotations as
    # an exponential map of the logarithms (=axis-angles)
    # of the absolute rotations
    R_absolute = so3_exponential_map(log_R_absolute * camera_mask)

    # get the current absolute cameras
    cameras_absolute = PerspectiveCameras(
        R = R_absolute,
        T = T_absolute * camera_mask,
        device = device,
    )

    # compute the relative cameras as a compositon of the absolute cameras
    cameras_relative_composed = \
        get_relative_camera(cameras_absolute, relative_edges)

    # compare the composed cameras with the ground truth relative cameras
    # camera_distance corresponds to $d$ from the description
    camera_distance = \
        calc_camera_distance(cameras_relative_composed, cameras_relative)

    # 3.2 SAVE LOSS VALUE TO PLOT LATER
    losses.append(camera_distance.item())

    # our loss function is the camera_distance
    camera_distance.backward()

    # apply the gradients
    optimizer.step()

    # plot and print status message
    if it % 200==0 or it==n_iter-1:
        status = 'iteration=%3d; camera_distance=%1.3e' % (it, camera_distance)
        plot_camera_scene(cameras_absolute, cameras_absolute_gt, status)

print('Optimization finished.')
```



iteration=1600; camera_distance=5.799e-03



iteration=1800; camera_distance=5.108e-03



3.1. Download the *cow mesh* and user the function `render_scene` (below) to render it using the cameras of the ground truth. Do the same using the initial values of the estimated cameras.

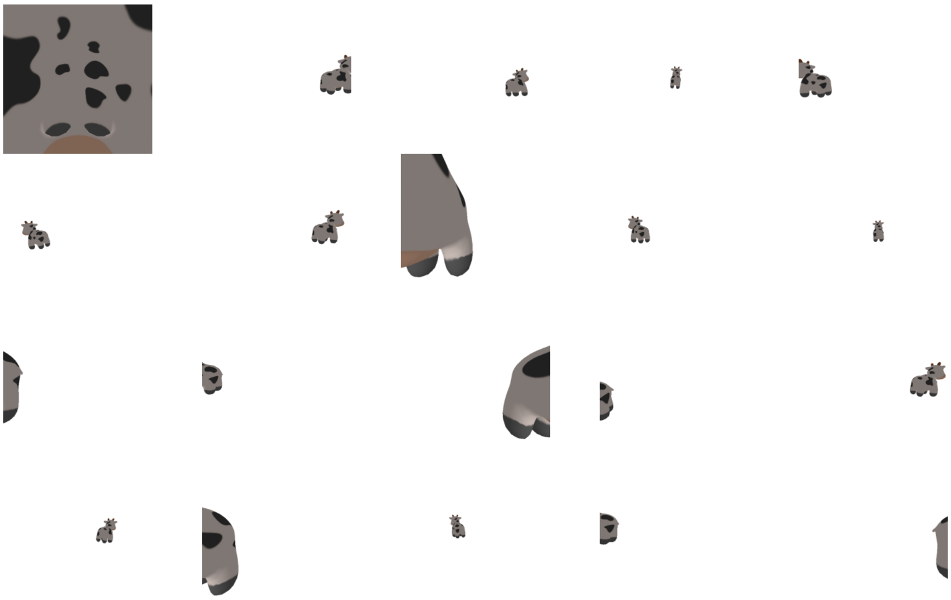
You don't need to understand how to set up a renderer now, we'll cover this later on the couser. For now, just focus on analyzing the results.

```
# download the cow mesh
!mkdir -p data/cow_mesh
!wget -P data/cow_mesh https://dl.fbaipublicfiles.com/pytorch3d/data/cow_mesh/cow.obj
!wget -P data/cow_mesh https://dl.fbaipublicfiles.com/pytorch3d/data/cow_mesh/cow.mtl
!wget -P data/cow_mesh https://dl.fbaipublicfiles.com/pytorch3d/data/cow_mesh/cow_texture.png
```

```
--2023-11-23 21:22:27--  https://dl.fbaipublicfiles.com/pytorch3d/data/cow_mesh/cow.obj
Resolving dl.fbaipublicfiles.com (dl.fbaipublicfiles.com)... 3.163.189.96, 3.163.189.14, 3.163.189.51, ...
Connecting to dl.fbaipublicfiles.com (dl.fbaipublicfiles.com)|3.163.189.96|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 330659 (323K) [text/plain]
Saving to: ‘data/cow_mesh/cow.obj’

cow.obj             100%[===================>] 322.91K  --.-KB/s    in 0.04s

2023-11-23 21:22:27 (9.00 MB/s) - ‘data/cow_mesh/cow.obj’ saved [330659/330659]

--2023-11-23 21:22:27--  https://dl.fbaipublicfiles.com/pytorch3d/data/cow_mesh/cow.mtl
Resolving dl.fbaipublicfiles.com (dl.fbaipublicfiles.com)... 3.163.189.96, 3.163.189.14, 3.163.189.51, ...
Connecting to dl.fbaipublicfiles.com (dl.fbaipublicfiles.com)|3.163.189.96|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 155 [text/plain]
Saving to: ‘data/cow_mesh/cow.mtl’

cow.mtl             100%[===================>]     155  --.-KB/s    in 0s
```

```
2023-11-23 21:22:27 (4.68 MB/s) - ‘data/cow_mesh/cow.mtl’ saved [155/155]

--2023-11-23 21:22:28--  https://dl.fbaipublicfiles.com/pytorch3d/data/cow_mesh/cow_texture.png
Resolving dl.fbaipublicfiles.com (dl.fbaipublicfiles.com)... 3.163.189.96, 3.163.189.14, 3.163.189.51, ...
Connecting to dl.fbaipublicfiles.com (dl.fbaipublicfiles.com)|3.163.189.96|:443... connected.
HTTP request sent, awaiting response... 200 OK
Length: 78699 (77K) [image/png]
Saving to: ‘data/cow_mesh/cow_texture.png’

cow_texture.png     100%[===================>]  76.85K  --.-KB/s    in 0.02s

2023-11-23 21:22:28 (4.33 MB/s) - ‘data/cow_mesh/cow_texture.png’ saved [78699/78699]
```

```
def render_scene(meshes, cameras, device):
  """
  Renders 3D meshes to a tensor of images.

  Args:
    meshes: a Meshes instance holding the meshes to be rendered
    cameras: a pytorch3D Cameras instance such as PerspectiveCameras
    device: a torch.device

  """
  if len(meshes) != len(cameras):
    meshes = meshes.extend(len(cameras))

  raster_settings = RasterizationSettings(
      image_size=512,
      blur_radius=0.0,
      faces_per_pixel=1,
  )
  lights = PointLights(device=device, location=[[0.0, 0.0, -3.0]])
  renderer = MeshRenderer(
      rasterizer=MeshRasterizer(
          cameras=cameras,
          raster_settings=raster_settings
      ),
      shader=SoftPhongShader(
          device=device,
          cameras=cameras,
          lights=lights
      )
  )
  return renderer(meshes).detach()

# you can visualize the images using the image_grid function:
# images = renderer(meshes, cameras, device)
# image_grid(images.cpu().numpy(), rows=4, cols=5, rgb=True)

# or you can choose a single image to check with matplotlib
# plt.figure(figsize=(10, 10))
# plt.imshow(images[0, ..., :3].cpu().numpy())
# plt.grid("off");
# plt.axis("off");


###############################################################################
# Code for 3.1
###############################################################################


cow_mesh = load_objs_as_meshes(["/content/data/cow_mesh/cow.obj"], device=device)


gtimages = render_scene(cow_mesh, cameras_absolute_gt, device)


image_grid(gtimages.cpu().numpy(), rows=4, cols=5, rgb=True)
```



```
# initialize the absolute log-rotations/translations with random entries
log_R_absolute_init = torch.randn(N, 3, dtype=torch.float32, device=device)
T_absolute_init = torch.randn(N, 3, dtype=torch.float32, device=device)
log_R_absolute_init[0, :] = 0.
T_absolute_init[0, :] = 0.

# instantiate a copy of the initialization of log_R / T
log_R_absolute = log_R_absolute_init.clone().detach()
T_absolute = T_absolute_init.clone().detach()

# get rotation matrices
R_absolute = so3_exponential_map(log_R_absolute)

# get the current absolute cameras
cameras_absolute = PerspectiveCameras(
    R = R_absolute,
    T = T_absolute,
    device = device,
)


rdm_images = render_scene(cow_mesh, cameras_absolute, device)


image_grid(rdm_images.cpu().numpy(), rows=4, cols=5, rgb=True)
```

3.2 Run the optimization loop and plot the *loss vs iteration* graph.

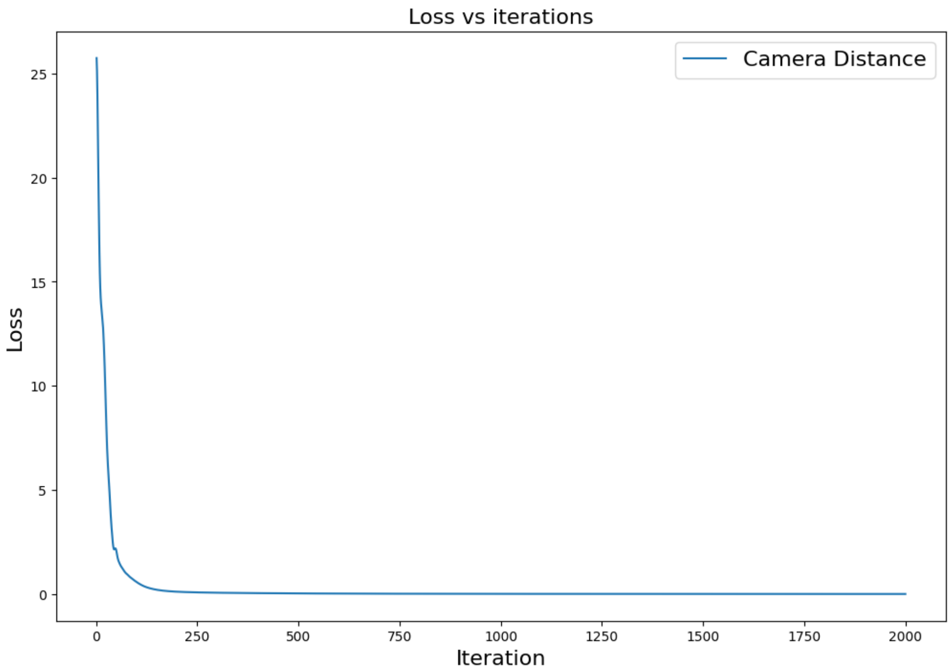
[Extra] E.1: Can you do better (improve the approximation)?

3.3 Render the images again, now using the ground truth cameras and the optimized cameras. Describe the results qualitatively.

[Extra] E.2: Use another representation for rotation matrices to solve the bundle adjustment problem.

```
#############################################################################
# Code and explanation for 3.2 - 3.3 (and extras)
#############################################################################
```

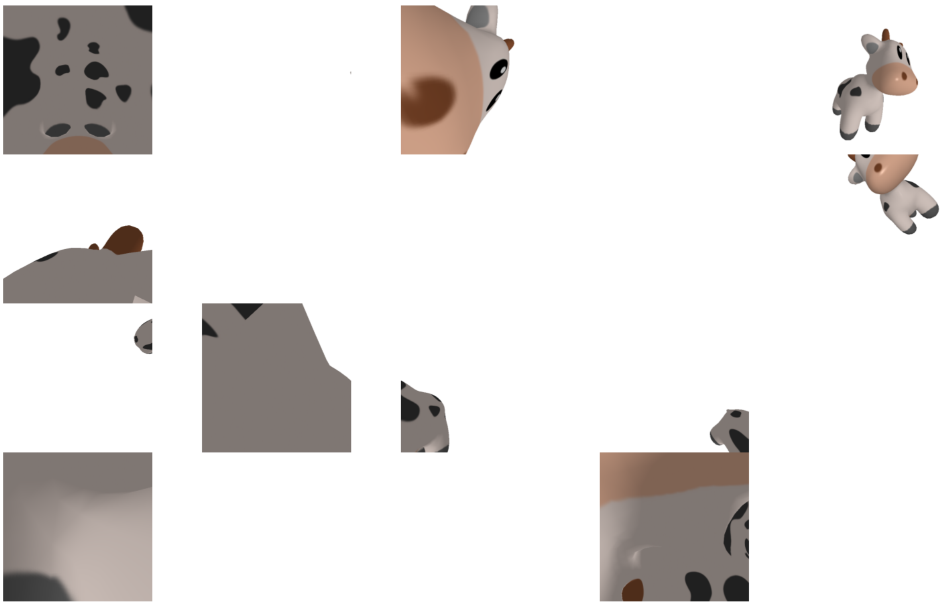
```
fig = plt.figure(figsize=(12, 8))
ax = fig.gca()
ax.plot([i for i in range(1, len(losses)+1)], losses, label="Camera Distance")
ax.legend(fontsize="16")
ax.set_xlabel("Iteration", fontsize="16")
ax.set_ylabel("Loss", fontsize="16")
ax.set_title("Loss vs iterations", fontsize="16");
```




```
opt_images = render_scene(cow_mesh, cameras_absolute, device)
```

```
image_grid(opt_images.cpu().numpy(), 4, 5, rgb=True)
```

```
image_grid(opt_images.cpu().numpy(), 4, 5, rgb=True)
```



A proposal for solving E.2

```
# Check: https://arxiv.org/abs/2006.14616
# @inproceedings{levinson20neurips,
#   title = {An Analysis of {SVD} for Deep Rotation Estimation},
#   author = {Jake Levinson, Carlos Esteves, Kefan Chen, Noah Snavely, Angjoo Kanazawa, Afshin Rostamizadeh, and
#   booktitle = {Advances in Neural Information Processing Systems 34},
#   year = {2020},
#   note = {To appear in}
# }
def symmetric_orthogonalization(x):
  """Maps 9D input vectors onto SO(3) via symmetric orthogonalization.

  x: should have size [batch_size, 9]

  Output has size [batch_size, 3, 3], where each inner 3x3 matrix is in SO(3).
  """
  m = x.view(-1, 3, 3)
  u, s, v = torch.svd(m)
  vt = torch.transpose(v, 1, 2)
  det = torch.det(torch.matmul(u, vt))
  det = det.view(-1, 1, 1)
  vt = torch.cat((vt[:, :2, :], vt[:, -1:, :] * det), 1)
  r = torch.matmul(u, vt)
  return r


def calc_camera_distance2(cam_1, cam_2):
    # rotation distance
    R_distance = ((cam_1.R - cam_2.R)**2).sum(1).mean()
    # translation distance
    T_distance = ((cam_1.T - cam_2.T)**2).sum(1).mean()
    # the final distance is the sum
    return R_distance + T_distance


# initialize the absolute rotations/translations with random entries
vec_R_absolute_init = torch.randn(N-1, 9, dtype=torch.float32, device=device)
```

```
    T_absolute_init = torch.randn(N-1, 3, dtype=torch.float32, device=device)

    # furthermore, we know that the first camera is a trivial one
    # (check exercise 1.1 above)
    # vec_R_absolute_init[0, :] = torch.tensor([1.,0.,0.,0.,1.,0.,0.,0.,1.])
    # T_absolute_init[0, :] = 0.

    # instantiate a copy of the initialization of R / T
    vec_R_absolute = vec_R_absolute_init.clone().detach()
    vec_R_absolute.requires_grad = True
    T_absolute = T_absolute_init.clone().detach()
    T_absolute.requires_grad = True

    # the mask the specifies which cameras are going to be optimized
    #     (since we know the first camera is already correct,
    #      we only optimize over the 2nd-to-last cameras)
    # camera_mask = torch.ones(N, 1, dtype=torch.float32, device=device)
    # camera_mask[0] = 0.

    # !! init the optimizer
    R0 = torch.tensor([1.,0.,0.,0.,1.,0.,0.,0.,1.]).unsqueeze(0).to(device)
    T0 = torch.tensor([0., 0, 0]).unsqueeze(0).to(device)
    optimizer = torch.optim.SGD([vec_R_absolute, T_absolute], lr=.1, momentum=0.9)

    lossesSVD = []
    # run the optimization
    n_iter = 14000  # fix the number of iterations
    for it in range(n_iter):
        # re-init the optimizer gradients
        optimizer.zero_grad()

        # compute the absolute camera rotations projecting on SO3 using SVD

        # !! R_absolute = so3_exponential_map(log_R_absolute * camera_mask)
        R_test = torch.concatenate([R0, vec_R_absolute])
        R_absolute = symmetric_orthogonalization(R_test)

        # get the current absolute cameras
        cameras_absolute = PerspectiveCameras(
            R = R_absolute,
            T = torch.concatenate([T0, T_absolute]), #T_absolute * camera_mask,
            device = device,
        )

        # compute the relative cameras as a compositon of the absolute cameras
        cameras_relative_composed = \
            get_relative_camera(cameras_absolute, relative_edges)

        # compare the composed cameras with the ground truth relative cameras
        # camera_distance corresponds to $d$ from the description
        camera_distance = \
            calc_camera_distance(cameras_relative_composed, cameras_relative)

        # 3.2 SAVE LOSS VALUE TO PLOT LATER
        lossesSVD.append(camera_distance.item())

        # our loss function is the camera_distance
        camera_distance.backward()

        # apply the gradients
        optimizer.step()

        # plot and print status message
        if it % 400==0 or it==n_iter-1:
            status = 'iteration=%3d; camera_distance=%1.3e' % (it, camera_distance)
            plot_camera_scene(cameras_absolute, cameras_absolute_gt, status)

    print('Optimization finished.')
```