## LaTeX

March 6, 2025

## 1 Algorithm

Algorithm 1 Process of Generating Data Using Vertical Federated Semisupervised Method

```
Require: X_{al}^{A}, X_{nl}^{A}, \mathcal{L}_{B}, \tau, n_{al}, n_{nl}

Ensure: X^{B_{predict}}

1: Initialization: X^{B_{predict}} = \emptyset, \mathcal{L}_{B}^{predict} = \{(\mu_{q}, \phi_{q}^{B}) \in \mathcal{L}_{B} \mid u_{q} > \tau\}

2: for (\mu_{q}, \phi_{q}^{B}) \in \mathcal{L}_{B}^{predict} do

3: X_{al}^{B_{predict}} = \{x_{i}^{B_{predict}}\}_{i=1}^{n_{al}}

4: X_{nl}^{B_{predict}} = \{x_{i}^{B_{predict}}\}_{i=n_{al}+1}^{n_{al}+n_{nl}}

5: p = \text{VFPU-M}(X_{al}^{A}, X_{nl}^{A}, X_{al}^{B_{predict}}, X_{nl}^{B_{predict}}, \phi_{q}^{B})

6: X^{B_{predict}} = X^{B_{predict}} \cup \{p\}

7: end for

8: return X^{B_{predict}}
```

## Algorithm 2 VFPU-M Algorithm

**Require:**  $X_{al}^A$ : Aligned data of party A,  $X_{nl}^A$ : Non-aligned data of party A,  $X_{al}^B$ : Aligned data of party B,  $X_{nl}^B$ : Non-aligned data of party B, y: Labels for the aligned data,  $\alpha$ : Confidence threshold, k: Proportion of data selected per iteration,T: Number of iterations.

```
Ensure: y<sup>pseudo</sup>
```

```
1: Initialize the federated model f_{\theta}
  2: \mathcal{D}_{L}^{(0)} = \{X_{al}^{A}, X_{al}^{B}, y\}

3: \mathcal{D}_{U}^{(0)} = \{X_{nl}^{A}, X_{nl}^{B}\}

4: for t = 1 to T do
                      Federated Model Training:
                     \theta^{(t)} \leftarrow \arg\min_{\theta} \sum_{(\mathbf{x}^A, \mathbf{x}^B, y) \in \mathcal{D}_L^{(t)}} \ell(f_{\theta}(\mathbf{x}^A, \mathbf{x}^B), y)
   6:
                     Confidence Calculation:
For \forall (\mathbf{x}_j^A, \mathbf{x}_j^B) \in \mathcal{D}_U^{(t)}, calculate the confidence:
   7:
   8:
                     s_j = \begin{cases} \max_c \mathbb{P}(y = c | f_{\theta^{(t)}}(\mathbf{x}_j^A, \mathbf{x}_j^B)) & \text{Classification task} \\ 1 - \frac{|\hat{y}_j - \mu_t|}{\sigma_t} & \text{Regression task} \end{cases} Where \mu_t, \sigma_t are the current predicted mean and standard deviation
   9:
 10:
 11:
                      Sample Selection:
                     \mathcal{C}^{(t)} = \{ (\mathbf{x}_j^A, \mathbf{x}_j^B) \in \mathcal{D}_U^{(t)} | s_j \geq \alpha \} Select the top k proportion of samples after sorting by confidence: \mathcal{S}^{(t)} = \text{TopK}(\mathcal{C}^{(t)}, k)
 12:
 13:
 14:
                    \forall (\mathbf{x}_j^A, \mathbf{x}_j^B) \in \mathcal{S}^{(t)}, \hat{y}_j = \begin{cases} \arg\max_c P(y = c | f_{\theta^{(t)}}(\mathbf{x}_j^A, \mathbf{x}_j^B)) \\ f_{\theta^{(t)}}(\mathbf{x}_j^A, \mathbf{x}_j^B) \end{cases}
                     Pseudo-label Generation:
 15:
                                                                                                                                                                                                        Classification
 16:
                                                                                                                                                                                                          Regression
                      \begin{array}{l} \textbf{Dataset Update:} \\ \mathcal{D}_{L}^{(t+1)} \leftarrow \mathcal{D}_{L}^{(t)} \cup \{(\mathbf{x}_{j}^{A}, \mathbf{x}_{j}^{B}, \hat{y}_{j})\}_{j \in \mathcal{S}^{(t)}} \\ \mathcal{D}_{U}^{(t+1)} \leftarrow \mathcal{D}_{U}^{(t)} \setminus \mathcal{S}^{(t)} \end{array} 
 17:
 19:
20: end for
21: \mathbf{y}^{\text{pseudo}} = \left[ y, \bigcup_{t=1}^{T} \{ \hat{y}_j \}_{j \in \mathcal{S}^{(t)}} \right] return \mathbf{y}^{\text{pseudo}}
```