LaTeX

March 3, 2025

1 Algorithm

Algorithm 1 Process of Generating Data Using Vertical Federated Semi-supervised Method

```
Require: X_{al}^{A}, X_{nl}^{A}, \mathcal{L}_{B}, \tau, n_{al}, n_{nl}

Ensure: X^{B_{predict}}

1: Initialization: X^{B_{predict}} = \emptyset, \mathcal{L}_{B}^{predict} = \{(\mu_{q}, \phi_{q}^{B}) \in \mathcal{L}_{B} \mid u_{q} > \tau\}

2: for (\mu_{q}, \phi_{q}^{B}) \in \mathcal{L}_{B}^{predict} do

3: X_{al}^{B_{predict}} = \{x_{i}^{B_{predict}}\}_{i=1}^{n_{al}}

4: X_{nl}^{B_{predict}} = \{x_{i}^{B_{predict}}\}_{i=n_{al}+1}^{n_{al}+n_{nl}}

5: p = \text{VFPU-M}(X_{al}^{A}, X_{nl}^{A}, X_{al}^{B_{predict}}, X_{nl}^{B_{predict}}, \phi_{q}^{B})

6: X^{B_{predict}} = X^{B_{predict}} \cup \{p\}

7: end for

8: return X^{B_{predict}}
```

Algorithm 2 VFPU-M Algorithm

```
Require: X_{al}^A, X_{nl}^A, X_{al}^B, X_{nl}^B, y, \alpha, k, T
Ensure: \mathbf{y}^{\text{pseudo}}
     1: Initialize the federated model f_{\theta}
     \begin{array}{l} \text{2:} \  \, \mathcal{D}_L^{(0)} = \{X_{al}^A, X_{al}^B, y\} \\ \text{3:} \  \, \mathcal{D}_U^{(0)} = \{X_{nl}^A, X_{nl}^B\} \\ \text{4:} \  \, \text{for} \  \, t = 1 \  \, \text{to} \  \, T \  \, \text{do} \\ \end{array} 
                            Federated Model Training:
                          \theta^{(t)} \leftarrow \arg\min_{\theta} \sum_{(\mathbf{x}^A, \mathbf{x}^B, y) \in \mathcal{D}_L^{(t)}} \ell(f_{\theta}(\mathbf{x}^A, \mathbf{x}^B), y)
Confidence Calculation:
For \forall (\mathbf{x}_j^A, \mathbf{x}_j^B) \in \mathcal{D}_U^{(t)}, calculate the confidence:
     6:
     7:
                        s_j = \begin{cases} \max_c \mathbb{P}(y = c | f_{\theta^{(t)}}(\mathbf{x}_j^A, \mathbf{x}_j^B)) & \text{Classification task} \\ 1 - \frac{|\hat{y}_j - \mu_t|}{\sigma_t} & \text{Regression task} \end{cases} Where \mu_t, \sigma_t are the current predicted mean and standard deviation
     9:
 10:
                            Sample Selection:
 11:
                           \mathcal{C}^{(t)} = \{(\mathbf{x}_j^A, \mathbf{x}_j^B) \in \mathcal{D}_U^{(t)} | s_j \geq \alpha \}  Select the top k proportion of samples after sorting by confidence:  \mathcal{S}^{(t)} = \text{TopK}(\mathcal{C}^{(t)}, k) 
 12:
  13:
  14:
                        Pseudo-label Generation: \forall (\mathbf{x}_{j}^{A}, \mathbf{x}_{j}^{B}) \in \mathcal{S}^{(t)}, \quad \hat{y}_{j} = \begin{cases} \arg \max_{c} \mathbb{P}(y = c | f_{\theta^{(t)}}(\mathbf{x}_{j}^{A}, \mathbf{x}_{j}^{B})) \\ f_{\theta^{(t)}}(\mathbf{x}_{j}^{A}, \mathbf{x}_{j}^{B}) \end{cases}
  15:
                                                                                                                                                                                                                                                          Classification
  16:
                                                                                                                                                                                                                                                           Regression
                          \begin{array}{l} \textbf{Dataset Update:} \\ \mathcal{D}_L^{(t+1)} \leftarrow \mathcal{D}_L^{(t)} \cup \{(\mathbf{x}_j^A, \mathbf{x}_j^B, \hat{y}_j)\}_{j \in \mathcal{S}^{(t)}} \\ \mathcal{D}_U^{(t+1)} \leftarrow \mathcal{D}_U^{(t)} \setminus \mathcal{S}^{(t)} \end{array} 
  17:
  18:
 20: end for 21: \mathbf{y}^{\text{pseudo}} = \left[y, \bigcup_{t=1}^{T} {\{\hat{y}_j\}_{j \in \mathcal{S}^{(t)}}}\right] return \mathbf{y}^{\text{pseudo}}
```