Feedback linearization of MIMO systems

Nonlinear system in state space form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u}$$

where:

- $\mathbf{x} \in \mathbb{R}^n$ is state of the system
- $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^n$ nonlinear smooth function of state evolution (vector field)
- $\mathbf{G}(\mathbf{x}) \in \mathbb{R}^{n \times m}$ nonlinear smooth input matrix

What was considered in class: focus on single-input ($\mathrm{m} = 1$):

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u$$

What I want to consider:

What if $m \neq 1$ as in example with vessel.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u}$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x})$$

Expected outcome:

Ideal Sympy routine (*system*) => (*linearization*)

Realistic Step by step solution how we can derive that $u = R^T(\theta)v$ will be inverse dynamics controller for vessel without knowledge that $R(\theta)R(\theta)^T = I$