

Appendix from M. Lu et al., “Beta Diversity Patterns Derived from Island Biogeography Theory” (Am. Nat., vol. 194, no. 3, p. E52)

Derivation of Expectation of Pairwise Jaccard Dissimilarity

$$\begin{aligned}
 E(\text{Jaccard}_{\text{dis}}) &= \sum_N \sum_k \sum_i \binom{M}{\gamma} \binom{\gamma}{k, i} p^{k+i} q^{k+j} (1-p)^{M-(k+i)} (1-q)^{M-(k+j)} \frac{\gamma - K}{\gamma} \\
 &= \sum_N \sum_k \sum_i \binom{M}{\gamma} \binom{\gamma-1}{k} \binom{\gamma-k}{i} p^{k+i} q^{k+j} (1-p)^{M-(k+i)} (1-q)^{M-(k+j)} \\
 &= \sum_N \sum_k \sum_i \binom{M}{\gamma} \binom{\gamma-1}{k} \binom{\gamma-k}{i} \left(\frac{p}{1-p}\right)^i \left(\frac{q}{1-q}\right)^j \left(\frac{p}{1-p}\right)^k \left(\frac{q}{1-q}\right)^k (1-p)^M (1-q)^M \\
 &= \sum_N \sum_k \binom{M}{\gamma} \binom{\gamma-1}{k} \left(\frac{p}{1-p} + \frac{q}{1-q}\right)^{N-k} \left(\frac{p}{1-p}\right)^k \left(\frac{q}{1-q}\right)^k (1-p)^M (1-q)^M \\
 &= \sum_N \sum_k \binom{M}{\gamma} \binom{\gamma-1}{k} (p+q-2pq)^{\gamma-k} (pq)^k (1-p)^{M-\gamma} (1-q)^{M-\gamma} \\
 &= \sum_N \sum_k \binom{M}{\gamma} \binom{\gamma-1}{k} (p+q-2pq)^{\gamma-k-1} (pq)^k (1-p)^{M-\gamma} (1-q)^{M-\gamma} (p+q-2pq) \\
 &= \sum_N \binom{M}{\gamma} (p+q-2pq+pq)^{\gamma-1} [(1-p)(1-q)]^{M-\gamma} (p+q-2pq) \\
 &= \sum_N \binom{M}{\gamma} (p+q-pq)^{\gamma} (1-p-q+pq)^{M-\gamma} \frac{(p+q-2pq)}{(p+q-pq)} \\
 &= [1 - (1-p)^M (1-q)^M] \frac{(p+q-2pq)}{(p+q-pq)}
 \end{aligned}$$

Because Jaccard is undefined at $\gamma = 0$, the above quantity should be normalized by the term $1 - (1-p)^M (1-q)^M$, which gives the expected pairwise dissimilarity conditioned on $\gamma > 0$, $((p+q-2pq)/(p+q-pq))$.

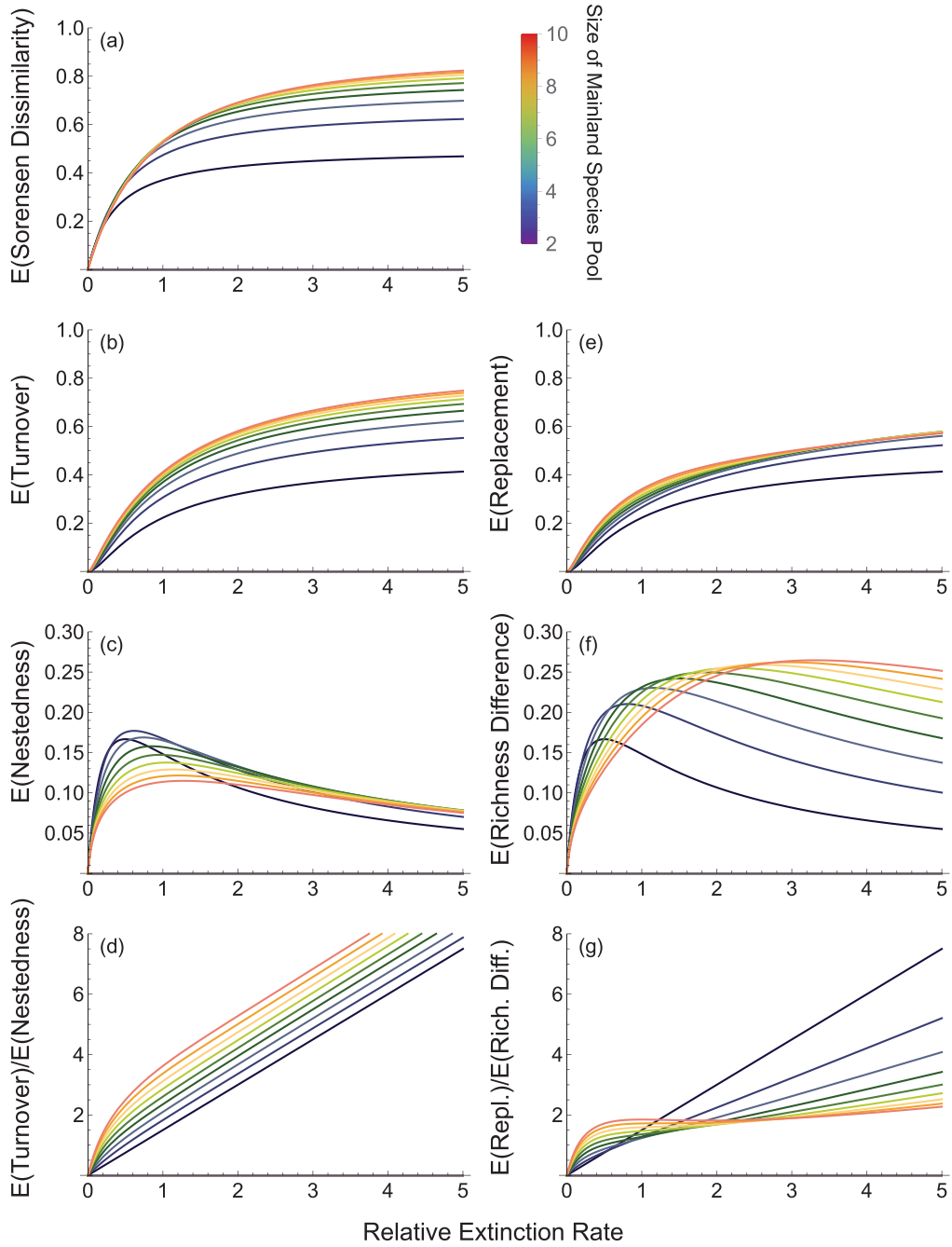


Figure A1: Expected pairwise Sørensen beta diversities (Y -axes) of two identical islands (same colonization and extinction rates), conditioned on both islands having species ($i > 0, j > 0$). Results are derived from the joint presence-absence distribution of two islands and shown for different mainland pool sizes. The relative extinction rate is the ratio of extinction rate and colonization rate.

The Effect of Interspecific Variation in Relative Extinction Rates

To examine the effect of interspecific variation in relative extinction rates, we conducted a simulation where the occurrence probability of the mainland species pool is drawn from a beta distribution:

$$\frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha, \beta)},$$

where $B(\alpha, \beta)$ is the beta function

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}.$$

The expectation of beta distribution is

$$E(p) = \frac{\alpha}{\alpha + \beta}.$$

We set

$$\begin{aligned}\alpha &= c = 1, \\ \beta &= e,\end{aligned}$$

so that the expected occurrence probability is just what is obtained from the equilibrium island biogeography theory. We assessed three different sizes of the mainland species pool: $M = 50$, $M = 100$, and $M = 300$. For each relative extinction rate, we simulated 1,000 pairs of communities and calculated the beta dissimilarity of the Jaccard family. We also compared it with the species-level neutral case where all species have the same relative extinction rates.

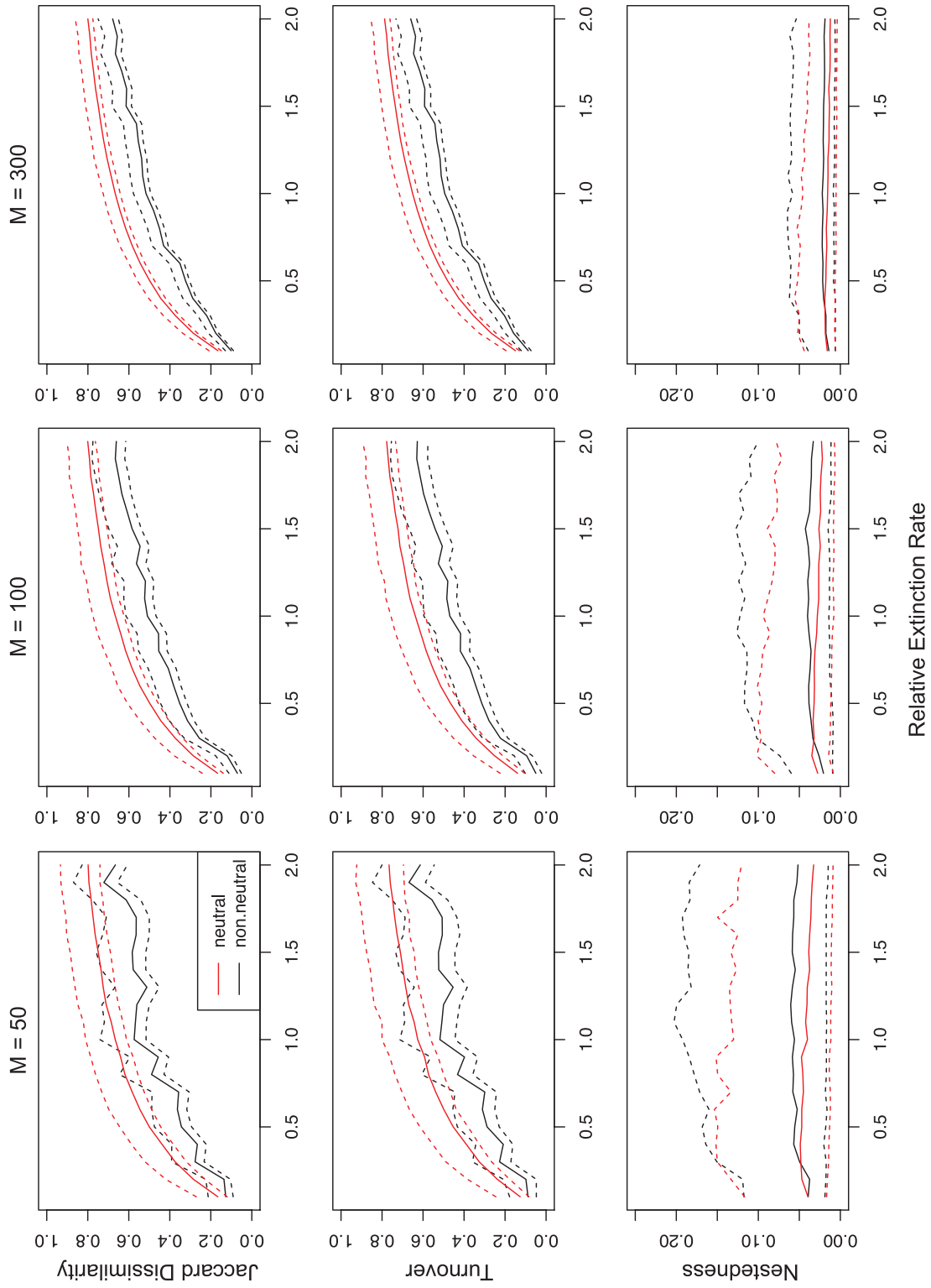


Figure A2: Expected pairwise Jaccard beta diversities of two identical islands, conditioned on both islands having species. The occurrence probabilities of species are randomly drawn from beta distributions with mean $E(p) = \alpha/(\alpha + \beta)$, where $\alpha = c = 1$ and $\beta = e$. The term M is the size of the mainland species pool.

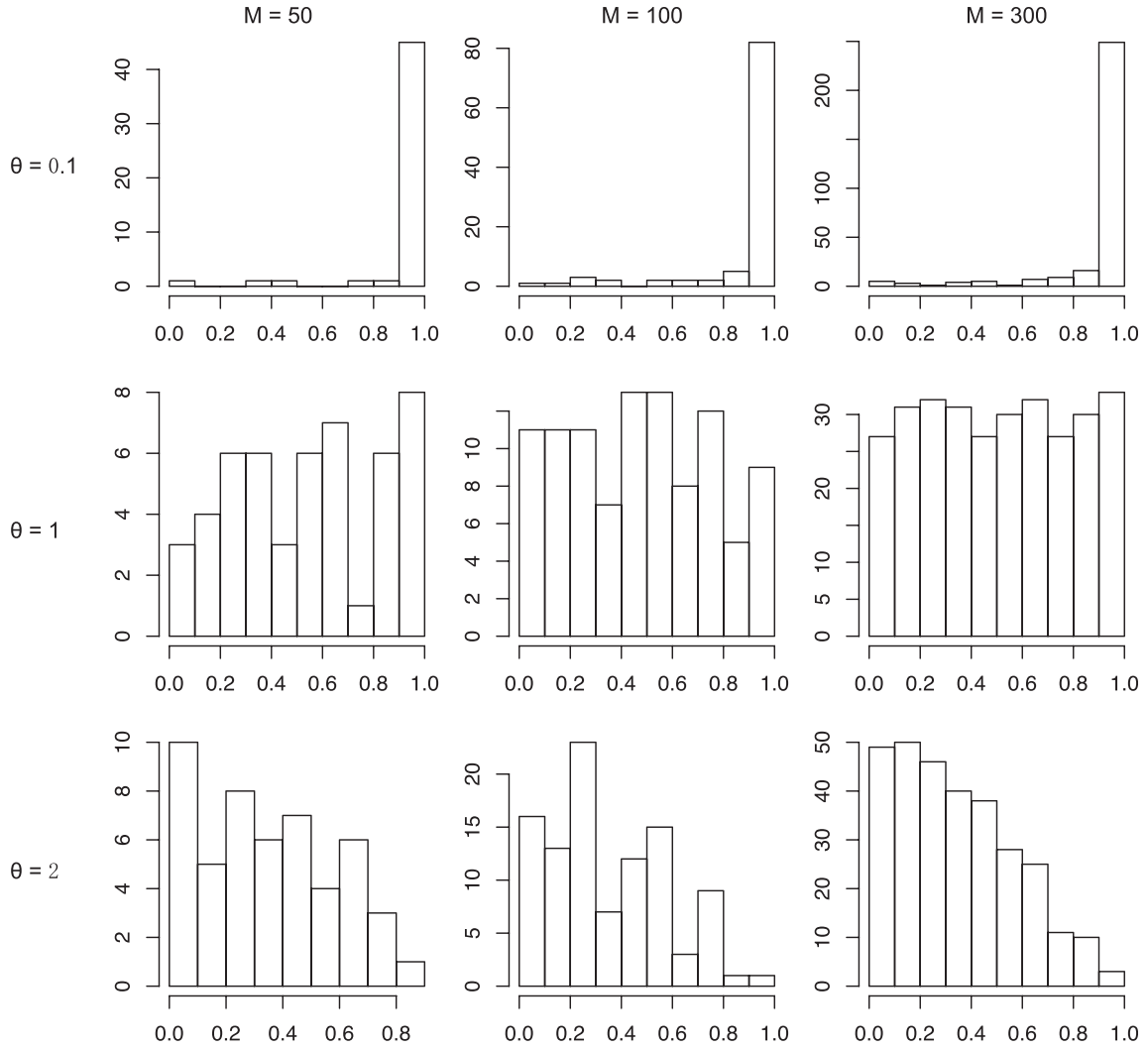


Figure A3: Frequency distributions of occurrence probabilities with different sizes of mainland species pool M and relative extinction rate θ .

The Effect of the Shape of Occurrence Probabilities Distributions

To assess the effect of the shape of occurrence probability distributions, we conducted a simulation with the probability drawn from a beta distribution with a fixed mainland species pool size of $M = 300$ and $\alpha = \beta = c = e$ so that the drawn probabilities have the same mean value of 0.5. This corresponds to the scenario of relative extinction rates equaling 1. But the occurrence probability distribution varies from the most uneven distribution, where there are a lot of low-occurrence species and high-occurrence species ($c = e = 0.1$) to the relatively uniform distribution ($c = e = 1$) to the most even distribution, where a lot of species have the same occurrence probability of 0.5. Again, we simulated 1,000 pairs of communities for each parameter combination. The results show that pairwise beta diversity decreases with the level of unevenness of the distribution.

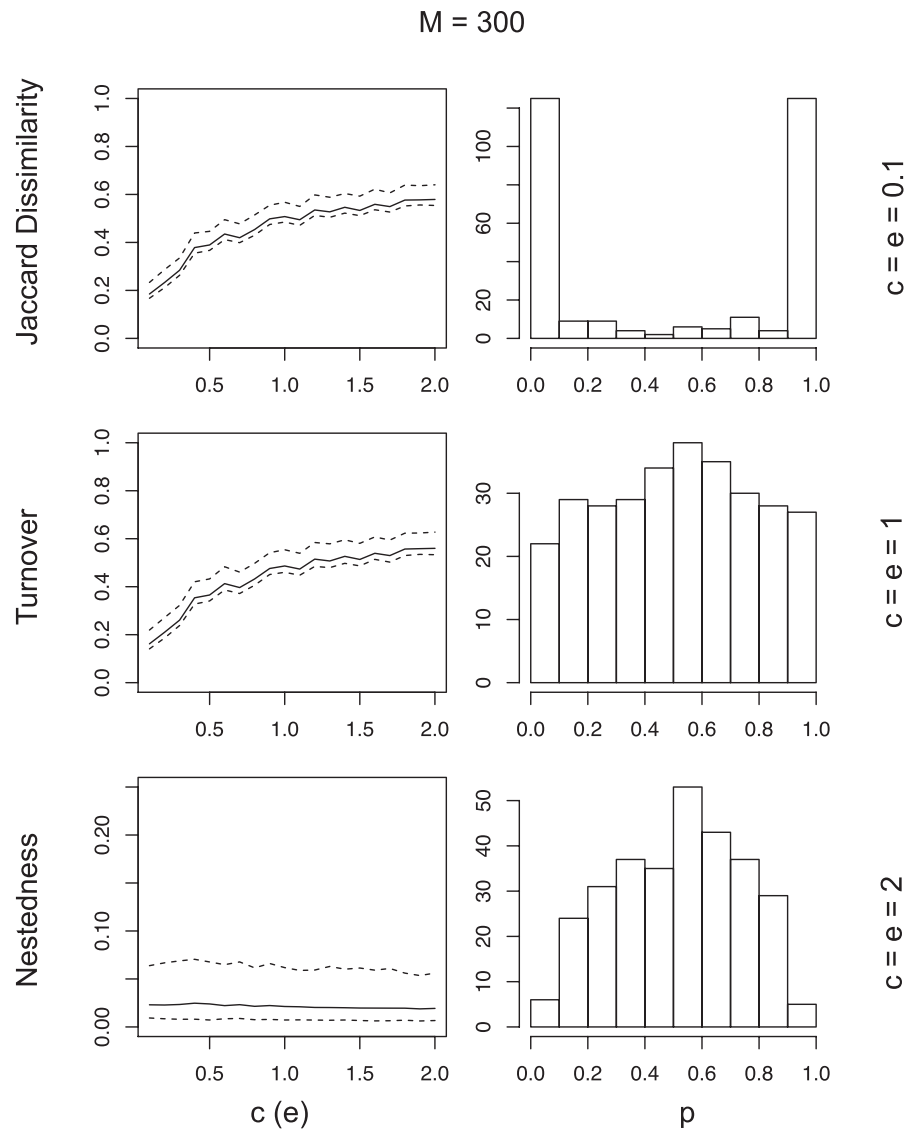


Figure A4: Expected pairwise Jaccard beta diversities of two identical islands with species occurrence probabilities drawn from beta distributions with the same mean but different shapes.