**The Theories and Principles behind Hardware**

**Implementation of Any-Point DFT**

There are several common forms of expression for DFT transformations. There is only a difference of a constant factor between these forms of expression, and the content they express is the same. In communications, the following formula is usually used to represent the *N*-point DFT transformation:

 （1）

Correspondingly, the *N*-point IDFT is represented as:

 （2）

It can be seen from equations (1) and (2) that by taking the complex conjugate of the input to the DFT operation and then taking the complex conjugate of the output result, the IDFT transformation result can be obtained from DFT. Therefore, all discussions related to the DFT can be naturally extended to the IDFT.

In fact, a larger *M*-point DFT can be used to calculate a smaller *N*-point DFT. If the transformation length of the larger *M*-point DFT is selected as , it can be implemented with FFT. The following explains how to obtain the *N*-point DFT result accurately from the *M*-point DFT (*M>N*). In the subsequent analysis, the constant factor in the DFT/IDFT transformation formula will not be considered.

According to the theory of digital signal processing, the discrete Fourier transform (DFT) is actually the result of taking one period of the Fourier series (DFS) of a discrete-time periodic signal. Since DFS is not very convenient due to issues such as sample rate conversion, this invention uses the Fourier transform (DTFT) of discrete-time periodic signals for related descriptions. The spectral lines of DTFT correspond in intensity to the DFS seriesfunction. In the following descriptions, constant factors in the formulas that do not affect the nature of the computational results will not be considered.

Figure 1 is a continuous periodic signal with a period of . The figure shows that the periodic signal is sampled to obtain a periodic discrete signal, with *N* evenly distributed sampling points per period and a sampling period of . Therefore, the following relationship holds:

 （3）

,  （4）



Figure 1

The result of performing DFT on these *N* points is consistent with the *N* spectral response values shown in the DFT window of the DTFT spectrum shown in Figure 2. The DFT result can be conveniently calculated using the following formula (Note: In the subsequent DFT formulas, the constant factor at the front will not be considered):

,  （5）

The frequency spacing between adjacent frequency points is:

 （6）



Figure 2

According to the sampling theorem, the DTFT shown in Figure 2 can be used to recover the continuous-time periodic signal shown in Figure 1 after passing through the low-pass filter shown in Figure 3. The remaining spectral lines after the spectrum of Figure 2 passes through the low-pass filter shown in Figure 3 can be obtained according to the DFT transformation result of equation (5). Therefore, according to the properties of the Fourier transform, the original continuous-time periodic signal can be reconstructed without distortion according to the following formula:

 （7）



Figure 3

Figure 4 shows the reconstructed continuous-time periodic signal of equation (7), which is the same as the signal shown in Figure 1. Now, upsample the signal of equation (7) relative to Figure 1, with *M* samples uniformly taken per period (*M>N*), and the sampling results are also shown in Figure 4.



Figure 4

The time domain samples in IDFT window shown in figure 4 can be obtained through the following expressions:

 （8）

,

 （9）

The *M*-point DFT of  is represented in the DFT window of Figure 5, and the corresponding frequency spacing is shown below:

 （10）

Therefore, there is a one-to-one relationship between the *N* spectrum lines in figure 2 and those in figure 5, and the frequency values of the corresponding frequency indices in the left side points are the same.  can also be obtained by IDFT according to the following formula.

,  （11）



Figure 5

 is the result of the DFT transformation of  (transformation length of *M*). Equations (11) and (9) represent the same set of data. By equating each item and utilizing the periodicity characteristic of the complex exponential function, the following relationship can be obtained (Note: The constant factor in front of the DFT/IDFT formula is omitted in the derivation process, which does not affect the correctness of the result).

 （12）

where,  is the result of *N*-point DFT.

In summary, in theory, it is completely possible to accurately calculate any *N*-point DFT smaller than *M* using an *M*-point DFT. If , then the *M*-point DFT can be implemented with FFT (Fast Fourier Transform). That is to say, after proper design, any transformation length of DFT can be completed through FFT.

According to the above theoretical analysis, using *M*-point DFT to calculate *N*-point DFT requires infinite periodic extension of the *N*-point input data and filtering with an ideal low-pass filter. In actual implementation, an ideal low-pass filter is impossible to realize, and it is also impossible to perform infinite periodic extension of the input data. However, we can use some digital signal processing techniques to make the *N*-point DFT results calculated using *M*-point DFT infinitely approximate to the DFT transformation results calculated according to equations (1) or (5).