

## 1 AMAT

Recall that AMAT stands for Average Memory Access Time. The main formula for it is:

$$\text{AMAT} = \text{Hit Time} + \text{Miss Rate} * \text{Miss Penalty}$$

In a multi-level cache, there are two types of miss rates that we consider for each level.

**Global:** Calculated as the number of accesses that missed at that level divided by the total number of accesses *to the cache system*.

**Local:** Calculated as the number of accesses that missed at that level divided by the total number of accesses *to that cache level*.

- 1.1 • An L2\$, out of 100 total accesses to the cache system, missed 20 times. What is the global miss rate of L2\$?

$$\frac{20}{100} = 20\%$$

- 1.2 If L1\$ had a miss rate of 50%, what is the local miss rate of L2\$?

$\frac{20}{50\% * 100} = \frac{20}{50} = 40\%$ . We know that L2\$ is accessed when L1\$ misses, so if L1\$ misses 50% of the time, that means we access L2\$ 50 times.

Suppose your system consists of:

1. An L1\$ that has a hit time of 2 cycles and has a local miss rate of 20%
2. An L2\$ that has a hit time of 15 cycles and has a global miss rate of 5%
3. Main memory where accesses take 100 cycles

- 1.3 What is the local miss rate of L2\$?

$$\text{L2\$ Local miss rate} = \frac{\text{Global Miss Rate}}{\text{L1\$ Miss Rate}} = \frac{5\%}{20\%} = 0.25 = 25\%$$

- 1.4 What is the AMAT of the system?

$$\text{AMAT} = 2 + 20\% \times 15 + 5\% \times 100 = 10 \text{ cycles (using global miss rates)}$$

$$\text{Alternatively, AMAT} = 2 + 20\% \times (15 + 25\% \times 100) = 10 \text{ cycles}$$

- 1.5 Suppose we want to reduce the AMAT of the system to 8 cycles or lower by adding in a L3\$. If the L3\$ has a local miss rate of 30%, what is the largest hit time that the L3\$ can have?

Let  $H$  = hit time of the cache. Using the AMAT equation, we can write:

$$2 + 20\% * (15 + 25\% * (H + 30\% * 100)) \leq 8$$

Solving for  $H$ , we find that  $H \leq 30$ . So the largest hit time is 30 cycles.

## 2 Coherency

The benefits of multi-threading programming come only after you understand concurrency. Here are two of the most common concurrency issues: **Cache-incoherence**: each hardware thread has its own cache, hence data modified in one thread may not be immediately reflected in the other. This can often be solved by bypassing the cache and writing directly to memory, i.e. using volatile keywords in many languages. **Read-modify-write**: Read-modify-write is a very common pattern in programming. In the context of multi-threading programming, the **interleaving** of R, M, W stages often produces a lot of issues.

In order to solve the problems created by Read-modify-write, we have to rely on the idea of **uninterrupted execution**, also known as atomic execution.

In RISC-V, we have two categories of atomic instructions:

1. **Load-reserve, store-conditional**: allows us to have uninterrupted execution across multiple instructions
2. **Amo.swap**: allows for uninterrupted memory operations within a single instruction

Both of these can be used to achieve atomic primitives. Here are examples for each:

### Test-and-set

```
Start:  addi      t0 x0 1 # Locked = 1
        amoswap.w.aq t1 t0 (a0)
        bne      t1 x0 Start
# If the lock is not free, retry

        ... # Critical section

        amoswap.w.rl x0 x0 (a0) # Release lock
```

### Compare-and-swap

```
# a0 holds address of memory location
# a1 holds expected value
# a2 holds desired value
# a0 holds return value, 0 if successful, !=0 otherwise

cas:
    lr.w t0, (a0) # Load original value.
    bne t0, a1, fail # Doesnt match, so fail.
    sc.w a0, a2, (a0) # Try to update.
    jr ra # Return.

fail:
    li a0, 1 # Set return to failure.
    jr ra # Return.
```

Instruction definitions:

1. **Load-reserve**: Loads the four bytes at  $M[R[rs1]]$ , writes them to  $R[rd]$ , sign-extending the result and registers a reservation on that word in memory.
2. **Store-conditional rd, rs2, (rs1)**: Stores the four bytes in register  $R[rs2]$  to  $M[R[rs1]]$ , provided there exists a load reservation on that memory address. Writes 0 to  $R[rd]$  if the store succeeded, or a nonzero error code otherwise.
3. **Amoswap rd, rs2, (rs1)**: Atomically, puts the sign-extended word located at  $M[R[rs1]]$  into  $R[rd]$  and puts  $R[rs2]$  into  $M[R[rs1]]$ .

Explanations for both methodologies:

1. **Test-and-set:** We have a lock stored at the address specified by `a0`. We utilize `amoswap` to put in 1 and get the old value. If the old value was a 1, we would not have changed the value of the lock and we will realize that someone currently has the lock. If the old value was a 0, we will have just "locked" the lock and can continue with the critical section. When we are done, we put a 0 back into the lock to "unlock" it.
2. **Compare-and-swap:** CAS tries to first reserve the memory and gets the value stored and compares it to the expected value. If the expected value and the value that was stored do not match, the entire process fails and we must restart to update based on the new information. Otherwise, we register a reservation on the memory and try to store the new value. If the exit code is nonzero, something went wrong with the store and we must retry the entire LR/SC process. Otherwise with a zero exit code, we continue into the critical section, then release the lock.

2.1 Why do we need special instructions for these operations? Why can't we use normal load and store for `lr` and `sc`? Why can't we expand `amoswap` to a normal load and store?

For `lr` and `sc`, after `lr`, other threads cannot write to the location marked reserve, hence the value loaded from memory will be unchanged between `lr` and `sc`. For `amoswap`, it does load and store in one single CPU cycle, hence the operation is atomic and uninterruptable.

2.2 Now that we have atomic operations, let's try to experiment with them. Let us try to implement an algorithm that enforces ordered thread execution. This means that if we have four threads, thread 0 goes first, thread 1 goes next, etc. For this problem assume that `a1` holds the location of a piece of memory we have access to for the entire duration of our algorithm. Also, we can assume there exists a label `get_thread_num` that returns the thread's number in `a0`, and that we save `ra` to the stack before line 1 and restore `ra` immediately before line 21. Try to fill in the blanks below. Please use LR/SC for this problem:

```

1      addi t0, x0, 0
2
3      _____ # Setup for the first (0-th) thread
4      ...
5      # Assume we now spawn 4 threads in this code
6      ...
7
8  Check: jal _____ # Get the current thread number
9              _____ # Get the ID of the next thread that should operate
10     _____ # (make sure this can't get interfered with)
11
12     _____
13  Done: addi t0, t0, 1
14
15     _____ # Set which thread is next to run
```

```

16
17     bne -----
18     ...
19     # Assume we now join the 4 threads in this code
20     ...
21     jr ra

1     addi t0, x0, 0
2     sw t0, 0(a1)
3     ...
4     # Assume we now spawn 4 threads in this code
5     ...
6 Check: jal ra, get_thread_num
7         lr.w t0, (a1)
8         bne a0, t0, Check
9 Done:   addi t0, t0, 1
10        sc.w a0, t0, (a1)
11        bne a0, x0, Check
12        ...
13        # Assume we now join the 4 threads in this code
14        ...
15        jr ra

```

### 3 Thread-Level Parallelism

As powerful as data level parallelization is, it can be quite inflexible, as not all applications have data that can be vectorized. Multithreading, or running a single piece of software on multiple hardware threads, is much more powerful and versatile.

OpenMP provides an easy interface for using multithreading within C programs. Some examples of OpenMP directives:

- The `parallel` directive indicates that each thread should run a copy of the code within the block. If a for loop is put within the block, **every** thread will run every iteration of the for loop.

```

#pragma omp parallel
{
    ...
}

```

NOTE: The opening curly brace needs to be on a newline or **else** there will be a compile-time error!

- The `parallel for` directive will split up iterations of a for loop over various threads. Every thread will run **different** iterations of the for loop. The following two code snippets are equivalent.

```

#pragma omp parallel for
for (int i = 0; i < n; i++) {
    ...
}

#pragma omp parallel
{
    #pragma omp for
    for (int i = 0; i < n; i++) { ... }
}

```

There are two functions you can call that may be useful to you:

- **int omp\_get\_thread\_num()** will return the number of the thread executing the code
- **int omp\_get\_num\_threads()** will return the number of total hardware threads executing the code

- 3.1 For each question below, state and justify whether the program is **sometimes incorrect**, **always incorrect**, **slower than serial**, **faster than serial**, or **none of the above**. Assume the default number of threads is greater than 1. Assume no thread will complete before another thread starts executing. Assume `arr` is an `int[]` of length `n`.

(a) `// Set element i of arr to i`  
`#pragma omp parallel`  
`{`  
`for (int i = 0; i < n; i++)`  
`arr[i] = i;`  
`}`

Slower than serial: There is no `for` directive, so every thread executes this loop in its entirety. `n` threads running `n` loops at the same time will actually execute in the same time as 1 thread running 1 loop. Despite the possibility of false sharing, the values should all be correct at the end of the loop. Furthermore, the existence of parallel overhead due to the extra number of threads could slow down the execution time.

(b) `// Set arr to be an array of Fibonacci numbers.`  
`arr[0] = 0;`  
`arr[1] = 1;`  
`#pragma omp parallel for`  
`for (int i = 2; i < n; i++)`  
`arr[i] = arr[i-1] + arr[i - 2];`

Always incorrect (when  $n > 4$ ): Loop has data dependencies, so the calculation of all threads but the first one will depend on data from the previous thread. Because we said “assume no thread will complete before another thread starts executing,” this code will always read incorrect values.

(c) `// Set all elements in arr to 0;`  
`int i;`  
`#pragma omp parallel for`  
`for (i = 0; i < n; i++)`  
`arr[i] = 0;`

Faster than serial: The `for` directive actually automatically makes loop variables (such as the index) private, so this will work properly. The `for` directive splits up the iterations of the loop into continuous chunks for each thread, so there will be no data dependencies or false sharing.

3.2 What potential issue can arise from this code?

```
1 // Decrements element i of arr. n is a multiple of omp_get_num_threads()
2 #pragma omp parallel
3 {
4     int threadCount = omp_get_num_threads();
5     int myThread = omp_get_thread_num();
6     for (int i = 0; i < n; i++) {
7         if (i % threadCount == myThread) arr[i] -= 1;
8     }
9 }
```

False sharing arises because different threads can modify elements located in the same memory block simultaneously. This is a problem because some threads may have incorrect values in their cache block when they modify the value `arr[i]`, invalidating the cache block.