Final Project

London Wagner, Erik Lovece, Carmen Canedo

2021-04-25

1 Introduction

Our final project analyzes the student performance dataset from the UCI Machine Learning Repository, originally gathered by Paulo Cortez from the University of Minho. This dataset measures the final student grade in a Portuguese class based on a variety of predictors. These predictors cover numerous aspects of not only students' academic lives, but also family life predictors such as parental employment, and personal predictors like whether or not they have home internet access and whether or not they are in a romantic relationship.

We seek to answer the question of what predictors have the greatest influence in how a student does in class. Conventional wisdom seems to dictate that high-achieving students have come from particularly favorable academic, filial, and personal environments, and previous studies have confirmed this. Our model, if properly constructed along the best machine learning practices, should corroborate this, although unexpected conclusions may also lie in store.

Our workflow for finding a sufficient model from which we will draw our conclusions is as follows:

1) Run a linear regression model with final grade as the response and all other variables as predictors
2) Use best subset, forward step, and backwards step to select variables for a reduced model
3) Use ridge, lasso, principal component regression, and partial least squares regression to conduct further dimension reduction
4) Use cross validation methods to determine which model predicts the final grade with the greatest accuracy
5) Make more definitive determinations based on the chosen model.

2 Loading Data

```r
student_por <- read_csv2("data/student-por.csv")

student_por
```

```
## # A tibble: 649 x 33
##    school sex     age address famsize Pstatus  Medu  Fedu Mjob  Fjob  reason
##    <chr>  <chr> <dbl> <chr>   <chr>   <chr>   <dbl> <dbl> <chr> <chr> <chr>
##  1 GP     F        18 U       GT3     A           4     4 at_h~ teac~ course
##  2 GP     F        17 U       GT3     T           1     1 at_h~ other course
##  3 GP     F        15 U       LE3     T           1     1 at_h~ other other
##  4 GP     F        15 U       GT3     T           4     2 heal~ serv~ home
##  5 GP     F        16 U       GT3     T           3     3 other other home
##  6 GP     M        16 U       LE3     T           4     3 serv~ other reput~
##  7 GP     M        16 U       LE3     T           2     2 other other home
##  8 GP     F        17 U       GT3     A           4     4 other teac~ home
##  9 GP     M        15 U       LE3     A           3     2 serv~ other home
## 10 GP     M        15 U       GT3     T           3     4 other other home
## # ... with 639 more rows, and 22 more variables: guardian <chr>,
## #   traveltime <dbl>, studytime <dbl>, failures <dbl>, schoolsup <chr>,
## #   famsup <chr>, paid <chr>, activities <chr>, nursery <chr>, higher <chr>,
## #   internet <chr>, romantic <chr>, famrel <dbl>, freetime <dbl>, goout <dbl>,
## #   Dalc <dbl>, Walc <dbl>, health <dbl>, absences <dbl>, G1 <dbl>, G2 <dbl>,
## #   G3 <dbl>
```

The student attributes and grades forming the predictors and response, quoted verbatim from a text file provided with the dataset, are as follows:

1 school - student's school (binary: "GP" - Gabriel Pereira or "MS" - Mousinho da Silveira)

2 sex - student's sex (binary: "F" - female or "M" - male)

3 age - student's age (numeric: from 15 to 22)

4 address - student's home address type (binary: "U" - urban or "R" - rural)

5 famsize - family size (binary: "LE3" - less or equal to 3 or "GT3" - greater than 3)

6 Pstatus - parent's cohabitation status (binary: "T" - living together or "A" - apart)

7 Medu - mother's education (numeric: 0 - none, 1 - primary education (4th grade), 2 – 5th to 9th grade, 3 – secondary education or 4 – higher education)

8 Fedu - father's education (numeric: 0 - none, 1 - primary education (4th grade), 2 – 5th to 9th grade, 3 – secondary education or 4 – higher education)

9 Mjob - mother's job (nominal: "teacher", "health" care related, civil "services" (e.g. administrative or police), "at_home" or "other")

10 Fjob - father's job (nominal: "teacher", "health" care related, civil "services" (e.g. administrative or police), "at_home" or "other")

11 reason - reason to choose this school (nominal: close to "home", school "reputation", "course" preference or "other")

12 guardian - student's guardian (nominal: "mother", "father" or "other")

13 traveltime - home to school travel time (numeric: 1 - <15 min., 2 - 15 to 30 min., 3 - 30 min. to 1 hour, or 4 - >1 hour)

14 studytime - weekly study time (numeric: 1 - <2 hours, 2 - 2 to 5 hours, 3 - 5 to 10 hours, or 4 - >10 hours)

15 failures - number of past class failures (numeric: n if 1<=n<3, else 4)

16 schoolsup - extra educational support (binary: yes or no)

17 famsup - family educational support (binary: yes or no)

18 paid - extra paid classes within the course subject (Math or Portuguese) (binary: yes or no)

19 activities - extra-curricular activities (binary: yes or no)

20 nursery - attended nursery school (binary: yes or no)

21 higher - wants to take higher education (binary: yes or no)

22 internet - Internet access at home (binary: yes or no)

23 romantic - with a romantic relationship (binary: yes or no)

24 famrel - quality of family relationships (numeric: from 1 - very bad to 5 - excellent)

25 freetime - free time after school (numeric: from 1 - very low to 5 - very high)

26 goout - going out with friends (numeric: from 1 - very low to 5 - very high)

27 Dalc - workday alcohol consumption (numeric: from 1 - very low to 5 - very high)

28 Walc - weekend alcohol consumption (numeric: from 1 - very low to 5 - very high)

29 health - current health status (numeric: from 1 - very bad to 5 - very good)

30 absences - number of school absences (numeric: from 0 to 93)

31 G1 - first period grade (numeric: from 0 to 20)

31 G2 - second period grade (numeric: from 0 to 20)

32 G3 - final grade (numeric: from 0 to 20, output target)

For all of our models, `G3` is our response and every other variable serves as a predictor (pending elimination).

3 Data Cleaning

When creating our models, some variables need be converting to factors in order to be properly interpreted by the `lm` function. We have converted the necessary variables, so now many of them (with special emphasis on binary variables) are now of type factor.

```
student_por
```

```
## # A tibble: 649 x 33
##    school sex     age address famsize Pstatus  Medu  Fedu Mjob  Fjob  reason
##    <fct>  <fct> <dbl> <fct>   <fct>   <fct>   <dbl> <dbl> <chr> <chr> <fct>
##  1 GP     F        18 U       GT3     A           4     4 at_h~ teac~ course
##  2 GP     F        17 U       GT3     T           1     1 at_h~ other course
##  3 GP     F        15 U       LE3     T           1     1 at_h~ other other
##  4 GP     F        15 U       GT3     T           4     2 heal~ serv~ home
##  5 GP     F        16 U       GT3     T           3     3 other other home
##  6 GP     M        16 U       LE3     T           4     3 serv~ other reput~
##  7 GP     M        16 U       LE3     T           2     2 other other home
##  8 GP     F        17 U       GT3     A           4     4 other teac~ home
##  9 GP     M        15 U       LE3     A           3     2 serv~ other home
## 10 GP     M        15 U       GT3     T           3     4 other other home
## # ... with 639 more rows, and 22 more variables: guardian <chr>,
## #   traveltime <dbl>, studytime <dbl>, failures <dbl>, schoolsup <fct>,
## #   famsup <fct>, paid <fct>, activities <fct>, nursery <fct>, higher <fct>,
## #   internet <fct>, romantic <fct>, famrel <dbl>, freetime <dbl>, goout <dbl>,
## #   Dalc <dbl>, Walc <dbl>, health <dbl>, absences <dbl>, G1 <dbl>, G2 <dbl>,
## #   G3 <dbl>
```

4 Exploratory Data Analysis & Checking Assumptions

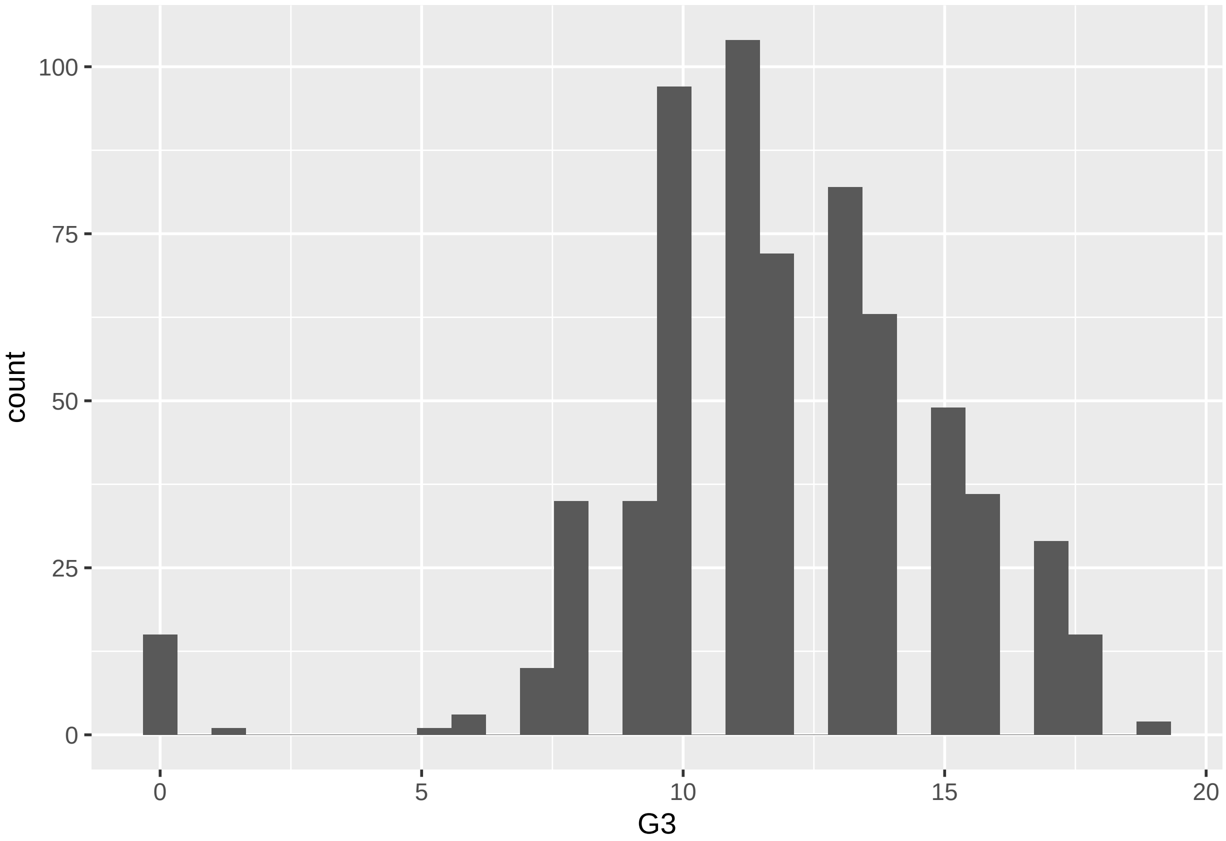
Before we begin our analysis, we wish to explore the distribution of the data and confirm it follows the typical assumptions of linear regression. Although we recognize that these assumptions are not as necessary in machine learning, they can be helpful when determining what kind of model we should use to predict our response variable, G3.

```r
summary(student_por)
```

```
##  school   sex          age        address famsize   Pstatus      Medu      
##  GP:423   F:383   Min.   :15.00   R:197   GT3:457   A: 80   Min.   :0.000  
##  MS:226   M:266   1st Qu.:16.00   U:452   LE3:192   T:569   1st Qu.:2.000  
##                   Median :17.00                             Median :2.000  
##                   Mean   :16.74                             Mean   :2.515  
##                   3rd Qu.:18.00                             3rd Qu.:4.000  
##                   Max.   :22.00                             Max.   :4.000  
##       Fedu          Mjob               Fjob                  reason   
##  Min.   :0.000   Length:649         Length:649         course    :285  
##  1st Qu.:1.000   Class :character   Class :character   home      :149  
##  Median :2.000   Mode  :character   Mode  :character   other     : 72  
##  Mean   :2.307                                         reputation:143  
##  3rd Qu.:3.000                                                         
##  Max.   :4.000                                                         
##    guardian           traveltime      studytime        failures      schoolsup
##  Length:649         Min.   :1.000   Min.   :1.000   Min.   :0.0000   no :581  
##  Class :character   1st Qu.:1.000   1st Qu.:1.000   1st Qu.:0.0000   yes: 68  
##  Mode  :character   Median :1.000   Median :2.000   Median :0.0000            
##                     Mean   :1.569   Mean   :1.931   Mean   :0.2219            
##                     3rd Qu.:2.000   3rd Qu.:2.000   3rd Qu.:0.0000            
##                     Max.   :4.000   Max.   :4.000   Max.   :3.0000            
##  famsup     paid     activities nursery   higher    internet  romantic 
##  no :251   no :610   no :334    no :128   no : 69   no :151   no :410  
##  yes:398   yes: 39   yes:315    yes:521   yes:580   yes:498   yes:239  
##                                                                        
##                                                                        
##                                                                        
##                                                                        
##      famrel         freetime        goout            Dalc            Walc     
##  Min.   :1.000   Min.   :1.00   Min.   :1.000   Min.   :1.000   Min.   :1.00  
##  1st Qu.:4.000   1st Qu.:3.00   1st Qu.:2.000   1st Qu.:1.000   1st Qu.:1.00  
##  Median :4.000   Median :3.00   Median :3.000   Median :1.000   Median :2.00  
##  Mean   :3.931   Mean   :3.18   Mean   :3.185   Mean   :1.502   Mean   :2.28  
##  3rd Qu.:5.000   3rd Qu.:4.00   3rd Qu.:4.000   3rd Qu.:2.000   3rd Qu.:3.00  
##  Max.   :5.000   Max.   :5.00   Max.   :5.000   Max.   :5.000   Max.   :5.00  
##      health         absences            G1              G2       
##  Min.   :1.000   Min.   : 0.000   Min.   : 0.0   Min.   : 0.00  
##  1st Qu.:2.000   1st Qu.: 0.000   1st Qu.:10.0   1st Qu.:10.00  
##  Median :4.000   Median : 2.000   Median :11.0   Median :11.00  
##  Mean   :3.536   Mean   : 3.659   Mean   :11.4   Mean   :11.57  
##  3rd Qu.:5.000   3rd Qu.: 6.000   3rd Qu.:13.0   3rd Qu.:13.00  
##  Max.   :5.000   Max.   :32.000   Max.   :19.0   Max.   :19.00  
##        G3       
##  Min.   : 0.00  
##  1st Qu.:10.00  
##  Median :12.00  
##  Mean   :11.91  
##  3rd Qu.:14.00  
##  Max.   :19.00  
```

Most noteworthy is how G1, G2, and G3's 1^{st} quartile of 10 is quite close to the median and mean on the 0-20 scale, yet the minimum in all three is zero. There is a significant gap between the lower and minimum behavior, and these lowest-end students may influence our subsequent analyses in some way.

```
## ‘stat_bin()‘ using ‘bins = 30‘. Pick better value with ‘binwidth‘.
```



```
lowest <- filter(student_por, G3 == 0)
num_lowest <- nrow(lowest)
num_lowest / nrow(student_por)
```

```
## [1] 0.02311248
```

```
lowest
```

```
## # A tibble: 15 x 33
##    school sex     age address famsize Pstatus  Medu  Fedu Mjob  Fjob  reason
##    <fct>  <fct> <dbl> <fct>   <fct>   <fct>   <dbl> <dbl> <chr> <chr> <fct> 
##  1 GP     M        18 U       LE3     T           1     1 other other course
##  2 MS     M        16 U       GT3     T           1     1 at_h~ serv~ home  
##  3 MS     M        16 R       GT3     T           2     1 other serv~ reput~
##  4 MS     M        17 U       GT3     T           2     2 other other course
##  5 MS     M        18 R       GT3     T           3     2 serv~ other course
##  6 MS     F        18 R       GT3     T           2     2 other other other 
##  7 MS     F        17 U       GT3     T           4     2 teac~ serv~ home  
##   8 MS     F        18 R       GT3     T           2     2 at_h~ other course
##   9 MS     F        18 R       LE3     A           4     2 teac~ other reput~
##  10 MS     F        19 U       GT3     T           1     1 at_h~ serv~ other
##  11 MS     F        19 R       GT3     A           1     1 at_h~ at_h~ course
##  12 MS     F        18 R       GT3     T           4     4 other teac~ other
##  13 MS     M        18 R       GT3     T           2     1 other other other
##  14 MS     M        19 R       GT3     T           1     1 other serv~ other
##  15 MS     M        18 R       GT3     T           4     2 other other home
## # ... with 22 more variables: guardian <chr>, traveltime <dbl>,
## #   studytime <dbl>, failures <dbl>, schoolsup <fct>, famsup <fct>, paid <fct>,
## #   activities <fct>, nursery <fct>, higher <fct>, internet <fct>,
## #   romantic <fct>, famrel <dbl>, freetime <dbl>, goout <dbl>, Dalc <dbl>,
## #   Walc <dbl>, health <dbl>, absences <dbl>, G1 <dbl>, G2 <dbl>, G3 <dbl>
```

As expected, the lowest-performing students left-skew the distribution of the final scores. However, it is not a couple isolated cases, but 2.3% of the student population in this class. Although there are some commonalities among these students (most of them attended nursery school but did not pay for extra educational support in this subject field, and they all had zero absences), we will later find that most of these factors in common are unimportant in our final models. For now, we run the model taking every student into account.

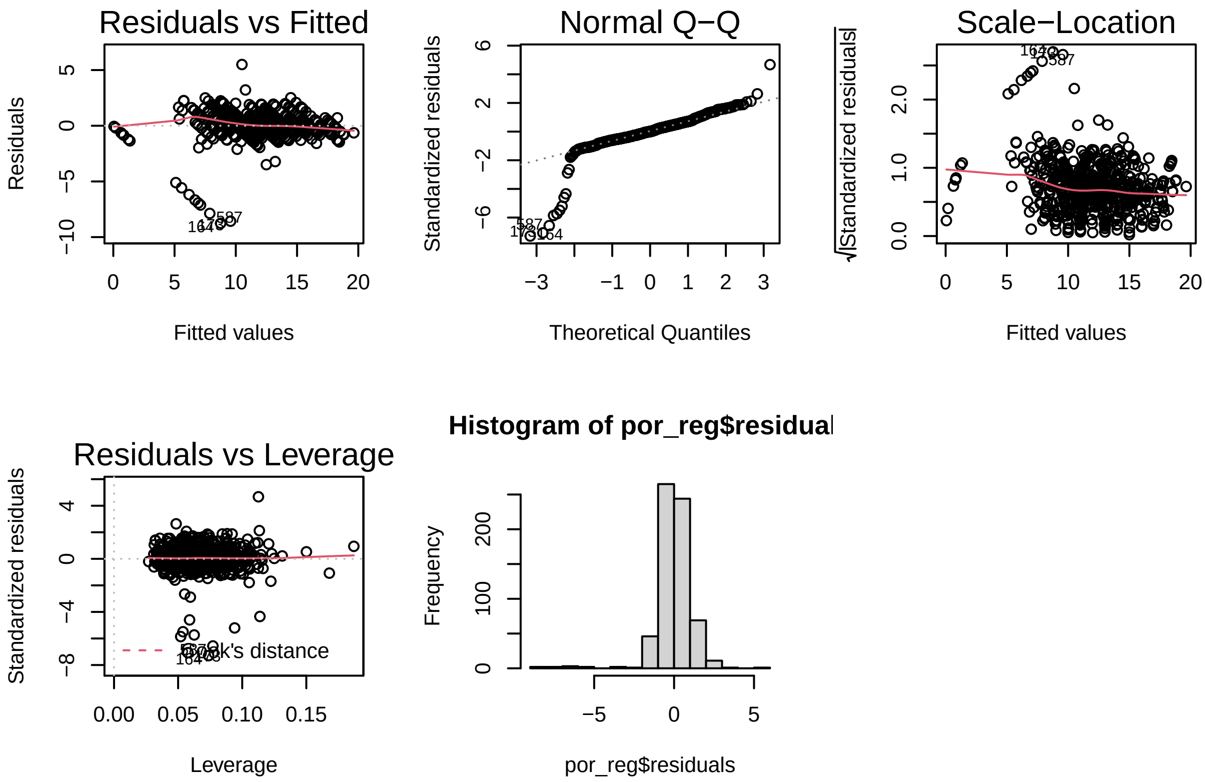
4.1 Running `lm`

```
por_reg <- lm(G3 ~ ., data = student_por)
summary(por_reg)
```

```
## 
## Call:
## lm(formula = G3 ~ ., data = student_por)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -8.7618 -0.5148  0.0038  0.6047  5.4973 
## 
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)    
## (Intercept)        0.63823    0.96361   0.662 0.508011    
## schoolMS          -0.19797    0.12783  -1.549 0.121992    
## sexM              -0.12258    0.11778  -1.041 0.298423    
## age                0.02869    0.04835   0.593 0.553208    
## addressU           0.11446    0.12277   0.932 0.351565    
## famsizeLE3         0.01560    0.11505   0.136 0.892197    
## PstatusT          -0.09746    0.16256  -0.600 0.549055    
## Medu              -0.09170    0.07097  -1.292 0.196799    
## Fedu               0.04962    0.06461   0.768 0.442773    
## Mjobhealth         0.26583    0.25225   1.054 0.292379    
## Mjobother         -0.09351    0.14208  -0.658 0.510720    
## Mjobservices       0.17255    0.17510   0.985 0.324808    
## Mjobteacher        0.22115    0.23558   0.939 0.348232    
## Fjobhealth        -0.44420    0.35256  -1.260 0.208189    
## Fjobother         -0.33805    0.21391  -1.580 0.114544    
## Fjobservices      -0.47121    0.22477  -2.096 0.036457 *  
## Fjobteacher      -0.54368    0.31611  -1.720 0.085958 .  
## reasonhome       -0.07885    0.13366  -0.590 0.555479    
## reasonother      -0.36174    0.17236  -2.099 0.036251 *  
## reasonreputation -0.16934    0.13990  -1.210 0.226584    
## guardianmother   -0.02513    0.12461  -0.202 0.840252    
## guardianother     0.21732    0.24922   0.872 0.383539    
## traveltime        0.13859    0.07459   1.858 0.063667 .  
## studytime         0.04965    0.06620   0.750 0.453569    
## failures         -0.25495    0.09900  -2.575 0.010254 *  
## schoolsupyes     -0.18419    0.17319  -1.064 0.287969    
## famsupyes         0.09456    0.10701   0.884 0.377230    
## paidyes          -0.19166    0.21664  -0.885 0.376663    
## activitiesyes     0.01208    0.10482   0.115 0.908275    
## nurseryyes       -0.09562    0.12722  -0.752 0.452553    
## higheryes         0.20749    0.18261   1.136 0.256285    
## internetyes       0.08517    0.12955   0.657 0.511152    
## romanticyes      -0.04209    0.10786  -0.390 0.696483    
## famrel           -0.01597    0.05471  -0.292 0.770469    
## freetime         -0.05002    0.05267  -0.950 0.342694    
## goout            -0.01889    0.05041  -0.375 0.708033    
## Dalc             -0.05194    0.07185  -0.723 0.469977    
## Walc             -0.01693    0.05553  -0.305 0.760521    
## health           -0.05522    0.03633  -1.520 0.129064    
## absences          0.01359    0.01173   1.158 0.247198    
## G1                0.12933    0.03762   3.438 0.000626 ***
## G2                0.87037    0.03495  24.906  < 2e-16 ***
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
## 
## Residual standard error: 1.249 on 607 degrees of freedom
## Multiple R-squared:   0.86,  Adjusted R-squared:  0.8506 
## F-statistic: 90.95 on 41 and 607 DF,  p-value: < 2.2e-16
```

Most variables in our many-variable linear model do not seem useful to us, prompting the use of best subset and dimension-reducing methods. To showcase other model deficiencies, we produce several plots of the residual distribution:

```
## 
##  Shapiro-Wilk normality test
## 
## data:  por_reg$residuals
## W = 0.78559, p-value < 2.2e-16
```



We see that the residuals are not randomly distributed per the residual plot and the QQ-plot yields residuals clearly left-skewed from normality. Shapiro-Wilk test run on the residuals gives us the utmost confidence that they do not have normal distribution. Future analysis may seek to analyze the possible high-leverage points and potential outliers as shown in the residuals vs. leverage plot, but this is outside the scope of this project and there are better-fitting methods to consider.

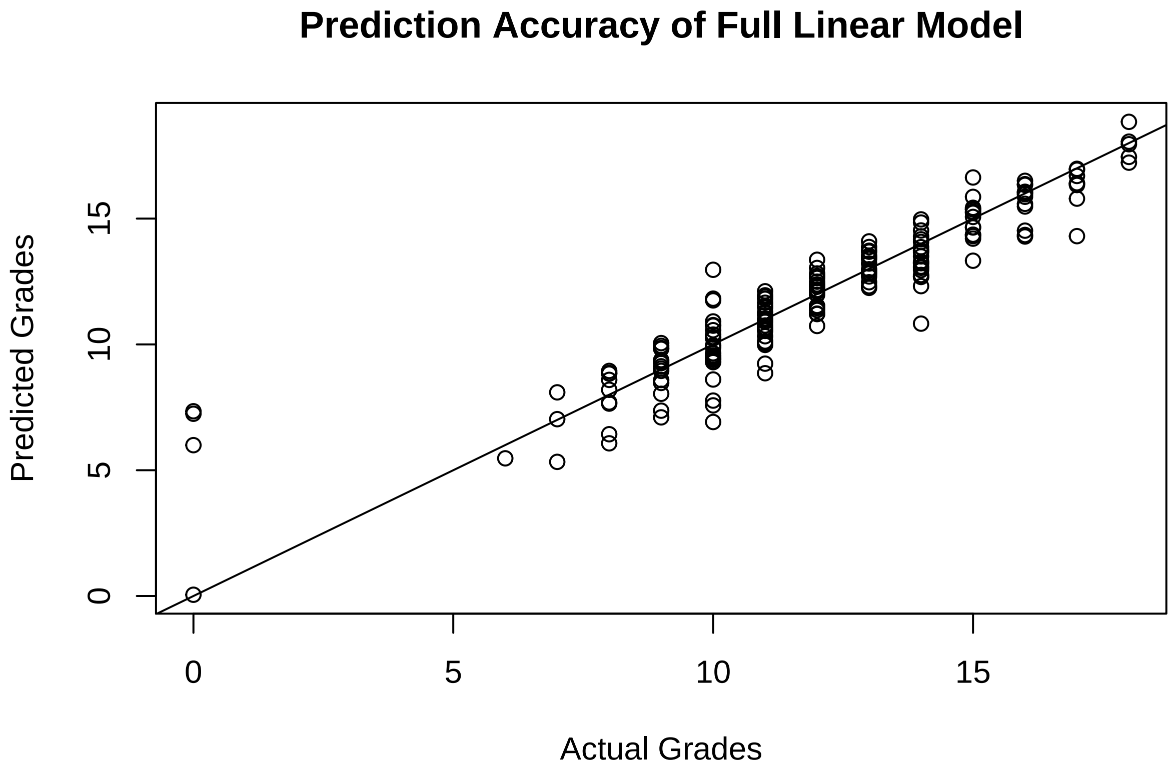
4.2 Finding Linear Regression MSE

Even though our initial model does not suit our needs, we can still calculate the prediction MSE as a baseline to compare future models to, anticipating that subsequent models will be more accurate.

```
set.seed(1)
n <- nrow(student_por)
Z <- sample(n, .7*n)

reg.fit <- lm(G3 ~ ., data = student_por, subset = Z)
```

```
g3_predicted <- predict(reg.fit, student_por)
```



We can see on the left side of the graph, when the actual G3 (final grade) is 0, the model tends to overestimate.

```
mse_lm <- mean((student_por$G3 - g3_predicted)[-Z]^2)
mse_lm
```

```
## [1] 1.549808
```

Despite its flaws, the linear model has a rather low MSE (1.550) for scores strictly on a 0-20 scale. When looking for a better model, we will use methods in an attempt to reduce the number of variables or the number of dimensions in the dataset. Therefore we will be using best subset, forward and backwards step, LASSO, Ridge, PCR, and PLS. First, we will start with Best Subset.

4.3 Best Subset

When initially running our best subset, we decided to cap the number of variables at 15. This was done in order to ensure the model has enough variables to predict G3 accurately without including unnecessary ones that would complicate the model.

```
# Takes a while to run

subsets <- regsubsets(G3 ~ ., data = student_por, nvmax = 15)
```

```
summary(subsets)$adjr2
```

```
##  [1] 0.8434889 0.8472902 0.8489562 0.8501271 0.8507762 0.8513288 0.8518147
##  [8] 0.8522527 0.8526228 0.8528667 0.8531441 0.8533129 0.8534618 0.8534808
## [15] 0.8535407
```

```r
summary(subsets)$cp
```

```
##  [1] 32.5841845 17.1053566 10.8932613  6.8369620  5.0413803  3.6686368
##  [7]  2.5898385  1.7227267  1.1515489  1.1240570  0.9573861  1.2563559
## [13]  1.6420508  2.5809380  3.3465888
```

```r
summary(subsets)$bic
```

```
##  [1] -1191.705 -1202.191 -1203.840 -1203.422 -1200.772 -1197.715 -1194.376
##  [8] -1190.835 -1187.002 -1182.618 -1178.385 -1173.676 -1168.881 -1163.512
## [15] -1158.327
```

```r
which.max(summary(subsets)$adjr2)
```

```
## [1] 15
```

```r
which.min(abs(summary(subsets)$cp - 1:15))
```

```
## [1] 5
```

```r
which.min(summary(subsets)$bic)
```

```
## [1] 3
```

When looking at the adjr2 value, we can tell that they are all relatively the same, ranging from 0.843 to 0.854. Because adding in more variables did not make a significant difference on this value, we are not going to propose a candidate model based on the highest adjr2. Instead, we will move forward with two candidate models, one with five variables as suggested by the Mallow's Cp and one with three variables as suggested by the BIC.

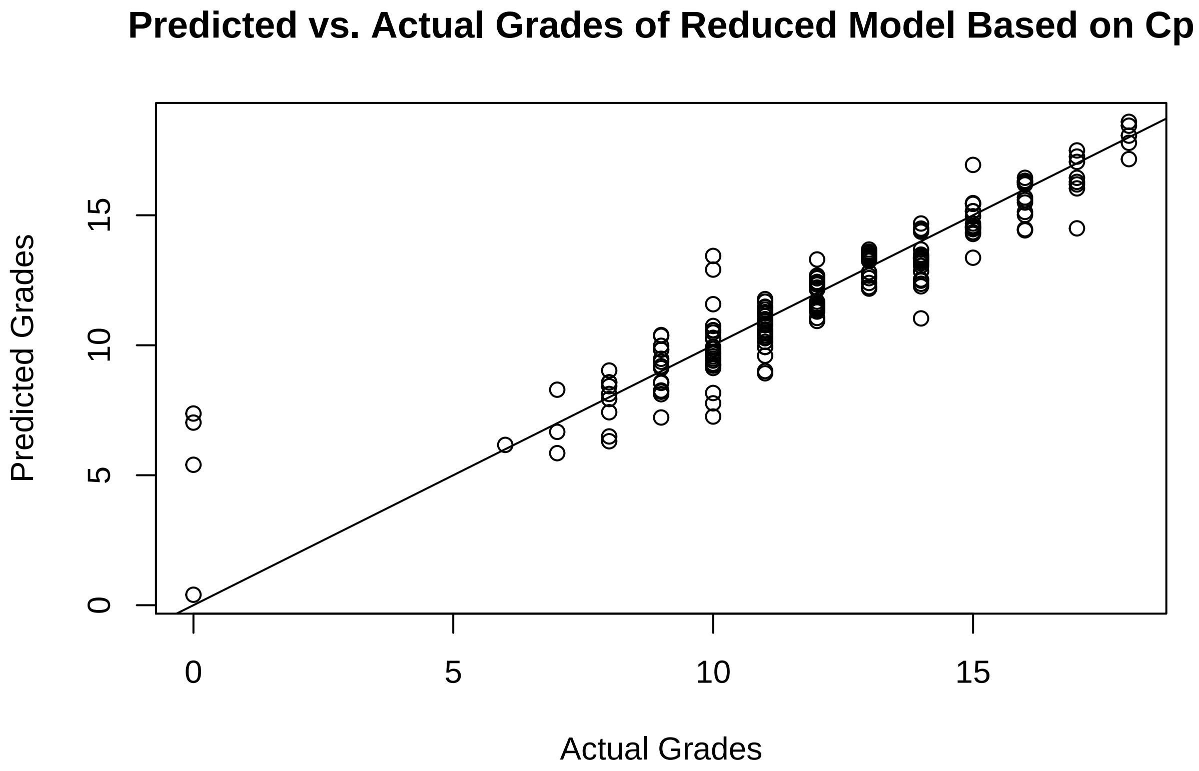
4.4 Model Based on Mallow's Cp

Based on the output from Best Subset, we are going to run a model that includes the variables:

- sex
- reason
- failures
- G1
- G2

```r
reg.bestsubCP <- lm(G3 ~ sex + reason + failures + G1 + G2, data = student_por, subset = Z)

g3_pred_bestsubCP <- predict(reg.bestsubCP, student_por)
```



Similar to the Predicted vs. Actual Grade plot for the initial linear regression, the Mallow's Cp model (with five variables) overestimates the final grade (G3) when the actual grade is 0. These data points are potential outliers, but removing them is outside of the scope of this project. Otherwise, the model fits the rest of the data well.

```r
mse_cp <- mean((student_por$G3 - g3_pred_bestsubCP)[-Z] ^ 2)
mse_cp
```

```
## [1] 1.474193
```

The MSE for the Best Subset Model based on Mallow's C is 1.474.

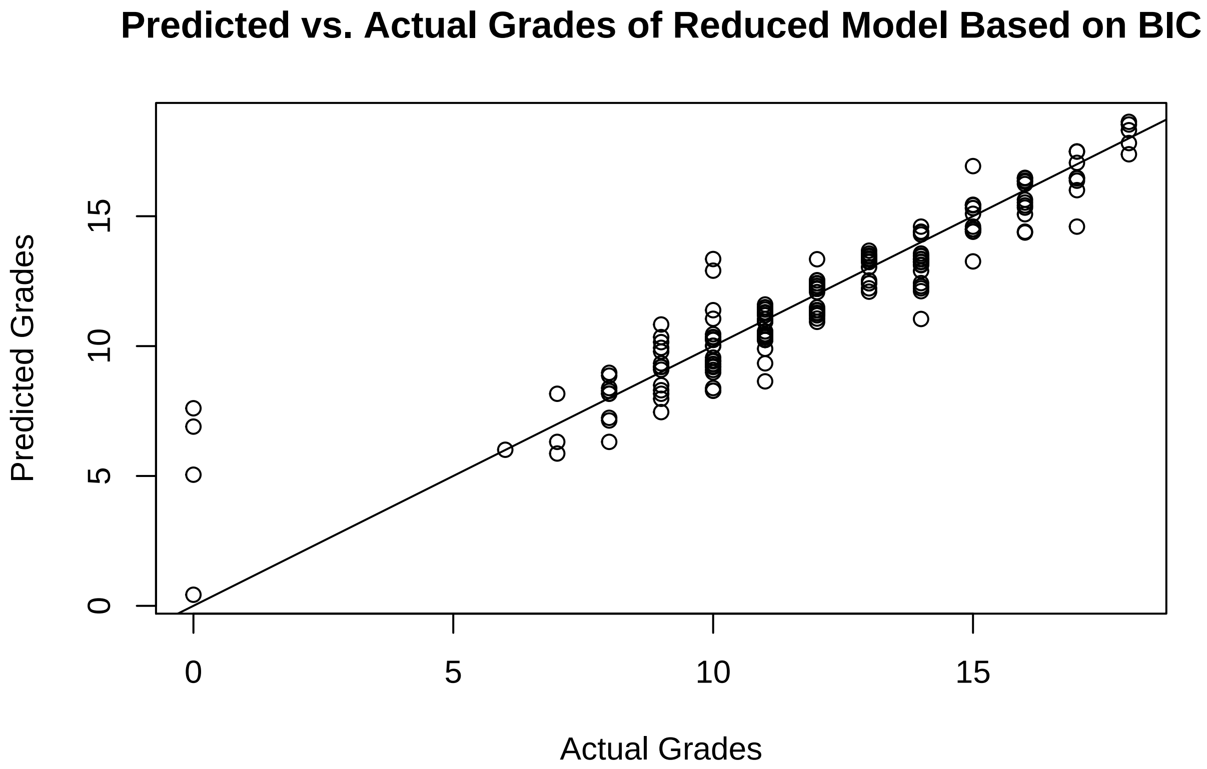
4.5 Model Based on BIC

Based on the output from Best Subset, we are going to run a model that includes the variables:

- reason
- G1
- G2

```r
reg.bestsubBIC <- lm(G3 ~ reason + G1 + G2, data = student_por, subset = Z)

g3_pred_bestsubBIC <- predict(reg.bestsubBIC, student_por)
```



Again, this plot is very similar to those for the previous models.

```
mse_bic <- mean((student_por$G3 - g3_pred_bestsubBIC)[-Z] ^ 2)
mse_bic
```

```
## [1] 1.427079
```

The MSE for the Best Subset Model based on BIC is 1.427.

4.6 Step Functions

Now we are going to be looking at forward and backward step functions.

```
summary(forward)
```

```
##
## Call:
## lm(formula = G3 ~ G2 + G1 + failures + reason + absences + sex +
##     school + traveltime + health, data = student_por)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.0833 -0.5178 -0.0053  0.6398  5.2097
##
```

```
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       0.44063    0.34169   1.290 0.197678
## G2                0.87996    0.03379  26.042  < 2e-16 ***
## G1                0.13706    0.03615   3.792 0.000164 ***
## failures         -0.24049    0.09074  -2.650 0.008244 **
## reasonhome       -0.09222    0.13010  -0.709 0.478659
## reasonother      -0.44994    0.16627  -2.706 0.006990 **
## reasonreputation -0.16537    0.13290  -1.244 0.213816
## absences          0.01623    0.01100   1.476 0.140522
## sexM             -0.20022    0.10191  -1.965 0.049894 *
## schoolMS         -0.22981    0.11621  -1.977 0.048419 *
## traveltime        0.11228    0.06839   1.642 0.101138
## health           -0.05394    0.03469  -1.555 0.120451
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 1.242 on 637 degrees of freedom
## Multiple R-squared:  0.8547, Adjusted R-squared:  0.8522
## F-statistic: 340.7 on 11 and 637 DF,  p-value: < 2.2e-16
```

```r
summary(backward)
```

```
##
## Call:
## lm(formula = G3 ~ school + sex + reason + traveltime + failures +
##     health + absences + G1 + G2, data = student_por)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.0833 -0.5178 -0.0053  0.6398  5.2097
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       0.44063    0.34169   1.290 0.197678
## schoolMS         -0.22981    0.11621  -1.977 0.048419 *
## sexM             -0.20022    0.10191  -1.965 0.049894 *
## reasonhome       -0.09222    0.13010  -0.709 0.478659
## reasonother      -0.44994    0.16627  -2.706 0.006990 **
## reasonreputation -0.16537    0.13290  -1.244 0.213816
## traveltime        0.11228    0.06839   1.642 0.101138
## failures         -0.24049    0.09074  -2.650 0.008244 **
## health           -0.05394    0.03469  -1.555 0.120451
## absences          0.01623    0.01100   1.476 0.140522
## G1                0.13706    0.03615   3.792 0.000164 ***
## G2                0.87996    0.03379  26.042  < 2e-16 ***
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 1.242 on 637 degrees of freedom
## Multiple R-squared:  0.8547, Adjusted R-squared:  0.8522
## F-statistic: 340.7 on 11 and 637 DF,  p-value: < 2.2e-16
```

Forward and backward step functions yield the exact same model; proceeding with forward step-generated model.

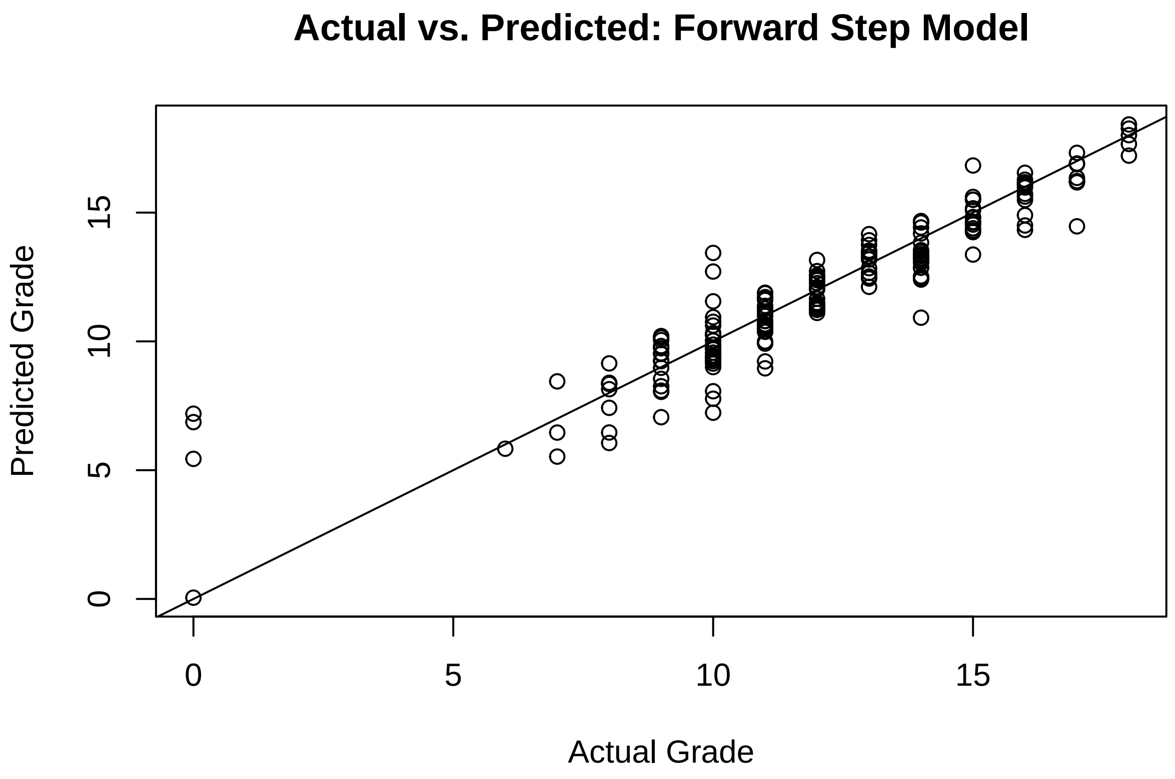
4.7 Set validation

Based on the output from Forward Step, we are going to run a model that includes the variables:

- failures
- reason
- absences
- sex
- school
- traveltime
- health
- G1
- G2

```
reg.forward <- lm(G3 ~ G2 + G1 + failures + reason +
                    absences + sex + school + traveltime +
                    health, data = student_por, subset = Z)

g3_pred_forward <- predict(reg.forward, student_por)
```



```
mse_valSet <- mean((student_por$G3 - g3_pred_forward)[-Z] ^ 2)
mse_valSet
```

```
## [1] 1.458564
```

The MSE for the Forward STep Model is 1.459.

4.8 Ridge Regression & LASSO Preparation

```
G3_test <- student_por$G3[-Z]

# Creating model matrix for rr and lasso calculations
x_col <- model.matrix(G3 ~ ., student_por)[, -1]
```

4.9 Ridge Regression

Now we are going to look at Ridge Regression.

```
set.seed(1)
cv.out1 <- cv.glmnet(x_col, student_por$G3, alpha = 0) # alpha = 0 ---> Ridge regression
predict(cv.out1, s = cv.out1$lambda.min, type = "coefficients")
```

```
## 42 x 1 sparse Matrix of class "dgCMatrix"
##                             1
## (Intercept)       0.647281475
## schoolMS         -0.203906036
## sexM             -0.140157325
## age               0.052338494
## addressU          0.134471281
## famsizeLE3        0.031162924
## PstatusT         -0.065759455
## Medu             -0.052585350
## Fedu              0.042315840
## Mjobhealth        0.214144623
## Mjobother        -0.125812519
## Mjobservices      0.107348800
## Mjobteacher       0.126800272
## Fjobhealth       -0.238572263
## Fjobother        -0.156282858
## Fjobservices     -0.300920468
## Fjobteacher      -0.249881588
## reasonhome       -0.066205753
## reasonother      -0.362017744
## reasonreputation -0.112276025
## guardianmother   -0.032801219
## guardianother     0.212481561
## traveltime        0.111686852
## studytime         0.060781281
## failures         -0.316944745
## schoolsupyes     -0.201688220
## famsupyes         0.087520158
## paidyes          -0.153298259
## activitiesyes     0.018965743
## nurseryyes       -0.084879671
## higheryes         0.280580244
## internetyes       0.099020055
## romanticyes      -0.087129209
## famrel            0.008539498
## freetime         -0.053552694
## goout            -0.025522281
## Dalc             -0.056693604
## Walc             -0.025883700
## health           -0.065912944
## absences          0.011871609
## G1                0.258399918
## G2                0.683417827
```

```
rr.mod <- glmnet(x_col[Z, ], student_por$G3[Z], alpha = 0, lambda = cv.out1$lambda.min)
rr.pred <- predict(rr.mod, s = cv.out1$lambda.min, newx = x_col[-Z, ])

mse_rr <- mean((rr.pred - student_por$G3[-Z])^2)
mse_rr
```

```
## [1] 1.596999
```

The MSE for Ridge Regression is 1.597.

$\lambda = .30$

4.10 LASSO

Next we are going to look at LASSO.

```
set.seed(1)
cv.out2 <- cv.glmnet(x_col, student_por$G3, alpha = 1)
predict(cv.out2, s = cv.out2$lambda.min, type = "coefficients")
```

```
## 42 x 1 sparse Matrix of class "dgCMatrix"
##                            1
## (Intercept)       0.46985582
## schoolMS         -0.03190401
## sexM             -0.01841156
## age               .
## addressU          .
## famsizeLE3        .
## PstatusT          .
## Medu              .
## Fedu              .
## Mjobhealth        .
## Mjobother         .
## Mjobservices      .
## Mjobteacher       .
## Fjobhealth        .
## Fjobother         .
## Fjobservices      .
## Fjobteacher       .
## reasonhome        .
## reasonother      -0.14557639
## reasonreputation  .
## guardianmother    .
## guardianother     .
## traveltime        .
## studytime         .
## failures         -0.09120067
## schoolsupyes      .
## famsupyes         .
## paidyes           .
## activitiesyes     .
## nurseryyes        .
## higheryes         .
## internetyes       .
## romanticyes       .
## famrel            .
## freetime          .
## goout             .
## Dalc              .
## Walc              .
## health            .
## absences          .
## G1                0.12252007
## G2                0.87247067
```

$\lambda = .10$

```
lasso.mod <- glmnet(x_col[Z, ], student_por$G3[Z], alpha = 1, lambda = cv.out2$lambda.min)
lasso.pred <- predict(lasso.mod, s = cv.out2$lambda.min, newx = x_col[-Z, ])

mse_lasso <- mean((lasso.pred - student_por$G3[-Z])^2)
mse_lasso
```

```
## [1] 1.527916
```

The MSE for Lasso is 1.528.

4.11 Principal Component Regression

Now we are going to look at principal component regression.

```
pcr.fit <- pcr(G3 ~ ., data = student_por, scale = TRUE, validation = "CV")
summary(pcr.fit)
```

```
## Data:    X dimension: 649 41
##  Y dimension: 649 1
## Fit method: svdpc
## Number of components considered: 41
##
```

```
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV           3.233    2.426    2.289    2.249    2.236    2.065    2.021
## adjCV        3.233    2.425    2.287    2.248    2.233    2.018    2.029
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV       1.877    1.866    1.863     1.869     1.851     1.827     1.795
## adjCV    1.857    1.853    1.858     1.865     1.859     1.825     1.797
##        14 comps  15 comps  16 comps  17 comps  18 comps  19 comps  20 comps
## CV        1.739     1.728     1.720     1.727     1.708     1.661     1.652
## adjCV     1.735     1.719     1.711     1.725     1.706     1.644     1.649
##        21 comps  22 comps  23 comps  24 comps  25 comps  26 comps  27 comps
## CV        1.639     1.632     1.600     1.578     1.542     1.534     1.522
## adjCV     1.632     1.632     1.603     1.575     1.531     1.529     1.518
##        28 comps  29 comps  30 comps  31 comps  32 comps  33 comps  34 comps
## CV        1.528     1.494     1.463     1.454     1.452     1.430     1.428
## adjCV     1.528     1.496     1.457     1.451     1.449     1.423     1.424
##        35 comps  36 comps  37 comps  38 comps  39 comps  40 comps  41 comps
## CV        1.405     1.407     1.404     1.407     1.410     1.292     1.293
## adjCV     1.400     1.403     1.400     1.402     1.406     1.287     1.289
##
## TRAINING: % variance explained
##     1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X     9.881    16.40    21.46    25.82    29.83    33.72    37.44    40.94
## G3   44.250    50.22    52.25    52.96    61.41    61.59    67.57    67.77
##     9 comps  10 comps  11 comps  12 comps  13 comps  14 comps  15 comps
## X     44.35     47.48     50.46     53.39     56.21     58.93     61.46
## G3    67.77     67.79     68.50     70.12     70.91     73.25     73.85
##     16 comps  17 comps  18 comps  19 comps  20 comps  21 comps  22 comps
## X      63.92     66.31     68.61     70.84     73.05     75.09     77.07
## G3     74.12     74.14     74.72     76.16     76.18     76.83     77.03
##     23 comps  24 comps  25 comps  26 comps  27 comps  28 comps  29 comps
## X      78.97     80.82     82.59     84.25     85.89     87.47     89.00
## G3     77.62     78.59     79.77     79.99     80.21     80.40     81.05
##     30 comps  31 comps  32 comps  33 comps  34 comps  35 comps  36 comps
## X      90.49     91.87     93.17     94.35     95.48     96.53     97.52
## G3     81.83     81.87     81.91     82.52     82.64     83.12     83.13
##     37 comps  38 comps  39 comps  40 comps  41 comps
## X      98.38     99.05     99.46     99.76       100
## G3     83.18     83.19     83.21     85.97        86
```

```
R2.pcr = as.numeric(R2(pcr.fit, estimate="train")$val)
mse.pcr = as.numeric(MSEP(pcr.fit, estimate="train")$val)

R2.pcr
```

```
##  [1] 0.0000000 0.4425048 0.5021809 0.5224846 0.5296433 0.6140847 0.6159433
##  [8] 0.6757365 0.6777191 0.6777401 0.6779114 0.6850131 0.7011920 0.7091014
## [15] 0.7324872 0.7384674 0.7411768 0.7413506 0.7472237 0.7615905 0.7617913
## [22] 0.7682921 0.7702811 0.7761945 0.7858873 0.7976724 0.7999222 0.8020673
## [29] 0.8040197 0.8104576 0.8183151 0.8186857 0.8191378 0.8251620 0.8264211
## [36] 0.8312160 0.8313354 0.8318378 0.8319371 0.8321284 0.8597183 0.8600091
```

```
mse.pcr
```

```
##  [1] 10.421058  5.809689  5.187802  4.976215  4.901614  4.021646  4.002277
##  [8]  3.379169  3.358508  3.358289  3.356504  3.282497  3.113895  3.031471
## [15]  2.787767  2.725447  2.697211  2.695401  2.634196  2.484479  2.482387
## [22]  2.414641  2.393914  2.332290  2.231281  2.108468  2.085022  2.062668
## [29]  2.042322  1.975232  1.893349  1.889487  1.884775  1.821997  1.808875
## [36]  1.758908  1.757664  1.752428  1.751393  1.749400  1.461884  1.458853
```

PCR attains the lowest prediction MSE = 1.458 when all 41 principal components are included. This result, which would be overly cumbersome to analyze within this project's scope, does not lend itself well to further analysis compared to more dimension-reduced models. If we were to compromise the number of principal components, we would still need to include 20+ to create a model with an MSE that is comparable to our previous models. Therefore, we will not be suggesting a candidate model based on principal component regression.

4.12 Partial Least Squares Regression

```
pls.fit <- plsr(G3 ~ ., data = student_por, scale = TRUE, validation = "CV")
summary(pls.fit)
```

```
## Data:    X dimension: 649 41
##  Y dimension: 649 1
## Fit method: kernelpls
## Number of components considered: 41
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV           3.233    1.869    1.482    1.410    1.378    1.362    1.345
## adjCV        3.233    1.867    1.476    1.404    1.372    1.355    1.338
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV       1.329    1.325    1.323     1.323     1.322     1.322     1.322
## adjCV    1.322    1.319    1.317     1.317     1.316     1.316     1.316
##        14 comps  15 comps  16 comps  17 comps  18 comps  19 comps  20 comps
## CV        1.322     1.322     1.322     1.322     1.322     1.322     1.322
## adjCV     1.316     1.316     1.316     1.316     1.316     1.316     1.316
##        21 comps  22 comps  23 comps  24 comps  25 comps  26 comps  27 comps
## CV        1.322     1.322     1.322     1.322     1.322     1.322     1.322
## adjCV     1.316     1.316     1.316     1.316     1.316     1.316     1.316
##        28 comps  29 comps  30 comps  31 comps  32 comps  33 comps  34 comps
## CV        1.322     1.322     1.322     1.322     1.322     1.322     1.322
## adjCV     1.316     1.316     1.316     1.316     1.316     1.316     1.316
##        35 comps  36 comps  37 comps  38 comps  39 comps  40 comps  41 comps
## CV        1.322     1.322     1.322     1.322     1.322     1.322     1.322
## adjCV     1.316     1.316     1.316     1.316     1.316     1.316     1.316
##
## TRAINING: % variance explained
##     1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X     9.252    13.87    18.53    22.04    24.79    27.69    30.06    32.51
## G3   68.458    81.36    83.60    84.60    85.24    85.70    85.94    85.99
##     9 comps  10 comps  11 comps  12 comps  13 comps  14 comps  15 comps
## X     35.36     37.87     40.78     42.82     44.51     46.38     48.35
## G3    86.00     86.00     86.00     86.00     86.00     86.00     86.00
##     16 comps  17 comps  18 comps  19 comps  20 comps  21 comps  22 comps
## X      50.64     52.41     54.81     56.98     58.71     60.58     62.49
## G3     86.00     86.00     86.00     86.00     86.00     86.00     86.00
##     23 comps  24 comps  25 comps  26 comps  27 comps  28 comps  29 comps
## X      64.48     66.68     69.15     71.33     73.58     75.51     77.53
## G3     86.00     86.00     86.00     86.00     86.00     86.00     86.00
##     30 comps  31 comps  32 comps  33 comps  34 comps  35 comps  36 comps
## X      79.56     81.64     83.31     85.29        87     88.77     90.56
## G3     86.00     86.00     86.00     86.00        86     86.00     86.00
##     37 comps  38 comps  39 comps  40 comps  41 comps
## X      92.63     94.45     96.32     98.35       100
## G3     86.00     86.00     86.00     86.00        86
```

```
R2.pls = as.numeric(R2(pls.fit, estimate="train")$val)
mse.pls = as.numeric(MSEP(pls.fit, estimate="train")$val)

R2.pls
```

```
##  [1] 0.0000000 0.6845804 0.8136119 0.8360447 0.8459948 0.8524414 0.8570038
##  [8] 0.8594111 0.8599011 0.8599891 0.8600032 0.8600060 0.8600076 0.8600085
## [15] 0.8600089 0.8600090 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091
## [22] 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091
## [29] 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091
## [36] 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091 0.8600091
```

```
mse.pls
```

```
##  [1] 10.421058  3.287006  1.942361  1.708588  1.604897  1.537717  1.490172
##  [8]  1.465085  1.459978  1.459062  1.458915  1.458886  1.458869  1.458860
## [15]  1.458856  1.458854  1.458854  1.458853  1.458853  1.458853  1.458853
## [22]  1.458853  1.458853  1.458853  1.458853  1.458853  1.458853  1.458853
## [29]  1.458853  1.458853  1.458853  1.458853  1.458853  1.458853  1.458853
## [36]  1.458853  1.458853  1.458853  1.458853  1.458853  1.458853  1.458853
```

```
mse.pls <- mse.pls[10]
```

PLS attains the lowest predict MSE = 1.458853 with 18 principal components. If we were going to consider one of these models as a candidate model, I would consider sacrificing a little prediction accuracy for simplicity/dimension reduction. I would recommend using the model with 10 principal components because the MSE is 1.459062 which is only slightly higher than that with 18 with 8 fewer principal components.

4.13 Comparing MSEs

Now that we have proposed all of our candidate models, we are going to take a look at the MSE to determine our final model.

```
## # A tibble: 7 x 2
##   method              MSE
##   <chr>              <dbl>
## 1 BIC-Minimized       1.43
## 2 AIC-Minimized       1.46
## 3 PLS                 1.46
## 4 Cp-Minimized        1.47
## 5 LASSO               1.53
## 6 Linear Regression   1.55
## 7 Ridge Regression    1.60
```

Based on the MSE, we are going to examine the top three models and determine which one has the best balance of number of predictors and accuracy.

```
reg.bestsubBIC
```

```
##
## Call:
## lm(formula = G3 ~ reason + G1 + G2, data = student_por, subset = Z)
##
## Coefficients:
##      (Intercept)        reasonhome       reasonother  reasonreputation
##         -0.01247          -0.08366          -0.33815          -0.10231
##               G1                G2
##          0.10995           0.92593
```

```
reg.forward # Picking this one
```

```
##
## Call:
## lm(formula = G3 ~ G2 + G1 + failures + reason + absences + sex +
##     school + traveltime + health, data = student_por, subset = Z)
##
## Coefficients:
##      (Intercept)                G2                G1          failures
##          0.50898           0.90144           0.09950          -0.37556
##       reasonhome       reasonother  reasonreputation          absences
##         -0.13167          -0.33802          -0.21118           0.01524
##             sexM          schoolMS        traveltime            health
##         -0.20738          -0.22059           0.16919          -0.04444
```

Picking forward-selected candidate model because best balance of number of predictors while sacrificing only a little accuracy.

5 Conclusion

After examining a variety of candidate models, we have decided to use the model generated by forward step selection. We believe that this model does not eliminate too many variables while maintaining a relatively low MSE of 1.459. We have run the model again with the full data set below.

```r
reg.forward_full <- lm(G3 ~ G1 + G2 + failures
                        + reason + absences + sex
                        + school + traveltime + health
                        , student_por)

summary(reg.forward_full)
```

```
##
## Call:
## lm(formula = G3 ~ G1 + G2 + failures + reason + absences + sex +
##     school + traveltime + health, data = student_por)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.0833 -0.5178 -0.0053  0.6398  5.2097
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       0.44063    0.34169   1.290 0.197678
## G1                0.13706    0.03615   3.792 0.000164 ***
## G2                0.87996    0.03379  26.042  < 2e-16 ***
## failures         -0.24049    0.09074  -2.650 0.008244 **
## reasonhome       -0.09222    0.13010  -0.709 0.478659
## reasonother      -0.44994    0.16627  -2.706 0.006990 **
## reasonreputation -0.16537    0.13290  -1.244 0.213816
## absences          0.01623    0.01100   1.476 0.140522
## sexM             -0.20022    0.10191  -1.965 0.049894 *
## schoolMS         -0.22981    0.11621  -1.977 0.048419 *
## traveltime        0.11228    0.06839   1.642 0.101138
## health           -0.05394    0.03469  -1.555 0.120451
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 1.242 on 637 degrees of freedom
## Multiple R-squared:  0.8547, Adjusted R-squared:  0.8522
## F-statistic: 340.7 on 11 and 637 DF,  p-value: < 2.2e-16
```

Our final model is as follows:

$$\begin{gathered}
\widehat{G3} = 0.441 + 0.137(G1) + 0.880(G2) \\
-0.240(failures) - 0.092(reasonhome) - \\
0.450(reasonother) - 0.165(reasonrep) + 0.016(absences) - \\
0.200(sexMale) - 0.230(schoolMS) + \\
0.112(traveltime) - 0.054(health)
\end{gathered}$$

Let’s look at each of these variables and what they mean.

- `G1` is the student’s grade during the first term
- `G2` is the student’s grade during the second term
- `failures` is the number of past class failures a student had

- `reason` is the reason a student attended a certain school (3 dummy variables, with "course" as the baseline)
- `absences` is the number of absences a student had
- `sex` is the sex of the student (1 dummy variable with "female" as the baseline)
- `school` is a the name of the school the student attended (1 dummy variable with "Gabriel Pereira" as the baseline)
- `traveltime` is the amount of time it took a student to get to school
- `health` is the current health status of the student on a numerical scale from 1 (worst) to 5 (best)

Based on our final model, these are the variables that are significant when attempting to predict a student's final grade in Portuguese class (G3).