Notes on Z4c

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Contents

I. DERIVATION

- $D_i D_j \alpha$:

$$\Gamma^k{}_{ij} = \frac{1}{2}\gamma^{kl}(\partial_i\gamma_{jl} + \partial_j\gamma_{li} - \partial_l\gamma_{ij}) \tag{1}$$

$$= \frac{1}{2}\tilde{\gamma}^{kl}\left[(\partial_i\tilde{\gamma}_{jl} - \partial_i\ln\chi\tilde{\gamma}_{jl}) + (\partial_j\tilde{\gamma}_{li} - \partial_j\ln\chi\tilde{\gamma}_{li}) - (\partial_l\tilde{\gamma}_{ij} - \partial_l\ln\chi\tilde{\gamma}_{ij})\right] \tag{2}$$

$$= \tilde{\Gamma}^k{}_{ij} - \frac{1}{2}(\partial_i\ln\chi\delta^k{}_j + \partial_j\ln\chi\delta^k{}_i - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi) \tag{3}$$

where $\partial_l\gamma_{ij} = \partial_l(\chi^{-1}\tilde{\gamma}_{ij}) = \chi^{-1}(\partial_l\tilde{\gamma}_{ij} - \chi^{-1}\partial_l\chi\tilde{\gamma}_{ij}) = \chi^{-1}(\partial_l\tilde{\gamma}_{ij} - \partial_l\ln\chi\tilde{\gamma}_{ij})$. Then,

$$D_iD_j\alpha = \partial_i\partial_j\alpha - \Gamma^k{}_{ij}\partial_k\alpha \tag{4}$$

$$= \partial_i\partial_j\alpha - \left[\tilde{\Gamma}^k{}_{ij} - \frac{1}{2}(\partial_i\ln\chi\delta^k{}_j + \partial_j\ln\chi\delta^k{}_i - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi)\right]\partial_k\alpha \tag{5}$$

$$= \partial_i\partial_j\alpha - \tilde{\Gamma}^k{}_{ij}\partial_k\alpha + \frac{1}{2}(\partial_i\ln\chi\partial_j\alpha + \partial_j\ln\chi\partial_i\alpha - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi\partial_k\alpha) \tag{6}$$

$$= \partial_i\partial_j\alpha - \tilde{\Gamma}^k{}_{ij}\partial_k\alpha + \partial_{(i}\ln\chi\partial_{j)}\alpha - \frac{1}{2}\tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi\partial_k\alpha \tag{7}$$

II. MORE

[1]

[1] Roger Alexander. Solving ordinary differential equations i: Nonstiff problems (e. hairer, sp norsett, and g. wanner). Siam Review, 32(3):485, 1990.

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