Notes on Z4c

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Contents

I. DERIVATION

- $D_i\beta^i$:

$$D_i\beta^i = \frac{1}{\sqrt{\gamma}}\partial_i\left(\sqrt{\gamma}\beta^i\right) = \chi^{3/2}\partial_i\left(\chi^{-3/2}\beta^i\right) = -\frac{3}{2}\chi^{-1}\partial_i\chi\beta^i + \partial_i\beta^i \tag{1}$$

where $\chi = \gamma^{-1/3}$.

- $D_iD_j\alpha$:

$$\begin{aligned}
\Gamma^k{}_{ij} &= \frac{1}{2}\gamma^{kl}(\partial_i\gamma_{jl} + \partial_j\gamma_{li} - \partial_l\gamma_{ij}) & (2)\\
&= \frac{1}{2}\tilde{\gamma}^{kl}\left[(\partial_i\tilde{\gamma}_{jl} - \partial_i\ln\chi\tilde{\gamma}_{jl}) + (\partial_j\tilde{\gamma}_{li} - \partial_j\ln\chi\tilde{\gamma}_{li}) - (\partial_l\tilde{\gamma}_{ij} - \partial_l\ln\chi\tilde{\gamma}_{ij})\right] & (3)\\
&= \tilde{\Gamma}^k{}_{ij} - \frac{1}{2}(\partial_i\ln\chi\delta^k{}_j + \partial_j\ln\chi\delta^k{}_i - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi) & (4)
\end{aligned}$$

where $\partial_l\gamma_{ij} = \partial_l(\chi^{-1}\tilde{\gamma}_{ij}) = \chi^{-1}(\partial_l\tilde{\gamma}_{ij} - \chi^{-1}\partial_l\chi\tilde{\gamma}_{ij}) = \chi^{-1}(\partial_l\tilde{\gamma}_{ij} - \partial_l\ln\chi\tilde{\gamma}_{ij})$. Then,

$$\begin{aligned}
D_iD_j\alpha &= \partial_i\partial_j\alpha - \Gamma^k{}_{ij}\partial_k\alpha & (5)\\
&= \partial_i\partial_j\alpha - \left[\tilde{\Gamma}^k{}_{ij} - \frac{1}{2}(\partial_i\ln\chi\delta^k{}_j + \partial_j\ln\chi\delta^k{}_i - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi)\right]\partial_k\alpha & (6)\\
&= \partial_i\partial_j\alpha - \tilde{\Gamma}^k{}_{ij}\partial_k\alpha + \frac{1}{2}(\partial_i\ln\chi\partial_j\alpha + \partial_j\ln\chi\partial_i\alpha - \tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi\partial_k\alpha) & (7)\\
&= \partial_i\partial_j\alpha - \tilde{\Gamma}^k{}_{ij}\partial_k\alpha + \partial_{(i}\ln\chi\partial_{j)}\alpha - \frac{1}{2}\tilde{\gamma}_{ij}\tilde{\gamma}^{kl}\partial_l\ln\chi\partial_k\alpha & (8)
\end{aligned}$$

- $\partial_k\tilde{A}^{ij}$:

$$\begin{aligned}
\partial_k\tilde{A}^{ij} &= \partial_k\left(\tilde{\gamma}^{il}\tilde{\gamma}^{jm}\tilde{A}_{lm}\right) = \partial_k\tilde{\gamma}^{il}\tilde{\gamma}^{jm}\tilde{A}_{lm} + \tilde{\gamma}^{il}\partial_k\tilde{\gamma}^{jm}\tilde{A}_{lm} + \tilde{\gamma}^{il}\tilde{\gamma}^{jm}\partial_k\tilde{A}_{lm} & (9)\\
&= -\tilde{\gamma}^{ip}\tilde{\gamma}^{lq}\partial_k\tilde{\gamma}_{pq}\tilde{\gamma}^{jm}\tilde{A}_{lm} - \tilde{\gamma}^{il}\tilde{\gamma}^{jp}\tilde{\gamma}^{mq}\partial_k\tilde{\gamma}_{pq}\tilde{A}_{lm} + \tilde{\gamma}^{il}\tilde{\gamma}^{jm}\partial_k\tilde{A}_{lm} & (10)\\
&= -\tilde{\gamma}^{ip}\tilde{A}^{jq}\partial_k\tilde{\gamma}_{pq} - \tilde{\gamma}^{jp}\tilde{A}^{iq}\partial_k\tilde{\gamma}_{pq} + \tilde{\gamma}^{il}\tilde{\gamma}^{jm}\partial_k\tilde{A}_{lm} & (11)\\
&= -2\tilde{\gamma}^{il}\tilde{A}^{jm}\partial_k\tilde{\gamma}_{lm} + \tilde{\gamma}^{il}\tilde{\gamma}^{jm}\partial_k\tilde{A}_{lm} & (12)
\end{aligned}$$

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II. MORE

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[1] Roger Alexander. Solving ordinary differential equations i: Nonstiff problems (e. hairer, sp norsett, and g. wanner). Siam Review, 32(3):485, 1990.