

Notes on Z4c

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I. DERIVATION

- $D_i D_j \alpha$:

$$\Gamma^k_{ij} = \frac{1}{2} \gamma^{kl} (\partial_i \gamma_{jl} + \partial_j \gamma_{li} - \partial_l \gamma_{ij}) \quad (1)$$

$$= \frac{1}{2} \tilde{\gamma}^{kl} [(\partial_i \tilde{\gamma}_{jl} - \partial_i \ln \chi \tilde{\gamma}_{jl}) + (\partial_j \tilde{\gamma}_{li} - \partial_j \ln \chi \tilde{\gamma}_{li}) - (\partial_l \tilde{\gamma}_{ij} - \partial_l \ln \chi \tilde{\gamma}_{ij})] \quad (2)$$

$$= \tilde{\Gamma}^k_{ij} - \frac{1}{2} (\partial_i \ln \chi \delta^k_j + \partial_j \ln \chi \delta^k_i - \tilde{\gamma}_{ij} \tilde{\gamma}^{kl} \partial_l \ln \chi) \quad (3)$$

where $\partial_l \gamma_{ij} = \partial_l (\chi^{-1} \tilde{\gamma}_{ij}) = \chi^{-1} (\partial_l \tilde{\gamma}_{ij} - \partial_l \ln \chi \tilde{\gamma}_{ij}) = \chi^{-1} (\partial_l \tilde{\gamma}_{ij} - \partial_l \ln \chi \tilde{\gamma}_{ij})$. Then,

$$D_i D_j \alpha = \partial_i \partial_j \alpha - \Gamma^k_{ij} \partial_k \alpha \quad (4)$$

$$= \partial_i \partial_j \alpha - \left[\tilde{\Gamma}^k_{ij} - \frac{1}{2} (\partial_i \ln \chi \delta^k_j + \partial_j \ln \chi \delta^k_i - \tilde{\gamma}_{ij} \tilde{\gamma}^{kl} \partial_l \ln \chi) \right] \partial_k \alpha \quad (5)$$

$$= \partial_i \partial_j \alpha - \tilde{\Gamma}^k_{ij} \partial_k \alpha + \frac{1}{2} (\partial_i \ln \chi \partial_j \alpha + \partial_j \ln \chi \partial_i \alpha - \tilde{\gamma}_{ij} \tilde{\gamma}^{kl} \partial_l \ln \chi \partial_k \alpha) \quad (6)$$

$$= \partial_i \partial_j \alpha - \tilde{\Gamma}^k_{ij} \partial_k \alpha + \partial_{(i} \ln \chi \partial_{j)} \alpha - \frac{1}{2} \tilde{\gamma}_{ij} \tilde{\gamma}^{kl} \partial_l \ln \chi \partial_k \alpha \quad (7)$$

- $\partial_k \tilde{A}^{ij}$:

$$\partial_k \tilde{A}^{ij} = \partial_k \left(\tilde{\gamma}^{il} \tilde{\gamma}^{jm} \tilde{A}_{lm} \right) = \partial_k \tilde{\gamma}^{il} \tilde{\gamma}^{jm} \tilde{A}_{lm} + \tilde{\gamma}^{il} \partial_k \tilde{\gamma}^{jm} \tilde{A}_{lm} + \tilde{\gamma}^{il} \tilde{\gamma}^{jm} \partial_k \tilde{A}_{lm} \quad (8)$$

$$= -\tilde{\gamma}^{ip} \tilde{\gamma}^{lq} \partial_k \tilde{\gamma}_{pq} \tilde{\gamma}^{jm} \tilde{A}_{lm} - \tilde{\gamma}^{il} \tilde{\gamma}^{jp} \tilde{\gamma}^{mq} \partial_k \tilde{\gamma}_{pq} \tilde{A}_{lm} + \tilde{\gamma}^{il} \tilde{\gamma}^{jm} \partial_k \tilde{A}_{lm} \quad (9)$$

$$= -\tilde{\gamma}^{ip} \tilde{A}^{jq} \partial_k \tilde{\gamma}_{pq} - \tilde{\gamma}^{jp} \tilde{A}^{iq} \partial_k \tilde{\gamma}_{pq} + \tilde{\gamma}^{il} \tilde{\gamma}^{jm} \partial_k \tilde{A}_{lm} \quad (10)$$

$$= -2 \tilde{\gamma}^{il} \tilde{A}^{jm} \partial_k \tilde{\gamma}_{lm} + \tilde{\gamma}^{il} \tilde{\gamma}^{jm} \partial_k \tilde{A}_{lm} \quad (11)$$

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II. MORE

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- [1] Roger Alexander. Solving ordinary differential equations i: Nonstiff problems (e. hairer, sp norsett, and g. wanner). Siam Review, 32(3):485, 1990.