Ray Triangle Intersection

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Given a triangle defined by points *p0*, *p1* and *p2* and a ray given by its origin *o* and direction *d*, the barycentrics of the hit point as well as the t-value of the hit can be obtained by solving the system:

$$\begin{bmatrix} \mathrm{e}_1 & \mathrm{e}_2 & -\mathrm{d} \end{bmatrix} \begin{bmatrix} u \\ v \\ t \end{bmatrix} = \mathrm{s}$$

Where:

$$\begin{aligned} \mathrm{e}_1 &= \mathrm{p}_1 - \mathrm{p}_0 \\ \mathrm{e}_2 &= \mathrm{p}_2 - \mathrm{p}_0 \\ \mathrm{s} &= \mathrm{o} - \mathrm{p}_0 \end{aligned}$$

This system can be solved by Cramer's Rule, yielding:

$$\begin{bmatrix} u \\ v \\ t \end{bmatrix} = \frac{1}{\begin{vmatrix} \mathrm{e}_1 & \mathrm{e}_2 & -\mathrm{d} \end{vmatrix}} \begin{bmatrix} \begin{vmatrix} \mathrm{s} & \mathrm{e}_2 & -\mathrm{d} \end{vmatrix} \\ \begin{vmatrix} \mathrm{e}_1 & \mathrm{s} & -\mathrm{d} \end{vmatrix} \\ \begin{vmatrix} \mathrm{e}_1 & \mathrm{e}_2 & \mathrm{s} \end{vmatrix} \end{bmatrix}$$

In the above, |*a b c*| denotes the determinant of the 3x3 with column vectors *a*,*b*, *c*.

Note that since the determinant is given by: $\begin{vmatrix} \mathrm{a} & \mathrm{b} & \mathrm{c} \end{vmatrix} = (\mathrm{a} \times \mathrm{b}) \cdot \mathrm{c} = (\mathrm{b} \times \mathrm{c}) \cdot \mathrm{a} = (\mathrm{c} \times \mathrm{a}) \cdot \mathrm{b}$, you can rewrite the above as:

$$\begin{bmatrix} u \\ v \\ t \end{bmatrix} = \frac{1}{(\mathrm{e}_1 \times \mathrm{d}) \cdot \mathrm{e}_2} \begin{bmatrix} -(\mathrm{s} \times \mathrm{e}_2) \cdot \mathrm{d} \\ (\mathrm{e}_1 \times \mathrm{d}) \cdot \mathrm{s} \\ -(\mathrm{s} \times \mathrm{e}_2) \cdot \mathrm{e}_1 \end{bmatrix}$$

Of which you should notice a few common subexpressions that, if exploited in an implementation, make computation of t, u, and v substantially more efficient.

A few final notes and thoughts:

If the denominator dot((*e1* x *d*), *e2*) is zero, what does that mean about the relationship of the ray and the triangle? Can a triangle with this area be hit by a ray? Given u and v, how do you know if the ray hits the triangle? Don't forget that the intersection point on the ray should be within the ray's `min_t` and `max_t` bounds.

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