Lecture 4:Semantics for predicate calculus

These semantics provide a formal basis for determining the meaning and truth value of well formed expressions aka well formed formulas(WFFs).

The truth of expressions depends on the mapping of constants, variables, predicates and functions into objects and relationships in the domain of discourse.

Semantics for predicate calculus

The truth of relationships in the domain determines the truth of the corresponding expressions, e.g. information about a person George and his friends Kate and Susie may be expressed by the following sentences:

friends(george, susie)

friends(george, kate)

If it is indeed true that George is a friend of Susie

Semantics for predicate calculus

And George is a friend of Kate, then these expressions would each have the truth value (assignment) T. If George is a friend of Susie and not a friend of Kate then the first expression would have assignment of T and the second would have a truth value F.

To use predicate calculus as a representation of problem solving, the objects and relations are described in the domain of interpretation with a set of well formed expressions(WFFs)

Semantics for predicate calculus

The terms and predicates of these expressions denote objects and relations in the domain. This becomes a database of predicate calculus expressions having truth value T and describing "the state of the world".

The description of George and his friends is a simple example of such a database.

Definition of semantics of Predicate calculus

Two definitions:

- Interpretation over Domain
- Use the interpretation

Interpretation

Let Domain D be an nonempty set

An interpretation over D assignment of the entities of D to each of the constants, variables, predicate and function symbols of a predicate calculus such that :

Definition of semantics of variable calculus

1) each constant is assigned an element of D

2) each variable is assigned a nonempty subset of D

3) each function of arity m is defined on m arguments of D and defines a mapping from D^m into D

4) each predicate of arity n is defined on n arguments from D and defines a mapping from D^n into {T, F}

EXAMPLE

If P(x) is a predicate and x has domain U, then the truth set of P(x) is a set of all elements t of U such that $P_n(t)$ is true i.e.{ t € U $P_n(t)$ is true i.e. Let U = {1, 2,3,4,5,6,7,8,9,10} and P(x): x is even. The truth set is:{2, 4, 6, 8, 10}

Truth value of Predicate calculus expressions

Assume an expression E and an Interpretation I for E over a nonempty domain D. The truth value for E is determined by:

1) The value of a constant which is the element of D it is assigned an interpretation to by I
2) The value of a variable which is the set of elements of D it is assigned by I
3) The value of a function expression which is that element of D obtained by evaluating the function for the parameter values assigned by the r

Truth value of Predicate calculus expressions

4)The value of truth symbol “true” is T and “false” F

5) The value of an atomic sentence is either T or F as determined by the interpretation of I

6)The value of the negation of a sentence is T if the value of the sentence is F and is F if the value of the sentence is T.

7) The value of the conjuction of two sentences is T if the value of both sentences is T and is F otherwise.

Truth value of Predicate calculus expressions

(8-10) The truth value of expressions using V,→ and ≡ is determined from the value of their operands

Finally for a variable X and a sentence S containing X"

11)The value ∀XS is true if S is T for all assignments to X under I and it is F otherwise.

12) The value of ∃XS is T if there is an assignment to X in the interpretation under which S is T, otherwise it is F.

Quantification of variables

Is a very important part of Predicate calculus semantics.

The Universal Quantifier ∀ means ‘for all’ or every as in Everyone means all

Main use is to quantify a predicate for example:

$\forall X \epsilon D$ P(X) For all values of x in the Domain(D), P(x) is true

Universal Quantifier(∀)

$\forall X \in D\ P(X)$

Every dog is a mammal(All dogs are mammals)

P(x): X is a mammal

Domain (D) = dogs

The Existential Quantifier ∃

∃ means there exists

Main use is to quantify predicates

$\exists x \epsilon D$, $P(x)$ means There exists x in the domain, such that $P(x)$ is true.

<u>Some</u> person in the world oldest.

$\exists x$ $x \epsilon D$ = (People) $P(x)$: x is the oldest

Quantification of variables

Likes(george, susie) and likes (george, kate).

The variable X stands for all constants that might appear as second parameter of the sentence.

The variable might be replaced by any other variable name such as Y or PEOPLE, without changing the meaning of the sentence.

In Predicate logic variables must be

Quantification of variables

A variable is said to be free when it is not within the scope of either ∀ universally)or ∃ is(existentially).

An expression is closed when all of its variables are quantified.

A ground expression Hs no variables at all.

Quantification of variables

Parenthesis are used to show the scope of quantification, that is the instances of a variable name over which a quantification holds example:

For the symbol indicating universal quantification(∀) $\forall(P(x) \vee Q(Y)) \rightarrow R(x)$ indicates that X is universally quantified in both P(X) and R(X).

Quantification of variables

Universal quantification introduces problems in computing the truth value of a sentence because all possible values of a variable symbol must be tested to see whether the expression true.

For example to test the truth value for the expression : $\forall x$ likes(george, X) where X ranges over a set of all humans, all possible value for X must be tested.

Quantification of variables

If the domain of an interpretation is finite, exhaustive testing of all substitutions to a universally quantified variable is computationally impossible.

The algorithm may never halt. Because of this problem, predicate calculus is said to be undecidable.

Because Propositional calculus does not support variables, sentences can only have a finite number of true assignments and all these assignments can be exaustively tested.

Quantification of variables

Variables can also be quantified existentially(∃).

The Algorithm may also never halt for example if there is an attempt to determine the truth of the expression by trying all substitutions until one is found that makes the expression true.

If the domain of the variable is infinite and the expression is false under all substitutions, the algorithm will never halt.

Self test activity

Consider the sentence:”Roofus is a mammal”.Note:(Roofus is the name of a dog)

From the above sentence produce:

(a)A statement (P)

(b)A predicate (P)

(c)A statement using the Universal Quantifier ($\exists$)