

# **README:**

A clustering diffused particle method

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iffused particle method calculates the scattered field from point particles with both electric and magnetic responses. The computational complexity is reduced by hierarchical clustering techniques to enable simulations with on the order of  $10^{10}$  particles. The calculation is based on a generalized Foldy-Lax equation in [2]. With so many particles we are able to see the transition to bulk media behavior of the fields. Thus the method is also used to verify a generalized effective medium theory [1].

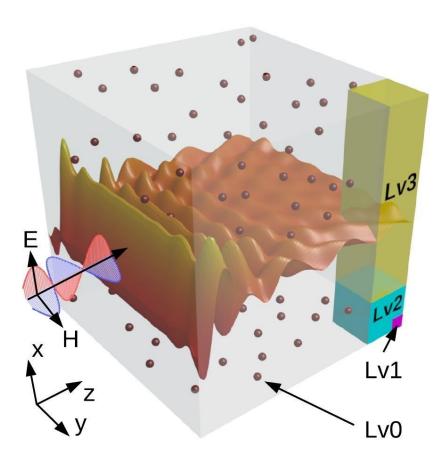


Figure 1: The simulation model of the diffused-particle method.

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# 1 Examples

# $1.1 \quad elec\_field.m$

The script *elec\_field.m* plots the electric field in the middle plane of the simulation region. It reproduces Fig. 5 in [1] and Fig. 4 in [2] with the input parameters given in the corresponding references. The input parameters are listed in the table below:

parameter	unit	description
k0	$\lambda^{-1}$	wave number
rho	$\lambda^3$	concentration of particles
alpha_e	$\lambda^3$	electric polarizability
alpha_m	$\lambda^3$	magnetic polarizability
Ys	λ	Y size of the sampling region
Zs	λ	Z size of the sampling region
n0		initial guess of refractive index n
L	λ	side length of the cubic simulation region
D1	λ	side length of a Lv1 voxel
D2	λ	side length of a Lv2 voxel

# $1.2 \quad sweep\_beta.m$

The script  $sweep\_beta.m$  plots the values of  $\varepsilon$ ,  $\mu$  and n with different  $\beta_e$  and  $\beta_m$  values, based on [2]. It reproduces Fig. 4 in [1] and Fig. 5 in [2](a) with the input parameters given in the corresponding references. The input parameters different from the script  $elec\_field.m$  are listed in the table below:

parameter	unit	description
alpha_e_sweep	$\lambda^3$	swept electric polarizability values
alpha_m_sweep	$\lambda^3$	swept magnetic polarizability values

# 1.3 $sweep\_n0.m$

The script  $sweep\_n0.m$  plots the number of iterations before the convergence of the algorithm with different  $n_0$  values, based on [2]. It reproduces Fig. 5 in [2](b) with the input parameters given in the corresponding references. The input parameters different from the script  $elec\_field.m$  are listed in the table below:

	parameter	unit	description
ĺ	n0_re		swept real part of $n_0$
ĺ	n0_im		swept imaginary part of $n_0$

# 2 Functions

## 2.1 Effective medium theories

These functions under the folder /functions/eff\_medium\_theory calculate theoretical macroscopic parameters  $\varepsilon$  and  $\mu$  given microscopic parameters  $\beta_{\rm e}$  and  $\beta_{\rm m}$ .

#### 2.1.1 CM.m

Computes permittivity and permeability by the Clausius-Mossotti relation. The input and output parameters are given below:

parameter	unit	description		
	Inputs	3		
be		$\beta_{\mathrm{e}}$		
bm		$\beta_{\mathrm{m}}$		
Outputs				
ер		ε		
mu		$\mu$		

#### 2.1.2 $GCM_m_branch.m$

Computes theoretical ("-" branch) effective permittivity and permeability based on cite1. The input and output parameters are the same as the function CM.m.

#### 2.1.3 $GCM_p_branch.m$

Computes theoretical ("+" branch) effective permittivity and permeability based on cite1. The input and output parameters are the same as the function CM.m.

#### 2.2 Green's functions

These functions under the folder /functions/Greens\_func calculate Green's functions.

#### 2.2.1 G2d.m

Calculates the 2D Green's function. The input and output parameters are given below:

parameter	unit	description			
	Inputs				
k0	$\lambda^{-1}$	free-space wave number			
rs	λ	3D location of the source			
rt $\lambda$		3D location of the observing point			
Outputs					
G the value of the 2d Green's func		the value of the 2d Green's function			

#### 2.2.2 G2d.m

Calculates the 3D Green's function  $\times 4\pi k_0^2$ . The input and output parameters are the same as the function G2d.m.

#### 2.2.3 Gxx.m

Calculates the xx component of 3D Green's function  $\times 4\pi k_0^2$ . The input and output parameters are the same as the function G2d.m.

# 2.3 Dipole and mesh generators

These functions under the folder  $/functions/cube\_creator$  generate 3D locations of the dipoles or the cubic mesh.

#### 2.3.1 $cube\_mesh.m$

Generates 3D cubic mesh in a cube. The input and output parameters are given below:

parameter unit description		description			
	Inputs				
ax $\lambda$ side length of the cube in x					
ay $\lambda$		side length of the cube in y			
az	λ	side length of the cube in z			
$dr$ $\lambda$		distance between neighbor mesh nodes			
Outputs					
r $\lambda$ 3D locations of mesh nodes					

#### 2.3.2 $ran\_cube.m$

Generates random 3D locations of dipoles in a cube. The input and output parameters are given below:

parameter unit		description			
	Inputs				
X	λ	side length of the cube in x			
Υ λ		side length of the cube in y			
$Z$ $\lambda$		side length of the cube in z			
n		number of dipoles			
Outputs					
r $\lambda$ random 3D locations of the dipole		random 3D locations of the dipoles			

# 2.4 Plot tools

These functions under the folder /functions/plot\_tool provide plot tool for different requirements.

#### $2.4.1 \quad plot\_DPM.m$

Plots the simulated values of epsilon and mu by diffused particle method, for example: Fig. 4 in [1]. The input parameters are given below:

parameter	unit	description
beta_e		$eta_{ m e}$
ер		$\varepsilon$
mu		$\mu$
fig_numb		figure number of the plot

## $2.4.2 \quad plot\_E\_colormap.m$

plots the electric field in the middle plane of the simulation region, for example Fig. 1 and Fig. 4 in [2]. The input parameters are given below:

parameter	unit	description	
L	λ	side length of the cubic simulation region	
D2	λ	side length of the lv2 voxel	
E		electric field distributed in the middle plane	

#### 2.4.3 $plot\_EMT.m$

Plots the theoretical values of  $\varepsilon$  and  $\mu$  vs.  $Re(\beta_e)$ , for example: Fig. 4 in [1]. The input parameters are given below:

parameter	unit	description
beta_e_start		start value of $\beta_{\rm e}$ array
beta_e_end		end value of $\beta_{\rm e}$ array
beta_m_start		start value of $\beta_{\rm m}$ array
beta_m_end		end value of $\beta_{\rm m}$ array
fig_numb		figure number of the plot

# References

- [1] L. Wang, I. L. Rasskazov, and P. S. Carney, Clausius-mossotti relation revisited: Media with electric and magnetic response, arXiv preprint: 2008.09178 (2020).
- [2] L. Wang, I. L. Rasskazov, and P. S. Carney, Clustering diffused-particle method for scattering from large ensembles of electromagnetically polarizable particles, arXiv preprint: 2008.09185 (2020) .