

VASCULAR SKELETON DEFORMATION EVALUATION BASED ON THE METRIC OF SINKHORN DISTANCE

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Summary

We customized Skeleton Matching Distance (SMD) for the structural consistency of vessel skeletons based on Sinkhorn distance[1][2].

We introduced the concept of degree-based weighted 'uniform' distribution and neighbor matching constraints, which allow our metric to smoothly evaluate the segmentation results, assessing a model's ability to capture the structural changes required for medical diagnosis effectively.

We conducted experiments comparing multiple evaluation metrics, including typical deformation examples and numerical analyses, demonstrating the superior characteristics of the introduced new metric.

Motivation

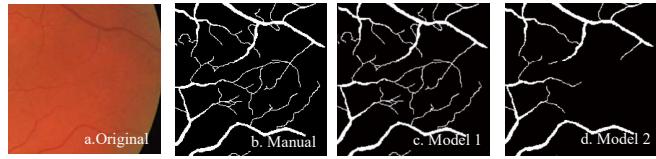


Fig. 1: Demonstration of two segmentation results for a same slice.
c) Dice(\uparrow): 0.79, SMD(\downarrow): 0.18. d) Dice: 0.81, SMD: 0.28.

Most existing metrics:

Not sensitive to the morphological differences in vascular prediction results, while these structures are essential visual markers of certain diseases.

Vascular skeletons:

The most representative features reflecting vascular morphological structure.

Method

SMD can be described as:

$$\begin{aligned} SMD(G_{pre}, G_{ref}) &= \min_{P_{mask} \in U(\alpha_{pre}, \beta_{ref})} \langle P_{mask}, D \rangle + \lambda H(P_{mask}) \\ &= \min_{P_{mask} \in U(d_{pre}/\sum d_{pre}, d_{ref}/\sum d_{ref})} \langle (M_r \vee M_c) \odot P, D \rangle + \lambda H(P_{mask}) \end{aligned}$$

1. Skeletons of vessels and reference vessels are graphically represented as G_{pre}, G_{ref}

2. Degree-weighted vectors $d_{pre}/\sum d_{pre}, d_{ref}/\sum d_{ref}$ represent weight distributions among Graphs' nodes

3. Euclidean distances between paired points generate a distance matrix D

4. Set the binarized mask matrix values to one at the smallest k values in each row/column of D to get M_r, M_c

5. Utilizing above properties, P_{mask} can be represented as

$$P_{ij} = u_i(M_r \vee M_c)_{ij} \exp(-D/\varepsilon)_{ij} v_j$$

where u, v are iterative variables in the form of $\alpha_{pre}, \beta_{ref}$

Code can be found at <https://github.com/lwannnn>.

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6. Using Sinkhorn iterations yield the plan matrix, from which SMD is calculated as the inner product of the distance matrix and plan matrix

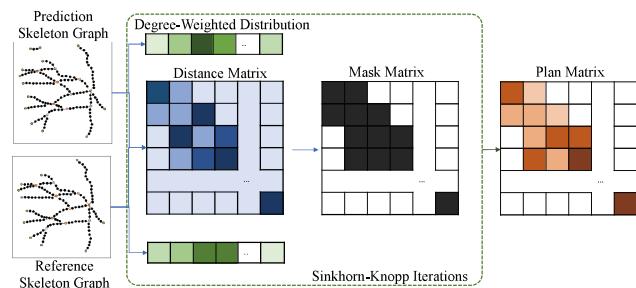


Fig. 2: The process of getting Plan Matrix.

Experiments

Comparison of Sensitivity between Existing Metrics and SMD

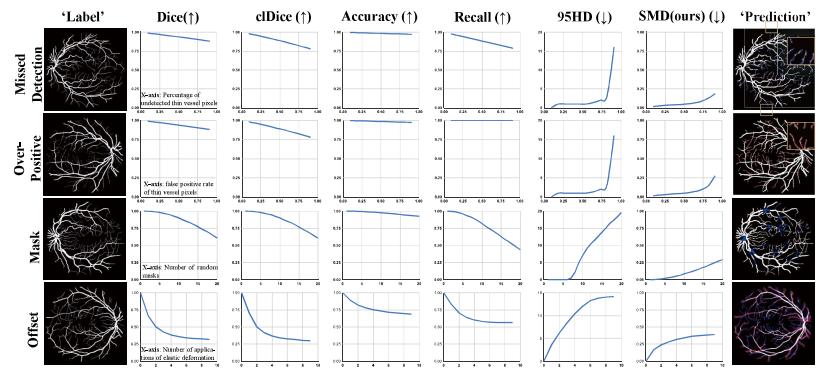
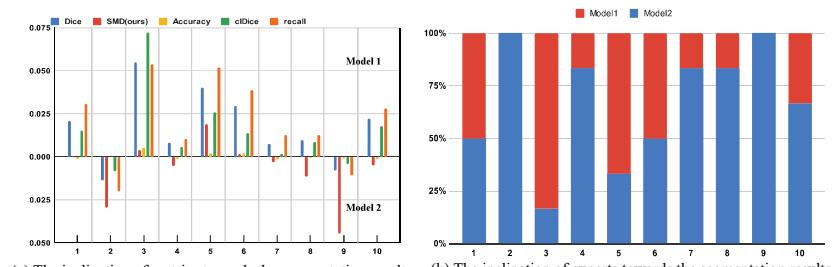


Fig. 3: The chart shows the curves of various metrics as the number of common errors increases. The direction of the arrow inside the parentheses indicates the trend of the metric values when the segmentation effect is improved. Correctly segmented pixels are white, blue indicates missed true vessel pixels, and red represents falsely identified vessel pixels.

SMD is sensitive to vascular disruption, curvature, and missed segments.

Comparison of Preferences between Expert Votes and Metric Values



(a) The inclination of metrics towards the segmentation results.

Fig. 4: In (a), downward bars favor Model 2, upward bars favor Model 1, with bar length indicating the difference in segmentation results between the models.

SMD is aligning well with the requirements of medical diagnosis with vascular segmentation results.

[1] Marco Cuturi, "Sinkhorn distances: Lightspeed computation of optimal transport," Advances in neural information processing systems, vol. 26, 2013.

[2] Jiyang Zhang, Xi Xiao, Long-Kai Huang, Yu Rong, and Yatao Bian, "Fine-tuning graph neural networks via graph topology induced optimal transport," arXiv preprint arXiv:2203.10453, 2022.