$$\widehat{\frac{\partial y}{\partial x_j^{(d)}}} = \frac{-2}{\sigma^2}\sum_{i=1}^{N} c_i e^{-\frac{\|x_i-x_j\|^2}{\sigma^2}} \left(x_i^{(d)} - x_j^{(d)}\right)$$

$$\frac{\partial y}{\partial x^{(d)}} = \frac{1}{N}\sum_{j=1}^{N}\left[\widehat{\frac{\partial y}{\partial x_j^{(d)}}}\right] = -\frac{2}{\sigma^2 N}\sum_j\sum_i c_i e^{-\frac{\|x_i-x_j\|^2}{\sigma^2}}\left(x_i^{(d)}-x_j^{(d)}\right)$$

Changement de variable j devient ~~la~~ l'index de la variable

i représente l'observation

il / l'observation variable

$$\left(\frac{\partial y}{\partial x_j}\right)_i = -\frac{2}{\sigma^2}\sum_{il=1}^{N} c_{il} e^{-\frac{\|x_{il}-x_i\|^2}{\sigma^2}}\left(x_{ilj}-x_{ij}\right)$$

$$= -\frac{2}{\sigma^2}\sum_{n=1}^{N} c_n e^{-\frac{\|x_n-x_i\|^2}{\sigma^2}}\left(x_{nj}-x_{ij}\right)$$

$$= -\frac{2}{\sigma^2}\left(\underline{c_1 e^{-\frac{\|x_1-x_i\|^2}{\sigma^2}}}\,\underline{\left(x_{1j}-x_{ij}\right)} + \underline{c_2 e^{-\frac{\|x_2-x_i\|^2}{\sigma^2}}}\,\underline{\left(x_{2j}-x_{ij}\right)} + \dots + \underline{c_N e^{-\frac{\|x_N-x_i\|^2}{\sigma^2}}}\left(x_{Nj}-x_{ij}\right)\right)$$

Soit $C = \begin{pmatrix} c_0 \\ \vdots \\ c_n \\ c_N \end{pmatrix}$ et $E_j = \begin{pmatrix} e^{-\frac{\|x_1-x_i\|^2}{\sigma^2}} \\ e^{-\frac{\|x_2-x_i\|^2}{\sigma^2}} \\ \vdots \\ e^{-\frac{\|x_N-x_i\|^2}{\sigma^2}} \end{pmatrix}$

$$C E_i^t = \begin{pmatrix} c_0 e^{-\frac{\|x_1-x_i\|^2}{\sigma^2}} & \dots & c_0 e^{-\frac{\|x_n-x_i\|^2}{\sigma^2}} & c_0 e^{-\frac{\|x_N-x_i\|^2}{\sigma^2}} \\ c_n e^{-\frac{\|x_1-x_i\|^2}{\sigma^2}} & \dots & c_n e^{-\frac{\|x_n-x_i\|^2}{\sigma^2}} & c_n e^{-\frac{\|x_N-x_i\|^2}{\sigma^2}} \\ \vdots & & \vdots & \\ c_N e^{-\frac{\|x_1-x_i\|^2}{\sigma^2}} & & c_N e^{-\frac{\|x_n-x_i\|^2}{\sigma^2}} & c_N e^{-\frac{\|x_N-x_i\|^2}{\sigma^2}} \end{pmatrix}$$

simple efficace

$$\underset{i}{\underline{\mathrm{diag}}}\left(C E_i^t\right) = \begin{pmatrix} c_0 e^{-\frac{\|x_0-x_i\|^2}{\sigma^2}} \\ \vdots \\ c_n e^{-\frac{\|x_n-x_i\|^2}{\sigma^2}} \\ \vdots \\ c_N e^{-\frac{\|x_N-x_i\|^2}{\sigma^2}} \end{pmatrix} = \mathrm{diag}_i$$

points $D_{iJ} = \begin{pmatrix} x_{1J} - x_{iJ} \\ x_{2J} - x_{iJ} \\ \vdots \\ x_{nJ} - x_{iJ} \\ \\ x_{NJ} - x_{iJ} \end{pmatrix}$

on a $\left(\frac{\partial \hat{y}}{\partial x_J}\right)_i = \frac{-2}{\sigma^2} (\text{diag})_i^T \cdot (D_{iJ})$

or $\frac{\partial \hat{y}}{\partial x_J} = \frac{1}{N}\sum_{i=1}^{N} \left(\frac{\partial y}{\partial x_J}\right)_i = \frac{1}{N}\sum_{i=1}^{N} -\frac{2}{\sigma^2} \text{diag}_i^T \cdot (D_{iJ})$

$= \frac{1}{N}\left(\frac{-2}{\sigma^2} \text{diag}_1^T D_{1J} + \dots + \frac{-2}{\sigma^2} \text{diag}_n^T D_{nJ} + \dots + \frac{-2}{\sigma^2} \text{diag}_N^T D_{NJ} \right)$

soit $\text{Diag} = \begin{pmatrix} \text{diag}_1^T \\ \vdots \\ \text{diag}_n^T \\ \vdots \\ \text{diag}_N^T \end{pmatrix}$ ✓

on a $\left[\frac{\partial y}{\partial x_J} = \left(\frac{1}{N}\right)\left(\frac{-2}{\sigma^2}\right)\left(\text{diag}\left[D_{\text{iag}} \times D_J\right]\right) \right]$

$D_J = (D_{1J} \dots D_{nJ} \dots D_{NJ})$

1 ~~écrire la fonction qui ...~~

1 → calculer C ✓

2 → écrire (D_{iJ}) ne pas factoriser!

3

4 → calculer B et B^T

5 → écrire diag of product ( )

6 → ~~écrire~~ calculer diag_i

7 → calculer ~~...~~ Diag.

3 → Calculer DJ

8 → calculer y from that ...