2-1 Solving One-Step Equations

Vocabulary

Review

1. Circle the *multiplicative inverse* of $\frac{1}{2}$. Underline the *additive inverse* of $\frac{1}{2}$.

2	1	$\frac{1}{2}$	$-\frac{1}{2}$	-2

2. Circle the *multiplicative inverse* of –3. Underline the *additive inverse* of -3.

3	1	$\frac{1}{3}$	$-\frac{1}{3}$	-3

Vocabulary Builder

isolate (verb) EYE suh layt

Main Idea: To **isolate** a variable in an equation means you get the variable with a coefficient of 1 alone on one side of the equation.

Other Word Forms: isolation (noun), isolated (adjective)

variable *isolated*
$x = 12$

variable NOT *isolated*
$5x = 60$

Use Your Vocabulary

3. Choose the correct form of the word *isolate* to complete each statement.

isolate isolation isolated

A very ill patient was placed in _?_, away from the other patients. isolation

In order to _?_ a variable, you may need to perform mathematical operations. isolate

A person living on a small island felt _?_ from the rest of the world. isolated

4. Circle the equations that show the variable *isolated*.

$4x + 1 = 13$	$x = 12 - 7$	$\frac{x}{3} = 10$	$\frac{2}{5} = x$

take note

Property Addition and Subtraction Properties of Equality

5. Complete the table.

Property	Algebra	Example
Addition Property of Equality	For any real numbers a, b, and c, if $a = b$, then $a + c = b +$ **c**.	$n - 7 = 12$ $n - 7 + 7 = 12 +$ **7**
Subtraction Property of Equality	For any real numbers a, b, and c, if $a = b$, then $a - c = b -$ **c**.	$n + 8 = 9$ $n + 8 -$ **8** $= 9 -$ **8**

Problem 1 Solving an Equation Using Subtraction

Got It? **What is the solution of $y + 2 = -6$? Check your answer.**

6. Underline the correct word to complete each sentence.

The equation $y + 2 = -6$ shows <u>addition</u> / subtraction.

The inverse of that operation is addition / <u>subtraction</u>.

7. Use the justifications to solve the equation.

$y + 2 = -6$	Write the original equation.
$y + 2 -$ **2** $=$ **−6** $-$ **2**	Subtract 2 from each side.
$y =$ **−8**	Simplify.

8. Check your answer by substituting it in the original equation for y. Then simplify.

Does **−8** $+ 2 = -6$? Yes / No

take note

Property Multiplication and Division Properties of Equality

9. Complete the table.

Property	Algebra	Example
Multiplication Property of Equality	For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot$ **c**.	$\frac{x}{5} = 10$ $\frac{x}{5} \cdot 5 = 10 \cdot$ **5**
Division Property of Equality	For any real numbers a, b, and c, such that $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{\mathbf{c}}$.	$6x = 30$ $\frac{6x}{6} = \frac{30}{\mathbf{6}}$

Problem 3 Solving an Equation Using Division

Got It? **What is the solution of $10 = 15x$? Check your answer.**

10. The equation is solved below. Write a justification for each step.

$10 = 15x$ Write the original equation.

$\frac{10}{15} = \frac{15x}{15}$ Divide each side by 15.

$\frac{2}{3} = x$ Simplify.

11. Check your answer.

$10 \stackrel{?}{=} 15 \cdot \frac{2}{3}$

$10 = 10$

Problem 4 Solving an Equation Using Multiplication

Got It? **What is the solution of $19 = \frac{r}{3}$?**

12. Underline the correct word or number to complete the sentence.

To isolate the variable, you should multiply / divide each side of the equation by 3 / 19.

13. When you isolate the variable, you obtain $r =$ 57.

Problem 5 Solving an Equation Using Reciprocals

Got It? **What is the solution of $12 = \frac{3}{4}x$? Check your answer.**

14. To solve the equation, divide / multiply both sides of the equation by the reciprocal of $\frac{3}{4}$.

15. **Multiple Choice** Choose the reciprocal of $\frac{3}{4}$.

A $\frac{3}{4}$ B $\frac{1}{4}$ C $\frac{4}{3}$ D 4

16. Use the reciprocal of $\frac{3}{4}$ to solve $12 = \frac{3}{4}x$ for x.

Answers may vary. Sample:

$12 = \frac{3}{4}x$

$\frac{4}{3}(12) = \frac{4}{3}\left(\frac{3}{4}x\right)$

$16 = x$

17. Now check your answer. Does $12 = \frac{3}{4} \cdot 16$? Yes / No

Problem 6 Using a One-Step Equation as a Model

Got It? **An online DVD rental company offers gift certificates that you can use to purchase rental plans. You have a gift certificate for $30. The plan you select costs $5 per month. How many months can you purchase with the gift certificate?**

18. Complete the model to solve the problem.

Relate	cost per month	times	number of months	is	amount of the gift certificate

Define Let m = the number of months you can purchase.

Write $\$\ 5 \cdot m = \$\ 30$

19. Solve the equation to find the number of months you can purchase.

$5m = 30$

$\frac{5m}{5} = \frac{30}{5}$

$m = 6$

You can purchase the rental plan for 6 months.

Lesson Check • Do you UNDERSTAND?

Vocabulary **Which property of equality would you use to solve $3 + x = -34$? Why?**

20. What operation does the equation $3 + x = -34$ show?

addition   division   multiplication   subtraction

21. Which property of equality would you use to solve $3 + x = -34$? Explain. Answers may vary.

Sample: Use the Subtraction Property of Equality to subtract 3 from each side of the equation because subtraction is the inverse operation of addition.

Math Success

Check off the vocabulary words that you understand.

- [ ] equivalent equations
- [ ] isolate
- [ ] inverse operations
- [ ] Addition Property
- [ ] Subtraction Property
- [ ] Multiplication Property
- [ ] Division Property

Rate how well you can *use the properties of equality*.

Need to review | 0 2 4 6 8 10 | Now I get it!

2-2 Solving Two-Step Equations

Vocabulary

Review

1. Circle the equation(s) in which the variable is *isolated*.

$6y = 36$ $\frac{x}{2} + 7 = 19$ $\boxed{8 + 4 = w}$ $\boxed{y = 3 - \frac{1}{2}}$

Draw a line from each equation in Column A to its *solution* in Column B.

Column A	Column B
2. $4x = 16$	$x = -6$
3. $5 + x = 17$	$x = 4$
4. $\frac{x}{3} = -2$	$x = 12$

Vocabulary Builder

deduce (noun) **dee DOOS**

Other Word Forms: deducible (adjective), deduction (noun)

Definition: When you **deduce** something, you reach a logical conclusion through reasoning.

Example: You find that when $a = 2$, $0a = 0$; when $a = \frac{1}{3}$, $0a = 0$; and when $a = -7$, $0a = 0$. You **deduce** that for any value of a, $0a$ will equal 0.

Use Your Vocabulary

Place a ✓ in the box if the statement is a logical *deduction*. Place an ✗ if it is NOT a logical *deduction*.

✓ **5.** A multiple of 5 always ends in 0 or 5. So, 240 is a multiple of 5.

✗ **6.** If a number is a whole number, it is also a rational number. So, all rational numbers must be whole numbers.

✓ **7.** Consider $a + b = 100$ for values of a and b. When a increases, b decreases. So, when b increases, a decreases.

A two-step equation involves two operations. To solve $2 \cdot x + 3 = 15$, undo the operations in the *reverse order* of the order of operations.

Order of Operations First multiply. Then add.

$$2 \cdot x + 3 = 15$$

Operations Used to Solve Equations Undo multiplication with division after you undo addition. First, undo addition with subtraction.

Circle the first operation you would undo in solving each equation. Then write the inverse operation you would use to undo the circled operation.

8. $3 \cdot r + 16 = 31$

subtraction

9. $\frac{1}{2} \cdot d - 7 = 10$

addition

10. $12 = -5y + 2$

subtraction

Problem 1 Solving a Two-Step Equation

Got It? **What is the solution of $5 = \frac{t}{2} - 3$?**

11. Circle the first operation you will undo.

addition subtraction multiplication division

12. Circle the second operation you will undo.

addition subtraction multiplication division

13. Which two operations, in order, will you use to solve the equation?

addition then multiplication

14. Now solve the equation.

Solutions may vary.
Sample:

$$5 = \frac{t}{2} - 3$$

$$5 + 3 = \frac{t}{2} - 3 + 3$$

$$8 = \frac{t}{2}$$

$$8 \cdot 2 = \frac{t}{2} \cdot 2$$

$$16 = t$$

Problem 2 Using an Equation as a Model

Got It? **You are making a bulletin board to advertise community service opportunities in your town. You plan to use one quarter of a sheet of construction paper for each ad and four full sheets for the title banner. You have 18 sheets of construction paper. How many ads can you make?**

15. Use the model to complete the equation.

Relate	number of sheets for the ads	plus	number of sheets for the title	is	total number of sheets
Define	Let a = the number of ads that you can make.				
Write	$\frac{1}{4}a$	+	4	=	18

16. Circle the operation you can use to undo multiplication by a fraction.

addition of the opposite   division by the reciprocal   multiplication by the reciprocal

17. Now solve the equation.

$$\frac{1}{4}a + 4 = 18$$
$$\frac{1}{4}a + 4 - 4 = 18 - 4$$
$$\frac{1}{4}a = 14$$
$$4 \cdot \frac{1}{4}a = 4 \cdot 14$$
$$a = 56$$

18. The number of ads that you can make is 56.

Problem 3 Solving With Two Terms in the Numerator

Got It? **What is the solution of $6 = \frac{x-2}{4}$?**

19. The equation has two operations: subtraction and _?_. division

To isolate x, use addition and _?_. multiplication

20. Use the justifications at the right to solve the equation.

$6 = \frac{x-2}{4}$	Write the original equation.
$6 \cdot 4 = \frac{x-2}{4} \cdot 4$	Multiply each side by 4.
$24 = x - 2$	Simplify.
$24 + 2 = x - 2 + 2$	Add 2 to each side.
$26 = x$	Simplify.

Problem 4 Using Deductive Reasoning

Got It? **What is the solution of $\frac{x}{3} - 5 = 4$? Justify each step.**

21. The equation $\frac{x}{3} - 5 = 4$ is solved below. Use one of the reasons from the box to justify each step.

Multiply each side by 3.
Simplify.
Add 5 to each side.

$\frac{x}{3} - 5 = 4$	Write the original equation.
$\frac{x}{3} - 5 + 5 = 4 + 5$	Add 5 to each side.
$\frac{x}{3} = 9$	Simplify.
$3 \cdot \frac{x}{3} = 3 \cdot 9$	Multiply each side by 3.
$x = 27$	Simplify.

Lesson Check • Do you UNDERSTAND?

What properties of equality would you use to solve $-8 = \frac{s}{4} + 3$? What operation would you perform first? Explain.

22. Circle the operations you will undo when you solve $-8 = \frac{s}{4} + 3$.

addition subtraction multiplication division

23. Which properties of equality would you use to undo these operations?

Sample: I would use the Subtraction Property of Equality to undo addition and the Multiplication Property of Equality to undo division.

24. What operation would you perform first? Explain.

Sample: I would subtract 3 from both sides. This isolates the addend containing *s*, which can be isolated by multiplying by 4.

Math Success

Check off the vocabulary words that you understand.

☐ isolated ☐ solution ☐ equation

Rate how well you can solve a *two-step equation.*

Need to review 0 2 4 6 8 10 Now I get it!

2-3 Solving Multi-Step Equations

Vocabulary

Review

1. Circle the *variable* or *variables* in each equation below.

$x - 11 = 35$ $\qquad$ $-2y + 6 + y = 6$ $\qquad$ $2t + 14 = t$ $\qquad$ $19 = 3 + 4b$

2. Find the *solution* of $19 = 3 + 4b$.

$$19 - 3 = 3 + 4b - 3$$
$$16 = 4b$$
$$\frac{16}{4} = \frac{4b}{4}$$
$$4 = b$$

Vocabulary Builder

term (noun) **turm**

Definition: A **term** is a number, a variable, or the product of a number and one or more variables. *Like terms* have exactly the same variable factors.

Main Idea: Combining *like terms* helps you solve equations.

terms

$-2x + 6 = 13x$

like terms

Use Your Vocabulary

3. Write the number of *terms* in each equation.

$14 = x + 6$ 3 $\qquad$ $2z + z - 5 = 10$ 4 $\qquad$ $9 = 6 + 2m - 7m$ 4

4. Look at the variables in each expression below. Write **Y** if the terms of each expression are *like terms*. Write **N** if they are NOT *like terms*.

$5x + (-2x)$ Y $\qquad$ $6w - 6z$ N $\qquad$ $\frac{m}{2} + \frac{m}{3}$ Y

Write T for *true* or F for *false*.

T **5.** Expressions with only numbers are always *like terms*.

T **6.** The expressions *xy* and *yx* are *like terms*.

Problem 1 Combining Like Terms

Got It? **What is the solution of $11m - 8 - 6m = 22$?**

7. Circle the like terms in the expression. $11m - 8 - 6m$

8. Underline the correct word to complete the sentence.

   I can rewrite the equation as $11m - 6m - 8 = 22$ using the

   Associative / <u>Commutative</u> Property of Addition.

9. Now solve the equation.

$$\begin{aligned} 11m - 8 - 6m &= 22 \\ 11m - 6m - 8 &= 22 \\ 5m - 8 &= 22 \\ 5m - 8 + 8 &= 22 + 8 \\ 5m &= 30 \\ \frac{5m}{5} &= \frac{30}{5} \\ m &= 6 \end{aligned}$$

Problem 2 Solving a Multi-Step Equation

Got It? **Noah and Kate are shopping for new guitar strings in a music store. Noah buys 2 packs of strings. Kate buys 2 packs of strings and a music book. The book costs \$16. Their total cost is \$72. How much is one pack of strings?**

10. Complete the model to write the equation.

Relate	amount Noah spent on strings	plus	amount Kate spent on strings and a music book	is	total amount spent by Noah and Kate
Define	Let c = cost of one pack of strings.				
Write	$2 \cdot c$	$+$	$2 \cdot c + 16$	$=$	72

11. Combine like terms to solve the equation.

$$\begin{aligned} 2c + 2c + 16 &= 72 \\ 4c + 16 &= 72 \\ 4c + 16 - 16 &= 72 - 16 \\ 4c &= 56 \\ c &= 14 \end{aligned}$$

12. The cost of one pack of strings is \$ 14 .

Problem 3 Solving an Equation Using the Distributive Property

Got It? **What is the solution of $18 = 3(2x - 6)$? Check your answer.**

13. Use the justifications at the right to solve the equation.

$18 = 3(2x - 6)$	Write the original equation.
$18 = 3 \cdot (2x) - 3 \cdot (6)$	Use the Distributive Property.
$18 = 6 \cdot (x) - 18$	Multiply.
$18 + 18 = 6x - 18 + 18$	Use the Addition Property of Equality.
$36 = 6x$	Add.
$\frac{36}{6} = \frac{6x}{6}$	Use the Division Property of Equality.
$6 = x$	Simplify.

14. Check your answer. $18 = 3(2x - 6)$

$18 \stackrel{?}{=} 3(2 \cdot 6 - 6)$

$18 \stackrel{?}{=} 3 \cdot (6)$

Problem 4 Solving an Equation That Contains Fractions

Got It? **What is the solution of $\frac{2b}{5} + \frac{3b}{4} = 3$? Why did you choose the method you used?**

15. Circle the first step you could use to solve the equation. Then underline the second step you could use.

Combine like terms. Divide each side by 5. Multiply each side by 4. Multiply each side by 20.

16. Suppose you began by writing the fractions with a common denominator. What would your second step be?

Combine like terms.

17. Now use one of the methods from Exercise 15 or Exercise 16 to solve the equation.

Solutions may vary.
Sample:

$20\left(\frac{2b}{5} + \frac{3b}{4}\right) = 20 \cdot 3$

$\frac{40b}{5} + \frac{60b}{4} = 60$

$8b + 15b = 60$

$23b = 60$

$b = \frac{60}{23}$

Problem 5 Solving an Equation That Contains Decimals

Got It? **What is the solution of $0.5x - 2.325 = 3.95$? Check your answer.**

18. Because the equation contains thousandths, multiply each side by 10^{3}, or 1000.

19. Rewrite the equation without decimals.

$500x - 2325 = 3950$

20. Now solve the equation.

$$\begin{aligned} 500x - 2325 &= 3950 \\ 500x &= 6275 \\ x &= 12.55 \end{aligned}$$

21. Check your answer.

Does $0.5 \cdot 12.55 - 2.325 = 3.95$? Yes / No

Lesson Check • Do you UNDERSTAND?

Reasoning **Ben solves the equation $-24 = 5(g + 3)$ by first dividing each side by 5. Amelia solves the equation by using the Distributive Property. Whose method do you prefer? Explain.**

22. Complete Ben's solution and Amelia's solution.

Ben's Solution

$$-24 = 5(g + 3)$$
$$\frac{-24}{5} = \frac{5(g + 3)}{5}$$
$$\frac{-24}{5} - 3 = g + 3 - 3$$
$$-7.8 = g$$

Amelia's Solution

$$-24 = 5(g + 3)$$
$$-24 = 5g + 15$$
$$\frac{-39}{5} = \frac{5g}{5}$$
$$-7.8 = g$$

23. Whose method do you prefer? Explain.

Explanations will vary. Some students may prefer to divide first; others may prefer to use the Distributive Property.

Math Success

Check off the vocabulary words that you understand.

☐ one-step equation ☐ multi-step equation

Rate how well you can *solve a multi-step equation.*

Need to review 0 2 4 6 8 10 Now I get it!

2-4 Solving Equations With Variables on Both Sides

Vocabulary

Review

Write the *like terms* in each expression or equation.

1. $5x + 2x + 6$

$5x$ $2x$

2. $1.5y - 1.2 + 1.2z + y$

$1.5y$ y

3. $\frac{1}{2}x + \frac{1}{2} = 3x$

$\frac{1}{2}x$ $3x$

Vocabulary Builder

identity (noun) eye DEN tuh tee

Main Idea: Any equation that is always true is an **identity**.

Examples: The equation $39 = 39$ is an **identity** because it is always true.
The equation $y + 3 = y + 3$ is an **identity** because it is true for all values of y.

Nonexample: The equation $x + 5 = 8$ is *not* an **identity** because it is *not* always true. It is true only when $x = 3$.

$m = m$ is an ***identity***.

$m = 15$ is NOT an ***identity***.

Use Your Vocabulary

Write a number or expression to make each equation an *identity*.

4. $25 +$ 0 $= 25$

5. $27 \cdot$ 1 $= 27$

6. $-5x + 3 =$ $-5x$ $+ 3$

7. Multiple Choice Which equation is NOT an *identity*?

(A) $0 + 7 = 7$ (B) $1 \cdot 9 = 9$ (C) $x + 3 = 3 + x$ (D) $x + 1 = x$

8. Draw a line from each equation in Column A to its description in Column B.

Column A	Column B
$x = x - 1$	always true
$x + x = 2x$	sometimes true
$5x = 15$	never true

Problem 1 Solving an Equation With Variables on Both Sides

Got It? **What is the solution of $7k + 2 = 4k - 10$?**

9. There is a variable on each side of the equation. Are they like terms? Yes / No

10. Use one of the reasons from the box to justify each step. You may use a reason more than once.

Division Property of Equality
Simplify.
Subtraction Property of Equality
Subtract.

$7k + 2 = 4k - 10$	Write the original equation.
$7k + 2 - 4k = 4k - 10 - 4k$	Subtraction Property of Equality
$3k + 2 = -10$	Subtract.
$3k + 2 - 2 = -10 - 2$	Subtraction Property of Equality
$3k = -12$	Subtract.
$\frac{3k}{3} = -\frac{12}{3}$	Division Property of Equality
$k = -4$	Simplify.

Problem 2 Using an Equation With Variables on Both Sides

Got It? **An office manager spent \$650 on a new energy-saving copier that will reduce the monthly electric bill for the office from \$112 to \$88. In how many months will the copier pay for itself?**

11. Complete the model below.

Relate	cost of the copier	plus	new monthly cost	times	number of months	is	old monthly cost	times	number of months

Define Let $m = \underline{?}$. Circle the correct answer.

Cost of the copier   Number of months   Amount of savings

Write	\$650	+	\$88	·	m	=	\$ 112	·	m

12. Now write and solve the equation.

$650 + 88m = 112m$

$650 + 88m - 88m = 112m - 88m$

$650 = 24m$

$\frac{650}{24} = \frac{24}{24}m$

$27.08 \approx m$

13. The copier will pay for itself in about 27 months.

Problem 3 Solving an Equation With Grouping Symbols

Got It? What is the solution of $4(2y + 1) = 2(y - 13)$?

14. Use the justifications at the right to solve the equation.

$4(2y + 1) = 2(y - 13)$	Write the original equation.
$4 \cdot (2y) + \boxed{4} \cdot (1) = \boxed{2} \cdot (y) - \boxed{2} \cdot (13)$	Distributive Property
$\boxed{8} \cdot (y) + \boxed{4} = \boxed{2} \cdot (y) - \boxed{26}$	Multiply.
$\boxed{8} \cdot (y) + \boxed{4} - 4 = \boxed{2} \cdot (y) - \boxed{26} - 4$	Subtraction Property of Equality
$\boxed{8} \cdot (y) = \boxed{2} \cdot (y) - \boxed{30}$	Subtract.
$\boxed{8} \cdot (y) - \boxed{2} \cdot (y) = \boxed{2} \cdot (y) - \boxed{30} - \boxed{2} \cdot (y)$	Subtraction Property of Equality
$\boxed{6} \cdot (y) = \boxed{-30}$	Subtract.
$\dfrac{\boxed{6y}}{\boxed{6}} = \dfrac{\boxed{-30}}{\boxed{6}}$	Division Property of Equality
$y = \boxed{-5}$	Simplify.

15. Check your answer by substituting it for y in the original equation.

$$4(2 \cdot -5 + 1) \stackrel{?}{=} 2(-5 - 13)$$
$$4(-10 + 1) \stackrel{?}{=} 2(-18)$$
$$4(-9) = -36$$

Problem 4 Identities and Equations With No Solution

Got It? What is the solution of $3(4b - 2) = -6 + 12b$?

16. Circle the first step you would take to isolate the variable. Underline the second step you would take.

Multiply each side by 3. Distribute the 3. <u>Subtract $12b$ from each side.</u>

17. Solve the equation.

$$3(4b - 2) = -6 + 12b$$
$$3(4b) - 3(2) = -6 + 12b$$
$$12b - 6 = -6 + 12b$$
$$12b - 12b - 6 = -6 + 12b - 12b$$
$$-6 = -6$$

18. Because $-6 = -6$ is always true, the original equation has

no solution / <u>infinitely many solutions</u>.

take note

Concept Summary Solving Equations

Remember to follow these steps when solving equations.

STEP 1 Use the Distributive Property to remove any grouping symbols. Use properties of equality to clear decimals and fractions.

STEP 2 Combine like terms on each side of the equation.

STEP 3 Use the properties of equality to get the variable terms on one side of the equation and the constants on the other.

STEP 4 Use the properties of equality to solve for the variable.

STEP 5 Check your solution in the original equation.

Lesson Check • Do you UNDERSTAND?

Vocabulary Tell whether each equation has *infinitely many solutions, one solution,* or *no solution.*

$3y - 5 = y + 2y - 9$ $2y + 4 = 2(y + 2)$ $2y - 4 = 3y - 5$

Write the steps to isolate the variable in each equation.

19. $3y - 5 = y + 2y - 9$

$3y - 5 = y + 2y - 9$
$3y - 5 = 3y - 9$
$-5 = -9$

20. $2y + 4 = 2(y + 2)$

$2y + 4 = 2(y + 2)$
$2y + 4 = 2y + 4$
$4 = 4$

21. $2y - 4 = 3y - 5$

$2y - 4 = 3y - 5$
$-4 = y - 5$
$1 = y$

22. Tell whether each equation has *infinitely many solutions, one solution,* or *no solution.*

$3y - 5 = y + 2y - 9$ no solution

$2y + 4 = 2(y + 2)$ infinitely many

$2y - 4 = 3y - 5$ one solution

Math Success

Check off the vocabulary words that you understand.

☐ like terms ☐ identity

Rate how well you can *solve equations with variables on both sides.*

Need to review 0 2 4 6 8 10 Now I get it!

2-5 Literal Equations and Formulas

Vocabulary

Review

Write the letter of each *formula* next to its description.

A. $C = 2\pi r$ B. $P = 2\ell + 2w$ C. $A = \frac{1}{2}bh$ D. $C = \frac{5}{9}(F - 32)$

B **1.** perimeter (P) of a rectangle with length (ℓ) and width (w)

C **2.** area (A) of a triangle with base (b) and height (h)

A **3.** circumference (C) of a circle with radius (r)

D **4.** temperature in degrees Celsius (C) given the same temperature in degrees Fahrenheit (F)

Vocabulary Builder

These **literal** equations relate distance (d), rate (speed, r), and time (t).

$$d = r \cdot t$$

$$\frac{d}{r} = t$$

$$\frac{d}{t} = r$$

literal (adjective) LIT ur ul

Related Words: letter (noun), literature (noun), literary (adjective)

Definition: When something is **literal**, it uses the exact, or primary, meaning of a word or words. It is also something that uses or is expressed by letters.

Math Usage: A **literal** equation is an equation that involves two or more letters (variables).

Example: The formula in the box is a **literal** equation with three variables. You can solve for any of the three variables in terms of the other two.

Nonexample: $2x + 5 = 12$ is *not* a **literal** equation because it does *not* contain two or more variables.

Use Your Vocabulary

Complete each statement with the appropriate word from the list.

letter   literature   literary   literal equation

5. The word *Boston* begins with the _?_ *B*. **letter**

6. A novel is an example of a _?_ work. **literary**

7. The equation $2x + 5 = 12$ is *not* an example of a _?_ because it has only one variable. **literal equation**

8. You study classic _?_ in English class. **literature**

Problem 1 Rewriting a Literal Equation

Got It? **Solve the equation $4 = 2m - 5n$ for m. What are the values of m when $n = -2, 0,$ and 2?**

Simplify.
Divide each side by 2.
Add $5n$ to both sides.

9. The equation is solved below. Choose a justification from the box for each step.

$4 = 2m - 5n$	Write the original equation.
$4 + 5n = 2m - 5n + 5n$	**Add $5n$ to each side.**
$4 + 5n = 2m$	**Simplify.**
$\frac{4 + 5n}{2} = \frac{2m}{2}$	**Divide each side by 2.**
$2 + \frac{5}{2}n = m$	**Simplify.**

10. Complete the table to find the value of m for each given value of n.

n	Substitute the value of n into the equation.	m
-2	$2 + \frac{5}{2} \cdot -2 = m$ $-3 = m$	-3
0	$2 + \frac{5}{2} \cdot 0 = m$ $2 = m$	2
2	$2 + \frac{5}{2} \cdot 2 = m$ $7 = m$	7

Problem 2 Rewriting a Literal Equation With Only Variables

Got It? **What equation do you get when you solve $-t = r + px$ for x?**

11. Use the justifications at the right to solve the equation.

$-t = r + px$	Write the original equation.
$-t - r = r + px - r$	Subtract the same amount from each side.
$-t - r = px$	Simplify.
$\frac{-t - r}{p} = \frac{px}{p}$	Divide each side by the same amount.
$-\frac{t}{p} - \frac{r}{p} = x$	Simplify.

Problem 3 Rewriting a Geometric Formula

Got It? **What is the height of a triangle that has an area of 24 in.^{2} and a base with a length of 8 in.?**

12. Circle the formula for the area of a triangle.

$A = \pi r^2$ $\quad$ $A = \frac{1}{2}bh$ $\quad$ $d = rt$ $\quad$ $A = \ell w$

13. Circle the rewritten formula you will use to find the height of the triangle.

$\ell = \frac{P - 2w}{2}$ $\quad$ $r = \sqrt{\frac{A}{\pi}}$ $\quad$ $t = \frac{d}{r}$ $\quad$ $h = \frac{2A}{b}$

14. Now find the height of a triangle with an area of 24 in.^{2} and a base of 8 in.

$h = \frac{2 \cdot A}{b}$

$= \frac{2 \cdot 24}{8}$

$= \frac{48}{8}$

$= 6$ in.

Problem 4 Rewriting a Formula

Got It? **Pacific gray whales migrate annually from the waters near Alaska to the waters near Baja California, Mexico, and back. The whales travel a distance of about 5000 mi each way at an average rate of 91 mi per day. About how many days does it take the whales to migrate one way?**

15. Write the formula that relates distance, rate, and time.

$d = r \cdot t$

16. Circle what you are asked to find in the problem.

distance $\quad$ rate $\quad$ time

17. Complete the reasoning model below.

Think	Write
To isolate t, I divide each side of the formula by r.	$\frac{d}{r} = \frac{rt}{r}$
Then I simplify.	$\frac{d}{r} = t$
Now I substitute 5000 for d and 91 for r.	$\frac{5000}{91} = t$

18. Simplify. The whales take about 55 days to migrate one way.

Lesson Check • Do you UNDERSTAND?

Compare and Contrast **How is the process of rewriting literal equations similar to the process of solving equations in one variable? How is it different?**

19. When you rewrite a literal equation, you are solving it for one of the variables. How is this process similar to solving an equation in one variable?

Explanations may vary. Sample: In each case, you need to isolate a variable and solve the equation for that variable.

20. Describe one difference between rewriting a literal equation and solving an equation in one variable.

Sample: When you rewrite a literal equation, you isolate one of the variables from other variables, instead of from numbers.

Math Success

Check off the vocabulary words that you understand.

☐ literal equation ☐ formula

Rate how well you can *rewrite literal equations*.

Need to review 0 2 4 6 8 10 Now I get it!

2-6 Ratios, Rates, and Conversions

Vocabulary

Review

1. Write a fraction with a *numerator* of 12 and a *denominator* of 13. $\frac{12}{13}$

2. Circle the fractions that are in simplest form.

$\frac{15}{30}$ $\quad$ $\frac{5}{18}$ $\quad$ $\frac{17}{42}$ $\quad$ $\frac{22}{33}$

3. Circle the *greatest common divisor* of the *numerator* and the *denominator* of a fraction that is in simplest form.

0 $\quad$ 1 $\quad$ 2 $\quad$ 3

Vocabulary Builder

rate (noun) rayt

Definition: A **rate** is a ratio that compares quantities measured in *different* units.

Examples: miles per gallon, cost per ounce, words per minute

Using Symbols: $\frac{23\text{ mi}}{1\text{ gal}}$, $\frac{\$1.32}{8\text{ oz}}$, $\frac{302\text{ words}}{5\text{ minutes}}$

You read the **rate**
45 mi/h
as
"45 miles per hour."

Use Your Vocabulary

Write a *rate* for each situation.

4. Chandler bicycles 20 miles per hour.

20 mi/h or $\frac{20\text{ mi}}{1\text{ hr}}$

5. Ann makes 80 bagels every 3 days.

$\frac{80\text{ bagels}}{3\text{ days}}$

6. So far, you have read 35 pages out of a 50-page assignment. Explain why the ratio 35 pages out of 50 pages is NOT a *rate*.

Sample: The ratio is not a rate because it compares two quantities in the same unit (pages). A rate compares two quantities in different units.

Problem 1 Comparing Unit Rates

Got It? **The prices for one shirt at three different stores are shown in the box at the right. If Store B lowers its price to $42 for four shirts, which store offers the best deal for one shirt? Explain.**

Price for 1 Shirt
Store A: $12.50
Store B: $11.25
Store C: $10

7. Circle the store that offered the best deal before Store B lowered its price.

Store A Store B Store C

8. Find Store B's new unit rate based on $42 for 4 shirts.

$$\frac{\text{cost of shirts}}{\text{number of shirts}} = \frac{\$42}{4} = \frac{\$10.50}{1 \text{ shirt}}$$

9. Circle the store that offers the best deal now.

Store A Store B Store C

10. Why does this store have the best deal?

Explanations may vary. Sample: Store C still has the best unit price because $10 per shirt is still the lowest price.

A *conversion factor* is a ratio of two equivalent measures in different units. A conversion factor is always equal to 1.

11. Complete each conversion factor.

$\frac{1 \text{ ft}}{12 \text{ in.}}$ $\frac{1 \text{ mi}}{5280 \text{ ft}}$ $\frac{1 \text{ m}}{100 \text{ cm}}$ $\frac{1 \text{ h}}{60 \text{ min}}$

Problem 2 Converting Units

Got It? **What is 1250 cm converted to meters?**

12. There are 100 centimeters in one meter.

Underline the correct word to complete each sentence.

13. When you convert from centimeters to meters, the number of meters will be greater than / <u>less than</u> the number of centimeters.

14. When you convert from centimeters to meters, the appropriate conversion factor will allow you to multiply / <u>divide</u> out the common units.

15. **Multiple Choice** Choose the conversion factor for converting centimeters to meters.

Ⓐ $\frac{1 \text{ m}}{100 \text{ cm}}$ Ⓑ $\frac{1 \text{ m}}{1000 \text{ cm}}$ Ⓒ $\frac{100 \text{ cm}}{1 \text{ m}}$ Ⓓ $\frac{1000 \text{ cm}}{1 \text{ m}}$

16. Complete the conversion.

$$1250 \text{ cm} \cdot \frac{1 \text{ m}}{100 \text{ cm}} = 12.5 \text{ m}$$

Problem 3 Converting Units Between Systems

Got It? The Sears Tower in Chicago, Illinois, is 1450 ft tall. How many meters tall is the tower? Use the fact that 1 m ≈ 3.28 ft.

17. Follow the steps to find how many meters tall the Sears Tower is.

1 Write the conversion factor as a ratio. Remember, the units to be divided out should be in the denominator.

$$\frac{1 \text{ m}}{3.28 \text{ ft}}$$

2 Find the height of the tower.

$$1450 \text{ ft} \cdot \frac{1 \text{ m}}{3.28 \text{ ft}} \approx 442 \text{ m}$$

3 The height of the Sears Tower is about 442 meters.

You can also convert rates. Because rates compare measures in two different units, you must multiply by two conversion factors to change both of the units.

Problem 4 Converting Rates

Got It? An athlete ran a sprint of 100 ft in 3.1 s. At what speed was the athlete running in miles per hour? Round to the nearest mile per hour.

18. Circle what you know. Underline what you want to find out.

speed of the athlete in feet per second   speed of the athlete in miles per hour

19. Underline the correct word to complete the sentence.

When writing a conversion factor, if the unit to be converted is in the numerator, then that unit should be in the numerator / denominator of the conversion factor.

20. You will need to perform two conversions to solve the problem. Circle the conversion factor you will use to convert to miles per second. Underline the conversion factor you will use to convert to miles per hour.

$\frac{1 \text{ mi}}{5280 \text{ ft}}$ $\frac{5280 \text{ ft}}{1 \text{ mi}}$ $\frac{3600 \text{ s}}{1 \text{ h}}$ $\frac{1 \text{ h}}{3600 \text{ s}}$

21. Use the conversion factors to solve the problem.

$$\frac{100 \text{ ft}}{3.1 \text{ s}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ s}}{1 \text{ h}} = \frac{360{,}000 \text{ mi}}{16{,}368 \text{ h}} \approx 22 \text{ mi/h}$$

Lesson Check • Do you UNDERSTAND?

Reasoning **Does multiplying by a conversion factor change the amount of what is being measured? How do you know?**

22. Circle the equations that are true.

$39 \cdot 1 = 39$ $\qquad 1 \cdot x = x$ $\qquad x \cdot 1 = x + 1$ $\qquad \frac{5}{5} \cdot x = x$

23. A conversion factor is always equal to **1**.

24. Underline the correct word, words, or number to complete the sentence.

Multiplying by a conversion factor changes / <u>does not change</u> what is being measured because you are multiplying by 0 / <u>1</u>.

Lesson Check • Do you UNDERSTAND?

Reasoning **If you convert pounds to ounces, will the number of ounces be *greater* or *less* than the number of pounds? Explain.**

25. There are **16** ounces in 1 pound.

26. Convert 2 pounds to ounces.

$$2 \text{ lb} \cdot \frac{16 \text{ oz}}{1 \text{ lb}} = 32 \text{ oz}$$

27. Convert 48 ounces to pounds.

$$48 \text{ oz} \cdot \frac{1 \text{ lb}}{16 \text{ oz}} = 3 \text{ lb}$$

28. Underline the correct word to complete the sentence.
If you convert pounds to ounces, the number of ounces will be <u>greater</u> / less than the number of pounds.

29. Explain your answer to Exercise 28.

Sample: When converting from a larger unit to a smaller unit, there will be a greater number of the smaller units.

Math Success

Check off the vocabulary words that you understand.

☐ ratio ☐ rate ☐ unit rate ☐ conversion factor ☐ unit analysis

Rate how well you can *compare and convert ratios and rates.*

Need to review 0 2 4 6 8 10 Now I get it!

2-7 Solving Proportions

Vocabulary

Review

Write each *unit rate* in words.

1. 65 mi/h — sixty-five miles per hour
2. 7 ft/day — seven feet per day
3. $3.99/lb — three dollars ninety-nine cents per pound
4. 11 km/s — eleven kilometers per second

Vocabulary Builder

A **proportion** always has an **equal sign**.

$$\frac{1}{5} = \frac{6}{30}$$

proportion (noun) pruh PAWR shun

Definition: A **proportion** is an equation that states that two ratios are equal.

What It Means: Any equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, is a **proportion**. You read a proportion "a is to b as c is to d."

Related Word: proportional (adjective)

Use Your Vocabulary

Complete each statement with the correct word from the list below.

proportion ratios proportional

5. A scaled map of the roads in a city is _?_ to the actual roads. proportional
6. When making fruit punch, you have to be sure that the amount of ginger ale is in _?_ to the amount of fruit juice. proportion
7. Because $\frac{5}{8}$ is not equal to $\frac{15}{20}$, the _?_ $\frac{5}{8}$ and $\frac{15}{20}$ do not form a proportion. ratios

Problem 1 Solving a Proportion Using the Multiplication Property

Got It? **What is the solution of the proportion $\frac{x}{7} = \frac{4}{5}$?**

8. Use the justifications at the right to solve the proportion.

$\frac{x}{7} = \frac{4}{5}$	Write the original equation.
$\boxed{7} \cdot \frac{x}{7} = \boxed{7} \cdot \frac{4}{5}$	Multiply each side by $\boxed{7}$.
$\boxed{x} = \frac{28}{5}$	Simplify.
$x = \boxed{5.6}$	Divide.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products ad and bc are called *cross products*. You can use the following property of cross products to solve proportions.

take note

Property Cross Products Property of a Proportion

9. Complete the table.

Words	The cross products of a proportion are equal.
Algebra	If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = \boxed{bc}$.
Example	$\frac{2}{3} = \frac{8}{12}$, so $2 \cdot \boxed{12} = 3 \cdot \boxed{8}$, or $24 = \boxed{24}$.

Problem 2 Solving a Proportion Using the Cross Products Property

Got It? **What is the solution of the proportion $\frac{y}{3} = \frac{3}{5}$?**

10. Use the model to help you find the cross products.

$5 \cdot \boxed{y} \longrightarrow \frac{y}{3} = \frac{3}{5} \longleftarrow 3 \cdot \boxed{3}$

11. Solve the proportion $\frac{y}{3} = \frac{3}{5}$.

$5y = 3(3)$
$5y = 9$
$y = 1.8$

Problem 3 Solving a Multi-Step Proportion

Got It? What is the solution of the proportion $\frac{n}{5} = \frac{2n+4}{6}$?

12. Complete the reasoning model below.

Think	Write
First I write the original proportion.	$\frac{n}{5} = \frac{2n+4}{6}$
Next I use the Cross Products Property.	$(6)n = 5(2n + 4)$
Then I use the Distributive Property.	$6 \cdot n = 10 \cdot n + 20$
I subtract $10n$ from each side.	$6n - 10n = 10n + 20 - 10n$
I simplify both sides.	$-4 \cdot n = 20$
Now I divide each side by -4.	$\frac{-4n}{-4} = \frac{20}{-4}$
Finally, I simplify.	$n = -5$

When you model a real-world situation with a proportion, you must write the proportion carefully. Be sure that the order of what is compared in each ratio is the same.

Correct: $\frac{100 \text{ mi}}{2 \text{ h}} = \frac{x \text{ mi}}{5 \text{ h}}$ **Incorrect:** $\frac{100 \text{ mi}}{2\text{h}} = \frac{5 \text{ h}}{x \text{ mi}}$

13. Suppose you can buy 3 pounds of meat for $12. Cross out the proportion below that will NOT help you find the cost of 5 pounds of meat.

$\frac{12 \text{ dollars}}{3 \text{ lb}} = \frac{x \text{ dollars}}{5 \text{ lb}}$ $\cancel{\frac{12 \text{ dollars}}{3 \text{ lb}} = \frac{5 \text{ lb}}{x \text{ dollars}}}$ $\frac{3 \text{ lb}}{12 \text{ dollars}} = \frac{5 \text{ lb}}{x \text{ dollars}}$

14. Suppose you need 9 pieces of wood to build 4 birdhouses. Cross out the proportion below that will NOT help you find the number of pieces of wood you will need to build 15 birdhouses.

$\frac{15 \text{ birdhouses}}{x \text{ pieces}} = \frac{4 \text{ birdhouses}}{9 \text{ pieces}}$ $\frac{9 \text{ pieces}}{4 \text{ birdhouses}} = \frac{x \text{ pieces}}{15 \text{ birdhouses}}$ $\cancel{\frac{9 \text{ pieces}}{15 \text{ birdhouses}} = \frac{x \text{ pieces}}{4 \text{ birdhouses}}}$

15. Suppose you can knit 3 scarves from 5 packages of yarn. Let $x =$ the number of scarves you can knit from 12 packages of yarn. Complete the proportion.

$\frac{5 \text{ packages}}{3 \text{ scarves}} = \frac{12 \text{ packages}}{x \text{ scarves}}$

Problem 4 Using a Proportion to Solve a Problem

Got It? **An 8-oz can of orange juice contains about 97 mg of vitamin C. About how many milligrams of vitamin C are there in a 12-oz can of orange juice?**

Selections may vary. Accept either of the circled proportions.

16. Let c = number of mg of vitamin C.

17. Circle the proportion you will use to solve this problem.

$\frac{8\text{ oz}}{12\text{ oz}} = \frac{c\text{ mg}}{97\text{ mg}}$ $\quad \frac{12\text{ oz}}{8\text{ oz}} = \frac{c\text{ mg}}{97\text{ mg}}$ $\quad \frac{8\text{ oz}}{97\text{ mg}} = \frac{12\text{ oz}}{c\text{ mg}}$ $\quad \frac{12\text{ oz}}{8\text{ oz}} = \frac{97\text{ mg}}{c\text{ mg}}$

18. Solve the problem using the proportion you chose.

Sample: $\frac{8\text{ oz}}{97\text{ mg}} = \frac{12\text{ oz}}{c\text{ mg}}$

$8c = 97(12)$

$8c = 1164$

$c = 145.5$

19. There are about 145.5 mg of vitamin C in a 12-oz can of orange juice.

Lesson Check • Do you UNDERSTAND?

Reasoning **When solving $\frac{x}{5} = \frac{3}{4}$, Lisa's first step was to write $4x = 5(3)$. Jen's first step was to write $20\left(\frac{x}{5}\right) = 20\left(\frac{3}{4}\right)$. Will both methods work? Explain.**

20. Circle the property that Lisa used. Underline the property that Jen used.

Multiplication Property $\quad$ Cross Products Property

21. Solve: $4x = 5(3)$.

$4x = 5(3)$

$4x = 15$

$x = 3.75$

22. Solve: $20\left(\frac{x}{5}\right) = 20\left(\frac{3}{4}\right)$.

$20\left(\frac{x}{5}\right) = 20\left(\frac{3}{4}\right)$

$4x = 15$

$x = 3.75$

23. Will both methods work? Explain.

Yes. Both methods give the same correct solution.

Math Success

Check off the vocabulary words that you understand.

☐ proportion ☐ cross products ☐ Cross Products Property

Rate how well you can *solve proportions*.

Need to review 0 2 4 6 8 10 Now I get it!

2-8 Proportions and Similar Figures

Vocabulary

Review

Do the ratios in each pair form a *proportion*? Explain.

1. $\frac{2}{5}$ and $\frac{10}{25}$

Yes. The ratios are equal.

2. $\frac{1}{3}$ and $\frac{30}{100}$

No. $\frac{1}{3} \neq \frac{30}{100}$.

Vocabulary Builder

similar figures

similar (adjective) SIM **uh lur**

Related Word: similarly (adverb)

Definition: Objects are **similar** if they are alike, but not necessarily identical.

Main Idea: In mathematics, **similar** figures have the same shape, but not necessarily the same size.

Use Your Vocabulary

3. Explain how a lion and a giraffe are *similar*. **Answers may vary. Samples are given.**

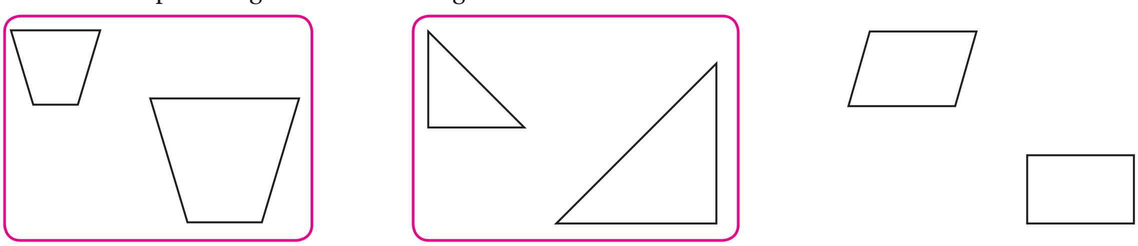
How is a lion like a giraffe?

A lion and a giraffe are similar because each is an animal.

How is a lion different from a giraffe?

They are different because one eats meat and the other eats plants.

4. Consider each pair of figures. Circle the figures that are *similar*.

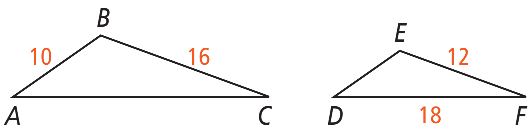


The symbol $\sim$ means "is similar to." In Problem 1 below, $\triangle ABC \sim \triangle DEF$.

In similar figures, the measures of corresponding angles are equal, and corresponding side lengths are in proportion. In Problem 1, the pairs of corresponding sides are $\overline{AB}$ and $\overline{DE}$, $\overline{AC}$ and $\overline{DF}$, and $\overline{BC}$ and $\overline{EF}$.

Problem 1 Finding the Length of a Side

Got It? **In the diagram, $\triangle ABC \sim \triangle DEF$. What is AC?**



5. Underline the correct word or words to complete the sentence.

Because the triangles are similar, the ratios of the corresponding sides are

equal / not equal .

6. Use the diagram above. Circle the ratio that forms a proportion with $\frac{BC}{EF}$.

$\frac{AC}{DE}$ $\frac{AC}{EF}$ $\frac{AC}{DF}$ $\frac{AC}{AB}$

7. Use the ratios from Exercise 6 to write a proportion. Solve your proportion for AC.

$\frac{BC}{EF} = \frac{AC}{DF}$

$\frac{16}{12} = \frac{AC}{18}$

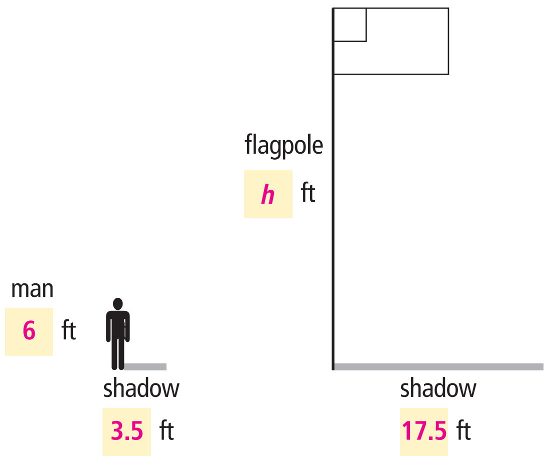
$12(AC) = 16(18)$

$AC = 24$

Problem 2 Applying Similarity

Got It? **A man who is 6 ft tall is standing next to a flagpole. The shadow of the man is 3.5 ft and the shadow of the flagpole is 17.5 ft. What is the height of the flagpole?**

8. Label the diagram. Let h = the height of the flagpole.



9. Complete the reasoning model below.

Think	Write
First I write a proportion to find the height, h, of the flagpole.	$\frac{6}{3.5} = \frac{h}{17.5}$
Then I use the Cross Products Property.	$3.5 \cdot h = 6 \cdot 17.5$
Then I simplify.	$3.5h = 105$
Now I divide each side by 3.5.	$\frac{3.5h}{3.5} = \frac{105}{3.5}$
And now I simplify.	$h = 30$
Finally I write a sentence to answer the question.	The height of the flagpole is 30 ft.

Problem 3 Interpreting Scale Drawings

Got It? On a map the scale is 1 in. : 110 mi. The distance from Jacksonville to Gainesville on the map is about 0.6 in. What is the actual distance from Jacksonville to Gainesville?

10. Let x = the actual distance from Jacksonville to Gainesville.

11. Use the given information to write and solve a proportion.

$$\frac{1}{110} = \frac{0.6}{x}$$
$$1(x) = 110 \cdot 0.6$$
$$x = 66$$

12. The actual distance from Jacksonville to Gainesville is 66 miles.

Problem 4 Using Scale Models

Got It? A scale model of a building is 6 in. tall. The scale of the model is 1 in. : 50 ft. How tall is the actual building?

13. Complete the equation in the model.

Relate scale of model equals $\dfrac{\text{model height}}{\text{actual height}}$

Define Let x = the actual height of the building.

Write $\dfrac{1}{50} = \dfrac{6}{x}$

14. Now write and solve a proportion.

$\frac{1}{50} = \frac{6}{x}$

$1(x) = 6(50)$

$x = 300$

15. The actual building is 300 ft tall.

Lesson Check • Do you UNDERSTAND?

Reasoning **Suppose $\triangle ABC \sim \triangle TUV$. Determine whether each pair of measures is equal.**

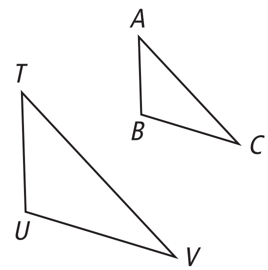
the measures of $\angle A$ and $\angle T$ the perimeters of the two triangles the ratios of the sides $\frac{BC}{UV}$ and $\frac{AC}{TV}$

Underline the correct word to complete each sentence.

16. In similar triangles, corresponding sides always have the same length / ratio.

17. In similar triangles, corresponding angles always have equal / unequal measures.

Use the triangles at the right. Write T for *true* or F for *false*.



T **18.** The measures of $\angle A$ and $\angle T$ are equal.

F **19.** The perimeters of the two triangles are equal.

T **20.** The ratios $\frac{BC}{UV}$ and $\frac{AC}{TV}$ are equal.

Math Success

Check off the vocabulary words that you understand.

☐ similar figures ☐ scale ☐ scale drawing ☐ scale model

Rate how well you can *use proportions to solve similar-figure problems*.

Need to review 0 2 4 6 8 10 Now I get it!

2-9 Percents

Vocabulary

Review

1. Circle each *rational* number.

$\sqrt{5}$ $\quad$ $\frac{19}{100}$ $\quad$ $\sqrt{64}$ $\quad$ -3.89

Write an *equivalent fraction* with a denominator of 100.

2. $\frac{4}{5}$ $\frac{80}{100}$ $\qquad$ **3.** $\frac{7}{25}$ $\frac{28}{100}$ $\qquad$ **4.** $\frac{8}{50}$ $\frac{16}{100}$

Vocabulary Builder

percent (noun) pur SENT

Related Words: cents (noun), century (noun), centimeter (noun)

Definition: A **percent** is a ratio that compares a number to 100.

Word Origin: **per** means "for every"; **-cent** means "hundred."
So, 39 **percent** means "39 for every hundred."

The symbol for **percent** is **%**.

Use Your Vocabulary

Complete each statement with the correct word from the list below.

percent $\qquad$ cents $\qquad$ century $\qquad$ centimeters

5. One dollar has the same value as 100 _?_. cents

6. There are 100 years in a _?_. century

7. There are 100 _?_ in 1 meter. centimeter

8. One part out of 100 is 1 _?_. percent

take note — Key Concept: The Percent Proportion and Percent Equation

You can represent "a is p percent of b" using either the percent proportion or the percent equation. In each case, b is the *base* and a is a *part* of *base* b.

The Percent Proportion

$$\frac{a}{b} = \frac{p}{100}$$

where base $b \neq 0$

The Percent Equation

$$a = p\% \cdot b$$

9. Complete the percent proportion and the percent equation by placing *part, whole,* and p in the correct places.

$$\frac{\text{part}}{\text{whole}} = \frac{p}{100} \qquad \text{part} = p\,\% \cdot \text{whole}$$

Problem 2 Finding a Percent Using the Percent Equation

Got It? Reasoning What percent of 84 is 63? Use the percent equation to solve. Then use the percent proportion. Compare your answers.

10. Solve the *percent equation* for p.

$$\text{part} = p\% \cdot \text{whole}$$
$$63 = p\% \cdot 84$$
$$\frac{63}{84} = p\%$$
$$0.75 = p\%$$
$$(0.75 \cdot 100)\% = p\%$$
$$75 = p$$

11. Solve the *percent proportion* for p.

$$\frac{\text{part}}{\text{whole}} = \frac{p}{100}$$
$$\frac{63}{84} = \frac{p}{100}$$
$$63 \cdot 100 = 84 \cdot p$$
$$6300 = 84p$$
$$75 = p$$

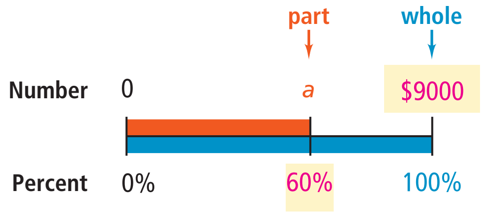
12. Compare your answers.

In each case, you find that 63 is 75% of 84.

Problem 3 Finding a Part

Got It? A family sells a car to a dealership for 60% less than they paid for it. They paid \$9000 for the car. For what price did they sell the car?

13. Complete the model. Then use the model to complete and solve the percent proportion.



$$\frac{a}{9000} = \frac{60}{100}$$

$$100a = 60 \cdot 9000$$

$$100a = 540{,}000$$

$$\frac{100a}{100} = \frac{540{,}000}{100}$$

$$a = 5400$$

14. Find the selling price of the car: 9000 − 5400 = 3600

15. The family sold the car for $ 3600 .

Problem 4 Finding a Base

Got It? **30% of what number is 12.5? Solve the problem using the percent equation. Then solve the problem using the percent proportion.**

16. In the problem, the unknown quantity is base *b* / part *a* .

17. Solve the problem using the percent equation and the percent proportion.

Percent Equation

$a = p\% \cdot b$

$$12.5 = 30\% \cdot b$$
$$12.5 = 0.30 \cdot b$$
$$\frac{12.5}{0.30} = \frac{0.30b}{0.30}$$
$$41.7 \approx b$$

Percent Proportion

$\frac{a}{b} = \frac{p}{100}$

$$\frac{12.5}{b} = \frac{30}{100}$$
$$12.5(100) = 30b$$
$$1250 = 30b$$
$$41.7 \approx b$$

18. 30% of 41.7 is about 12.5.

take note

Key Concept Simple Interest Formula

Simple interest is interest you earn on only the principal in an account. The simple interest formula is given below, where I is the interest, P is the principal, r is the annual interest rate, written as a decimal, and t is the time in years.

$$I = Prt$$

19. You invest $100 at a simple interest rate of 2.5% per year for 6 years. Write an equation to show how much interest you will earn.

First, write the interest rate, 2.5%, as a decimal.

Remember to insert leading zeros. 2.5% = 0.025

Now write the equation. $I = 100 \cdot 0.025 \cdot 6$

Problem 5 Using the Simple Interest Formula

Got It? **You deposited $125 in a savings account that earns a simple interest rate of 1.75% per year. You earned a total of $8.75 in interest. For how long was your money in the account?**

20. Complete the model.

Relate	interest	is	principal	times	annual interest rate	times	time in years
Define	Let t = time in years.						
Write	$8.75	=	$125	·	1.75 %	·	t

21. As a decimal, 1.75% = 0.0175.

22. Now solve for t.

$8.75 = 125(1.75\%)(t)$

$8.75 = 125(0.0175)(t)$

$8.75 = 2.1875(t)$

$4 = t$

23. Your money was in the account for 4 years.

Lesson Check • Do you UNDERSTAND?

Open-Ended **Give an example of a percent problem where the part is greater than the base.**

24. Place a ✓ if the situation has a part greater than the whole. Place an ✗ if the situation does NOT have a part greater than the whole.

✗ The green marbles in a jar of red, green, and blue marbles

✓ Your math test score when you answer every question and the extra credit question correctly

✗ The part of chicken stew that is chicken

Math Success

Check off the vocabulary words that you understand.

- ☐ percent
- ☐ part
- ☐ base

Rate how well you can ***solve percent problems***.

Need to review 0 2 4 6 8 10 Now I get it!

2-10 Change Expressed as a Percent

Vocabulary

Review

1. What is a *percent*? Use the term *ratio* in your definition.

Answers may vary. Sample: A percent is a ratio that compares a number to 100.

2. Write the *percent* of a dollar each coin represents.

penny	nickel	dime	quarter
1 %	5 %	10 %	25 %

Vocabulary Builder

change (noun) chaynj

Main Idea: When a quantity increases or decreases, it undergoes a **change**.

Examples: If the temperature of a room **changes** from 78°F to 75°F, the **change** is a *decrease* of 3°F. If the temperature of the room **changes** from 65°F to 69°F, the **change** is an *increase* of 4°F.

Use Your Vocabulary

Describe each *change* as an *increase* or a *decrease*.

3. 72 to 84

increase

4. 25 to 16

decrease

5. \$.99 to \$1.02

increase

You can find the percent change when you know the original amount and how much it has changed.

If the new amount is greater than the original amount, the percent change is a *percent increase*. If the new amount is less than the original amount, the percent change is a *percent decrease*.

take note

Key Concept Percent Change

Percent change is the ratio of the **amount of change** to the original amount. The **amount of change** is the amount of increase or decrease.

Percent Change

$$p\% = \frac{\text{amount of increase or decrease}}{\text{original amount}}$$

6. Draw a line from each phrase in Column A to the correct subtraction expression in Column B.

Column A	Column B
amount of increase	original amount − new amount
amount of decrease	new amount − original amount

Problem 1 Finding a Percent Decrease

Got It? The average monthly precipitation for Chicago, Illinois, peaks in June at 4.1 in. The average precipitation in December is 2.8 in. What is the percent decrease from June to December?

7. Write an expression to show the change in temperature from June to December.

4.1 − 2.8

8. Complete the equation.

Percent change $= \dfrac{4.1 - 2.8}{4.1}$ Substitute.

$= \dfrac{1.3}{4.1}$ Simplify.

$\approx$ 31.7 % Write as a percent.

9. The percent decrease in precipitation is about 32 %.

Problem 2 Finding a Percent Increase

Got It? In one year, the toll for passenger cars to use a tunnel rose from $3 to $3.50. What was the percent increase?

10. The new amount of the toll is $ 3.50 .

The original amount of the toll is $ 3.00 .

11. Explain how you know you are finding a percent increase.

Sample: The new amount of the toll is greater than the original amount of the toll. Therefore, I will be finding a percent increase.

12. Substitute the values you know into the Percent Change formula.

$$\text{Percent change} = \frac{3.5 - 3}{3}$$

13. Now solve the equation. Write the result as a percent.

Percent change $= \frac{3.5 - 3}{3}$

Percent change $= \frac{0.5}{3}$

Percent change $\approx$ 0.166 or about 17%

14. The price of the toll increased by about 17 %.

Problem 3 Finding Percent Error

Got It? You think that the distance between your house and a friend's house is 5.5 mi. The actual distance is 4.75 mi. What is the percent error in your estimation?

15. In the percent error ratio, you find an absolute value in the numerator. The absolute value of a number is always negative / nonnegative.

16. Complete the steps to solve the problem.

$$\text{percent error} = \frac{|\text{estimated value} - \text{actual value}|}{\text{actual value}}$$ Write the ratio.

$$= \frac{|5.5 - 4.75|}{4.75}$$ Substitute.

$$= \frac{0.75}{4.75}$$ Simplify.

$\approx$ 0.158, or about 16 % Write the result as a percent.

Problem 5 Finding the Greatest Possible Percent Error

Got It? The diagram at the right shows the dimensions of a gift box to the nearest inch. Its measured volume is 360 in.^{3}, and the greatest possible error in volume is about 24%. If the gift box's dimensions were taken to the nearest half inch, how would the greatest possible error be affected?

17. The greatest possible error in each measurement is half the unit of measure. Find the least and greatest possible measurements for each dimension.

$5 - 0.25 =$ 4.75 $\quad$ $6 - 0.25 =$ 5.75 $\quad$ $12 -$ 0.25 $=$ 11.75

$5 + 0.25 =$ 5.25 $\quad$ $6 +$ 0.25 $=$ 6.25 $\quad$ 12 $+$ 0.25 $=$ 12.25

18. Find the minimum and maximum possible volumes. Use $V = \ell wh$.

$V = (11.75) \cdot (5.75) \cdot$ **4.75** $\approx$ **320.922** in.^{3}

$V = (12.25) \cdot (6.25) \cdot$ **5.25** $\approx$ **401.953** in.^{3}

19. Now find the differences and circle the greater difference.

minimum volume difference

360 − **320.922** = **39.078**

maximum volume difference

401.953 − 360 = **41.953**

20. Complete the equation to determine the greatest possible percent error.

$$\frac{\text{greater difference in volume}}{\text{measured volume}} = \frac{41.953}{360} \approx 0.117$$, or about **12** %

21. Compare your answer from Exercise 20 to the original greatest possible error of about 24%. How is greatest possible error affected if you measure to the nearest half inch rather than to the nearest inch?

Answers may vary. Sample: It is about half the amount.

Lesson Check • Do you UNDERSTAND?

Vocabulary **Determine whether each situation involves a *percent increase* or a *percent decrease*.**

A hat that originally cost $12 sold for $9.50.

You buy a CD for $10 and sell it for $8.

A store buys glasses wholesale for $2 per glass. The store sells them for $4.50.

Underline the correct word to complete each sentence.

22. When the new amount is greater than the original amount, the percent change is a percent increase / decrease .

23. When the new amount is less than the original amount, the percent change is a percent increase / decrease .

24. The price of the hat went down, so it is a percent increase / decrease .

25. The price of the CD went down, so it is a percent increase / decrease .

26. The price of the glasses went up / down , so it is a percent increase / decrease .

Math Success

Check off the vocabulary words that you understand.

- ☐ percent change
- ☐ percent increase
- ☐ percent decrease
- ☐ percent error

Rate how well you can *solve percent increase and decrease problems*.

Need to review 0 2 4 6 8 10 Now I get it!