1-1 Additional Vocabulary Support

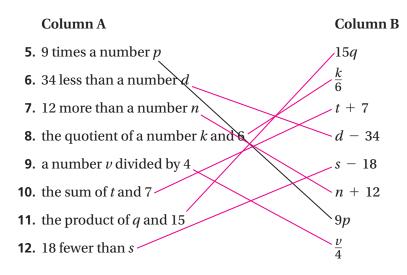
Variables and Expressions

difference	divided by	less	more than
product	quotient	sum	times

Use the list to write two words or word phrases that represent each operation.

1. Addition	sum; more than
2. Subtraction	less; difference
3. Multiplication	times; product
4. Division	quotient; divided by

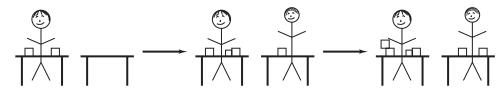
For Exercises 5–12, draw a line from each phrase in Column A to a matching algebraic expression in Column B. The first one is done for you.



1_1 Think About a Plan

Variables and Expressions

Volunteering Serena and Tyler are wrapping gift boxes at the same pace. Serena starts first, as shown in the diagram. Write an algebraic expression that represents the number of boxes Tyler will have wrapped when Serena has wrapped *x* boxes.



Think

1. Since Serena started first she will always have more boxes than Tyler. How many boxes did Serena wrap before Tyler started?

2

Plan

2. Examine the situation. What phrase in the situation could be rewritten as an algebraic symbol? What is the associated symbol?

"two fewer boxes" can be rewritten using subtraction; -

Solve

3. When Serena has wrapped *x* boxes, how many boxes has Tyler wrapped?

x - 2

4. Could this situation be expressed in another manner? Explain and give an example to prove your point.

Yes; you can express the number of boxes Serena has wrapped in relation to Tyler. This would be expressed as x + 2.

Practice

Form G

Variables and Expressions

Write an algebraic expression for each word phrase.

1. 10 less than *x*

$$x - 10$$

3. 7 minus *f*

$$7 - f$$

5. *x* multiplied by 6

7. one fourth of a number n

$$n \div 4$$

9. the quotient of 15 and y

11. 3 plus the product of 2 and h

$$3 + 2 \cdot h$$

2. 5 more than *d*

$$5+d$$

4. the sum of 11 and k

$$11 + k$$

6. a number *t* divided by 3

8. the product of 2.5 and a number t

$$2.5 \cdot t$$

10. a number q tripled

$$q \cdot 3$$

12. 3 less than the quotient of 20 and x

$$20 \div x - 3$$

Write a word phrase for each algebraic expression.

13. n + 6

the sum of *n* and 6

14. 5 - c

5 less than c

15. 11.5 + y

the sum of 11.5 and y

16. $\frac{x}{4} - 17$

17 less than the quotient of x and 4 **17.** 3x + 10

10 more than the product of 3 and x **18.** 10x + 7z

the sum of 10x and 7z

Write a rule in words and as an algebraic expression to model the relationship in each table.

19. The local video store charges a monthly membership fee of \$5 and \$2.25 per video.

	Videos (v)	Cost (c)
	1	\$7.25
	2	\$9.50
	3	\$11.75
٦		

\$5 plus \$2.25 times the number of videos; 5 + 2.25v

1_1 Practice (continued)

Form G

Variables and Expressions

20. Dorothy gets paid to walk her neighbor's dog. For every week that she walks the dog, she earns \$10.

	Weeks (w)	Pay (<i>p</i>)
(4	\$40.00
	5	\$50.00
(6	\$60.00
τ		

\$10 times the number of weeks; 10w

Write an algebraic expression for each word phrase.

- **21.** 8 minus the quotient of 15 and $y = 8 15 \div y$
- 22. a number q tripled plus z doubled 3q + 2z
- 23. the product of 8 and z plus the product of 6.5 and y 8z + 6.5y
- **24.** the quotient of 5 plus d and 12 minus $w = \frac{5+d}{12-w}$
- **25. Error Analysis** A student writes $5y \cdot 3$ to model the relationship *the sum of 5y and 3*. Explain the error.

The word "sum" indicates that addition should be used and not multiplication. The student has used the multiplication symbol instead of the +.

26. Error Analysis A student writes *the difference between 15 and the product of 5 and y* to describe the expression 5y - 15. Explain the error.

The number 15 should be first and the expression should be written 15 - 5y.

27. Jake is trying to mail a package to his grandmother. He already has *s* stamps on the package. The postal worker tells him that he's going to have to double the number of stamps on the package and then add 3 more. Write an algebraic expression that represents the number of stamps that Jake will have to put on the package.

2s + 3

Practice

Form K

Variables and Expressions

Write an algebraic expression for each word phrase.

1. 11 more than *y*

$$y + 11$$

2. 5 less than *n*

$$n-5$$

3. the sum of 15 and w

$$w + 15$$

4. 22 minus *k*

$$22 - k$$

5. a number *b* divided by 8

6. *q* multiplied by 2

7. the product of 3.3 and a number x

8. one third of a number m

$$\frac{1}{3}m$$

Write a word phrase for each algebraic expression.

9. 8 - a

8 minus a number a

10. v + 9

the sum of a number v and 9

11. $\frac{y}{5} - 10$

the quotient of a number y and 5 minus 10

12. 1.9 + n

the sum of 1.9 and a number n

13. 5h + 3k

the sum of 5 times a number h and 3 times a number k

14. 2x + 1

the sum of 2 times number x and 1

Write a rule in words and as an algebraic expression to model the relationship.

15. The cost of beverages in a vending machine is shown. y = 1.25x

Beverages	Cost
1	\$1.25
2	\$2.50
3	\$3.75

1_1 Practice (continued)

Form K

Variables and Expressions

16. Jordan gets paid to mow his neighbor's lawn. For every week that he mows the lawn, he earns \$20. Write a rule as an algebraic expression to model the relationship.

y = 20x

Write an algebraic expression for each word phrase.

17. 14 minus the quotient of 25 and p

 $14 - \frac{25}{p}$

18. a number w tripled plus t quadrupled

3w + 4t

19. the product of 13 and m plus the product of 2.7 and n

13m + 2.7n

20. the product of 2 times a and 5 times b

2a(5b), or 10ab

21. Error Analysis A student writes *the sum of 7 times a number n plus 5* to describe the expression 7(n + 5). Explain the error.

It should be 7 times the sum of a number n plus 5.

22. Sarah is going to pay for an item using gift cards. The clerk tells her that she will need 2 gift cards and an additional \$3 to pay for the item. Write an algebraic expression to model the situation using the variable g for the amount of the gift cards to pay her total bill, t.

t=2g+3

Standardized Test Prep

Variables and Expressions

Multiple Choice

For Exercises 1-7, choose the correct letter.

1. The word *minus* corresponds to which symbol? **B**

A. +

B. —

 $D. \times$

2. The phrase *product* corresponds to which symbol? **F**

F. X

G. +

3. The word *plus* corresponds to which symbol? **B**

A. -

B. +

C. <

D. ÷

4. What is an algebraic expression for the word phrase 10 more than a number f?

F. 10 - f

G. $\frac{10}{f}$

H. $10 \times f$

5. What is an algebraic expression for the word phrase the product of 11 and a number s? B

A. $\frac{11}{s}$

B. $11 \times s$

C. 11 + s

D. 11 - s

6. Hannah and Tim collect stamps. Tim is bringing his stamps to Hannah's house so that they can compare. Hannah has 60 stamps. Which expression represents the total number of stamps that they will have if t represents the number of stamps Tim has? H

F. $60 \times t$

G. $60 \div t$

H. 60 + t

1. 60 - t

7. Hershel's bakery sells donuts by the box. There are d donuts in each box. Beverly is going to buy 10 boxes for a class field trip. Which expression represents the total number of donuts that Beverly is going to get for her field trip? A

A. $10 \times d$

B. $10 \div d$

C. 10 - d

D. 10 + d

Short Response

8. There are 200 people interested in playing in a basketball league. The leaders of the league are going to divide all of the people into *n* teams. What algebraic expression represents the number of players on each team? 200 \div n

[2] Question answered correctly.

[1] Answer is incomplete.

[0] Answer is wrong.

1_1 Enrichment

Variables and Expressions

An equation is used to set an expression and a constant, or two expressions, equal to each other.

Write the phrase *a number h plus 3 is equal to 8* as an equation.

The phrase *a number h plus 3 is equal to 8,* written as an algebraic equation, is h + 3 = 8.

Write an algebraic equation for each word phrase.

- 1. The sum of 10 and a number y is equal to 18. 10 + y = 18
- **2.** 15 less than a number *g* is equal to 45. g 15 = 45
- 3. The product of 25 and a number f is 5. 25 \times f = 5
- **4.** The quotient of 49 and x is 7. **49** \div x = **7**
- **5.** The sum of *t* and 2 is equal to 5 less than *t*. t + 2 = t 5
- **6.** The quotient of 6 + n and 3 f is 11. $\frac{6 + n}{3 f} = 11$

Write an algebraic equation to model the relationship expressed.

7. Jane tried to fly her kite but discovered that the kite string was too short. If she doubles the length of the string, it will be 28 feet long.

$$2 \times I = 28$$

8. Raul is saving money to buy a car. He decides to withdraw \$50 from his savings account for books. The amount left in his account after the withdrawal is \$200.

$$b - 50 = 200$$

Reteaching

Variables and Expressions

You can represent mathematical phrases and real-world relationships using symbols and operations. This is called an algebraic expression.

For example, the phrase 3 plus a number n can be expressed using symbols and operations as 3 + n.

Problem

What is the phrase 5 minus a number d as an algebraic expression?

The phrase 5 minus a number d, rewritten as an algebraic expression, is 5 - d.

The left side of the table below gives some common phrases used to express mathematical relationships, and the right side of the table gives the related symbol.

Phrase	Symbol
sum	+
difference	_
product	×
quotient	÷
less than	_
more than	+

Exercises

Write an algebraic expression for each word phrase.

- 1. 5 plus a number d 5 + d
- **3.** 11 fewer than a number f = f 11
- **5.** the quotient of 20 and t **20** ÷ t
- 2. the product of 5 and $g \times g$
- **4.** 17 less than h 17
- **6.** the sum of 12 and 4 12 + 4

Write a word phrase for each algebraic expression.

- **7.** h + 6the sum of h and 6
- 10. $\frac{35}{r}$ the quotient of 35 and r
- 5 less than a number m
- 11. h + mthe sum of h and m
- 9. $q \times 10$ the product of q and 10
- **12.** 5*n* the product of 5 and n

Reteaching (continued)

Variables and Expressions

Multiple operations can be combined into a single phrase.

Problem

What is the phrase 11 minus the product of 3 and d as an algebraic expression?

The phrase 11 minus the product of 3 and a number d, rewritten as an algebraic expression, is 11 - 3d.

Exercises

Write an algebraic expression for each phrase.

- 13. 12 less than the quotient of 12 and a number $z = 12 \div z 12$
- **14.** 5 greater than the product of 3 and a number q 5 + 3 × q
- **15.** the quotient of 5 + h and $n + 3 = \frac{5 + h}{n + 3}$
- **16.** the difference of 17 and $\frac{22}{t}$ **17** $-\frac{22}{t}$

Write an algebraic expression or equation to model the relationship expressed in each situation below.

- 17. Jane is building a model boat. Every inch on her model is equivalent to 3.5 feet on the real boat her model is based on. What would be the mathematical rule to express the relationship between the length of the model, m_i and the length of the boat, b?
 - 3.5m = b
- 18. Lyn is putting away savings for his college education. Every time Lyn puts money in his fund, his parents put in \$2. What is the expression for the amount going into Lyn's fund if Lyn puts in L dollars?

L + 2

Additional Vocabulary Support Order of Operations and Evaluating Expressions

Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Definition	Picture or Example
power	A <i>power</i> has two parts, a <i>base</i> and an <i>exponent</i> .	10^3
exponent	The exponent tells you how many times to use the base as a factor.	103
base	The exponent tells you how many times to use the <i>base</i> as a factor.	→ 10 ³
simplify	To simplify is to write an expression in simplest form.	$10^3 = 1,000$
evaluate	You <i>evaluate</i> an algebraic expression by replacing each variable with a given number.	Evaluate the expression $(xy)^2$ for $x = 3$ and $y = 4$. $(3 \cdot 4)^2 = 144$

Think About a Plan

Order of Operations and Evaluating Expressions

Salary You earn \$10 for each hour you work for a canoe rental shop. Write an expression for your salary for working h hours. Make a table to find how much you earn for working 10, 20, 30, and 40 hours.

Think

1. What word or phrase indicates the operation that should be used to help you solve this problem?

"for each hour you work"

Plan

2. Using your response from Exercise 1, write an expression that will tell you how much you earn for every *h* hours you work.

10 \times h, where h is the number of hours worked

Solve

3. Use your expression from Exercise 2 to find the amount that you will earn for working 10, 20, 30, and 40 hours.

\$100; \$200; \$300; \$400

4. Make a table summarizing your results.

	Hours (h)	Money (\$)	١
	10	100	ľ
	20	200	ľ
	30	300	ľ
	40	400	ľ
٦			Г

Practice

Form G

Order of Operations and Evaluating Expressions

Simplify each expression.

4.
$$\left(\frac{5}{6}\right)^2 \left(\frac{25}{36}\right)$$

5.
$$(1+3)^2$$
 16

6.
$$(0.1)^3$$
 0.001

8.
$$\left(\frac{16}{2}\right) - 4(5)$$
 -12

8.
$$\left(\frac{16}{2}\right) - 4(5)$$
 -12 9. $4^4(5) + 3(11)$ **1313**

10.
$$17(2) - 4^2$$
 18

11.
$$\left(\frac{20}{5}\right)^3 - 10(3)^2$$
 -26 12. $\left(\frac{27-12}{8-3}\right)^3$ **27**

12.
$$\left(\frac{27-12}{8-3}\right)^3$$
 27

14.
$$2^5 - 4^2 \div 2^2$$
 28 15. $\left(\frac{3(6)}{17 - 5}\right)^4$ **81**

15.
$$\left(\frac{3(6)}{17-5}\right)^4$$
 $\frac{81}{16}$

Evaluate each expression for s = 2 and t = 5.

16.
$$s + 6$$
 8

17.
$$5 - t$$
 0

18. 11.5 +
$$s^2$$
 15.5

19.
$$\frac{s^4}{4} - 17$$
 -13

20.
$$3(t)^3 + 10$$
 385 21. $s^3 + t^2$ **33**

21.
$$s^3 + t^2$$
 33

22.
$$-4(s)^2 + t^3 \div 5$$
 23. $\left(\frac{s+2}{5t^2}\right)^2$

23.
$$\left(\frac{5t^2}{5t^2}\right)$$

24.
$$\left(\frac{3s(3)}{11-5(t)}\right)^2$$

$$\frac{16}{15,625}$$
 or 0.001024

25. Every weekend, Morgan buys interesting clothes at her local thrift store and then resells them on an auction website. If she brings \$150.00 and spends s, write an expression for how much change she has. Evaluate your expression for s = \$27.13 and s = \$55.14.

150 - s; \$122.87; \$94.86

Practice(continued)

Form G

Order of Operations and Evaluating Expressions

26. A bike rider is traveling at a speed of 15 feet per second. Write an expression for the distance the rider has traveled after s seconds. Make a table that records the distance for 3.0, 5.8, 11.1, and 14.0 seconds.

	d=15.0s			
	Time (s)	Distance (ft)		
(3.0	45.0		
(5.8	87.0		
(11.1	166.5		
(14.0	210.0		
1		J		

Simplify each expression.

27.
$$4[(12 + 5) - 4^4]$$

28.
$$3[(4-6)^2+7]^2$$

27.
$$4[(12+5)-4^4]$$
 28. $3[(4-6)^2+7]^2$ **29.** $2.5[13-\left(\frac{36}{6}\right)^2]$ **30.** $[(48 \div 8)^3-7]^3$ **31.** $\left(\frac{4(-4)(3)}{11-5(1)}\right)^3$ **32.** $4[11-(55-3^5)]^3$

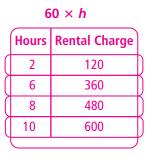
30.
$$[(48 \div 8)^3 - 7]^3$$

9.129.329

31.
$$\left(\frac{4(-4)(3)}{11-5(1)}\right)^3$$

32.
$$4[11 - (55 - 3^5) \div 3]$$

- **33.** a. If the tax that you pay when you purchase an item is 12% of the sale price, write an expression that gives the tax on the item with a price p. Write another expression that gives the total price of the item, including tax. $0.12 \times p$; 0.12p + p;
 - b. What operations are involved in the expressions you wrote? multiplication and addition
 - c. Determine the total price, including tax, of an item that costs \$75. \$84
 - **d.** Explain how the order of operations helped you solve this problem. First you have to multiply 0.12 by p to determine the tax, then you have to add the tax to the original sale price.
- **34.** The cost to rent a hall for school functions is \$60 per hour. Write an expression for the cost of renting the hall for h hours. Make a table to find how much it will cost to rent the hall for 2, 6, 8, and 10 hours.



Evaluate each expression for the given values of the variables.

35.
$$4(c+5) - f^4$$
; $c = -1, f = 4$

35.
$$4(c+5) - f^4$$
; $c = -1, f = 4$ **36.** $-3[(w-6)^2 + x]^2$; $w = 5, x = 6$

37.
$$3.5[h^3 - \left(\frac{3j}{6}\right)^2]$$
; $h = 3$, $j = -4$

37.
$$3.5[h^3 - \left(\frac{3j}{6}\right)^2]$$
; $h = 3$, $j = -4$ **38.** $x[y^2 - (55 - y^5) \div 3]$; $x = -6$, $y = 6$

Practice

Form K

Order of Operations and Evaluating Expressions

Simplify each expression.

3.
$$\left(\frac{7}{8}\right)^2$$
 $\frac{49}{64}$

4.
$$(4+3)^2$$
 49

6.
$$\left(\frac{21}{3}\right) - 2(3)$$
 1

7.
$$11(3) - 3^2$$
 24

8.
$$\left(\frac{15}{5}\right)^3 - 6(2)^2$$
 3

10.
$$3^4 - 2^4 \div 2^2$$
 77

Evaluate each expression for x = 3 and y = 2.

11.
$$x + 7$$
 10

12.
$$8 - y$$
 6

13.
$$\frac{x^3}{3} - 8$$
 1

14.
$$5(y)^3 - 6$$
 34

15.
$$-6(x)^2 + y^3 - 8$$
 -54

16.
$$\left(\frac{x+1}{y^2}\right)^2$$
 1

Practice (continued)

Form K

Order of Operations and Evaluating Expressions

17. George is driving at an average speed of 62 miles per hour. Write an expression that would give his distance traveled for h hours. Make a table that records his distance for 3, 5.5, 7, and 8.5 hours.

	d = 62h			
	Time (hr)	Distance (mi)		
1	3	186		
J	5.5	341		
Ī	7	434		
I	8.5	527		
Ţ				

Simplify each expression.

18.
$$5[(4 + 8) - 3^3]$$

19.
$$2[(7-10)^2+5]^2$$

20.
$$[(32 \div 4)^3 - 500]^3$$

21.
$$\left(\frac{2(-2)(4)}{12-4(2)}\right)^3$$

22. The cost to rent a car is \$30 per day. Write an expression for the cost of renting a car for d days. Make a table to find how much it will cost to rent a car for 3, 5, 7, and 10 days.

	c = 30d		
	Time (days)	Cost (\$)	
	3	90	
	5	150	
	7	210	
	10	300	
1			

Evaluate each expression for the given values of the variables.

23.
$$2(m+1) - n^3$$
; $m = -2$, $n = 3$

23.
$$2(m+1) - n^3$$
; $m = -2$, $n = 3$ **24.** $-3[(a-3)^2 + b]^2$; $a = 4$, $b = 6$ **-147**

25.
$$-1\left[x^3 - \left(\frac{2y}{4}\right)^2\right]; x = 5, y = -\frac{124}{4}$$

25.
$$-1\left[x^3 - \left(\frac{2y}{4}\right)^2\right]; x = 5, y = -2$$
 26. $t[v^2 - (23 - v^2) + 3]; t = -2, v = 2$

27. Reasoning Show that the expressions $3m^2n^2$ and $5m^3 + 13m^2n$ are equal when m = 2 and n = 5.

$$3m^2n^2 = 3(2^2)(5^2) = 3(4)(25) = 300$$

 $5m^3 + 13m^2n = 5(2^3) + 13(2^2)(5) = 5(8) + 13(4)(5) = 300$

1_2 Standardized Test Prep

Order of Operations and Evaluating Expressions

Gridded Response

Solve each exercise and enter your answer on the grid provided. Round your answers to the nearest hundredth if necessary.

1. What is the simplified form of $(3.2)^4$? 104.86

2. What is the simplified form of $(6^2 + 4) - 15$? 25

3. What is the simplified form of 4 \times 6² \div 3 + 7? 55

4. What is the value of $-4d^2 + 15d^2 \div 5$ for d = 1? **-1**

5. What is the value of $(5x^2)^3 + 16y \div 4y$ for x = 2 and y = 3? **8004**

Enrichment

Order of Operations and Evaluating Expressions

The order of operations must be applied even when you are working with variables.

Simplify the expression $4[(x^2)^3 + 5(x + 3x)]$.

 $4[(x^2)^3 + 5(4x)]$ Begin with the parentheses inside the brackets.

 $4[x^6 + 5(4x)]$ Then simplify the exponents inside the brackets.

 $4[x^6 + 20x]$ Then multiply.

 $4x^6 + 80x$ Then distribute the 4 inside the brackets.

The completely simplified form of $4\lceil (x^2)^3 + 5(x+3x) \rceil$ is $4x^6 + 80x$.

Simplify each expression.

1.
$$x^4(x) + 3(x) x^5 + 3x$$

2.
$$(d^4)^4 + (4d)(5d) d^{16} + 20d^2$$

3.
$$\left(\frac{x^4}{x^2}\right)^2 x^4$$

4.
$$x[(4x-x)^2+7]$$
 9 x^3+7x

5.
$$5x[(8x \div 2)^3 - x]$$
 320 $x^4 - 5x^4$

5.
$$5x[(8x \div 2)^3 - x]$$
 320 $x^4 - 5x^2$ 6. $h[11h - (12h - 9h^5) \div 3]$ 7 $h^2 + 3h^6$

Evaluate each expression for the given values of the variables.

7.
$$4k(k+4k)^3+5-d^4$$
; $d=2$, $k=4$

7.
$$4k(k+4k)^3+5-d^4$$
; $d=2$, $k=4$
8. $-3[(z-6z)^2+4(g+5g)]^2$; $z=5$, $g=6$
127.989

9.
$$7.5[(l^2)^3 - (\frac{4n}{12n})^2]; l = -1, n = 9$$

9.
$$7.5[(l^2)^3 - \left(\frac{4n}{12n}\right)^2]$$
; $l = -1$, $n = 9$
10. $r[r^2 - (55 - s^5) - 3s^5]$; $r = -2$, $s = 8$
131,174

11. Myra drove at a speed of 60 miles per hour. How far had she traveled after 1 hour? What about after 4, 6, and 7 hours? Use a table to organize your information. Examine the information in the table. How long did it take her to drive 540 miles?

60 mi; 240 mi; 360 mi; 420 mi

Time (hr)	Distance (mi)
1	60
4	240
6	360
7	420

9 hr

Reteaching

Order of Operations and Evaluating Expressions

Exponents are used to represent repeated multiplication of the same number. For example, $4 \times 4 \times 4 \times 4 \times 4 = 4^5$. The number being multiplied by itself is called the base; in this case, the base is 4. The number that shows how many times the base appears in the product is called the exponent; in this case, the exponent is 5. 4⁵ is read four to the fifth power.

Problem

How is $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ written using an exponent?

The number 6 is multiplied by itself 7 times. This means that the base is 6 and the exponent is 7. $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ written using an exponent is 6^7 .

Exercises

Write each repeated multiplication using an exponent.

1.
$$4 \times 4 \times 4 \times 4 \times 4$$

3.
$$1.1 \times 1.1 \times 1.1 \times 1.1 \times 1.1$$

1.1⁵

5.
$$(-7) \times (-7) \times (-7) \times (-7)$$

2.
$$2 \times 2 \times 2$$

4.
$$3.4 \times 3.4 \times$$

6.
$$11 \times 11 \times 11$$
11³

Write each expression as repeated multiplication.

7.
$$4^3 4 \times 4 \times 4$$

8.
$$5^4$$
 5 × 5 × 5 × 5

9.
$$1.5^2$$
 1.5 × 1.5

10.
$$\left(\frac{2}{7}\right)^4 \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right)$$

11.
$$x^7 \times x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

12.
$$(5n)^5$$
 5n × 5n × 5n × 5n × 5n

13. Trisha wants to determine the volume of a cube with sides of length s. Write an expression that represents the volume of the cube. s³

Reteaching (continued)

Order of Operations and Evaluating Expressions

The order of operations is a set of guidelines that make it possible to be sure that two people will get the same result when evaluating an expression. Without this standard order of operations, two people might evaluate an expression differently and arrive at different values. For example, without the order of operations, someone might evaluate all expressions from left to right, while another person performs all additions and subtractions before all multiplications and divisions.

You can use the acronym P.E.M.A. (Parentheses, Exponents, Multiplication and Division, and Addition and Subtraction) to help you remember the order of operations.

Problem

How do you evaluate the expression $3 + 4 \times 2 - 10 \div 5$?

$$3 + 8 - 10 \div 5$$

= $3 + 8 - 2$

There are no parentheses or exponents, so first, do any multiplication or division from left to right.

Do any addition or subtraction from left to right.

Exercises

Simplify each expression.

14.
$$(5 + 3)^2$$
 64

15.
$$(8-5)(14-6)$$
 24

16.
$$(15-3) \div 4$$
 3

17.
$$\left(\frac{22+3}{5}\right)$$
 5

18.
$$40 - 15 \div 3$$
 35

19.
$$20 + 12 \div 2 - 5$$
 21

20.
$$(4^2 + 5^2)^2$$
 1681

21.
$$4 \times 5 - 3^2 \times 2 \div 6$$
 17

Write and simplify an expression to model the relationship expressed in the situation below.

22. Manuela has two boxes. The larger of the two boxes has dimensions of 15 cm by 25 cm by 20 cm. The smaller of the two boxes is a cube with sides that are 10 cm long. If she were to put the smaller box inside the larger, what would be the remaining volume of the larger box?

$$15 \times 25 \times 20 - 10^3 = 6500 \text{ cm}^3$$

Additional Vocabulary Support

Real Numbers and the Number Line

Concept List

inequalities	integers	irrational numbers
natural numbers	perfect square	radical
radicand	square root	whole numbers

Choose the concept from the list above that best represents the item in each box.

1. $\sqrt{64}$	2. π , $\sqrt{3}$	3. {0, 1, 2, 3, }
1		
radicand	irrational numbers	whole numbers
4. $\sqrt{1.44} = 1.2$	5. $\longrightarrow \sqrt{64}$	6. $8^2 = 64$
square root	radical	perfect square
7. {, -2, -1, 0, 1, 2, 3,}	8. {1, 2, 3, }	9 . < , > , ≤ , ≥
integers	natural numbers	inequalities

$1-3 \frac{\text{Thin}}{}$

Think About a Plan

Real Numbers and the Number Line

Home Improvement If you lean a ladder against a wall, the length of the ladder should be $\sqrt{(x)^2 + (4x)^2}$ ft to be considered safe. The distance x is how far the ladder's base is from the wall. Estimate the desired length of the ladder when the base is positioned 5 ft from the wall. Round your answer to the nearest tenth.

Think

1. What does *x* represent in the given expression? What value is given for *x*? the distance from the base of the ladder to the wall; **5** ft

Plan

- 2. What is the expression when the given value is substituted for x? $\sqrt{5^2 + (20)^2}$
- 3. How do you simplify the expression under the square root symbol? Square each of 5 and 20, then add the results.
- **4.** What is the value of the expression under the square root symbol? Is this number a perfect square?

425; no

Solve

5. What is an estimate for the desired length of the ladder? Round your answer to the nearest tenth.

20.6 ft

Practice

Form G

Real Numbers and the Number Line

Simplify each expression.

1.
$$\sqrt{4}$$
 2

2.
$$\sqrt{36}$$
 6

3.
$$\sqrt{25}$$
 5

4.
$$\sqrt{81}$$
 9

5.
$$\sqrt{121}$$
 11

6.
$$\sqrt{169}$$
 13

7.
$$\sqrt{625}$$
 25

8.
$$\sqrt{225}$$
 15

9.
$$\sqrt{\frac{64}{9}} \frac{8}{3}$$

10.
$$\sqrt{\frac{25}{81}} \frac{5}{9}$$

11.
$$\sqrt{\frac{225}{169}}$$
 $\frac{1}{1}$

12.
$$\sqrt{\frac{1}{625}}$$
 $\frac{1}{25}$

13.
$$\sqrt{0.64}$$
 0.8

14.
$$\sqrt{0.81}$$
 0.9

15.
$$\sqrt{6.25}$$
 2.5

Estimate the square root. Round to the nearest integer.

16.
$$\sqrt{10}$$
 3

17.
$$\sqrt{15}$$
 4

18.
$$\sqrt{38}$$
 6

19.
$$\sqrt{50}$$
 7

20.
$$\sqrt{16.8}$$
 4

21.
$$\sqrt{37.5}$$
 6

22.
$$\sqrt{67.5}$$
 8

23.
$$\sqrt{81.49}$$
 9

24.
$$\sqrt{121.86}$$
 11

Find the approximate side length of each square figure to the nearest whole unit.

- **25.** a rug with an area of 64 ft² **8 ft**
- **26.** an exercise mat that is 6.25 m^2 **2.5 m**
- 27. a plate that is 49 cm^2 7 cm

Practice (continued)

Form G

Real Numbers and the Number Line

Name the subset(s) of the real numbers to which each number belongs.

28.
$$\frac{12}{18}$$

31.
$$\sqrt{2}$$

rational

rational; integer

irrational

irrational

33.
$$\sqrt{13}$$

34.
$$-\frac{4}{3}$$

35.
$$\sqrt{61}$$

rational; integer; whole; natural

irrational

rational

irrational

Compare the numbers in each exercise using an inequality symbol.

36.
$$\sqrt{25}$$
, $\sqrt{64}$ $\sqrt{25} < \sqrt{64}$

37.
$$\frac{4}{5}$$
, $\sqrt{1.3}$

38.
$$\pi$$
, $\frac{19}{6}$

39.
$$\sqrt{81}$$
, $-\sqrt{121}$ $\sqrt{81} > -\sqrt{121}$

40.
$$\frac{27}{17}$$
, 1.7781356 **41.** $-\frac{14}{15}$, $\sqrt{0.8711}$ $\frac{27}{17} < 1.7781356$ $-\frac{14}{15} < \sqrt{0.871}$

41.
$$-\frac{14}{15}$$
, $\sqrt{0.8711}$ $-\frac{14}{15} < \sqrt{0.8711}$

Order the numbers from least to greatest.

42. 1.875,
$$\sqrt{64}$$
, $-\sqrt{121}$ $-\sqrt{121}$, 1.875, $\sqrt{64}$

43.
$$\sqrt{0.8711}$$
, $\frac{4}{5}$, $\sqrt{1.3}$ $\frac{4}{5}$, $\sqrt{0.8711}$, $\sqrt{1.3}$

42.
$$1.875$$
, $\sqrt{64}$, $-\sqrt{121}$

$$-\sqrt{121}$$

$$-\sqrt{121}$$
, 1.875 , $\sqrt{64}$,
$$\frac{4}{5}$$
, $\sqrt{0.8711}$, $\frac{4}{5}$, $\sqrt{1.3}$

$$\frac{64.56}{8.477}$$
, $\sqrt{67.4698}$, $\frac{64.56}{8.477}$, $\frac{67.4698}{8.477}$, $\frac{64.56}{8.477}$,

45.
$$-\frac{14}{15}$$
, 5.587, $\sqrt{81}$ **46.** $\frac{100}{22}$, $\sqrt{25}$, $\frac{27}{17}$ $-\frac{14}{15}$, 5.587, $\sqrt{81}$ $\frac{27}{17}$, $\frac{100}{22}$, $\sqrt{25}$

46.
$$\frac{100}{22}$$
, $\sqrt{25}$, $\frac{27}{17}$

47.
$$\pi$$
, $\sqrt{10.5625}$, $-\frac{15}{5.8}$ $-\frac{15}{5.8}$, π , $\sqrt{10.5625}$

48. Marsha, Josh, and Tyler are comparing how fast they can type. Marsha types 125 words in 7.5 minutes. Josh types 65 words in 3 minutes. Tyler types 400 words in 28 minutes. Order the students according to who can type the fastest. Josh, Marsha, Tyler

Practice

Form K

Real Numbers and the Number Line

Simplify each expression.

1. $\sqrt{144}$ 12

2. $\sqrt{25}$ 5

3. $\sqrt{169}$ 13

4. $\sqrt{49}$ **7**

5. $\sqrt{256}$ 16

6. $\sqrt{400}$ 20

7. $\sqrt{\frac{9}{49}}$ $\frac{3}{7}$

8. $\sqrt{\frac{196}{144}}$ $\frac{7}{6}$

9. $\sqrt{0.01}$ 0.1

10. $\sqrt{0.49}$ **0.7**

Estimate the square root. Round to the nearest integer.

11. $\sqrt{38}$ 6

12. $\sqrt{65}$ **8**

13. $\sqrt{99}$ 10

14. $\sqrt{145.5}$ **12**

15. $\sqrt{23.75}$ **5**

16. $\sqrt{64.36}$ **8**

Find the approximate side length of each square figure to the nearest whole

- 17. a tabletop with an area 25 ft^2
 - 5 ft
- **18.** a wall that is 105 m^2
 - 10 m

Practice (continued)

Form K

Real Numbers and the Number Line

Name the subset(s) of the real numbers to which each number belongs.

19. $\frac{3}{4}$

20. -8

21. 2π

rational

rational, integer

irrational

22. 45,368

23. $\sqrt{11}$

24. $-\frac{2}{3}$

rational, natural, whole, integer

irrational

rational

Compare the numbers in each exercise using an inequality symbol.

25. $\sqrt{36}$, $\sqrt{49}$

 $\sqrt{36} < \sqrt{49}$

26. $\frac{1}{3}$, $\sqrt{1.25}$

 $\frac{1}{3} < \sqrt{1.25}$

27. $\sqrt{100}$, $-\sqrt{169}$

 $\sqrt{100} > -\sqrt{169}$

28. $\frac{34}{19}$, 1.8

 $\frac{34}{19} < 1.8$

Order the numbers in each exercise from least to greatest.

29. 2.75, $\sqrt{25}$, $-\sqrt{36}$

 $-\sqrt{36}$, 2.75, $\sqrt{25}$

30. 1.25, $-\frac{1}{3}$, $\sqrt{1.25}$

 $-\frac{1}{2}$, $\sqrt{1.25}$, 1.25

31. $\frac{3}{5}$, -0.6, $\sqrt{1}$

 $-0.6, \frac{3}{5}, \sqrt{1}$

32. $\frac{80}{25}$, $\sqrt{9}$, $\frac{30}{9}$

 $\sqrt{9}$, $\frac{80}{25}$, $\frac{30}{9}$

33. Kate, Kevin, and Levi are comparing how fast they can run. Kate was able to run 5 miles in 47.5 minutes. Kevin was able to run 8 miles in 74 minutes. Levi was able to run 4 miles in 32 minutes. Order the friends from the fastest to the slowest.

Levi, Kevin, Kate

Standardized Test Prep

Real Numbers and the Number Line

Multiple Choice

For Exercises 1-6, choose the correct letter.

1. To which subset of the real numbers does -18 not belong? \triangle

A. irrational

B. rational

C. integer

D. negative integers

2. To which subset of the real numbers does $\sqrt{2}$ belong? **F**

F. irrational

G. rational

H. integer

I. whole

3. You can tell that π is an irrational number because it has a what? D

A. non-repeating decimal

C. repeating decimal

B. non-terminating decimal

D. non-repeating and a non-terminating decimal

4. What is $\sqrt{324}$? **6**

F. 15

G. 18

H. 19

I. 24

5. What is $\sqrt{196}$? A

A. 14

B. 0

C. 4

D. 19

6. What is $\sqrt{36x^6y^4}$? **G**

F. $6x^6y^4$ **G.** $6x^3y^2$

H. $18x^3y^2$

1. $24x^6v^4$

Short Response

7. Why is 8.8 classified as a rational number?

8.8 can be classified as a rational number because it can be rewritten as the fraction $\frac{88}{10}$.

- [2] Question answered correctly.
- [1] Answer is incomplete.
- [0] Answer is wrong.

Enrichment

Real Numbers and the Number Line

You can find the square root of a variable in the same way that you can find the square root of a number.

$$\sqrt{x^2} = x$$
 because $x \cdot x = x^2$

The same rules hold true for the square roots of expressions as well.

$$\sqrt{(x+1)^2} = (x+1)$$
 because $(x+1)(x+1) = (x+1)^2$

Exercises

Simplify each expression.

1.
$$\sqrt{64}$$

8

2.
$$\sqrt{121}$$

11

3.
$$\sqrt{x^4}$$

4.
$$\sqrt{y^{12}}$$

*v*⁶

5.
$$\sqrt{x^4y^8}$$

$$6. \sqrt{\left(\frac{x^4}{x^2}\right)}$$

7.
$$\sqrt{(x+1)^2}$$

x + 1

8.
$$\sqrt{(45x+89)^2}$$

45x + 89

9.
$$\sqrt{(-23x^4+81)^8}$$

 $(-23x^4 + 81)^4$

10.
$$\sqrt{(11g+81)^6(25h-16)^4}$$

 $(11q + 81)^3(25h - 16)^2$

11.
$$\sqrt{\sqrt{x^8}}$$

12. The formula for finding the area of a circle is $A = \pi r^2$. You are building a target for practicing archery. The area of the target is 706.5 cm². Use 3.14 as an approximation for π and determine the radius of the target.

15 cm

Reteaching

Real Numbers and the Number Line

A number that is the product of some other number with itself, or a number to the second power, such as $9 = 3 \times 3 = 3^2$, is called a perfect square. The number that is raised to the second power is called the square root of the product. In this case, 3 is the square root of 9. This is written in symbols as $\sqrt{9} = 3$. Sometimes square roots are whole numbers, but in other cases, they can be estimated.

Problem

What is an estimate for the square root of 150?

There is no whole number that can be multiplied by itself to give the product of 150.

$$10 \times 10 = 100$$

$$11 \times 11 = 121$$

$$12 \times 12 = 144$$

$$13 \times 13 = 169$$

You cannot find the exact value of $\sqrt{150}$, but you can estimate it by comparing 150 to perfect squares that are close to 150.

150 is between 144 and 169, so $\sqrt{150}$ is between $\sqrt{144}$ and $\sqrt{169}$.

$$\sqrt{144} < \sqrt{150} < \sqrt{169}$$

$$12 < \sqrt{150} < 13$$

The square root of 150 is between 12 and 13. Because 150 is closer to 144 than it is to 169, we can estimate that the square root of 150 is slightly greater than 12.

Exercises

Find the square root of each number. If the number is not a perfect square, estimate the square root to the nearest integer.

10. A square mat has an area of 225 cm². What is the length of each side of the mat? 15 cm

Real Numbers and the Number Line

Reteaching (continued)

The real numbers can be separated into smaller, more specific groups, called subsets. Each of these subsets has certain characteristics. For example, a rational number can be expressed as a fraction of two integers, with the denominator of the fraction not equal to 0. Irrational numbers cannot be expressed as a fraction of two integers.

Every real number belongs to at least one subset of the real numbers. Some real numbers belong to multiple subsets.

Problem

To which subsets of the real numbers does 17 belong?

17 is a natural number, a whole number, and an integer.

But 17 is also a rational number because it can be written as $\frac{17}{1}$, a fraction of two integers with the denominator not equal to 0.

A number cannot belong to both the subset of rational numbers and the subset of irrational numbers, so 17 is not an irrational number.

Exercises

List the subsets of the real numbers to which each of the given numbers belongs.

11. 5

rational, whole, natural, integer

12. 116

rational, whole, natural, integer

14. 17.889

rational

rational

15. -25

rational, integer

- **13.** $\sqrt{3}$ irrational
- **16.** -68

rational, integer

whole, integer

- - **18.** 0 rational, whole, integer
- **19.** $\sqrt{16}$ rational, natural,

20. $\sqrt{20}$ irrational **21.** $\sqrt{6.25}$

rational

rational

Additional Vocabulary Support

Properties of Real Numbers

Use the list below to complete the graphic organizer.

Associative Property of Multiplication Identity Property of Addition Multiplication Property of -1

Commutative Property of Addition Identity Property of Multiplication Zero Property of Multiplication

Addition

Multiplication

Commutative Property

of Addition

$$a + b = b + a$$

Associative Property of Addition

$$(a + b) + c$$

= $a + (b + c)$

Identity Property

of Addition

$$\alpha + 0 = \alpha$$

Commutative Property of Multiplication

$$a \times b = b \times a$$

Associative Property

of Multiplication

$$(a \times b) \times c$$

= $a \times (b \times c)$

Identity Property

of Multiplication

$$a \times 1 = a$$

Zero Property of

Multiplication

$$a \times 0 = 0$$

Multiplication

Property of −1

$$-1 \times a = -a$$

Name	Class	Date	

Think About a Plan

Properties of Real Numbers

Travel It is 235 mi from Tulsa to Dallas. It is 390 mi from Dallas to Houston.

- a. What is the total distance of a trip from Tulsa to Dallas to Houston?
- b. What is the total distance from Houston to Dallas to Tulsa?
- **c.** Explain how you can tell whether the distances described in parts (a) and (b) are equal by using reasoning.

Think

- 1. What operation(s) will you use to solve the problem?
 Addition
- 2. Which of the properties of real numbers involve the operations identified in part (a)?

Commutative Property of Addition, Associative Property of Addition, Additive Identity

Plan

- **3.** Write expressions that can be simplified to solve parts (a) and (b).
 - A. 235 + 390
- B. 390 + 235
- **4.** How are the two expressions similar? How are those similarities related to the situation as described?

The numbers are the same. Distances between cities are the same, regardless of which direction I am going in.

5. How are the expressions different? How are those differences related to the situation as described?

The numbers are added in different order. The first has you going in one direction, the second has you returning the other direction.

Solve

6. Find the total distances asked for in parts (a) and (b). What do you notice about the answers?

625 miles, 625 miles; They are the same.

- 7. Which of the properties of real numbers best explains your results?
 Commutative property of addition
- 8. Discuss how that property explains your results.

The property tells us that the order of the addends does not affect the sum.

Practice

Form G

Properties of Real Numbers

Name the property that each statement illustrates.

- 1. 12 + 917 = 917 + 12Commutative Property of Addition
- 3. $35 \cdot x = x \cdot 35$ Commutative Property of Multiplication
- 5. m + 0 = mIdentity Property of Addition

Use mental math to simplify each expression.

2.
$$74.5 \cdot 0 = 0$$

Zero Property of Multiplication

- 4. $3 \cdot (-1 \cdot p) = 3 \cdot (-p)$ Multiplication Property of -1
- **6.** 53.7 · 1 = 53.7 **Identity Property of Multiplication**

8.
$$19.2 + 0.6 + 12.4 + 0.8$$
 33

Simplify each expression. Justify each step.

13.
$$(2 + 7m) + 5$$

= $(7m + 2) + 5$ Comm. Prop. of Add.
= $7m + (2 + 5)$ Assoc. Prop. of Add.
= $7m + 7$ Combine like terms.

12.
$$5(16p)$$

= $(5 \cdot 16)p$ Assoc. Prop. of Mult.
= $80p$ Simplify.

14.
$$\frac{12st}{4t}$$
 $\frac{12}{4} \cdot s \cdot \frac{1}{t} \cdot t$ Prop. of Mult.
 $= \frac{12}{4} \cdot s \cdot 1$ Mult. Ident.
 $= \frac{12}{4} \cdot 1 \cdot s$ Assoc. Prop. of Mult.
 $= 3s$ Simplify.

Tell whether the expressions in each pair are equivalent.

15.
$$7x$$
 and $7x \cdot 1$ **Equivalent**

17.
$$(12 - 7) + x$$
 and $5x$ **Not equivalent**

19.
$$\frac{24xy}{2x}$$
 and $12y$

16.
$$4 + 6 + x$$
 and $4 \cdot x \cdot 6$
Not equivalent

18.
$$p(4-4)$$
 and 0 Equivalent

20.
$$\frac{27m}{(3+9-12)}$$
 and $27m$ **Not equivalent**

- **21.** You have prepared 42 mL of distilled water, 18 mL of vinegar and 47 mL of salt water for an experiment.
 - a. How many milliliters of solution will you have if you first pour the distilled water, then the salt water, and finally the vinegar into your beaker?
 - **b.** How many milliliters of solution will you have if you first pour the salt water, then the vinegar, and finally the distilled water into your beaker? **107 ml**
 - c. Explain why the amounts described in parts (a) and (b) are equal. Assoc. Prop. of Add.

Practice (continued)

Form G

Properties of Real Numbers

Use deductive reasoning to tell whether each statement is *true* or *false*. If it is false, give a counterexample.

- **22.** For all real numbers a and b, a b = -b + a. true
- **23.** For all real numbers p, q and r, p-q-r=p-r-q. true
- **24.** For all real numbers x, y and z, (x + y) + z = z + (x + y). true
- **25.** For all real numbers m and n, $\frac{m}{m} \cdot n = \frac{n}{n} \cdot m$. false; $\frac{5}{5} \times 3 \neq \frac{3}{3} \times 5$
- **26. Writing** Explain why the commutative and associative properties don't hold true for subtraction and division but the identity properties do.

```
Examples: 5 - 0 = 5; 5 \div 1 = 5; Counterexamples: 5 - 3 \ne 3 - 5; (5 - 3) - 2 \ne 5 - (3 - 2); 6 \div 3 \ne 3 \div 6; (24 \div 6) \div 2 \ne 24 \div (6 \div 2)
```

27. Reasoning A recipe for brownies calls for mixing one cup of sugar with two cups of flour and 4 ounces of chocolate. They are all to be mixed in a bowl before baking. Will the brownies taste different if you add the ingredients in different orders? Relate your answer to a property of real numbers. no; Like the Comm. Prop. of Add., the order doesn't matter. Like the Assoc. Prop. of Add., it doesn't matter if the flour and sugar are added and then the chocolate, or if the sugar and chocolate are added and then the flour or any other combination.

Simplify each expression. Justify each step.

28.
$$(6^7)(5^3 + 2)(2 - 2)$$
 0

29.
$$(m-16)(-7 \div -7)$$
 m – **16**

- **30. Open-Ended** Provide examples to show the following.
 - a. The associative property of addition holds true for negative integers.
 - **b.** The commutative property of multiplication holds true for non-integers.
 - **c.** The multiplicative property of negative one holds true regardless of the sign of the number on which the operation is performed.
 - **d.** The commutative property of multiplication holds true if one of the factors is zero.

Answers may vary. Samples:

a.
$$[-3 + (-4)] + (-1) = -7 + (-1) = -8$$
; $-3 + [-4 + (-1)] = -3 + (-5) = -8$
b. $(\frac{1}{2} \cdot \frac{2}{3}) \cdot \frac{3}{4} = \frac{1}{4}$; $\frac{1}{2} \cdot (\frac{2}{3} \cdot \frac{3}{4}) = \frac{1}{4}$
c. $-1 \cdot 5 =$ the opposite of $5 = -5$; $-1 \cdot -5 =$ the opposite of $-5 = 5$
d. $3 \cdot 0 = 0 \cdot 3 = 0$

Practice

Form K

Properties of Real Numbers

Match statements 1-8 with the property, a - h, that the statement illustrates.

- a Commutative Property of Addition:
- **b.** Commutative Property of Multiplication:
- c. Additive Identity:
- d. Multiplicative Identity:
- e. Associative Property of Addition:
- f. Associative Property of Multiplication:
- g. Zero Property of Multiplication:
- h. Multiplicative Property of -1:

$$a + b = b + a$$

$$a \cdot b = b \cdot a$$

$$a + 0 = a$$

$$a \cdot 1 = a$$

$$(a + b) + c = a + (b + c)$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

$$a \cdot 0 = 0$$

$$-1 \cdot a = -a$$

1.
$$12 + 917 = 917 + 12$$

2.
$$5 \cdot 0 = 0$$
 q

3.
$$35 \cdot x = x \cdot 35$$
 b

4.
$$(x \cdot 3) \cdot 4 = x \cdot (3 \cdot 4)$$
 f

5.
$$m + 0 = m$$
 c

6.
$$25 \cdot 1 = 25$$
 d

7.
$$(15 + 9) + 11 = 15 + (9 + 11)$$
 e 8. $-1 \cdot 6 = -6$ h

8.
$$-1 \cdot 6 = -6$$
 h

Simplify each expression. Justify each step that has not been justified.

9.
$$5 + (3x + 2) = 5 + (2 + 3x)$$

$$= (5 + 2) + 3x$$

$$= 7 + 3x$$

Commutative Property of Addition

Associative Property of Addition

Combine like terms.

10.
$$3 \cdot (x \cdot 6) = 3 \cdot (6 \cdot x)$$

$$= (3 \cdot 6) \cdot x$$

$$= 18x$$

Commutative Property of Multiplication

Associative Property of Multiplication

Multiply.

Class Date

ate ___

1-4

Practice (continued)

Form K

Properties of Real Numbers

Simplify each expression. Justify each step.

11.
$$(2 + 7m) + 5$$

$$= (7m + 2) + 5$$
Commutative Property of Addition
$$= 7m + (2 + 5)$$
Associative Property of Addition
$$= 7m + 7$$
Commutative Property of Multiplication
$$= (9 \cdot 21) \cdot r$$
Associative Property of Multiplication
$$= 189r$$
Multiply.

Tell whether the expressions in each pair are equivalent.

13. 2x and $2x \cdot 1$ equivalent

14. $(5 - 2) \cdot x$ and 3x equivalent

15. 8 + 6 + b and 8 + 6b **not equivalent**

- **16.** $5 \cdot (4 4)$ and 0 equivalent
- **17.** You have prepared 40 mL of vanilla, 20 mL of chocolate, and 50 mL of milk for a milkshake.
 - **a.** How many milliliters of milkshake will you have if you first pour the vanilla, then the chocolate, and finally the milk into your glass? **110 mL**
 - **b.** How many milliliters of milkshake will you have if you first pour the chocolate, then the vanilla, and finally the milk into your glass? **110 mL**
 - **c.** Explain how you can tell whether the amounts of milkshake described in parts (a) and (b) are equal. **Commutative Property of Addition**

Use deductive reasoning to tell whether each statement is *true* or *false*. If it is false, give a counterexample.

18. For all real numbers a and b, a - b = b - a.

False $7 - 3 \neq 3 - 7$

19. For all real numbers p, q, and r, p - q - r = p - r - q.

20. For all real numbers *x*, *y*, and *z*, (x + y) + z = z + (x + y).

- True
- **21.** For all real numbers n, n + 1 = n.

False 8 + 1 ≠ 8

22. Writing Explain why the commutative and associative properties do not hold true for subtraction and division.

Answers will vary. Counterexamples: $5 - 3 \neq 3 - 5$; $(5 - 3) - 2 \neq 5 - (3 - 2)$; $6 \div 3 \neq 3 \div 6$; $(24 \div 6) \div 2 \neq 24 \div (6 \div 2)$

Standardized Test Prep

Properties of Real Numbers

Multiple Choice

For Exercises 1-5, choose the correct letter.

1. Which of the following statements is *not* always true? **B**

A.
$$a + (-b) = -b + a$$

C.
$$(a + b) + (-c) = a + [b + (-c)]$$

B.
$$a - (-b) = (-b) - a$$

D.
$$-(-a) = a$$

2. Which pair of expressions are equivalent? I

F.
$$18m \cdot 0 \text{ and } 1$$

H.
$$(12 - 5) + \pi$$
 and 7π

G.
$$6 + r + 11$$
 and $6 \cdot r \cdot 11$

1.
$$x(3 - 3)$$
 and 0

3. What property is illustrated by the equation (8 + 2) + 7 = (2 + 8) + 7?

A. Commutative Property of Addition

B. Associative Property of Addition

C. Distributive Property

D. Identity Property of Addition

4. Which expression is equivalent to $-a \cdot b$? **F**

F.
$$a \cdot (-b)$$

G.
$$b-a$$

H.
$$(-a)(-b)$$
 I. $-a + b$

$$1. -a + b$$

5. Which is an example of an identity property? B

$$\mathbf{A.}\ a\boldsymbol{\cdot} 0=0$$

$$\mathbf{B.} \ x \cdot 1 = x$$

C.
$$(-1)x = -x$$

A.
$$a \cdot 0 = 0$$
 B. $x \cdot 1 = x$ **C.** $(-1)x = -x$ **D.** $a + b = b + a$

Short Response

6. The fact that changing the grouping of addends does not change the sum is the basis of what property of real numbers?

Assoc. Prop. of Add.

[2] Question answered correctly.

[1] Answer is incomplete.

[0] Answer is wrong.

	Name _		Class	Date _	
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1-4

Enrichment

Properties of Real Numbers

Which of the properties of real numbers are illustrated by the following situations? Explain your reasoning.

1. One team scores 3 runs in the first inning and 2 runs in the fourth inning. The other team scores 2 runs in the first inning and 3 runs in the fourth. In the fifth inning, the score is tied.

Commutative Property of Addition

2. Your friend gets a job making \$9.50 per hour. One week she takes a vacation and does not work. She makes no money that week.

Zero Property of Multiplication

3. In putting together a mixture of fertilizer, a gardener mixes nitrogen and phosphorus before adding potassium. The next day the gardener mixes phosphorus and potassium before adding nitrogen. The two mixtures are exactly the same.

Associative Property of Addition

4. A restaurant received two orders from the apartment managers of two different apartment buildings. The first apartment manager said he was ordering 3 meals each for the occupants of 4 different apartments. The second said he was ordering 4 meals each for the occupants of 3 different apartments. The apartment managers ordered the same number of meals.

Commutative Property of Multiplication

5. The owner of a theater checked how much money was in the box office 10 minutes before a show began. No tickets were purchased in the last 10 minutes, so the owner was not surprised that the final amount of money was the same as when when he previously checked.

Additive Identity

6. Usually, when Marty makes pancakes for his kids, he changes the amount of each ingredient depending on how many servings he is making. Since he was making the exact number of servings the recipe called for, he was able to use the numbers published in the cook book.

Multiplicative Identity

Reteaching

Properties of Real Numbers

Equivalent algebraic expressions are expressions that have the same value for all values for the variable(s). For example x + x and 2x are equivalent expressions since, regardless of what number is substituted in for x, simplifying each expression will result in the same value. Certain properties of real numbers lead to the creation of equivalent expressions.

Commutative Properties

The commutative properties of addition and multiplication state that changing the order of the addends does not change the sum and that changing the order of factors does not change the product.

Addition: a + b = b + a

Multiplication: $a \cdot b = b \cdot a$

To help you remember the commutative properties, you can think about the root word "commute." To commute means to move. If you think about commuting or moving when you think about the commutative properties, you will remember that the addends or factors move or change order.

Problem

Do the following equations illustrate commutative properties?

a.
$$3 + 4 = 4 + 3$$

b.
$$(5 \times 3) \times 2 = 5 \times (3 \times 2)$$

c.
$$1 - 3 = 3 - 1$$

3 + 4 and 4 + 3 both simplify to 7, so the two sides of the equation in part (a) are equal. Since both sides have the same two addends but in a different order, this equation illustrates the Commutative Property of Addition.

The expression on each side of the equation in part (b) simplifies to 30. Both sides contain the same 3 factors. However, this equation does not illustrate the Commutative Property of Multiplication because the terms are in the same order on each side of the equation.

1-3 and 3-1 do not have the same value, so the equation in part (c) is not true. There is not a commutative property for subtraction. Nor is there a commutative property for division.

Associative Properties

The associative properties of addition and multiplication state that changing the grouping of addends does not change the sum and that changing the grouping of factors does not change the product.

Addition: (a + b) + c = a + (b + c) Multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Reteaching (continued)

Properties of Real Numbers

Problem

Do the following equations illustrate associative properties?

a.
$$(1+5)+4=1+(5+4)$$

b.
$$4 \times (2 \times 7) = 4 \times (7 \times 2)$$

(1+5)+4 and 1+(5+4) both simplify to 10, so the two sides of the equation in part (a) are equal. Since both sides have the same addends in the same order but grouped differently, this equation illustrates the Associative Property of Addition.

The expression on each side of the equation in part (b) simplifies to 56. Both sides contain the same 3 factors. However, the same factors that were grouped together on the left side have been grouped together on the right side; only the order has changed. This equation does not illustrate the Associative Property of Multiplication.

Other properties of real numbers include:

d. Multiplicative property of negative one:
$$-1 \cdot a = -a$$

$$a + 0 = 0$$

$$12 + 0 = 12$$

$$a \cdot 1 = a \qquad \qquad 32 \cdot 1 = 32$$

$$a \cdot 0 = 0$$

$$6 \cdot 0 = 0$$

$$1 \cdot a = -a$$

$$-1 \cdot 7 = -7$$

Exercises

What property is illustrated by each statement?

1.
$$(m + 7.3) + 4.1 = m + (7.3 + 4.1)$$

Associative Property of Addition

3.
$$12x + 4y + 0 = 12x + 4y$$

Additive Identity

5.
$$17 + (-2) = (-2) + 17$$

Commutative Property of Addition

Simplify each expression. Justify each step.

7.
$$(12 + 8x) + 13$$

=
$$(8x + 12) + 13$$
 Comm. Prop. of Add.
= $8x + (12 + 13)$ Assoc. Prop. of Add.
= $8x + 25$ Combine like terms.

$$= 8x + 25$$

9.
$$(7-7) + 12$$

= 0 + 12 Add. Ident.
= 12 Simplify.

2.
$$5p \cdot 1 = 5p$$

Multiplicative Identity

4.
$$(3r)(2s) = (2s)(3r)$$

Commutative Property of Multiplication

6.
$$-(-3) = 3$$

Multiplicative Property of −1

8.
$$(5 \cdot m) \cdot 7$$

= 35m

$$= (m \cdot 5) \cdot 7$$
$$= m \cdot (5 \cdot 7)$$

Comm. Prop. of Mult. Assoc. Prop. of Mult. Comm. Prop. of Mult.

1-5

Additional Vocabulary Support

Adding and Subtracting Real Numbers

Problem

A diver dives 50 ft and then rises 12 ft to look at a fish. Then he dives down 7 ft to look at some coral. Next, he rises 20 ft to take a photograph. What is his location in relation to sea level? Justify your steps. Then check your answer.

$$0 - 50 + 12 - 7 + 20$$

$$= 0 + (-50) + 12 + (-7) + 20$$

$$= 0 + 12 + 20 + (-50) + (-7)$$

$$= 0 + (12 + 20) + [(-50) + (-7)]$$

$$= 32 + (-57)$$

Write an expression.

Rule for subtracting real numbers

Commutative Property of Addition

Group addends with the same sign and add.

Identity Property of Addition

Rule for adding numbers with different signs

Exercises

= -25

A roller coaster rises 50 ft, falls 20 ft, rises 70 ft and falls 60 ft. What is the final location of the roller coaster in relation to its starting elevation? Justify your steps. Then check your answer.

$$0 + 50 - 20 + 70 - 60$$

$$= 0 + 50 + (-20) + 70 + (-60)$$

$$= 0 + 50 + 70 + (-20) + (-60)$$

$$= 0 + (50 + 70) + [(-20) + (-60)]$$

$$= 0 + 120 + (-80)$$

$$= 120 + (-80)$$

$$= 40$$

Write an expression.

Rule for subtracting real numbers

Commutative Property of Addition

Group addends with the same sign.

Add inside grouping symbols.

Identity Property of Addition

Rule for adding numbers with different signs

A stock price per share was \$45.00 last week. The price changed by gaining \$4, losing \$6, losing \$5, and gaining \$7. What was the ending stock price? Justify your steps. Then check your answer.

$$45 + 4 - 6 - 5 + 7$$

$$45 + 4 + (-6) + (-5) + 7$$

$$= 45 + 4 + 7 + (-6) + (-5)$$

$$= (45 + 4 + 7) + [(-6) + (-5)]$$

$$= (56) + (-11)$$

$$= 45$$

Write an expression.

Rule for subtracting real numbers

Commutative Property of Addition

Group addends with the same sign.

Add inside grouping symbols.

Rule for adding numbers with different signs

Think About a Plan

Adding and Subtracting Real Numbers

Meteorology Weather forecasters use a barometer to measure air pressure and make weather predictions. Suppose a standard mercury barometer reads 29.8 in. The mercury rises 0.02 in. and then falls 0.09 in. The mercury falls again 0.18 in. before rising 0.07 in. What is the final reading on the barometer?

Think

- **1.** What operation does "rise" suggest?
- 2. What operation does "fall" suggest? ____subtraction

Plan

3. Write either *plus* or *minus* in each box so that the following represents the problem.

plus minus minus plus 29.8 0.02 0.09 0.18 0.07

4. Write an expression to represent the problem.

$$29.8 + 0.02 - 0.09 - 0.18 + 0.07$$

Solve

- 5. What is the value of the expression you wrote in Exercise 4? _____ 29.62
- **6.** What is the final reading on the barometer? 29.62 in.

Practice

Form G

Adding and Subtracting Real Numbers

Use a number line to find each sum.

3.
$$9 + (-4)$$
 5

4.
$$-6 + (-2)$$
 -8

5.
$$-6 + 3 - 3$$

6.
$$5 + (-10)$$
 -5

7.
$$-7 + (-7)$$
 -14

8.
$$9 + (-9)$$
 0

9.
$$-8 + 0$$
 -8

Find each sum.

10.
$$22 + (-14)$$
 8

11.
$$-36 + (-13)$$
 -49

12.
$$-15 + 17$$
 2

14.
$$19 + (-30)$$
 -11

15.
$$-18 + (-18)$$
 -36

16.
$$-1.5 + 6.1$$
 4.6

17.
$$-2.2 + (-16.7)$$
 18. $5.3 + (-7.4)$ **-2.1**

18.
$$5.3 + (-7.4)$$
 -2.1

19.
$$-\frac{1}{9} + \left(-\frac{5}{9}\right) - \frac{2}{3}$$
 20. $\frac{3}{4} + \left(-\frac{3}{8}\right) \frac{3}{8}$

20.
$$\frac{3}{4} + \left(-\frac{3}{8}\right)$$

21.
$$-\frac{1}{5} + \frac{7}{10} \frac{1}{2}$$

- **22. Writing** Explain how you would use a number line to find 6 + (-8). Answers may vary. Sample: Start at 0. Move 6 spaces to the right and then 8 spaces to the left. The answer is -2.
- 23. Open-Ended Write an addition equation with a positive addend and a negative addend and a resulting sum of -8.

Answers may vary. Sample: -10 + 2 = -8

24. The Bears football team lost 7 yards and then gained 12 yards. What is the result of the two plays?

a gain of 5 yd

Practice (continued)

Form G

Adding and Subtracting Real Numbers

Find each difference.

26.
$$-8 - 12$$
 -20

32.
$$-4.5 - 5.8 - 10.3$$

33.
$$-3.7 - (-4.2)$$
 0.5

34.
$$-\frac{7}{8} - \left(-\frac{1}{8}\right) - \frac{3}{4}$$
 35. $\frac{2}{3} - \frac{1}{2} \cdot \frac{1}{6}$

35.
$$\frac{2}{3} - \frac{1}{2}$$

36.
$$\frac{4}{9} - \left(-\frac{2}{3}\right)$$
 1 $\frac{1}{9}$

Evaluate each expression for m = -4, n = 5, and p = 1.5.

37.
$$m - p$$
 -5.5

38.
$$-m + n - p$$
 7.5 39. $n + m - p$ **-0.5**

39.
$$n + m - n$$
 -0.5

40. At 4:00 A.M., the temperature was -9° F. At noon, the temperature was 18° F. What was the change in temperature?

27 degrees

41. A teacher had \$57.72 in his checking account. He made a deposit of \$209.54. Then he wrote a check for \$72.00 and another check for \$27.50. What is the new balance in his checking account?

\$167.76

which is greater than -47 + (-33).

- **42.** A scuba diver went down 20 feet below the surface of the water. Then she dove down 3 more feet. Later, she rose 7 feet. What integer describes her depth? -16
- **43. Reasoning** Without doing the calculations, determine whether -47 (-33)or -47 + (-33) is greater. Explain your reasoning. -47 - (-33) is greater; -47 - (-33) is the same as -47 + 33

Practice

Form K

Adding and Subtracting Real Numbers

Use a number line to find each sum.

2.
$$-4 + 6$$

2.
$$-4 + 6$$
 2 3. $7 + (-3)$ **4**

4.
$$-5 + (-1)$$
 -6 5. $-4 + 2$ **-2 6.** $5 + (-8)$ **-3**

6.
$$5 + (-8)$$
 -3

- **7.** Is the sum of two negative numbers *positive* or *negative*? negative
- **8. Writing** Is the sum -4 + 2 *positive* or *negative*? How do you know? negative; Answers may vary. Sample: |-4| is greater than |2| so the sum is negative.

Find each sum.

9.
$$12 + (-4)$$

9.
$$12 + (-4)$$
 8 10. $-22 + (-10)$ **-32 11.** $-25 + 27$ **2**

11.
$$-25 + 27$$
 2

13.
$$15 + (-20) -5$$

12.
$$21 + 43$$
 64 13. $15 + (-20)$ **-5 14.** $-25 + (-25)$ **-50**

15
$$-15 + 36$$
 2

16.
$$-2.2 + (-16.7)$$
 -18.9

15.
$$-1.5 + 3.6$$
 2.1 16. $-2.2 + (-16.7)$ **-18.9 17.** $-\frac{1}{7} + \left(-\frac{4}{7}\right)$ **-5**

Practice (continued)

Form K

Adding and Subtracting Real Numbers

18. Which addition problem is equivalent to -5 - (-8)?

A.
$$5 + 8$$

C.
$$5 + (-8)$$

B.
$$-5 + 8$$

D.
$$-5 + (-8)$$

Find each difference.

20.
$$-5 - 6$$
 -11

20.
$$-5 - 6$$
 -11 21. $-7 - (-10)$ **3**

25. 1.2 - (-1.3) **2.5 26.**
$$-\frac{7}{9} - \left(-\frac{2}{9}\right)$$
 $-\frac{5}{9}$ **27.** $\frac{1}{2} - \frac{1}{4}$ $\frac{1}{4}$

27.
$$\frac{1}{2} - \frac{1}{4}$$
 $\frac{1}{4}$

28. A football team gained 5 yards and then lost 7 yards. What real number represents the team's position relative to its original position?

29. The temperature at 6:00 P.M. was 3° C. At midnight, the temperature was -2° C. What real number represents the change in temperature?

30. Rose had \$60 in her checking account. She deposited a check for \$20 that she received from her grandfather. Then she wrote a check for \$35. What is the balance in her checking account?

\$45

Standardized Test Prep

Adding and Subtracting Real Numbers

Multiple Choice

For Exercises 1-5, choose the correct letter.

1. Which expression is equivalent to 17 + (-15)?

A.
$$-17 + 15$$

C. 17 - 15

B.
$$-17 - 15$$

D. 17 + 15

2. Which number could be placed in the square to make the equation true? F

$$-5 - \Box = 14$$

F. −19

I. 19

3. Which expression has the greatest value? B

A.
$$-14 - (-5)$$

C.
$$-14 - 5$$

B.
$$-5 - (-14)$$

D. -5 - 14

4. The wheel was invented about 2500 BC. The gasoline automobile was invented in AD 1885. How many years passed between the invention of the wheel and the invention of the automobile? G

F. 1615 years

H. 1725 years

G. 4385 years

I. 5385 years

5. If r = -18, s = 27, and t = -15, what is the value of r - s - t? **B**

A. -60

B. -30

C. -6

D. 6

Short Response

6. In golf, there is a number of strokes assigned to each hole, called the par for that hole. If you get the ball in the hole in fewer strokes than par, you are under par for the hole. If it takes you more strokes than the par, you are over par for the hole. On the first 9 holes of golf, Avery had a par, 1 over par, 2 under par, another par, 1 under par, 1 over par, 3 over par, 2 under par, and 1 under par.

a. What addition expression would represent all 9 holes?

$$0+1+(-2)+0+(-1)+1+3+(-2)+(-1)$$

b. What is Avery's score relative to par?

-1

[2] Both parts answered correctly.

[1] One part answered correctly.

[0] Neither part answered correctly.

Enrichment

Adding and Subtracting Real Numbers

A number square is a square where the numbers in any row, column, or diagonal have the same sum. Notice that in the square at the right, the sum of each row, each column, and each diagonal is 15.

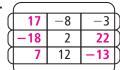
1				L
T	2	9	4	
\mathbb{I}	7	5	3	
\mathbb{I}	6	1	8	
τ			$\overline{}$	

Complete each number square.



۱.)
(-3	11	1
(7	3	-1
(5	-5	9
	$\overline{}$		

1				١
	-7	13	-15	D
	-11	-3	5	D
	9	-19	1	D
7				Γ



1.	$\overline{}$		
(-1.6	-0.1	-0.4
(0.5	-0.7	-1.9
(-1	-1.3	0.2
-	$\overline{}$		

5.

—20	8	6	-14
2	-10	-8	-4
-6	-2	0	-12
(4	-16	-18	10
$\overline{}$			

1				
	-1.3	0.7	-0.9	3.5
	2.7	-0.1	1.5	-2.1
$\left(\right]$	3.1	-0.5	1.1	-1.7
	-2.5	1.9	0.3	2.3
τ				

Name ______ Date _____

1_5 Reteaching

Adding and Subtracting Real Numbers

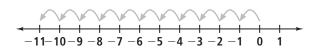
You can add real numbers using a number line or using the following rules.

Rule 1: To add two numbers with the same sign, add their absolute values. The sum has the same sign as the addends.

Problem

What is the sum of -7 and -4?

Use a number line.



Start at zero.

Move 7 spaces to the left to represent -7. Move another 4 spaces to the left to represent -4.

The sum is -11.

Use the rule.

$$-7 + (-4)$$

The addends are both negative.

$$|-7| + |-4|$$

Add the absolute values of the addends.

$$7 + 4 = 11$$

$$|-7| = 7$$
 and $|-4| = 4$.

$$-7 + (-4) = -11$$

The sum has the same sign as the addends.

Rule 2: To add two numbers with different signs, subtract their absolute values. The sum has the same sign as the addend with the greater absolute value.

Problem

What is the sum of -6 and 9?

Use the rule.

$$9 + (-6)$$

The addends have different signs.

$$|9| - |-6|$$

Subtract the absolute values of the addends.

$$9 - 6 = 3$$

$$|9| = 9$$
 and $|-6| = 6$.

$$9 + (-6) = 3$$

The positive addend has the greater absolute value.

Reteaching (continued)

Adding and Subtracting Real Numbers

Exercises

Find each sum.

1.
$$-4 + -12$$
 -16

3.
$$-9 + 1 - 8$$

4.
$$13 + (-7)$$
 6

5.
$$8 + (-14)$$
 -6

6.
$$-11 + (-5)$$
 -16

7.
$$4.5 + (-1.1)$$
 3.4

8.
$$-5.1 + 8.3$$
 3.2

Addition and subtraction are inverse operations. To subtract a real number, add its opposite.

Problem

What is the difference -5 - (-8)?

$$-5 - (-8) = -5 + 8$$
 The opposite of -8 is 8.

$$= 3$$

Use Rule 2.

The difference -5 - (-8) is 3.

Exercises

Find each difference.

10.
$$8 - 20 - 12$$

12.
$$-4 - 9$$
 -13

13.
$$-8 - (-14)$$
 6

14.
$$-11 - (-4)$$
 -7

17.
$$-1.5 - (-1.5)$$
 0

18.
$$-1.7 - 5.4$$
 -7.1

- **19**. The temperature was 5°C. Five hours later, the temperature had dropped 10° C. What is the new temperature? -5° C
- **20. Reasoning** Which is greater, 52 + (-77) or 52 (-77)? Explain.

52 - (-77) is greater. It is the same as 52 + 77 which is a positive number. The sum of 52 + (-77) is a negative number.

1-6 Additional Vocabulary Support

Multiplying and Dividing Real Numbers

Use the list below to complete the graphic organizer.

- Dividing by a fraction is equivalent to multiplying by the _____ of the fraction.
- describes the relationship between a number and its multiplicative inverse
- The reciprocal of a number is its _____.
- For every nonzero real number a, there is a multiplicative inverse $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$.
- $\bullet \quad -5\left(-\frac{1}{5}\right) = 1$
- $\frac{a}{b} \rightarrow \frac{b}{a}$
- $-\frac{2}{3}(-\frac{3}{2}) = 1$
- $\frac{a}{h} \div \frac{c}{d} = \frac{a}{h} \times \frac{d}{c}$

Inverse Property of Multiplication

For every nonzero real number a, there is a multiplicative inverse $\frac{1}{a}$ such that $a\frac{1}{a} = 1$.

$$-\frac{2}{3}\left(-\frac{3}{2}\right)=1$$

describes the relationship between a number and its multiplicative inverse

Multiplicative Inverse

 $-5\left(-\frac{1}{5}\right)=1$

Dividing by a fraction is equivalent to multiplying by of the fraction

The reciprocal of a number is its

Reciprocal

 $\frac{a}{b} \rightarrow \frac{b}{a}$

Dividing by a fraction is equivalent to multiplying by of the fraction

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Name Class Date	
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1-6

Think About a Plan

Multiplying and Dividing Real Numbers

Farmer's Market A farmer has 120 bushels of beans for sale at a farmer's market. He sells an average of $15\frac{3}{4}$ bushels each day. After 6 days, what is the change in the total number of bushels the farmer has for sale at the farmer's market?

Understanding the Problem

- 1. How does the number of bushels the farmer has change each day?

 The number of bushels decreases.
- 2. Should the change be a positive or a negative number? How do you know? negative: The amount the farmer has is less.

Planning the Solution

3. What expression represents the total number of bushels sold in 6 days?

$$6\cdot \left(-15\frac{3}{4}\right)$$

Getting an Answer

4. Evaluate your expression in Exercise 3 to determine the change in the total number of bushels the farmer has for sale at the farmer's market.

$$-94\frac{1}{2}$$
 bushels

5. Is your answer reasonable? Explain.

yes; the change is negative and the absolute value of the change must be less than 120 because the farmer cannot have a negative amount of beans.

Practice

Form G

Multiplying and Dividing Real Numbers

Find each product. Simplify, if necessary.

4.
$$(-9)^2$$

5.
$$-3 \times 12$$

81

7.
$$-3(2.3)$$

8.
$$(-0.6)^2$$

$$-6.9$$

11. $-\frac{2}{5}\left(-\frac{5}{8}\right)$

$$-19.2$$

10.
$$-\frac{3}{4} \cdot \frac{2}{9}$$

12.
$$\left(\frac{2}{3}\right)^2$$

 $-\frac{1}{6}$

- 13. After hiking to the top of a mountain, Raul starts to descend at the rate of 350 feet per hour. What real number represents his vertical change after $1\frac{1}{2}$ hours? -525 ft
- 14. A dolphin starts at the surface of the water. It dives down at a rate of 3 feet per second. If the water level is zero, what real number describes the dolphin's location after $3\frac{1}{2}$ seconds?

$$-10\frac{1}{2}$$
 ft

Simplify each expression.

15.
$$\sqrt{1600}$$

16.
$$-\sqrt{625}$$

17.
$$\pm \sqrt{10,000}$$

40

-25

18. $-\sqrt{0.81}$

19.
$$\pm \sqrt{1.44}$$

20. $\sqrt{0.04}$

-0.9

±1.2

0.2

21.
$$\pm \sqrt{\frac{4}{9}}$$

22.
$$-\sqrt{\frac{16}{49}}$$

23.
$$\sqrt{\frac{10}{12}}$$

 $\pm \frac{2}{3}$

$$-\frac{4}{7}$$

Practice (continued)

Form G

Practice (continued)

Multiplying and Dividing Real Numbers

24. Writing Explain the differences among $\sqrt{25}$, $-\sqrt{25}$, and $\pm\sqrt{25}$.

There are 2 square roots of 25, 5 and -5. $\sqrt{25}$ represents the positive square root and $-\sqrt{25}$ represents the negative square root, and $\pm\sqrt{25}$ represents both square roots.

25. Reasoning Can you name a real number that is represented by $\sqrt{-36}$? Explain.

no; There is no number that can be multiplied by itself and have a negative product.

Find each quotient. Simplify, if necessary.

27.
$$-250 \div (-25)$$
 10 28. $98 \div 2$ **49**

30.
$$-93 \div (-3)$$
 31

29.
$$84 \div (-4)$$
 -21 30. $-93 \div (-3)$ **31 31.** $\frac{-105}{5}$ **-21**

32.
$$14.4 \div (-3)$$
 -4.8

32.
$$14.4 \div (-3)$$
 -4.8 33. $-1.7 \div (-10)$ **0.17 34.** $-8.1 \div 3$ **-2.7**

35.
$$17 \div \frac{1}{3}$$
 51

35.
$$17 \div \frac{1}{3}$$
 51 36. $-\frac{3}{8} \div \left(-\frac{9}{10}\right)$ $\frac{5}{12}$ **37.** $-\frac{5}{6} \div \frac{1}{2}$ $-1\frac{2}{3}$

37.
$$-\frac{5}{6} \div \frac{1}{2}$$
 $-1\frac{2}{3}$

Evaluate each expression for $a = -\frac{1}{2}$, $b = \frac{3}{4}$, and c = -6.

38.
$$-ab = \frac{3}{8}$$

39.
$$b \div c -\frac{1}{8}$$
 40. $\frac{c}{a}$ **12**

40.
$$\frac{c}{a}$$
 12

41. Writing Explain how you know that -5 and $-\frac{1}{5}$ are multiplicative inverses.

Because $-5 \times \frac{1}{5} = -1$, the two numbers are multiplicative inverses.

42. At 6:00 P.M., the temperature was 55°F. At 11:00 P.M. that same evening, the temperature was 40°F. What real number represents the average change in temperature per hour?

-3° F/h

Practice

Form K

Multiplying and Dividing Real Numbers

- **1. Writing** Is the product $-8 \times (-5)$ *positive* or *negative*? How do you know? positive; Answers may vary. Sample: -8 and -5 have the same sign, so their product will be positive.
- 2. **Open-Ended** Write a multiplication problem with a negative product. How do you know the product will be negative?

Answers may vary. The answer should have one positive factor and one negative factor. Sample: -5(4)

Find each product. Simplify, if necessary.

3.
$$-2(-3)$$
 6

6.
$$(-5)^2$$
 25

7.
$$-3 \times 7$$
 -21

9.
$$-3(1.2)$$
 -3.6

10.
$$-\frac{1}{2} \cdot \frac{1}{3} - \frac{1}{6}$$

9.
$$-3(1.2)$$
 -3.6 10. $-\frac{1}{2} \cdot \frac{1}{3}$ - $\frac{1}{6}$ 11. $-\frac{2}{5} \left(-\frac{1}{4}\right)$ $\frac{1}{10}$

- 12. A scuba diver descends in the water at the rate of 40 feet per minute. What real number describes the diver's location with respect to the water level after the first 3 minutes of his dive? -120 ft
- 13. A football team has three 15-yard penalties. What real number describes the change in yardage from these penalties? -45 yd

Practice (continued)

Form K

Multiplying and Dividing Real Numbers

Simplify each expression.

14.
$$\sqrt{16}$$
 4

15.
$$-\sqrt{25}$$
 -5

15.
$$-\sqrt{25}$$
 -5 16. $\pm\sqrt{100}$ \pm **10**

17.
$$-\sqrt{36}$$
 -6

17.
$$-\sqrt{36}$$
 -6 18. $\pm\sqrt{0.64}$ **\pm 0.8**

19.
$$\pm \sqrt{\frac{4}{25}}$$
 $\pm \frac{2}{5}$

20. Writing Explain the differences among $\sqrt{4}$, $-\sqrt{4}$, and $\pm\sqrt{4}$. Answers may vary. Sample: There are 2 square roots of 4, 2 and -2. $\sqrt{4}$ represents the positive square root, $-\sqrt{4}$ represents the negative square root, and $\pm \sqrt{4}$ represents both square roots.

Find each quotient. Simplify, if necessary.

22.
$$-25 \div (-5)$$
 5

24.
$$24 \div (-8)$$
 -3 25. $-27 \div (-3)$ **9**

25.
$$-27 \div (-3)$$

26.
$$\frac{-35}{5}$$
 -7

27.
$$4.4 \div (-2)$$
 -2.2

27.
$$4.4 \div (-2)$$
 28. $-\frac{1}{8} \div \left(-\frac{1}{2}\right) \frac{1}{4}$

29.
$$-\frac{3}{4} \div \frac{1}{5}$$
 $-3\frac{3}{4}$

30. The population of Centerville has decreased by 500 people in the last 5 years. What real number describes the average change in population per year? -100 people/yr

Standardized Test Prep

Multiplying and Dividing Real Numbers

Multiple Choice

For Exercises 1-5, choose the correct letter.

1. Which expression has a negative value? C

A.
$$(-2)^2$$

B.
$$(-5)(-7)$$

C.
$$(-3)^3$$

D.
$$0 \times (-5)$$

2. If $x = -\frac{3}{4}$ and $y = \frac{1}{6}$, what is the value of -2xy?

F.
$$-\frac{1}{4}$$

F.
$$-\frac{1}{4}$$
 G. $-\frac{1}{6}$

H.
$$\frac{1}{6}$$

1.
$$\frac{1}{4}$$

3. Which expression has the same value as $-\frac{1}{7} \div \left(-\frac{2}{3}\right)$? **A**

A.
$$\frac{1}{7} imes \frac{3}{2}$$

$$\mathbf{B.} \ -\!\!\left(\frac{1}{7} \times \frac{3}{2}\right)$$

c.
$$\frac{7}{1} \times \frac{2}{3}$$

A.
$$\frac{1}{7} \times \frac{3}{2}$$
 B. $-(\frac{1}{7} \times \frac{3}{2})$ **C.** $\frac{7}{1} \times \frac{2}{3}$ **D.** $-(\frac{7}{1} \times \frac{2}{3})$

4. ABC stock sold for \$64.50. Four days later, the same stock sold for \$47.10. What is the average change per day? F

5. The formula $C = \frac{5}{9}(F - 32)$ converts a temperature reading from the Fahrenheit scale F to the Celsius scale C. What is the temperature 5°F measured in Celsius? B

A.
$$\left(-20\frac{5}{9}\right)^{\circ}$$
 C

D.
$$(20\frac{5}{9})^{\circ}$$
 C

Short Response

6. A clock loses 2 minutes every 6 hours. At 3:00 P.M., the clock is set to the correct time and allowed to run without interference.

a. What integer would describe the time loss after exactly 3 days? -24 min

b. What would the clock read at 3:00 P.M. three days later? 2:36 P.M.

[2] Both parts answered correctly.

[1] One part answered correctly.

[0] Neihter part answered correctly.

Enrichment

Multiplying and Dividing Real Numbers

A matrix is a rectangular array of numbers. Some examples of matrices are given at the right.

$$\begin{bmatrix} 5 & 9 \\ 3 & 4 \\ 7 & 0 \end{bmatrix} \qquad \begin{bmatrix} -4 & 3 & 0 \\ -8 & 6 & 3 \\ 4 & -1 & 2 \end{bmatrix}$$

You can perform operations using matrices. One operation is called scalar multiplication. In scalar multiplication, each number in the matrix is multiplied by the number outside the matrix. The products are listed in another matrix in the same order.

Complete each scalar multiplication.

1.
$$5\begin{bmatrix} 3 & 10 & 4 \\ 7 & 6 & 11 \end{bmatrix}$$
 $\begin{bmatrix} 15 & 50 & 20 \\ 35 & 30 & 55 \end{bmatrix}$

1.
$$5\begin{bmatrix} 3 & 10 & 4 \\ 7 & 6 & 11 \end{bmatrix}$$
 $\begin{bmatrix} 15 & 50 & 20 \\ 35 & 30 & 55 \end{bmatrix}$ 2. $-7\begin{bmatrix} 0 & -2 & 4 & 2 & -8 \\ 10 & -6 & 5 & 4 & -1 \\ 3 & -7 & 9 & -4 & -5 \end{bmatrix}$

3.
$$10\begin{bmatrix} -3 & 5 & -8 \\ 2 & 10 & -1 \\ 0 & 4 & 4 \end{bmatrix} \begin{bmatrix} -30 & 50 & -80 \\ 20 & 100 & -10 \\ 0 & 40 & 40 \end{bmatrix}$$

4.
$$0 \begin{bmatrix} 27 & -84 \\ 16 & -76 \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

The matrix at the right compares the prices of 3 different digital cameras at 3 different stores.

Camera A	Camera B	Camera C
\$153.00	\$207.00	\$255.00
\$142.50	\$212.00	\$251.00
\$192.00	\$209.50	\$249.50

5. If the sales tax is 6%, each number in the matrix must be multiplied by 1.06 to determine the total cost of each camera. Write a scalar multiplication problem that could be used to determine the total cost of the cameras.

	\$153.00	\$207.00	\$255.00
1.06	\$142.50	\$207.00 \$212.00	\$251.00
	_\$192.00	\$209.50	\$249.50

6. Complete the scalar multiplication you wrote in Exercise 5.

\$162.18	\$219.42	\$270.30	
\$151.05	\$224.72	\$266.06	
\$203.52	\$222.07	\$264.47	

- 7. Use the matrix you found in Exercise 6 to determine the difference in the total cost if you bought Camera C from the Discount Store rather than the Camera Store. \$4.24
- 8. What is the difference between the greatest total cost and the least total cost for Camera A? \$52.47

Reteaching

Multiplying and Dividing Real Numbers

You need to remember two simple rules when multiplying or dividing real numbers.

- 1. The product or quotient of two numbers with the same sign is positive.
- 2. The product or quotient of two numbers with different signs is negative.

Problem

What is the product -6(-30)?

$$-6(-30) = 180$$

-6 and -30 have the same sign so the product is positive.

Problem

What is the quotient $72 \div (-6)$?

$$72 \div (-6) = -12$$

 $72 \div (-6) = -12$ 72 and -6 have different signs so the quotient is negative.

Exercises

Find each product or quotient.

1.
$$-5(-6)$$
 30

3.
$$-3 \times 22$$
 -66

5.
$$81 \div (-9)$$
 -9

6.
$$-55 \div (-11)$$
 5

9.
$$(-6)^2$$
 36

11.
$$-7.7 \div (-11)$$
 0.7 12. $-1.4(-2)$ **2.8**

13.
$$-\frac{1}{2} \times \frac{1}{3}$$
 $-\frac{1}{6}$ **14.** $-\frac{2}{3}(-\frac{3}{5})$ $\frac{2}{5}$

14.
$$-\frac{2}{3}\left(-\frac{3}{5}\right)$$

15.
$$\frac{3}{4} \cdot \left(-\frac{1}{3}\right) -\frac{1}{4}$$

- **16.** The temperature dropped 2°F each hour for 6 hours. What was the total change in temperature? -12° F
- 17. Reasoning Since $5^2 = 25$ and $(-5)^2 = 25$, what are the two values for the square root of 25? 5 and -5

1-6 Reteaching (continued)

Multiplying and Dividing Real Numbers

The product of 7 and $\frac{1}{7}$ is 1. Two numbers whose product is 1 are called reciprocals. To divide a number by a fraction, multiply by its reciprocal.

Problem

What is the quotient $\frac{2}{3} \div \left(-\frac{5}{7}\right)$?

$$\frac{2}{3} \div \left(-\frac{5}{7}\right) = \frac{2}{3} \times \left(-\frac{7}{5}\right)$$
$$= -\frac{14}{15}$$

 $\frac{2}{3} \div \left(-\frac{5}{7}\right) = \frac{2}{3} \times \left(-\frac{7}{5}\right)$ To divide by a fraction, multiply by its reciprocal.

 $=-\frac{14}{15}$ The signs are different so the answer is negative.

Exercises

Find each quotient.

18.
$$\frac{1}{2} \div \frac{1}{3}$$
 1 $\frac{1}{2}$

19.
$$-6 \div \frac{2}{3}$$
 -9

18.
$$\frac{1}{2} \div \frac{1}{3}$$
 19. $-6 \div \frac{2}{3}$ **9 20.** $-\frac{2}{5} \div \left(-\frac{2}{3}\right)$ **3**

21.
$$\frac{1}{2} \div \left(-\frac{1}{4}\right)$$
 -2

21.
$$\frac{1}{2} \div \left(-\frac{1}{4}\right)$$
 -2 22. $\left(-\frac{5}{7}\right) \div \left(-\frac{1}{2}\right)$ **13** $\frac{3}{7}$ **23.** $-\frac{2}{3} \div \frac{1}{4}$ **-2** $\frac{2}{3}$

23.
$$-\frac{2}{3} \div \frac{1}{4}$$
 -2 $\frac{2}{3}$

24. Writing Another way of writing $\frac{a}{b}$ is $a \div b$. Explain how you could evaluate $\frac{\frac{1}{2}}{1}$.

What is the value of this expression?

Change the problem to the equivalent division problem $\frac{1}{2} \div \frac{1}{6}$. To find this quotient, change this division problem to the multiplication problem $\frac{1}{2} \times \frac{6}{1}$. The answer is 3.

Additional Vocabulary Support The Distributive Property

Complete the vocabulary chart by filling in the missing information.

Word or Word Phrase	Definition	Picture or Example
coefficient	a numerical factor of a term that contains a variable	$4a^2 - 3ab + 2b - 8$ Coefficients: 4, -3 , and 2
constant	a term that has no variable	4a ² - 3ab + 2b - 8 Constant: -8
Distributive Property	For real numbers, a, b, and c the product of a and (b + c) is ab + bc	$7(3+2) = 7 \cdot 5 = 35$ $7 \cdot 3 + 7 \cdot 2 = 21 + 14 = 35$
like terms	terms that have exactly the same variable factors raised to the same power	-8x and 5x
term	a number, a variable, or the product of a number and one or more variables	$4a^2 - 3ab + 2b - 8$ Terms: $4a^2$, $-3ab$, $2b$, and -8 .

Name	Class	Date

1-7

Think About a Plan

The Distributive Property

Exercise The recommended heart rate for exercise, in beats per minute, is given by the expression 0.8(200 - y) where y is a person's age in years. Rewrite this expression using the Distributive Property. What is the recommended heart rate for a 20-year-old person? For a 50-year-old person? Use mental math.

Understanding the Problem

1. What relationship does the given expression represent? What does the variable in the expression represent?

It gives the recommended heart rate for exercise, in beats per minute, for people of different ages. The variable is the age of the person being evaluated.

- What does it mean to rewrite the expression using the Distributive Property?The 0.8 must be distributed by multiplying it by each term inside the parentheses.
- 3. What does it mean to use mental math?Calculations that can be done mentally without work being shown.

Planning the Solution

4. How do you determine the recommended heart rate for people of different ages? You can substitute their ages in for *y* of the given expression.

Getting an Answer

 $\textbf{5.} \ \ Rewrite the expression using the \ Distributive \ Property.$

160 - 0.8*y*

 $\textbf{6.} \ \ \textbf{What is the recommended heart rate for a 20-year-old person? Show your work.}$

144 beats/min

7. What is the recommended heart rate for a 50-year-old person? Show your work.

120 beats/min

Practice

Form G

The Distributive Property

Use the Distributive Property to simplify each expression.

1.
$$3(h-5)$$

2.
$$7(-5 + m)$$

7m - 35

3.
$$(6 + 9\nu)6$$

2.
$$7(-5+m)$$
 3. $(6+9v)6$ **4.** $(5n+3)12$

$$54v + 36$$

$$60n + 36$$

$$-20a + 160$$

$$42x + 84$$

5.
$$20(8-a)$$
 6. $15(3y-5)$ 7. $21(2x+4)$ 8. $(7+6w)6$

$$-20a+160$$
 45y-75 42x+84 36w+42

9.
$$(14 - 9p)1.1$$

10.
$$(2b - 10)3.2$$

11.
$$\frac{1}{3}(3z + 12)$$

9.
$$(14 - 9p)1.1$$
 10. $(2b - 10)3.2$ **11.** $\frac{1}{3}(3z + 12)$ **12.** $4(\frac{1}{2}t - 5)$

$$-9.9p + 15.4$$

$$6.4b - 32$$
 $z + 4$

13.
$$(-5x - 14)(5.1)$$

$$-25.5x - 71.4 \qquad \qquad -\frac{1}{2}r - \frac{5}{7}$$

13.
$$(-5x - 14)(5.1)$$
 14. $1(-\frac{1}{2}r - \frac{5}{7})$ **15.** $10(6.85j + 7.654)$ **16.** $\frac{2}{3}(\frac{2}{3}m - \frac{2}{3})$

$$68.5j + 76.54$$

$$\frac{4}{9}m - \frac{4}{9}$$

Write each fraction as a sum or difference.

17.
$$\frac{3n+5}{7}$$
 $\frac{3n}{7}$ + $\frac{5}{7}$

18.
$$\frac{14-6x}{19}$$
 $\frac{14}{19} - \frac{6x}{19}$

19.
$$\frac{3d+5}{6}$$
 $\frac{d}{2}$ + $\frac{5}{6}$

17.
$$\frac{3n+5}{7}$$
 $\frac{3n}{7}$ + $\frac{5}{7}$ 18. $\frac{14-6x}{19}$ $\frac{14}{19}$ - $\frac{6x}{19}$ 19. $\frac{3d+5}{6}$ $\frac{d}{2}$ + $\frac{5}{6}$ 20. $\frac{9p-6}{3}$ 3p - 2

21.
$$\frac{18+8z}{6}$$
 3 + $\frac{4z}{3}$

22.
$$\frac{15n-42}{14}$$
 $\frac{15n}{14}$ - 3

21.
$$\frac{18+8z}{6}$$
 3 + $\frac{4z}{3}$ 22. $\frac{15n-42}{14}$ $\frac{15n}{14}$ 3 23. $\frac{56-28w}{8}$ 7 - $\frac{7w}{2}$ 24. $\frac{81f+63}{9}$ 9f + 7

24.
$$\frac{81f + 63}{9}$$
 9f + 7

Simplify each expression.

25.
$$-(14 + x)$$

28.
$$-(-r+1)$$

29.
$$-(4m - 6n)$$

30.
$$-(5.8a + 4.2b)$$
 $-5.8a - 4.2b$

25.
$$-(14 + x)$$
 26. $-(-8 - 6t)$ 27. $-(6 + d)$ 28. $-(-r + 1)$ 7. $-(-7 + 1)$ 29. $-(4m - 6n)$ 30. $-(5.8a + 4.2b)$ 31. $-(-x + y - 1)$ 32. $-(f + 3g - 7)$ 31. $-(-x + y - 1)$ 32. $-(f + 3g - 7)$ 33. $-(f - 3g + 7)$ 34. $-(f - 3g + 7)$ 35. $-(f - 3g + 7)$ 36. $-(f - 3g + 7)$ 37. $-(f - 3g + 7)$ 38. $-(f - 3g + 7)$ 39. $-(f - 3g + 7)$

32.
$$-(f + 3g - 7)$$
 $-f - 3g + 7$

Use mental math to find each product.

37.
$$4.2 \times 5$$
 21 38. 4×10.1 **40.4**

39.
$$8.25 \times 4$$
 33

40.
$$11 \times 4.1$$
 45.1

- **41.** You buy 75 candy bars at a cost of \$0.49 each. What is the total cost of 75 candy bars? Use mental math. \$36.75
- 42. The distance around a track is 400 m. If you take 14 laps around the track, what is the total distance you walk? Use mental math. 5600 m
- **43.** There are 32 classmates that are going to the fair. Each ticket costs \$19. What is the total amount the classmates spend for tickets? Use mental math. \$608

Practice (continued)

Form G

The Distributive Property

Simplify each expression by combining like terms.

44.
$$4t + 6t$$
 10t

45.
$$17y - 15y$$
 2y

46.
$$-11b^2 + 4b^2$$
 -7 b^2

47.
$$-2y - 5y - 7y$$

48.
$$14n^2 - 7n^2$$
 7n²

49.
$$8x^2 - 10x^2$$
 -2 x^2

50.
$$2f + 7g - 6 + 8g$$

 $2f + 15g - 6$

51.
$$8x + 3 - 5x - 9$$

52.
$$-5k - 6k^2 - 12k + 10$$

 $-6k^2 - 17k + 10$

Write a word phrase for each expression. Then simplify each expression.

53.
$$2(n+1)$$

54.
$$-5(x-7)$$

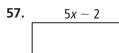
55.
$$\frac{1}{2}(4m - 8)$$

two times the sum of a number and one; 2n + 2 negative five times the difference of a number and seven; -5x + 35

one-half the difference of four times a number and eight; 2m - 4

56. The tax a plumber must charge for a service call is given by the expression 0.06(35 + 25h) where h is the number of hours the job takes. Rewrite this expression using the Distributive Property. What is the tax for a 5 hour job and a 20 hour job? Use mental math. 2.1 + 1.5h; \$9.60; \$32.10

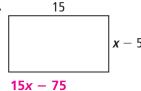
Geometry Write an expression in simplified form for the area of each rectangle.



20x - 8

$$-2n + 17$$

-48n + 408



Simplify each expression.

60.
$$4ik - 7ik + 12ik$$
 9jk

61.
$$-17mn + 4mn - mn + 10mn$$
 -4mn

62.
$$8xy^4 - 7xy^3 - 11xy^4 - 3xy^4 - 7xy^3$$
 63. $-2(5ab - 6) - 10ab + 12$

63.
$$-2(5ab - 6)$$
 -10 $ab + 12$

64.
$$z + \frac{2z}{5} - \frac{4z}{5}$$
 $\frac{3z}{5}$

65.
$$7m^2n + 4m^2n^2 - 4m^2n - 5m^3n^2 - 5mn^2$$

 $3m^2n + 4m^2n^2 - 5m^3n^2 - 5mn^2$

66. Reasoning Demonstrate why
$$\frac{12x-6}{6} \neq 2x-6$$
. Show your work. $\frac{12x-6}{6} = \frac{1}{6}(12x-6) = \frac{1}{6}(12x) - \frac{1}{6}(6) = 2x-1$; $2x-1 \neq 2x-6$

Simplify each expression.

67.
$$4(2h+1) + 3(4h+7)$$
 68. $5(n-8) + 6(7-2n)$ **69.** $7(3+x) - 4(x+1)$ **20 69.** $7(3+x) - 4(x+1)$ **3 3 x 17**

68.
$$5(n-8) + 6(7-2n)$$
 -7n + 2

69.
$$7(3 + x) - 4(x + 1)$$

70.
$$6(y+5)-3(4y+2)$$
 71. $-(a-3b+27)$ **72.** $-2(5-4s+6t)-5s+t$

-6v + 24

71.
$$-(a - 3b + 27)$$

72.
$$-2(5-4s+6t)-5s+t$$

3s - 11t - 10

Practice

Form K

The Distributive Property

Use the Distributive Property to simplify each expression.

1.
$$7(z-4)$$
 7z - 28

2.
$$3(2 + w)$$
 6 + 3w

3.
$$(2h-4)11$$
 22h - 44

4.
$$(6y - 3)5$$
 30y - 15

5.
$$17(2b + 3)$$
 34b + 51

6.
$$12(4-8p)$$
 48 - 96p

7.
$$7(11 - n)$$
 77 – 7n

8.
$$(1 - 11j)4$$
 4 – 44j

Write each fraction as a sum or difference.

9.
$$\frac{2x+3}{3}$$
 $\frac{2}{3}x+1$

10.
$$\frac{11n-14}{9}$$
 $\frac{11}{9}n-\frac{14}{9}$

11.
$$\frac{5t-12}{10}$$
 $\frac{1}{2}t-\frac{6}{5}$

12.
$$\frac{24k+18}{6}$$
 4k + 3

Simplify each expression.

13.
$$-1(p+6)$$
 $-p-6$

14.
$$-(-9-4y)$$
 9 + 4y

15.
$$-(a-15)$$
 -a + 15

16.
$$-(-z-12)$$
 z + **12**

Use mental math to find each product.

17.
$$2.1 \times 6$$
 12.6

18.
$$12 \times 6.8$$
 81.6

19.
$$49 \times 7$$
 343

- 21. You buy 125 tickets to an amusement park that each cost \$19.50. What is the total cost of the 125 tickets? Use mental math. \$2437.50
- 22. There are 12 sections in the stadium. Each section of the stadium can seat 1500 people. What is the total seating capacity of the stadium? Use mental math. 18,000 people

Practice (continued)

Form K

The Distributive Property

Simplify each expression by combining like terms.

23.
$$9y + 11y$$
 20y

24.
$$23b - 19b$$
 4b

25.
$$35t - 42t$$
 -7t

26.
$$-4p + 2p$$
 -2p

27.
$$-10x^2 - 14x^2$$
 -24x²

28.
$$-5k^2 + 6k^2 k^2$$

29.
$$7w^2 - 14w^2$$
 -7w²

30.
$$6a - 7 + 4 - a$$
 5a - 3

Write a word phrase for each expression. Then simplify each expression.

31.
$$3(c + 5)$$

three times the sum of a number and five; 3c + 15

32.
$$-7(n-1)$$

negative seven times the difference of a number and one; -7n + 7

33. The profit a company receives is given by the expression 0.15(855p - 315)where *p* is the number of products sold. Rewrite this expression using the Distributive Property. What is the profit for 25 products sold and 150 products sold? Use mental math. 128.25p - 47.25; \$3159; \$19,190.25

Simplify each expression.

34.
$$7xy - xy + 3xy$$
 9 xy

36.
$$5m^2n - 3mn^2 - 7m^2n$$

 $-3mn^2 - 2m^2n$

38.
$$-vw^2 + vw - v^2w - 3vw^2 + 2v^2w$$

 $vw - 4vw^2 + v^2w$

35.
$$25pq + 13pq - 6 - 35pq + 4$$
 3pq - 2

37.
$$3(-6fg - 4)$$

-18fg - 12

39.
$$x + \frac{x}{4} - \frac{3x}{4}$$

40. Reasoning Demonstrate why $\frac{15x-5}{5} \neq 3x-5$. Show your work.

$$\frac{15x - 5}{5} = \frac{1}{5}(15x - 5)$$
$$= \frac{1}{5}(15x) - \frac{1}{5}(5)$$
$$= 3x - 1$$

Simplify each expression.

41.
$$3(x + 4) + 2(5x + 2)$$

13x + 16

41.
$$3(x + 4) + 2(5x + 2)$$
 42. $3(2n - 7) + 7(4 - 2n)$ **43.** $5(5 + t) - 3(t - 6)$ **13x + 16 2t + 43**

43.
$$5(5+t)-3(t-6)$$

Standardized Test Prep

The Distributive Property

Multiple Choice

For Exercises 1–6, choose the correct letter.

1. What is the simplified form of the expression 6(4x-7)?

A.
$$10x - 1$$

B.
$$24x - 7$$

C.
$$24x - 42$$

D.
$$24x + 42$$

2. What is the simplified form of the expression -2(-5x - 8)?

F.
$$-7x - 10$$

G.
$$10x - 8$$

H.
$$10x - 16$$

I.
$$10x + 16$$

3. What is the simplified form of the expression $14mn + 6mn^2 - 8mn - 7m^2n + 5m^2n$?

A.
$$10m^2n^2$$

B.
$$6mn - 4m^2n$$

C.
$$6mn + 5m^2n - 1mn^2$$

D.
$$6mn - 2m^2n + 6mn^2$$

4. Concert tickets cost \$14.95 each. Which expression represents the total cost of 25 tickets? F

F.
$$25(15 - 0.05)$$

F.
$$25(15 - 0.05)$$
 G. $25(15 + 0.05)$

H.
$$15(25 - 10.05)$$

I.
$$25(15) - 0.05$$

5. Which expression represents 7 times the sum of a number and 8? **B**

A.
$$7n + 8$$

B.
$$7(n + 8)$$

C.
$$8(n + 7)$$

D.
$$n + 56$$

6. There are 297 students in a senior class. The cost of the senior trip is \$150 per student. Which expression represents the total cost of the senior trip? H

G.
$$300(150 - 3)$$

H.
$$150(300 - 3)$$

I.
$$150(300) - 3$$

Short Response

7. The profit Samantha's company makes is given by the expression 0.1(1000 + 300m)where m is total number of sales. Rewrite this expression using the Distributive Property. What is the profit if her company sells 50 pieces of merchandise? Use mental math.

100 + 30m: \$1600

- [2] Both parts answered correctly.
- [1] One part answered correctly.
- [0] Neither part answered correctly.

Enrichment

The Distributive Property

The Distributive Property can be used more than once in the same expression. In this lesson, you learned that basic multiplication calculations can be completed using mental math.

$$3 \cdot 84 = 3(80 + 4) = 3(80) + 3(4) = 240 + 12 = 252$$

The same process can be used when both numbers are two-digit numbers. However, the Distributive Property must be used more than once. Look at the following example.

$$(49)(26) = (40 + 9)(20 + 6)$$
Rewrite 49 as $40 + 9$ and 26 as $20 + 6$.
$$= (40)(20 + 6) + (9)(20 + 6)$$

$$= (40)(20) + (40)(6) + (9)(20) + (9)(6)$$
Distribute 40 and 9 into $(20 + 6)$.
$$= 800 + 240 + 180 + 54$$
Multiply.
$$= 1274$$
Add.

Exercises

Use the Distributive Property to find each product. Show your work.

This same procedure can be utilized for simplifying algebraic expressions. Instead of (20 + 2)(30 + 1), the expression might be (x + 2)(x + 1).

$$(x + 2)(x + 1) = (x)(x + 1) + (2)(x + 1)$$
 $(x + 2)$ can be distributed into $(x + 1)$.
 $= (x)(x) + (x)(1) + (2)(x) + (2)(1)$ Distribute x and x into x and x into x into x into x and x into x

Exercises

Use the Distributive Property to find each product. Show your work.

4.
$$(x + 3)(x + 4)$$
 5. $(x + 1)(x + 8)$ **6.** $(x + 4)(x + 2)$ $x^2 + 7x + 12$ $x^2 + 9x + 8$ $x^2 + 6x + 8$ **7.** $(x + 1)^2$ (Hint: Remember that $(x + 1)^2 = (x + 1)(x + 1)$.)

$$x^2 + 2x + 1$$

Reteaching

The Distributive Property

The Distributive Property states that the product of a sum and another factor can be rewritten as the sum of two products, each term in the sum multiplied by the other factor. For example, the Distributive Property can be used to rewrite the product 3(x + y) as the sum 3x + 3y. Each term in the sum x + y is multiplied by 3; then the new products are added.

Problem

What is the simplified form of each expression?

a.
$$4(x + 5)$$

= $4(x) + 4(5)$ Distributive Property
= $4x + 20$ Simplify.

b.
$$(2x - 3)(-3)$$

= $2x(-3) - 3(-3)$ Distributive Property
= $-6x + 9$ Simplify.

The Distributive Property can be used whether the factor being multiplied by a sum or difference is on the left or right.

The Distributive Property is sometimes referred to as the Distributive Property of Multiplication over Addition. It may be helpful to think of this longer name for the property, as it may remind you of the way in which the operations of multiplication and addition are related by the property.

Exercises

Use the Distributive Property to simplify each expression.

1.
$$6(z + 4)$$
 6z + 24

2.
$$2(-2 - k)$$

3.
$$(5x + 1)4$$

2.
$$2(-2-k)$$
 3. $(5x+1)4$ **4.** $(7-11n)10$ **-4-2k 20x+4 70-110n**

5.
$$(3 - 8w)4.5$$

6.
$$(4p + 3)2.$$

6.
$$(4p + 5)2.6$$
 7. $4(y + 4)$

8.
$$6(q-2)$$

$$10.4p + 13$$

Write each fraction as a sum or difference.

9.
$$\frac{2m-5}{9}$$

10.
$$\frac{8 + 7z}{11}$$
 $\frac{8}{11} + \frac{7z}{11}$

11.
$$\frac{24f + 15}{9}$$

9.
$$\frac{2m-5}{9}$$
 10. $\frac{8+7z}{11}$ 11. $\frac{24f+15}{9}$ 12. $\frac{12d-16}{6}$ $\frac{2m}{9}-\frac{5}{9}$ $\frac{8}{11}+\frac{7z}{11}$ $\frac{8f}{3}+\frac{5}{3}$ 2d $-\frac{8}{3}$

Simplify each expression.

14.
$$-(-9h - 4)$$
 15. $-(-n + 11)$ **16.** $-(6 - 8f)$

$$9h + 4$$

$$-6 + 8f$$

1-7

Reteaching (continued)

The Distributive Property

The previous problem showed how to write a product as a sum using the Distributive Property. The property can also be used to go in the other order, to convert a sum into a product.

Problem

How can the sum of like terms 15x + 6x be simplified using the Distributive Property?

Each term of 15x + 6x has a factor of x. Rewrite 15x + 6x as 15(x) + 6(x). Now use the Distributive Property in reverse to write 15(x) + 6(x) as (15 + 6)x, which simplifies to 21x.

Exercises

Simplify each expression by combining like terms.

17.
$$16x + 12x$$

28x

18.
$$25n - 17n$$

8n

19.
$$-4p + 6p$$

2p

20.
$$-15a - 9a$$

21.
$$-9k^2 - 5k^2$$

22.
$$12t^2 - 20t^2$$

By thinking of or rewriting numbers as sums or differences of other numbers that are easier to use in multiplication, the Distributive Property can be used to make calculations easier.

Problem

How can you multiply 78 by 101 using the Distributive Property and mental math?

78×101	Write the product.
10 ^ 101	Wille the bioduct.

 $78 \times (100 + 1)$ Rewrite 101 as sum of two numbers that are easy to use in multiplication.

78(100) + 78(1) Use the Distributive Property to write the product as a sum.

7800 + 78 Multiply. 7878 Simplify.

Exercises

Use mental math to fintd each product.

23.
$$5.1 \times 7$$

24.
$$24.95 \times 4$$

26.
$$12 \times 95$$

1-8

Additional Vocabulary Support

An Introduction to Equations

Problem

Is x = 5 a solution of the equation 20 = 2x + 10? Justify and explain your work.

Explain	Work	Justify
First, write the equation.	20 = 2x + 10	Original equation
Second, substitute 5 for x.	20 = 2(5) + 10	Substitute.
Then, simplify.	20 = 20	Simplify.
Finally, answer the question asked.	Yes, 5 is a solution of the equation $20 = 2x + 10$.	State the original question with the correct answer.
	Solution	

Yes

yes

Exercise

Is n = 4 a solution of the equation 16 = 3n + 4? Justify and explain your work.

Explain	Work	Justify
First, write the equation.	16 = 3n + 4	Original equation
Second, substitute 4 for n.	16 = 3(4) + 4	Substitute.
Then, simplify.	16 = 16	Simplify.
Finally, answer the question asked.	Yes, 4 is a solution of the equation $16 = 3n + 4$.	State the original question with the correct answer.
	Solution	

Think About a Plan

An Introduction to Equations

Deliveries The equation 25 + 0.25p = c gives the cost c in dollars that a store charges to deliver an appliance that weighs p pounds. Use the equation and a table to find the weight of an appliance that costs \$55 to deliver.

Understanding the Problem

1. What information are you given about the situation? What is the relationship between the delivery charge and the weight of an appliance?

the relationship between cost and weight; 25 + 0.25p = c

2. What are you being asked to determine?

the weight of an appliance that costs \$55 to deliver

Planning the Solution

- 3. How can you determine the cost to deliver an appliance that weighs 50 pounds? Substitute 50 for p in the given equation and simplify.
- **4.** Make a table that shows the delivery charge for appliances of various weights. Your table should include the weight of an appliance that produces the desired delivery cost.

	Weight (lbs.)	Delivery Charge (\$)
(10	27.50
(60	40
	100	50
	120	55
- 1		

Getting an Answer

5. What is the weight of an appliance that costs \$55 to deliver?

120 pounds

Practice

Form G

An Introduction to Equations

Tell whether each equation is true, false, or open. Explain.

- **1.** $45 \div x 14 = 22$ open; it contains a variable
- 3. 3(-6) + 5 = 26 3false; 3(-6) + 5 = -13
- 5. -14n 7 = 7open; it contains a variable
- 7. 10 + (-15) 5 = -5false: 10 + (-15) - 5 = -10

- **2.** -42 10 = -52
- **4.** $(12 + 8) \div (-10) = -12 \div 6$ true
- **6.** 7k 8k = -15open; it contains a variable
- **8.** $32 \div (-4) + 6 = -72 \div 8 + 7$

Tell whether the given number is a solution of each equation.

- **9.** 3b 8 = 13; -7 **no 10.** -4x + 7 = 15; -2 **yes 11.** 12 = 14 2f; -1 **no**

- **12.** -6 = 14 11n; 2 no **13.** 7c (-5) = 26; 3 yes **14.** 25 10z = 15; -1 no

- **15.** -8a 12 = -4; 1 no **16.** $20 = \frac{1}{2}t + 25$; -10 yes **17.** $\frac{2}{3}m + 2 = \frac{7}{3}$; $\frac{1}{2}$ yes

Write an equation for each sentence.

- **18.** The difference of a number and 7 is 8. n 7 = 8
- **19.** 6 times the sum of a number and 5 is 16. 6(n + 5) = 16
- 20. A computer programmer works 40 hours per week. What is an equation that relates the number of weeks w that the programmer works and the number of hours h that the programmer spends working? h = 40w
- 21. Josie is 11 years older than Macy. What is an equation that relates the age of Josie J and the age of Macy M? J = M + 11

Use mental math to find the solution of each equation.

- **22.** t-7=10 **17 23.** 12=5-h **-7 24.** 22+p=30 **8 25.** 6-g=12 **-6**

- **26.** $\frac{x}{4} = 3$ **12 27.** $\frac{v}{8} = -6$ **-48 28.** 4x = 36 **9 29.** 12b = 60 **5**

Practice (continued)

Form G

An Introduction to Equations

Use a table to find the solution of each equation.

30.
$$4m - 5 = 11$$
 4 31. $-3d + 10 = 43$ **32.** $2 = 3a + 8$ **-2 33.** $5h - 13 = 12$ **5**

31.
$$-3d + 10 = 43$$

32.
$$2 = 3a + 8 - 2$$

33.
$$5h - 13 = 12$$
 5

34.
$$-8 = 3y - 2$$
 -2

35.
$$8n + 16 = 24$$

36.
$$35 = 7z - 7$$

34.
$$-8 = 3y - 2$$
 35. $8n + 16 = 24$ **1 36.** $35 = 7z - 7$ **6 37.** $\frac{1}{4}p + 6 = 8$ **8**

Use a table to find two consecutive integers between which the solution lies.

38.
$$7t - 20 = 33$$

39.
$$7.5 = 3.2 - 2.1n$$

40.
$$37d + 48 = 368$$

between 7 and 8

- **41.** The population of a particular village can be modeled by the equation y = 110x + 56, where x is the number of years since 1990. In what year were there 1706 people living in the village? 2005
- **42. Open-Ended** Write four equations that all have a solution of -10. The equations should consist of one multiplication, one division, one addition, and one

$$-2x = 20; \frac{x}{2} = -5; x - 4 = -14; x + 3 = -7$$

43. There are 68 members of the marching band. The vans the band uses to travel to games each carry 15 passengers. How many vans does the band need to reserve for each away game? 5 vans

Find the solution of each equation using mental math or a table. If the solution lies between two consecutive integers, identify those integers.

44.
$$d + 8 = 10$$

45.
$$3p - 14 = 9$$

45.
$$3p - 14 = 9$$
 46. $8.3 = 4k - 2.5$ **47.** $c - 8 = -12$ between 7 and 8 between 2 and 3 -4

47.
$$c - 8 = -12$$

48.
$$6y - 13 = -13$$

49.
$$15 = 8 + (-a)$$

48.
$$6y - 13 = -13$$
 49. $15 = 8 + (-a)$ **50.** $-3 = -\frac{1}{3}h - 10$ **51.** $21 = 7x + 8$

51.
$$21 = 7x + 8$$

52. Writing Explain the difference between an expression and an equation.

An equation has two different quantities that are equal to each other and an expression does not. An expression can only be simplified whereas an equation can be solved.

Practice

Form K

An Introduction to Equations

Tell whether each equation is true, false, or open. Explain.

1.
$$13 + (-12) - 3 = -2$$
 true

3.
$$-72 \div 12 + 6 = 14 - 8$$

false; $-72 \div 12 + 6 = 0$
and $14 - 8 = 6$

5.
$$(10 + 5)(-4) - 10 = -80 - 10$$

false; $(10 + 5)(-4) - 10 = -70$
and $-80 - 10 = -90$

2.
$$7 - 8x = -15$$
 open; it contains a variable

4.
$$6(-5) \div (-2) = -3(-5)$$

true; $6(-5) \div (-2) = 15$ and $-3(-5) = 15$

6.
$$-9(-5 + 8) = \frac{1}{3}(-9)(9)$$

true; $-9(-5 + 8) = -27$ and $\frac{1}{3}(-9)(9) = -27$

Tell whether the given number is a solution of each equation.

7.
$$5x + 10 = -35$$
; -9 yes $5(-9) + 10 = -35$

8.
$$8p - 3 = 13; -2$$
 no

9.
$$-16 - h = 20;4$$
 no

10.
$$-24 = -6m + 6$$
; 5 yes

11.
$$-49 + 7t = 21; -10$$
 no

12.
$$32 = 7z - 10$$
; 6 yes

- 13. The distance in miles a family has traveled so far on their trip can be modeled by the equation y = 0.6x - 75, where x is the number of minutes of driving today. How many minutes has the family been traveling today when they have traveled 201 miles? 460 min
- 14. There are 325 students that need to take Algebra 1 this year. Each class is limited to at most 30 students. How many classes need to be offered? 11 classes

Use mental math to find the solution of each equation.

15.
$$n + 9 = 15$$
 6

16.
$$20 = 12 - a$$
 -8

17.
$$44 + v = 22$$
 -22

18.
$$14 - y = 16$$
 -2

1_Q Practice (continued)

Form K

An Introduction to Equations

Write an equation for each sentence.

- **19.** The sum of a number and 11 is -12. n + 11 = -12
- 20. -4 times the sum of 6 times a number and 3 is 15 -4(6n + 3) = 15
- **21.** Sara is 5 years younger than Geoff. What is an equation that relates the age of Sara *S* and the age of Geoff *G*? G = S + 5, or S = G 5
- **22. Open-Ended** Write four equations that all have a solution of 6. The equations should consist of one multiplication, one division, one addition, and one subtraction equation.

Answers may vary. Sample: 2x = 12; $\frac{x}{2} = 3$; x + 3 = 9; x - 3 = 3

Use a table to find the solution of each equation.

23.
$$2d - 4 = -8$$
 -2

24.
$$12 - 5x = -13$$
 5

25.
$$18 = -w + 14$$
 -4

26.
$$6z - 14 = -8$$
 1

Use a table to find two consecutive integers between which the solution lies.

27.
$$4k - 13 = 12$$
 between 6 and 7

28.
$$24.1 = 15q - 32.5$$
 between **3** and **4**

Find the solution of each equation using mental math or a table. If the solution lies between two consecutive integers, identify these integers.

29.
$$j - 18 = 2$$
 20

30.
$$6y + 5 = 36$$
 between **5** and **6**

31.
$$2.2 - 2n = 12.2$$
 -5

32.
$$-7b + 15 = -6$$
 3

33. Writing Explain how you can determine if a number is a solution of an equation.

Answers may vary. Sample: You can substitute the number into the equation for the variable and simplify. If both sides are the same, then the number is a solution.

Standardized Test Prep

An Introduction to Equations

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Which equation is true? **B**

A.
$$25 - (-18) = 7$$

B.
$$\frac{1}{3}(-9) - 6 = -9$$

C.
$$25(-2) + 7 = -39 + 4$$

D.
$$-19 + 8(-2) = -7(-5)$$

2. Which equation has a solution of -6?

F.
$$15x - 20 = 70$$

G.
$$14 = 6x - 22$$

F.
$$15x - 20 = 70$$
 G. $14 = 6x - 22$ **H.** $3x - 8 = -10$ **I.** $\frac{1}{2}x - 8 = -11$

$$1. \ \frac{1}{2}x - 8 = -1$$

3. Which equation has a solution of $\frac{1}{2}$?

A.
$$13x - 12 = 14$$

B.
$$9x + 15 = 20$$

C.
$$-6x - 18 = -21$$

A.
$$13x - 12 = 14$$
 B. $9x + 15 = 20$ **C.** $-6x - 18 = -21$ **D.** $-11x = 12x + 12$

4. The money a company received from sales of their product is represented by the equation y = 45x - 120, where y is the money in dollars and x is the number of products sold. How many products does the company need to sell in order to receive \$3705? **G**

5. Mrs. Decker walks for 30 minutes each day as often as possible. What is an equation that relates the number of days d that Mrs. Decker walks and the number of minutes m that she spends walking? A

A.
$$m = 30d$$

B.
$$d = 30m$$

C.
$$d = m + 30$$

D.
$$m = d + 30$$

Short Response

6. There are 450 people travelling to watch a playoff football game. Each bus can seat up to 55 people. Write an equation that represents the number of buses it will take to transport the fans. Use a table to find a solution. 55b = f; 9 buses

- [2] Both parts answered correctly.
- [1] One part answered correctly.
- [0] Neither part answered correctly.

Enrichment

An Introduction to Equations

The relationships you have examined in this lesson are all called linear — that is, the graph of the relation forms a straight line. Linear relationships are convenient because, once you know the equation or rule, the pattern is predictable.

The relationships shown in the tables are linear. Fill in the missing cells in each table.

	X	y	
	1	6	
	2	12	
	3	18	
	4	24	
	5	30	
7			Γ

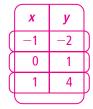
2.

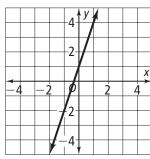
	x	y	
	0	-2	`
	2	1	`
	4	4	
	6	7	`
	8	10	
1			

3.

	x	у	
	1	1	
	3	5	
	5	9	
	7	13	
	9	17	
1			Γ.

- **4.** What is the rule or equation for the relationship represented in the table in Exercise 1? y = 6x
- **5.** What is the rule or equation for the relationship represented in the table in Exercise 2? $y = \frac{3}{2}x - 2$
- **6.** What is the rule or equation for the relationship represented in the table in Exercise 3? y = 2x - 1
- 7. Make a table of coordinate pairs from the graph shown at the right.





- 8. What is the rule or equation for the relationship represented in the graph? y = 3x + 1
- **9.** What do you notice about the equation and where the line crosses the *y*-axis on the graph? The line crosses the y-axis at y = 1 and there is +1 in the equation.

Reteaching

An Introduction to Equations

An equation is a mathematical sentence with an equal sign. An equation can be true, false, or open. An equation is true if the expressions on both sides of the equal sign are equal, for example 2 + 5 = 4 + 3. An equation is false if the expressions on both sides of the equal sign are not equal, for example 2 + 5 = 4 + 2.

An equation is considered open if it contains one or more variables, for example x + 2 = 8. When a value is substituted for the variable, you can then decide whether the equation is true or false for that particular value. If an open sentence is true for a value of the variable, that value is called a solution of the equation. For x + 2 = 8, 6 is a solution because when 6 is substituted in the equation for x, the equation is true: 6 + 2 = 8.

Problem

Is the equation true, false, or open? Explain.

a.
$$15 + 21 = 30 + 6$$

The equation is true, because both expressions equal 36.

b.
$$24 \div 8 = 2 \cdot 2$$

The equation is false, because $24 \div 8 = 3$ and $2 \cdot 2 = 4$; $3 \neq 4$.

c.
$$2n + 4 = 12$$

The equation is open, because there is a variable in the expression on the left side.

Tell whether each equation is true, false, or open. Explain.

1.
$$2(12) - 3(6) = 12$$

$$2(12) - 3(6) = 12$$
 2. $3x + 12 = -19$ **3.** false; $2(12) - 3(6) = 6$ **2.** open; it contains a variable

4.
$$2(-8) + 4 = 12$$

false; $2(-8) + 4 = -12$

5.
$$7 - 9 + 3 = x$$
 open; it contains a variable

6.
$$(28 + 12) \div -2 = -20$$
 true

7.
$$14 - (-8) - 14 = 8$$
 true

8.
$$(13 - 16) \div 3 = 1$$

false; $(13 - 16) \div 3 = -1$

9.
$$42 \div 7 + 3 = 9$$
 true

3. 14 - 19 = -5

Problem

Is x = -3 a solution of the equation 4x + 5 = -7?

$$4x + 5 = -7$$

$$4(-3) + 5 = -7$$

Substitute -3 for x.

$$-7 = -7$$

Simplify.

Since -7 = -7, -3 is a solution of the equation 4x + 5 = -7.

Tell whether the given number is a solution of each equation.

10.
$$4x - 1 = -27; -7$$

11. 18 - 2n = 14; 2yes

12.
$$21 = 3p - 5$$
; 9

13.
$$k = (-6)(-8) - 14$$
; -62 **14.** $20\nu + 36 = -156$; -6

15.
$$8y + 13 = 21; 1$$

16.
$$-24 - 17t = -58; 2$$
 17. $-26 = \frac{1}{3}m + 5; -7$ **18.** $\frac{1}{4}g - 8 = \frac{3}{2}; 38$ **yes 19.** $\frac{1}{4}g - 8 = \frac{3}{2}; 38$

17.
$$-26 = \frac{1}{3}m + 5; -7$$

18.
$$\frac{1}{4}g - 8 = \frac{3}{2}$$
; 38

1_Q Reteaching (continued)

An Introduction to Equations

A table can be used to find or estimate a solution of an open equation. You will have to choose a value to begin your table. If you choose the value that makes the equation true, you have found the solution and are done. If your choice is not the solution, make another choice based on the values of both sides of the equation for your first choice. If you choose one value that makes one side of the equation too high and then another value that makes that same side too low, you know that the solution must lie between the two values you chose. It may not be possible to determine an exact solution for each equation; estimating the solution to be between two integers may be all that is possible in some cases.

Problem

What is the solution of 6n + 8 = 28?

If n = 2, then the left side of the equation is 6(2) + 8 or 20, which is too low.

If n = 5, then the left side of the equation is 6(5) + 8 or 38, which is too high.

The solution must lie between 2 and 5, so keep trying values between them.

If n = 3, then the left side of the equation is 6(3) + 8 or 26, which is too low.

If n = 4, then the left side of the equation is 6(4) + 8 or 32, which is too high.

The solution must lie between 3 and 4, but there are no other integers between 3 and 4.

You can give an estimate for the solution of 6n + 8 = 28 as being between the integers 3 and 4.

Write an equation for each sentence.

19. 13 times the sum of a number and 5 is 91. 13(n + 5) = 91

20. Negative 8 times a number minus 15 is equal to 30. -8n - 15 = 30

21. Jared receives \$23 for each lawn he mows. What is an equation that relates the number of lawns w that Jared mows and his pay p? p = 23w

22. Shariff has been working for a company 2 years longer than Patsy. What is an equation that relates the years of employment of Shariff *S* and the years of employment of Patsy *P*? S = P + 2

Use mental math to find the solution of each equation.

23.
$$h + 6 = 13$$
 7 24. $-11 = n + 2$ **-13 25.** $6 - k = 14$ **-8 26.** $5 = -8 + t$ **13**

27.
$$\frac{z}{5} = -2$$
 -10 28. $\frac{j}{-6} = 12$ **-72 29.** $8c = -48$ **-6 30.** $-15a = -45$ **3**

Use a table to find the solution of each equation.

31.
$$-3b - 12 = 15$$
 32. $15y + 6 = 21$ **33.** $-8 = 5y + 22$ **34.** $6t - 1 = -49$ **-8**

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Additional Vocabulary Support

Patterns, Equations, and Graphs

For Exercises 1–4, draw a line from each word or phrase in Column A to its definition in Column B. The first one is done for you.

Column B inductive reasoning to increase solution of a two variable equation reasoning from cause to effect extend process of reaching a conclusion based on an observed pattern deductive reasoning any ordered pair (x, y) that makes the equation true

For Exercises 5–8, draw a line from each word or phrase in Column A to its match in column B based on the situation described. The first one is done for you.

Sophia made one necklace. Each additional hour she can make two more.

Column A Column B 5. table y = 2x + 16. equation When x equals 4, y will equal 9.

- viteri x equals 4, y will equa
- **7.** ordered pair (1, 3)
- 8. prediction | Input (x) | Output (y) | 1 | 3 | 2 | 5 | | 3 | 7

4

?

Think About a Plan

Patterns, Equations, and Graphs

Air Travel Use the table below. How long will the jet take to travel 5390 miles?

Passenger Jet Travel				
Hours, h 1 2 3	4			
Miles, <i>m</i> 490 980 1470	1960			

Understanding the Problem

1. What does the information in the table represent?

the distance the jet travels in miles and the time in hours it is in flight

2. What is the pattern in each row of numbers?

The hours increase by 1. The distance increases by 490.

Planning the Solution

- 3. What is a general equation that represents the relationship between hours and miles? m = 490h
- 4. How can you determine the number of hours it will take for the jet to travel 5390 miles? Substitute 5390 for *m* and solve the equation.

Getting an Answer

5. How long will the jet take to travel 5390 miles? Show your work.

5390 = 490h; 11 hours

6. Besides the distance the jet travels and the time it is in flight, what else could be determined from the information in the table?

the speed of the jet

Practice

Form G

Patterns, Equations, and Graphs

Tell whether the given equation has the ordered pair as a solution.

1.
$$y = x - 4$$
; (5, 1) yes

2.
$$y = x + 8$$
; (8, 0) no

3.
$$y = -x - 2$$
; (2, -4) ves

4.
$$y = -3x$$
; $(2, -6)$ yes **5.** $y = x + 1$; $(1, 0)$ no **6.** $y = -x$; $(-7, 7)$ yes

5.
$$y = x + 1$$
; (1, 0) no

6.
$$y = -x$$
; $(-7, 7)$ yes

7.
$$y = x + \frac{1}{2}$$
; $(1, \frac{1}{2})$ no

7.
$$y = x + \frac{1}{2}$$
; $(1, \frac{1}{2})$ no 8. $y = x - \frac{2}{5}$; $(-2, -2\frac{2}{5})$ yes 9. $\frac{x}{-3} = y$; $(2, -6)$ no

9.
$$\frac{x}{-3} = y$$
; (2, -6) no

Use a table, an equation, and a graph to represent each relationship.

10. Petra earns \$22 per hour.

	Hours	Dollars	١
	1	22	
	2	44	
	3	66	
	4	88	
	5	110	
1			ľ



$$y = 22x$$

11. The calling plan costs \$0.10 per minute.

	Minute (s)	Cost (\$)
	1	0.10
(5	0.50
(10	1
	15	1.50
	20	2
٦)

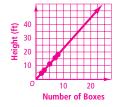


$$y = 0.1x$$

Use the table to draw a graph and answer the question.

12. The table shows the height in feet of a stack of medium sized moving boxes. What is the height of a stack of 14 boxes?

	Boxes	Height (ft)
	2	4.5
	3	6.75
	5	11.25
	7	15.75
	8	18
٦		J



31.5ft

13. The table shows the number of pages Dustin read in terms of hours. How many pages will Dustin read in 12 hours?

	Hours	Pages
	1	23
	2	46
	3	69
1	4	92
	5	115
٦		7



276 pages

Practice (continued)

Form G

Patterns, Equations, and Graphs

Use the table to write an equation and answer the question.

- **14.** The table shows the amount earned for washing cars. How much is earned for washing 25 cars?
- **15.** The table shows the distance in terms of hours Jerry and Michelle have traveled on the way to visit their family. They take turns driving for 12 hours. What distance will they travel in that time?

	Cars	Money Earned (\$)	
	3	13.50	
	6	27	
	9	40.50	
	12	54	
τ			ſ

y = 4.50x; \$112.50

Hours	Miles
1	67
2	134
3	201
4	268
5	335

16. A worker finds that it takes 9 tiles to cover one square foot of floor. Make a table and draw a graph to show the relationship between the number of tiles and the number of square feet of floor covered. How many square feet of floor will be covered by 261 tiles?

Floo	or (ft.²)	Tiles	
	1	9	
	3	27	
	5	45	
	7	63	
	9	81	
	11	99	



29 ft²

Tell whether the given equation has the ordered pair as a solution.

- **17.** y = 3x 2; (-1, -5)ves
- **18.** y = -5x + 7; (1, -2)
- **19.** y = -4x 3; (1, 1)

- **20.** y = 13 + 6x; (-1, 7)yes
- **21.** $-\frac{2}{3}x 5 = y$; (9, -11) **22.** $y = 10 \frac{x}{2}$; (5, $\frac{15}{2}$)
- 23. Writing Explain what inductive reasoning is. Include in your explanation what inductive reasoning can be used for.

Inductive reasoning is the process of reaching a conclusion based on an observed pattern. You can use inductive reasoning to predict values.

Practice

Form K

Patterns, Equations, and Graphs

Tell whether the given ordered pair is a solution of the equation.

1.
$$y = x + 8$$
; (-3, 5) yes $5 = -3 + 8$

2.
$$y = x - 6$$
; (4, -2) **yes**

3.
$$y = x - 12$$
; (3, 15) no

4.
$$y = -7x$$
; $(-3, -21)$ no

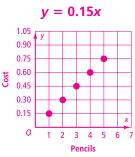
5.
$$y = x + 8; (-1, 7)$$
 yes

6.
$$y = -x + 4$$
; $(-5, -1)$ no

Use a table, an equation, and a graph to represent each relationship.

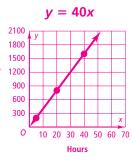
7. Zachary sells pencils for \$0.15 each.

	Pencils	Cost (\$)
1	1	0.15
	2	0.30
	3	0.45
1	4	0.60
	5	0.75



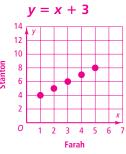
8. Jerry earns \$40 per hour.

Hours	Earnings (\$)
5	200
10	400
20	800
30	1200
40	1600

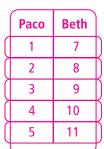


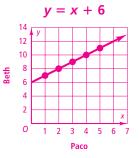
9. Stanton can mow 3 more lawns per day than Farah.

	Farah	Stanton
(1	4
$(\!\!\lceil$	2	5
${\Bbb C}$	3	6
$(\![$	4	7
$(\![$	5	8
τ		

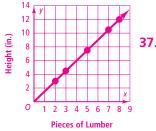


10. Beth has been with the company 6 years longer than Paco.





11. The table shows the height in inches of a stack of lumber. Draw a graph of the situation. What is the height of a stack of 25 pieces of lumber?



37.5 in.

Pieces Height (in.) 2 3 4.5 5 7.5 7 10.5 8 12

Practice (continued)

Form K

Patterns, Equations, and Graphs

Use the table to write an equation and answer the question.

12. The table shows the cost of staying at a certain hotel. How much does 30 nights cost?

y = 79x; \$2370

Nights	Cost (\$)
3	237
6	474
9	711
12	948
)

13. The table shows the number of words a secretary types in terms of minutes. How many words can he type in 60 minutes?

y = 62x; 3720 words

	Minutes	Words
	5	310
	10	620
	15	930
(20	1240
	25	1550
- 1		

Tell whether the given ordered pair is a solution of the equation.

14.
$$y = -6x + 1; (-1, -5)$$
 no

15.
$$y = -\frac{1}{2}x + 4$$
; (-8, 8) **yes**

16.
$$y = -0.1x + 5$$
; (10, 4) yes

17.
$$y = 0.25 + 5.5x$$
; $(-1, -5.75)$ **no**

18.
$$\frac{2}{5}x + 5 = y$$
; $(-15, -1)$ yes **19.** $y = 12 - \frac{3x}{4}$; $(-8, 6)$ no

19.
$$y = 12 - \frac{3x}{4}$$
; (-8, 6) **no**

20. Writing Explain how you can use an equation to make predictions about a particular relationship.

Answers may vary. Sample: You can choose the value you are using for your prediction, substitute it in for the appropriate variable, and solve for the other variable.

Standardized Test Prep

Patterns, Equations, and Graphs

Multiple Choice

For Exercises 1–5, choose the correct letter.

1. If x = -3 and y = -5, what does 3x - 2y equal? C

A. -19

B. -1

C. 1

D. 19

2. Which ordered pair is a solution of y = 6x - 1?

F. (-3, -17)

G. (-1, 7)

H. (1, 7)

I. (3, 17)

3. Which ordered pair is a solution of -x = y?

A. (-1, -1)

B. (1, -1)

C. (1, 1)

D. (-1, -2)

4. Which equation represents the table shown? **F**

F. y = 8.5x

G. y = 8.5x + 12.50

H. y = 15x

I. y = 15x + 12.50

	Hours	Money (\$)
(15	127.50
(25	212.50
${\Bbb C}$	35	297.50

5. Sally is 3 years younger than Ralph. Which equation represents this relationship? **D**

A. R = 3S

B. R = S - 3

C. S = R + 3

D. S = R - 3

Extended Response

- **6.** Justin earns \$19.50 per hour working as a store manager.
 - **a.** Use a table to represent this relationship.

	Hours	Money (\$)
	1	19.50
(2	39
	3	58.50

- [2] All parts answered correctly.
- [1] Some parts answered correctly.
- [0] No part answered correctly.
- **b.** Use an equation to represent this relationship. y = 19.50x
- **c.** Use a graph to represent this relationship.
- d. What will Justin earn for working 40 hours?

\$780



Enrichment

Patterns, Equations, and Graphs

Marissa decided to sell woodworking crafts that she makes. First she invests \$585 in tools. Each item costs \$35 in wood and other supplies.

1. Write an equation that relates total costs *y* to the number of crafts created *x*.

$$y = 35x + 585$$

2. Fill in the table to show Marissa's cost for making 5, 10, 20, 50, and 100 crafts.

- Crafts
 Cost(\$)

 5
 760

 10
 935

 20
 1285

 50
 2335

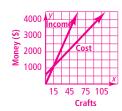
 100
 4085
- **3.** Graph the relation on the graph at the bottom of this page. Label the line *Cost*.

See graph in Exercise 6.

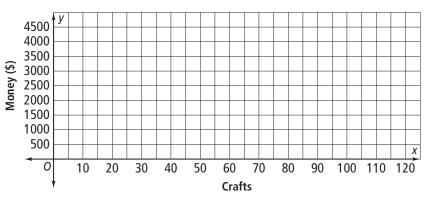
- **4.** Marissa sells her crafts for \$75 each on the internet. Write an equation that relates her income y to the number of crafts sold x. y = 75x
- **5.** Fill in the table to show Marissa's income for making 5, 10, 20, 50, and 100 crafts.

Crafts	Income(\$)
5	375
10	750
20	1500
50	3750
100	7500

6. Graph the relation on the graph at the bottom of this page. Label the line *Income*.



7. How can you identify the break-even point, where expenses are paid and profits begin? Explain. What is the break-even point for Marissa?



Answers may vary. Sample: The break-even point is the point where the cost and the income lines intersect. The break-even point is approximately 15 crafts.

Reteaching

Patterns, Equations, and Graphs

Tables, equations, and graphs are some of the ways that a relationship between two quantities can be represented. You can use the information provided by one representation to produce one of the other representations; for example, you can use data from a table to produce a graph. You can also use any of the representations to draw conclusions about the relationship.

Problem

Are (2, 11) and (5, 3) solutions of the equation y = 3x + 5?

For each ordered pair, you can substitute the *x*- and *y*- coordinates into the equation for *x* and *y* and then simplify to see if the values satisfy the equation.

For (2, 11): For (5, 3):

11 = 3(2) + 5 Substitute for x and y. 3 = 3(5) + 5

11 = 11 Multiply and then add. $3 \neq 20$

Since both sides of the equation have the same value, the ordered pair (2, 11) is a solution of the equation y = 3x + 5. Since the two sides of the equation have different values, the ordered pair (5, 3) is not a solution of the equation y = 3x + 5.

Problem

The table shows the relationship between the number of hours Kaya works at her job and the amount of pay she receives. Extend the pattern. How much money would Kaya earn if she worked 40 hours?

Method 1: Write an equation.

y = 12.50x Kaya earns \$12.50 per hour.

= 12.50(40) Substitute 40 for x.

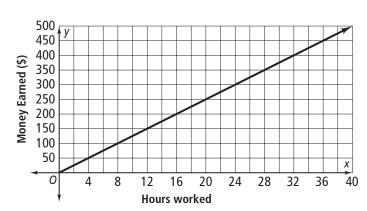
= 500 Simplify.

She would earn \$500 in 40 hours.





She would earn \$500 in 40 hours.



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Reteaching (continued)

Patterns, Equations, and Graphs

Exercises

Tell whether the equation has the given ordered pair as a solution.

1.
$$y = x - 7$$
; (2, -5) yes

2.
$$y = x + 6$$
; (-5, 11) **n**

1.
$$y = x - 7$$
; $(2, -5)$ yes **2.** $y = x + 6$; $(-5, 11)$ no **3.** $y = -x + 1$; $(-1, 0)$ no

4.
$$y = -5x$$
; $(-3, -15)$ no

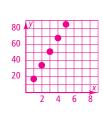
5.
$$y = x - 8$$
; $(7, -1)$ yes

4.
$$y = -5x$$
; $(-3, -15)$ no **5.** $y = x - 8$; $(7, -1)$ yes **6.** $y = x + \frac{3}{4}$; $(-1, -\frac{1}{4})$ yes

Use a table, an equation, and a graph to represent each relationship.

7. Tickets to the fair cost \$17.

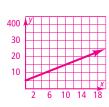
	Tickets	Cost (\$)
	1	17
	2	34
	3	51
	4	68
	5	85
٦		



$$y = 17x$$

8. Brian is 5 years older than Sam.

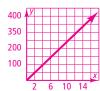
	Sam (yrs)	Brain (yrs)
(0	5
(3	8
	6	11
	9	14
	12	17
٦		



$$y = x + 5$$

Use the table to draw a graph and answer the question.

9. The table shows Jake's earnings for the number of cakes he baked. What are his earnings for baking 75 cakes?



\$1800

Cakes	Earnings (\$)
5	120
10	240
15	360
	·

Use the table to write an equation and answer the question.

10. The table shows the number of miles that Kate runs on a weekly basis while training for a race. How many total miles will she have run after 15 weeks?

	Training Weeks	Miles Run
(1	40
(2	80
(3	120
1		J

y = 40x; 600 mi

11. The table shows the amount of money Kevin receives for items that he sells. How much will he earn if he sells 30 items?

	Items Sold	Earnings (\$)
	15	1125
	20	1500
	25	1875
٦		

y = 75x; \$2250

Chapter 1 Quiz 1

Form G

Lessons 1-1 through 1-4

Do you know HOW?

Write an algebraic expression for each phrase.

- 1. a number x plus 11 \times + 11
- 2. 15 less than the product of 2 and r = 2r 15
- 3. the quotient of h and 4 plus 10 $\frac{h}{4}$ + 10
- **4.** the product of 6 and t divided by 7 $\frac{6t}{7}$

Simplify each expression.

5.
$$18 \div (5 + 2^2)$$
 2 6. $\sqrt{169}$ **13**

6.
$$\sqrt{169}$$
 13

7.
$$5 + 4^2 - 3(7) + 3^2$$
 9

8.
$$25 \div (42 + 2^3)$$

8.
$$25 \div (42 + 2^3) \frac{1}{2}$$
 9. $-16 + 8y + (-3) 8y - 19 10. (\frac{5}{6} \cdot 0)(21) 0$

Evaluate each expression for the given values of the variables.

11.
$$4t + 2u^2 - u^3$$
; $t = 2$ and $u = 1$

11.
$$4t + 2u^2 - u^3$$
; $t = 2$ and $u = 1$ **9 12.** $(2a)^2 - (b^3 - a^2)$; $a = -3$ and $b = 2$ **37**

13.
$$5y + 6z^2 - y^3$$
; $y = -4$ and $z = 5$ **194**

13.
$$5y + 6z^2 - y^3$$
; $y = -4$ and $z = 5$ **194 14.** $(2h)^3 - (k^3 - h^2)$; $h = -1$ and $k = -3$ **20**

15. Name the subset(s) of the real numbers to which each number belongs. Then order the numbers from least to greatest.

$$-14$$
, $1\frac{3}{4}$, $\sqrt{2}$ $\sqrt{2}$: irrational; -14 : rational, integer; $1\frac{3}{4}$: rational; -14 , $\sqrt{2}$, $1\frac{3}{4}$

- **16.** Estimate $\sqrt{35}$ to the nearest integer. **6**
- 17. Which property is illustrated by $6 \times 5 = 5 \times 6$? comm. prop. of mult.

Do you UNDERSTAND?

- **18. Writing** What word phrases represent the expressions 5 + (-3x) and -3x + 5? Are the two expressions equivalent? Explain. the sum of 5 and negative 3 times x; yes, because of the comm. prop. of add.
- **19**. **Reasoning** Use grouping symbols to make the following equation true.

$$5^3 \div 5 + 20 = 5$$
 $5^3 \div (5 + 20) = 5$

Chapter 1 Quiz 2

Form G

Lessons 1-5 through 1-9

Do you know HOW?

- 1. Is the ordered pair (-3, 2) a solution to the equation 5x + 2y = -11? Show your work. yes; 5(-3) + 2(2) = -15 + 4 = -11
- **2.** Is the ordered pair (-2, 7) a solution to the equation 18 4x = -2x 12? Show your work.

no;
$$18 - 4(-2) = -2(-2) - 12$$
; $26 \neq -8$

3. Is the ordered pair (-1, 7) a solution to the equation 7x + y = y - 7? Show your work.

yes;
$$7(-1) + 7 = 7 - 7$$
; $0 = 0$

Evaluate each expression for m = 3 and n = -2.

4.
$$2n + 6$$
 2

5.
$$-3m - n$$
 -7

6.
$$(mn)^2$$
 36

Simplify each expression.

7.
$$6f^2g - 10f^2g$$
 -4 f^2g **8. -5 - 8 **-13****

9.
$$2.5 - (-4.2)$$
 6.7

10.
$$-(-7y + 12)$$
 7 $y - 12$

10.
$$-(-7y + 12)$$
 7y - 12 11. $\frac{2}{3}[9n - (-15)]$ **6n + 10 12.** $(-a + 100)\frac{1}{5}$ $-\frac{1}{5}a$ + 20

12.
$$(-a + 100)\frac{1}{5} - \frac{1}{5}a + 20$$

Do you UNDERSTAND?

- **13. Reasoning** Are $12x^2y^3z$ and $-45zy^3x^2$ like terms? Explain. Yes, the variables and their exponents are the same and the order does not matter because multiplication is commutative.
- **14. Writing** Describe the process for adding two numbers with different signs. Subtract the absolute values of both numbers and keep the sign of the number with the greatest absolute value.
- **15. Reasoning** Is the following statement true or false? If the sum of three numbers is negative, then all three numbers are negative. If false, give a counterexample. false; Answers may vary. Sample: -10 + 5 + 2 = -3

Chapter 1 Test

Form G

Do you know HOW?

Write an algebraic expression for each phrase.

- 1. a number p minus 19 p-19
- **2.** 12 more than 5 times c **5c** + **12**
- **3.** 1 less than the quotient of a number *n* and 6 $\frac{n}{6}$ **1**
- **4.** 9 times the sum of a number t and 3 9(t + 3)
- **5.** 12 times the quantity 15 minus a number d 12(15 d)

Simplify each expression.

6.
$$22 + (3^2 - 4^2)$$
 15 7. $\sqrt{625}$ **25**

7.
$$\sqrt{625}$$
 25

8.
$$(3^3 - 9)^2$$
 324

9.
$$-10 - (-2) \cdot (-4^3)$$
 -138 10. $\left(-\frac{1}{4}\right)^3$ **-1**

11.
$$5^2 \div 2$$
 12.5

Evaluate each expression for the given values of the variables.

12.
$$5x + 2y^2 - y^3$$
; $x = 2$ and $y = 4$ -22

12.
$$5x + 2y^2 - y^3$$
; $x = 2$ and $y = 4$ **-22 13.** $(5m)^2 - (2n - 3m)^3$; $m = -3$ and $n = 5$
-6634 14. $u + 3v^2 - 2u^3$; $u = -1$ and $v = -3$ **28 15.** $(3c)^3 - (c - 4d)^2$; $c = -2$ and $d = 5$

14.
$$u + 3v^2 - 2u^3$$
; $u = -1$ and $v = -3$ **28**

15.
$$(3c)^3 - (c - 4d)^2$$
; $c = -2$ and $d = 3c - 700$

16. Name the subset(s) of the real numbers to which each number belongs. Then order the numbers from least to greatest.

$$\sqrt{1.1}$$
, -1 , $\frac{1}{2}$ $\sqrt{1.1}$: irrational; -1 : rational, integer; $\frac{1}{2}$: rational; -1 , $\frac{1}{2}$, $\sqrt{1.1}$

- 17. Estimate $\sqrt{120}$ to the nearest integer. 11
- **18.** Which property is illustrated by -8 + 0 = -8? Identity prop. of add.
- 19. Are the following expressions equivalent? Explain.

$$\frac{28mn}{7n}$$
 and $4m$ yes; $\frac{28mn}{7n} = \frac{4 \cdot 7mn}{7n} = 4m$

Chapter 1 Test (continued)

Form G

- **20.** Is the ordered pair (-8, -7) a solution to the equation 3x + 10 = 2y? Show your work. yes; 3(-8) + 10 = 2(-7)
- **21.** Is the ordered pair (5, 0) a solution to the equation 4x + 20 = 12y? Show your work. no; $40 \neq 0$
- **22.** Order the numbers $\frac{3}{4}$, $-1\frac{3}{4}$, $-\frac{5}{4}$, and $-\frac{1}{4}$ from least to greatest. $-1\frac{3}{4}$, $-\frac{5}{4}$, $-\frac{1}{4}$, $\frac{3}{4}$

Evaluate each expression for a = -1 and b = 5.

23.
$$5a - 7$$
 -12

24.
$$-2b - 3a$$
 -7 25. $(ab)^2$ **25**

25.
$$(ab)^2$$
 25

Simplify each expression.

26.
$$9xy^2 - 11xy^2 - 2xy^2$$

27.
$$15 - (-3) - 2^2$$
 14

26.
$$9xy^2 - 11xy^2$$
 27. $15 - (-3) - 2^2$ **14 28.** $-\frac{1}{4}(-4 + 2p)$ **1** $-\frac{1}{2}p$

Do you UNDERSTAND?

29. Open-Ended Write an equation that can be solved correctly in two different ways. Demonstrate both methods.

Method 1:

$$\frac{2(x+1)}{2}=\frac{8}{2}$$

$$2x+2=8$$

$$x + 1 - 1 = 4 - 1$$
 $2x + 2 - 2 = 8 - 2$ $x = 3$ $\frac{2x}{2} = \frac{6}{2}$

$$2 - 2 = 8 - 2$$

- **30. Reasoning** Find the value of $22 \div 2 + 9 4^2 + 18$. Then change one operation sign and add one set of grouping symbols so that the value of the expression is 36.
 - 22; 22 \div (2 + 9) + 4² + 18
- **31. Writing** Describe the difference between the set of whole numbers and the set of natural numbers.

The natural numbers do not include 0, while the whole numbers do.

32. Writing Describe the process for finding the product or quotient of two numbers with the same sign and the product or quotient of two numbers with different signs. If both numbers have the same sign, find the product or quotient and the result is positive. If the numbers have different signs, find the product or quotient of the numbers and the result is negative.

Chapter 1 Quiz 1

Form K

Lessons 1-1 through 1-4

Do you know HOW?

Write an algebraic expression for each phrase.

- **1.** a number *t* times 6 **6***t*
- 2. 6 more than the product of 8 and $n \cdot 8n + 6$

Simplify each expression.

3.
$$36 \div (4 + 2^3)$$
 3

4.
$$\sqrt{121}$$
 11

5.
$$14 + (9^3 - 5^2)$$
 718

6.
$$\sqrt{289}$$
 17

Evaluate each expression for the given values of the variable.

7.
$$5m + 6n^2 - n^3$$
; $m = 2$ and $n = 4$

7.
$$5m + 6n^2 - n^3$$
; $m = 2$ and $n = 4$
8. $(3x)^2 - (x^3 - y^2)$; $x = -3$; $y = -5$

9. Name the subset(s) of real numbers to which each number belongs. Then order the numbers from least to greatest.

$$\sqrt{10}$$
, -11 , $\frac{9}{10}$ $\sqrt{10}$: irrational; -11 : rational, integer; $\frac{9}{10}$: rational; -11 , $\frac{9}{10}$, $\sqrt{10}$

- **10.** Estimate $\sqrt{150}$ to the nearest integer. **12**
- **11.** Which property is illustrated by 5 + n = n + 5? commutative property of addition

Do you UNDERSTAND?

12. Reasoning Tell whether $\sqrt{0.25}$ is *rational* or *irrational*. Explain.

rational; $\sqrt{0.25}$ can be simplified to 0.5, which can be written as the fraction $\frac{1}{2}$

Chapter 1 Quiz 2

Form K

Lessons 1-5 through 1-9

Do you know HOW?

1. Is the ordered pair (2, 1) a solution to the equation 4x - 2y = 5? Show your work. No;

$$4(2) - 2(1) \stackrel{?}{=} 5$$

 $8 - 2 \stackrel{?}{=} 5$
 $6 \neq 5$

2. Is the ordered pair (-2, 2) a solution to the equation 2 + 6x = -5y? Show your work. Yes;

$$2 + 6(-2) \stackrel{?}{=} -5(2)$$

 $2 - 12 \stackrel{?}{=} -10$
 $-10 = -10$

Simplify each expression.

3.
$$6n^2 - 5n^2$$
 n^2

4.
$$18 \div \left(-\frac{3}{5}\right)$$
 -30

5.
$$-(-5 + 10x)$$
 5 - 10x

6.
$$-4(p-(-5))$$
 $-4p-20$

- **7.** Write an equation for the sentence: the difference of 12w and -9 is -22. 12w - (-9) = -22
- **8.** Order the numbers $-\frac{7}{4}$, $\frac{3}{4}$, $\frac{4}{3}$, and $-2\frac{1}{4}$ from least to greatest. $-2\frac{1}{4}, -\frac{7}{4}, \frac{3}{4}, \frac{4}{3}$

Do you UNDERSTAND?

- **9. Open-Ended** Suppose you used the Distributive Property to get the expression 12x + 3y - 9. With what expression could you have started? Answers may vary. Sample: 3(4x + y - 3)
- **10. Reasoning** Find the value of $12 + 9 \div 3 + 8^2 3^3$. Then change two operation signs so that the value of the expression is -22. 52: $12 + 9 \div 3 - 8^2 + 3^3$

Chapter 1 Test

Form K

Do you know HOW?

Write an algebraic expression for each phrase.

- **1.** 11 more than the product of 9 and a 9a + 11
- 2. 5 less than the quotient of a number p and 4 $\frac{p}{4}$ 5
- **3.** Write a word phrase for 5d 12.

12 less than the product of 5 and d

4. Katie went to the driving range to hit some golf balls. While she was there, she decided to take a 30-minute lesson. The table shows how the total cost of the practice session depends on the number of buckets of golf balls Katie hits. What is a rule for the total cost of the practice session? Give the rule in words and as an algebraic expression.

Golf Number of Buckets **Total Cost** 2 $(5 \cdot 2) + 40$ $(5 \cdot 4) + 40$ 4 6 $(5 \cdot 6) + 40$

five times the number of buckets plus 40; 5b + 40

Simplify each expression.

5.
$$-15 - (-6) \cdot (-2^3)$$
 -63

6.
$$\left(-\frac{1}{3}\right)^3$$
 $-\frac{1}{27}$

7.
$$2ab^2 - 7ab^2$$
 -5ab²

8.
$$36 \div \left(-\frac{9}{10}\right)$$
 -40

9.
$$-\frac{1}{2}(-6 + 12x)$$
 3 - 6x

10.
$$-8(2t - (4 - 7))$$
 -16t - 24

12.
$$\sqrt{\frac{25}{49}}$$
 $\frac{5}{7}$

Evaluate each expression for the given values of the variables.

13.
$$3b + 4b^2 - a^3$$
; $a = 3$ and $b = -5$ **14.** $(3f)^2 - (f - g)^3$; $f = -2$; $g = 7$

14.
$$(3f)^2 - (f - g)^3$$
; $f = -2$; $g = 7$

15. Which property is illustrated by (x + y) + z = x + (y + z)? associative property of addition

Chapter 1 Test (continued)

Form K

16. Name the subset(s) of real numbers to which each number belongs. Then order the numbers from least to greatest.

$$-5, \frac{6}{11}, \sqrt{6}$$
 -5: rational, integer; $\frac{6}{11}$: rational; $\sqrt{6}$: irrational; -5, $\frac{6}{11}$, $\sqrt{6}$

- 17. Estimate $\sqrt{83}$ to the nearest integer. 9
- **18.** Is the ordered pair (2, 4) a solution to the equation 2x + 3y = 10? Show your work. No;

$$2(2) + 3(4) \stackrel{?}{=} 10$$

 $4 + 12 \stackrel{?}{=} 10$
 $16 \neq 10$

19. Is the ordered pair (-3, -1) a solution to the equation -8x - 19 = -5y? Show your work. **Yes**;

$$-8(-3) - 19 \stackrel{?}{=} -5(-1)$$

24 - 19 $\stackrel{?}{=}$ 5
5 = 5

20. Order the numbers $\sqrt{15}$, -1, $-\frac{5}{4}$, and $\frac{2}{3}$ from least to greatest.

$$-\frac{5}{4}$$
, -1, $\frac{2}{3}$, $\sqrt{15}$

Do you UNDERSTAND?

21. Reasoning Which of (2, -3), (4, -1), (6, 1), and (9, 4) are solutions of y = x - 5? What is the pattern in the solutions of y = x - 5?

All are solutions of the equation. The pattern is you subtract 5 from the value of x to find the value of y.

22. Writing Describe two different ways for finding the value of 4(10 - 7). Show your work using both methods.

You can either simplify the parentheses and then multiply or you can use the Distributive Property and simplify.

$$4(10-7)=4(3)=12$$

$$4(10-7)=40-28=12$$

Performance Tasks

Chapter 1

TASK 1

In each sentence below, circle the key words or phrases that indicate a mathematical operation and write the symbol for the operation above the words or phrases. Write an equation for each sentence.

- a. A number multiplied by 8 and divided by four gives 7 more than the number. $\frac{8x}{4} = 7 + x$
- **b.** Five times a number decreased by eight is equal to thirty-two. 5x 8 = 32
- c. The sum of the square of a number and a second number is forty-two. $x^2 + y = 42$
- d. One-third of a number added to itself equals three times the difference of the number and seven. $\frac{1}{3}x + x = 3(x - 7)$
 - [4] Student gives correct equations.
 - [3] Student gives equations that may contain some minor errors.
 - [2] Student answers one part correctly and the other part has major errors.
 - [1] Student gives equations that may contain major errors or omissions.
 - [0] Student makes little or no effort.

TASK 2

Two students write the following expressions to answer an exercise:

$$7 + 4(5-3)^2 + \frac{9}{3}$$

$$7 + 4(5-3)^2 + \frac{9}{3}$$
 and $\frac{9}{3} + (5-3)^2 \cdot 4 + 7$

- a. Simplify the two expressions. List each step you use. 26; Check students' work.
- b. Explain the similarities in the steps. In either case, the same order of operations must be followed.
- c. Make up another expression that uses the same numbers and operations, but has a different value. Then simplify the expression, listing each step. Check students' work.
 - [4] Student shows understanding of the task, completes all portions of the task appropriately with no errors in computation, and fully supports work with appropriate explanations.
 - [3] Student shows understanding of the task, completes all portions of the task appropriately with one error in computation, and supports work with appropriate explanations.
 - [2] Student shows understanding of the task, but makes errors in computation resulting in incorrect answer(s), or need to explain better.
 - [1] Student shows minimal understanding of the task or offers little explanation.
 - [0] Student shows no understanding of the task and offers no explanation.

Name Date	Name	Class	Date
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Performance Task (continued)

Chapter 1

TASK 3

- a. A friend is having trouble comparing rational numbers. Write an explanation that will tell your friend how to decide if a rational number is greater than, less than, or equal to another rational number. Consider positive and negative numbers in your answer. Check students' work.
- **b.** Mason works for \$5 per hour on weekends doing yard work. Write a rule for the relationship between hours worked and total income. A = 5h
 - [4] Student shows understanding of the task, completes all portions of the task appropriately, and fully supports work with appropriate explanations.
 - [3] Student shows understanding of the task, completes all portions of the task appropriately, and supports work with appropriate explanations with a minor error.
 - [2] Student shows understanding of the task, but need to explain better.
 - [1] Student shows minimal understanding of the task or offers little explanation.
 - [0] Student shows no understanding of the task and offers no explanation.

TASK 4

A friend has asked you to explain commutative properties to him. After you explain the commutative properties for addition and multiplication, he asks you about commutative properties for subtraction and division.

a. Use examples to show that the operations of subtraction and division are not commutative.

Answers may vary. Sample: $3 - 1 \neq 1 - 3$; $5 \div 1 \neq 1 \div 5$

b. For your example that shows subtraction is not commutative, rewrite it as addition so that the order of the terms can be changed without affecting the result.

Answers may vary. Sample: 3 + (-1) = (-1) + 3

- [4] Student shows understanding of the task, completes all portions of the task appropriately, and fully supports work with appropriate explanations.
- [3] Student shows understanding of the task, completes all portions of the task appropriately, and supports work with appropriate explanations with a minor error.
- [2] Student shows understanding of the task, but need to explain better.
- [1] Student shows minimal understanding of the task or offers little explanation.
- [0] Student shows no understanding of the task and offers no explanation.

Cumulative Review

Chapter 1

Multiple Choice

For Exercises 1-10, choose the correct letter.

1. Which property is illustrated by (3 + 5) + 7 = 3 + (5 + 7)? **B**

A. Additive Identity

C. Commutative Property of Addition

B. Associative Property of Addition

D. Distributive Property

2. Which algebraic expression represents the statement "4 more than the product of 6 and a number"? I

F. 4n + 6

H. 4 - 6n

G. 6 - 4n

I. 6n + 4

3. What is the value of $-9 + (2^3 - 3^2)$? **B**

A. -26

B. -10

C. -8

D. -4

4. What is the value of $\sqrt{49}$? **G**

F. $\sqrt{7}$

G. 7

H. 14

I. 49

5. What is the order of the numbers $\sqrt{12}$, -3.5, $\frac{5}{3}$, $-\frac{2}{3}$ from least to greatest? D

A. $\sqrt{12}$, -3.5, $\frac{5}{3}$, $-\frac{2}{3}$

C. $-3.5, -\frac{2}{3}, \sqrt{12}, \frac{5}{3}$

B. $\sqrt{12}$, $\frac{5}{3}$, $-\frac{2}{3}$, -3.5

D. $-3.5, -\frac{2}{3}, \frac{5}{3}, \sqrt{12}$

6. Which ordered pair is not a solution of y = 2x + 1?

F. (3, 7)

G. (0, 1)

H. (-1, 1)

I. (-3, -5)

7. Which expression is equivalent to -3.2(2x - 2.1)?

A. -6.4x + 6.72

C. 6.4x + 6.72

B. -6.4x - 6.72

D. -6.4x + 2.1

8. Toby purchased 5 tickets online for a show. The tickets cost \$12 each plus there was a \$3.50 service fee for the order. How much money did Toby spend for the tickets?

F. \$15.50

G. \$51.50

H. \$60

I. \$63.50

9. What is the value of $3^3 - (4^2 - 2^3)$?

A. −1

B. 7

C. 19

D. 35

10. Which expression is equivalent to 4(2x + 1) - (-6x)?

F. 14x + 4

G. 8x - 2

H. 2x + 4

1. -14x - 4

Cumulative Review (continued)

Chapter 1

- 11. In the absence of predators, the rabbit population in a forest has grown to 5^6 over the past 5 years. What is the rabbit population in the forest? 15,625
- 12. Cherie is laying square tiles on her square kitchen floor. She buys the tiles for \$2 per square foot tile. If her total estimated cost for the tiles is \$288, what is the length of her floor in feet? 12 ft
- **13.** Simplify $8^2 \div 4 + 3(6-3) + 2^3$. **33**
- **14.** What is the value of 3 + |x 2| for x = -3? **8**
- **15.** Evaluate $x(y-z)^2$ for x=-1, y=5, and z=-3. **-64**
- **16.** Write an equation for the sentence: the difference of 6n and -5 is -13. 6n + 5 = -13
- **17. Vocabulary** What type of number can be written in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$? rational number
- **18.** Simplify $(x^2 + 6) (3x^2 2x 5)$. $-2x^2 + 2x + 11$
- **19.** What is the solution of the equation 9x + 12 = 39?
- **20.** Jack is taking his family to the fair. He plans to take \$5 for each admission ticket plus \$35 for food. Write an equation that models the amount of money Jack takes to the fair. d = 5t + 35
- **21.** What is the value of the expression (-7)(3) (5)(-3)? **-6**

Chapter 1 Project Teacher Notes: Checks and Balances

About the Project

The project gives students an opportunity to learn how to use equations to model a personal budget. Students write equations to model budget plans. They also develop spreadsheets to analyze their spending and saving habits. Students will display and present their budgets using circle graphs.

Introducing the Project

- Ask students to work with partners or in small groups. Ask groups to list important or expensive items that any group member bought recently.
- Now ask groups to list items they would buy if they could spend \$150. Have each group list possible ways to find out costs.
- Ask each student to select one item from the list as his or her goal.

Activity 1: Researching

Ask groups of students to think of several items that they would like to buy for less than \$150 a piece. Have each group price the items they select using ads or by visiting stores. Each group then selects one item as a goal. Check students' work.

Activity 2: Modeling

Ask students to write and to solve equations to discover how much they need to save each week to make their purchases. In Geraldo's equation, x represents the amount of money he should save per week. The expression 40 + 16x represents the total savings over 4 months, or 16 weeks, given that he has already saved \$40. Since Geraldo wants to determine how much to save each month so that his total savings over 16 weeks is \$129, he should solve 40 + 16x = 129 for x; check students' work.

Activity 3: Organizing

Ask students to make spreadsheets to help organize and analyze their budget plans. Check students' work.

Activity 4: Graphing

Ask students to make circle graphs for the budgets they created. Check students' work.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage groups to explain their processes as well as their results.

- Have students review the equations, graphs, and explanations that they needed for the project.
- Ask groups to share their insights that resulted from completing the project, such as any shortcuts they found for making circle graphs or spreadsheets, or ways to manage their own money.

Name	Class	Date	

Chapter 1 Project: Checks and Balances

Beginning the Chapter Project

When there is something you really want to buy, do you already have money saved for it? Or, do you put money aside each week until you can afford it? Maybe you just dream about it! A budget for your money can help you make dreams become reality.

As you work through the activities, you will use equations to help model your personal finances. You will develop spreadsheets to analyze your weekly budget, including regular savings. You will use percents to make graphs. Then you will display and present your budget plan using graphs and spreadsheets.

List of Materials

- · Newspapers or catalogs
- Calculator

- Protractor
- · Graph paper

Activities

Activity 1: Researching

Think of several items you would like to buy for less than \$150 apiece, such as a CD player, sports equipment, or clothing.

- Find the prices of these items using ads or by visiting several stores. What factors other than price should you consider? Explain.
- After completing your research, choose one item that you would like to buy. Explain your decision.
- If you can find the item on sale for 25% off, how much would you save? What would the item cost?

Activity 2: Modeling

To write a successful budget, you need to consider savings.

- Geraldo has already saved \$40 and wants to buy a CD player for \$129 about four months from now. To find how much he should save each week between now and then, he wrote 40 + 16x = 129. Explain his equation.
- In Activity 1, you chose one item to purchase as the goal for your project. How much does it cost? When do you want to buy this item?
- Write and solve an equation to find how much you should save per week to achieve this goal.
- Suppose you earn \$15 per week. What percent of your weekly earnings will you need to save?
- What if you earn \$125 per week? What percent of your weekly earnings will you need to save? Is this more or less than the percent you would save if you only earned \$15 per week?

Name	Class	Date	

Chapter 1 Project: Checks and Balances (continued)

Activity 3: Organizing

A spreadsheet can help you organize your information.

- Begin your budget by recording the amount of money you earn, the amount of money you save, and the amount of money you spend for two weeks.
- Analyze your expenses to plan how much you can spend each week while still meeting your savings goal.
- Design a spreadsheet to show all of the important categories in your budget plan. Include a column or row to show the total you will have saved by the end of each week.
- Will you reach your savings goal when you planned? Enter dollar amounts on your spreadsheet and verify that your budget works. What percent of your budget is allocated to savings?
- What percent of your budget is allocated to other activities?

Activity 4: Graphing

Make a circle graph for the personal budget you wrote in Activity 3. In a table, show the dollar amounts, percents, and degree measures of the angles you used to draw the graph.

Finishing the Project

The answers to the four activities should help you complete your project. Assemble all the parts of your project, including the research on what you would like to buy, your expenses record, your spreadsheet, and your circle graph. Are the expenses you recorded for two weeks typical for you? Does your budget support your purchase goal? Summarize the strengths and weaknesses of your budget.

Reflect and Revise

Present your budget and purchase goal to a small group of classmates. Compare your decisions to theirs. Present to the group two of your equations. Check each other's work (including the circle graphs) for reasonableness and accuracy. Use the group's comments and suggestions to revise and improve your project.

Extending the Project

Maintain your budget for several weeks. Are your savings on track? If not, what expenses can you reduce? Can you increase your income?

Name _	Class	Date

Chapter 1 Project Manager: Checks and Balances

Getting Started

Read the project. As you work on the project, you will need newspapers or catalogs, a protractor, a calculator, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist	Suggestions
☐ Activity 1: pricing items	\square Find prices in ads, catalogs, or stores.
☐ Activity 2: writing and solving equations	\square Solve Geraldo's equation to see that it is reasonable.
☐ Activity 3: preparing a budget	\square Include all income and expenses for two weeks.
☐ Activity 4: making a circle graph	\square Recall that there are 360° in a circle.
□ budget spreadsheet	☐ Does your spreadsheet accurately reflect your income and expenses? Has preparing a budget had an impact on how you spend money? What circumstances would require significant changes to your spreadsheet?

Scoring Rubric

- 4 Equations are correct and calculations are accurate. The spreadsheet shows detail, a good understanding of budget planning, is easy to follow, and is accurate. The circle graph is accurate and labeled carefully. Clear and correct explanations show good reasoning. A complete and accurate data table is made.
- 3 Equations are correct with minor calculation errors. The spreadsheet is correctly laid out but contains minor errors. The circle graph is neat but some of the labels are incorrect. Explanations show good reasoning but some sentences are unclear. The data table is mostly accurate.
- 2 Equations are incorrect. The spreadsheet could be better organized and show more detail. The graph is incorrectly labeled. Explanations are incomplete and incorrect.
- 1 Major elements of the project are incomplete or missing.
- **0** Project is not turned in or shows no efforts.

Your Evaluation of Project Evaluate your work, based on the Scoring Rubric.