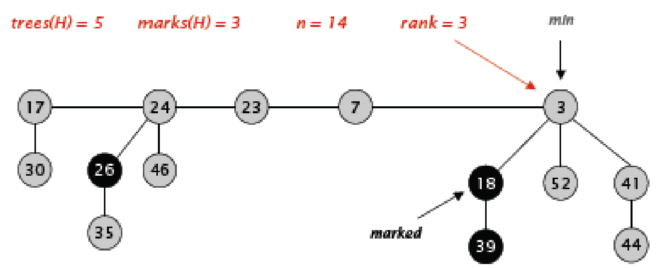
CS 430 Lecture 22 Activities

Fibonacci Heaps

Fibonacci heaps which support heap operations that do not delete elements in constant amortized time. From a theoretical standpoint, Fibonacci heaps are especially desirable when the number of Extract-Min and Delete operations is small relative to the number of other operations performed. This situation arises in many graph algorithms.

In essence, a Fibonacci heap is a "lazy" binomial heap in which the necessary housekeeping is delayed until the last possible moment: deletion.

- Set of Heap ordered trees (each parent smaller than children).
- Maintain pointer to minimum element (Find-Min takes $O(1)$ time).
- Set of marked nodes (if one of its children has been removed).
- n - number of nodes in the heap.
- Rank(x) – number of children of node x.
- Rank(H) – Max rank of any node in heap H.
- Trees(H) – Number of trees in heap H.
- Marks(H) – number of marked nodes in H.



1. See https://www.cs.usfca.edu/~galles/JavascriptVisual/FibonacciHeap.html and https://www.cs.princeton.edu/~wayne/cs423/fibonacci/FibonacciHeapAnimation.html to help describe how each operation is done, and a rough estimate on its run time:

Make-Heap : $O(1)$ Makes a single node heap.

Insert : $O(1)$ Make single node heap, put in the (bidirectional linked) list to left of current min. Update min pointer if necessary.

Minimum : $O(1)$ We maintain a pointer to the min.

Union : $O(1)$ Link the two root lists together[1], and then update the min pointer to point to the smaller of the two previous mins.

[1] You don't need to walk either to do that.

Extract-Min : Find min is $O(\log n)$[2] because we have a pointer to the min. Remove min from root list (saving the value to return). If the min had children, put those children in the root list. CONSOLIDATE the root list[3]

Decrease-Key : $O(1)$ Reduce value at that node, move it and its subtree to root list, check if new min value pointer. Breaking that binomial heap by removing a subtree[4]. Wait for a marked node to lose a 2nd child, move to root list, don't CONSOLIDATE yet.

Delete :

Operation	Binary Heap	Binomial Heap	Fibonacci Heap
MAKE-HEAP	$\theta(1)$	$\theta(1)$	$\theta(1)$
INSERT	$\theta(\log n)$	$\theta(\log n)$	$\theta(1)$
MINIMUM	$\theta(1)$	$\theta(\log n)$	$\theta(1)$
EXTRACT-MIN	$\theta(\log n)$	$\theta(\log n)$	$\theta(\log n)$
UNION	$\theta(n)$	$\theta(\log n)$	$\theta(1)$
DECREASE-KEY	$\theta(\log n)$	$\theta(\log n)$	$\theta(1)$
DELETE	$\theta(\log n)$	$\theta(\log n)$	$\theta(\log n)$

Note that the times indicated for the Fibonacci heap are amortized times while the times for binary and binomial heaps are worst-case per-operation times.

[2]The name Fibnoacci comes from the fact that the base in the $\log n$ is the golden ratio. The maximum degree of any node in the root list is also related to Fibonacci.

[3]Use array of rank on last node

[4]Do not CONSOLIDATE yet.