CS 430 Lecture 24 Activities

Opening Questions

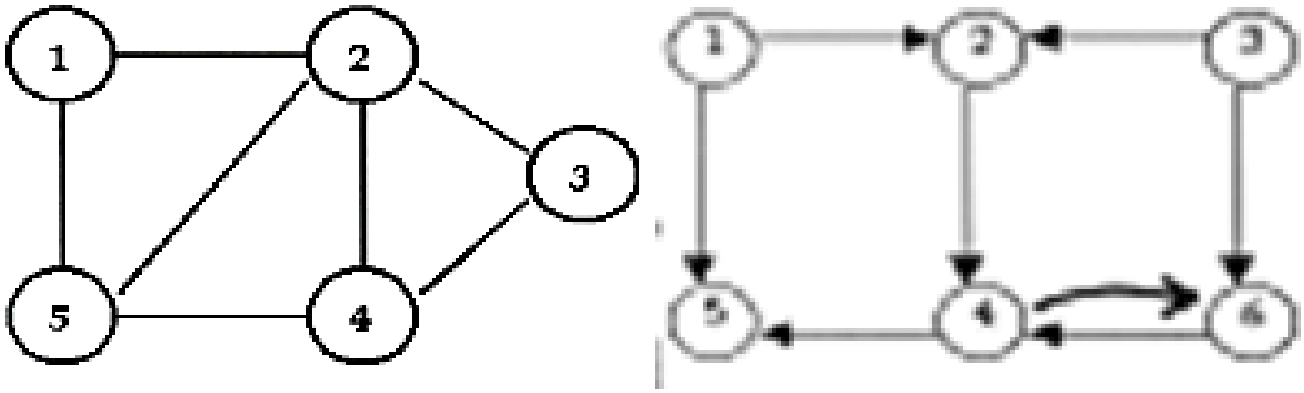
1. Give an example NOT discussed in the video lecture of a problem that can be represented by a graph.
2. If there is a path in a graph from a vertex back to itself, that is called a _____.
3. Which representation of a graph, adjacency-list and adjacency-matrix, usually uses more memory and why?

Graphs

1. Draw the graph: A directed graph $G = (V, E)$ where $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{(1, 2), (2, 2), (2, 4), (2, 5), (4, 1), (4, 5), (5, 4), (6, 3)\}$. What is the edge $(2, 2)$ called?
2. Draw the graph: An undirected graph $G = (V, E)$ where $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{\{1, 2\}, \{1, 5\}, \{2, 5\}, \{3, 4\}\}$. What is vertex 4 called? What is the difference about how an edge set E is denoted for an undirected graph? Are self-loops allowed in an undirected graph?
3. Define these terms:
   - Vertex v is adjacent to vertex u in an undirected graph.
   - Vertex v is adjacent to vertex u in a directed graph.
   - The degree of a vertex in an undirected graph.
   - The degree of a vertex in a directed graph.
   - A path in an undirected graph.
   - A path in a directed graph.
   - The length of a path.
   - v is reachable from u.
   - A simple path.
   - A cycle in an undirected graph. What about a simple cycle?
   - A cycle in a directed graph. What about a simple cycle?
   - Acyclic graph.
   - Connected undirected graph.
   - Connected directed graph.
   - Bipartite Graph.

Graph Implementations

4. What is the adjacency list implementation of these two graphs?



5. What is the adjacency matrix implementation of the above two graphs?
6. How do the two implementations handle a weighted graph?
7. Two different representations of the graph data structure are discussed in the book, adjacency-list and adjacency-matrix. Please briefly discuss the runtime (in terms of $|V|$ and $|E$ of these graph operations/algorithms using each implementation.) Assume vertices are labeled as integers.
    - What is the worst-case big-O runtime for checking to see if an edge from vertex u to vertex v exists?
    - How long does it take to compute the out-degree of every vertex of a directed graph?
    - How long does it take to compute the in-degree of every vertex of a directed graph?

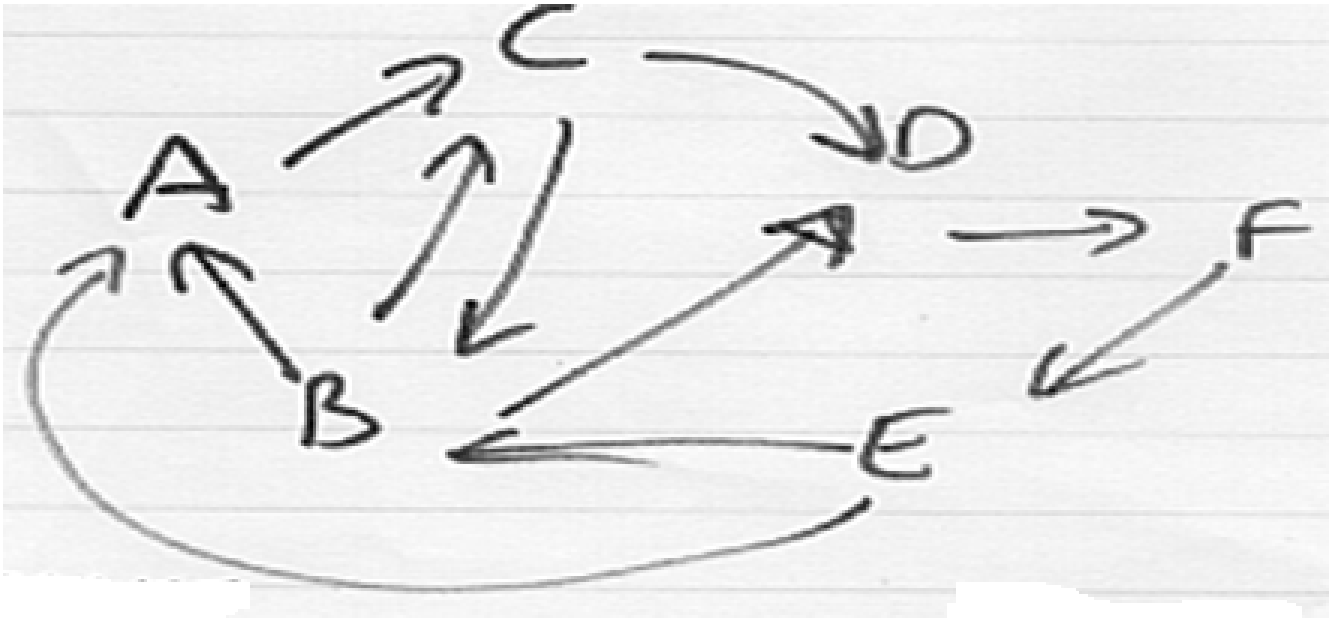
Graph Traversals

A way to search/visit all the vertices in a graph. There is not a unique answer usually.

- Undirected graph - if connected, all vertices will be visited.
- Directed graph - Must be strongly connected to be able to visit all vertices.

Breadth first - visit vertices one edge from a given (or random) source, two edges from source, etc. Uses a queue and some way to make a vertex (white initially, gray when first visited and put in queue, black when out of queue), label a vertex how far from the source, and label a vertex with how its predecessor vertex was during the traversal.

8. Perform a breadth first search on this graph.



Algorithm 24.1 Breadth-First Search for Graphs

```
1:  function BFS(G, s)
2:      for all vertex u ∈ V[G] − {s} do
3:          Color(u) ← WHITE                          ▷ Unvisited
4:          d[u] ← ∞
5:          π[u] ← NIL
6:      end for
7:      Color(s) ← GRAY                               ▷ First time seen, put in queue
8:      d[s] ← 0
9:      π[s] ← NIL
10:     Q ← ∅
11:     Enqueue(Q, s)
12:     while Q ≠ ∅ do
13:         u ← Dequeue(Q)
14:         for all v ∈ Adj(u) do
15:             if Color(v) == WHITE then
16:                 Color(v) ← GRAY
17:                 d[v] ← d[u] + 1
18:                 π[v] ← u
19:                 Enqueue(Q, v)
20:             end if
21:         end for
22:         Color(u) ← BLACK                          ▷ Last time seen, out of queue
23:     end while
24: end function
```