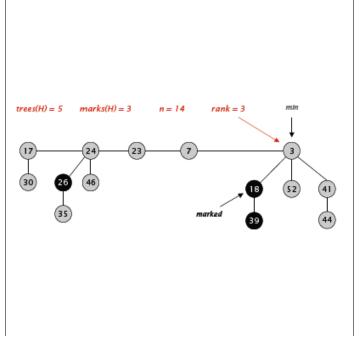
- Set of Heap ordered trees (each parent smaller than children).
- Maintain pointer to minimum element (FIND-MIN takes O(1) time).
- Set of marked nodes (if one of its children has been removed).
- n number of nodes in the heap.
- Rank(x) number of children of node x.
- Rank(H) Max rank of any node in heap H.
- Trees(H) Number of trees in heap H.
- Marks(H) number of marked nodes in H.



CS 430 Lecture 22 Activities

Fibonacci Heaps

Fibonacci heaps which support heap operations that do not delete elements in constant amortized time. From a theoretical standpoint, Fibonacci heaps are especially desirable when the number of Extract-Min and Delete operations is small relative to the number of other operations performed. This situation arises in many graph algorithms.

In essence, a Fibonacci heap is a "lazy" binomial heap in which the necessary housekeeping is delayed until the last possible moment: deletion.

1. See https://www.cs.usfca.edu/~galles/JavascriptVisual/FibonacciHeap.html and https://www.cs.princeton.edu/~wayne/cs423/fibonacci/FibonacciHeapAnimation.html to help describe how each operation is done, and a rough estimate on its run time:

Make-Heap : O(1) Makes a single node heap.

Insert: O(1) Make single node heap, put in the (bidirectional linked) list to left of current min. Update min pointer if necessary.

Minimum: O(1) We maintain a pointer to the min.

Union: O(1) Link the two root lists together¹, and then update the min pointer to point to the smaller of the two previous mins.

¹You don't need to walk either to do that.

Operation	Binary Heap	Binomial Heap	Fibonacci Heap
MAKE-HEAP	$\theta(1)$	$\theta(1)$	$\theta(1)$
Insert	$\theta(\log n)$	$\theta(\log n)$	$\theta(1)$
MINIMUM	$\theta(1)$	$\theta(\log n)$	$\theta(1)$
EXTRACT-MIN	$\theta(\log n)$	$\theta(\log n)$	$\theta(\log n)$
Union	$\theta(n)$	$\theta(\log n)$	$\theta(1)$
DECREASE-KEY	$\theta(\log n)$	$\theta(\log n)$	$\theta(1)$
DELETE	$\theta(\log n)$	$\theta(\log n)$	$\theta(\log n)$

Note that the times indicated for the Fibonacci heap are amortized times while the times for binary and binomial heaps are worst-case per-operation times.

Extract-Min: Find min is $O(\log n)^2$ because we have a pointer to the min. Remove min from root list (saving the value to return). If the min had children, put those children in the root list. Consolidate the root list³

Decrease-Key: O(1) Reduce value at that node, move it and its subtree to root list, check if new min value pointer. Breaking that binomial heap by removing a subtree⁴. Wait for a marked node to lose a 2nd child, move to root list, don't CONSOLIDATE yet.

Delete:

²The name Fibnoacci comes from the fact that the base in the $\log n$ is the golden ratio. The maximum degree of any node in the root list is also related to Fibonacci.

³Use array of rank on last node

⁴Do not Consolidate yet.