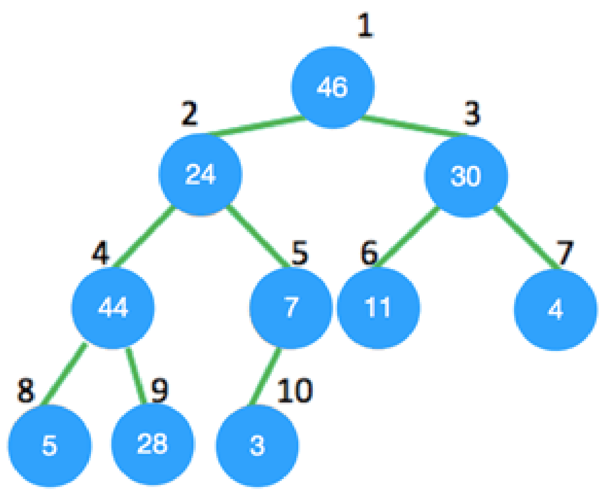
After Lecture 07 & 08 – Answer any questions on HW2 (due today)
Practice Problems (all taken from previous exams)

1. Which one of the following is false?
   a) Heap sort is an in-place algorithm.
   b) Heap sort has $O(n \log n)$ average case time complexity.
   c) Heap sort is a stable sort. Heap sort is an in-place algorithm as it needs $O(1)$ auxiliary space.
   d) Heap sort is a comparison-based sorting algorithm.

2. Consider the max heap shown below, the node with value 24 violates the max-heap property. Once heapify procedure is applied to it, which position will it be in?



   a) 5
   b) 8
   c) 9
   d) You cannot call heapify at the node with value 24

3. Counting sort can be used on any numeric data.
   a) TRUE, it can be used, but it is a very bad idea. Counting sort requires a very large range of numbers, counting sort requires a very large array. This reduces its memory efficiency and increases space consumption. So while it possible to be used, it isn't a good idea to do so on any numeric data.
   b) FALSE

4. Which of the following is not true about all comparison based sorting algorithms?
   a) The minimum possible runtime growth on a random input is $O(n \log n)$.
   b) Can be made stable by also using position when two elements are compared.
   c) Counting Sort is not a comparison-based sorting algorithm.

d) Merge Sort is a comparison-based sorting algorithm.

5. The BUILD-MAX-HEAP discussed in class and shown to be $O(n)$ uses this process. Call Heapify from heap index position $\lfloor \frac{heapsize}{2} \rfloor$ down to heap index position 1. Building a heap can also be implemented by starting with an empty heap and repeatedly using MAX-HEAP-INSERT to insert the elements into the heap. Consider the following implementation:

function BUILD-MAX-HEAP1(A)
    $H \leftarrow$ empty heap (of max size $A.length$)
    for $i = 1$ to $A.length$ **do**
        H.MAX-HEAP-INSERT($A[i]$)
    end for
end function

a) Do the procedures BUILD-MAX-HEAP and BUILD-MAX-HEAP1 always create the same heap when run on the same input array? Prove that they do, or provide a counterexample. The procedures do not always create the same heap when run on the same input array.

b) Show that in the worst case, BUILD-MAX-HEAP1 requires $\Theta(n \lg n)$ time to build an n-element heap. Since all but the last level is always filled, the height h of an n element heap is boundend because $\sum_{i=0}^{h} 2^i = 2^{h+1} - 1 = \ldots$

6. The operation HEAP-DELETE(A, i) deletes the item in node i from heap A. Give an implementation of HEAP-DELETE that runs in $O(\lg n)$ time for an n-element max-heap.

Algorithm 4.1 HEAP-DELETE

function HEAPDELETE(A, i)
    swap A[i] with A[heapsize] ▷ move the item to be deleted to the last position in the heap
    decrement heapsize
    key $\leftarrow$ A[i]
    if key $\leq$ A[PARENT(i)] **then**
        HEAPIFY(A, i)
    end if
    ...
end function

7. Professor Fermat has the policy of giving A's to the top $n^{\frac{1}{2}} = \sqrt{n}$ students of his class, where n is the number of students. The algorithm that he uses to determine the top $n^{\frac{1}{2}}$ students first sorts the list of students by their numerical, real-valued grade, and then picks the top $n^{\frac{1}{2}}$ students from the sorted list. This algorithm has time-complexity $O(n \log n)$ because of the sort. Can you suggest a more efficient algorithm that has time-complexity $O(n)$? Describe your algorithm informally in English and justify its time-complexity. You can use a MaxHeap to effectively sort the students based off of the students grades which takes $O(n)$. Now perform RemoveMax $\sqrt{n}$ times. Each RemoveMax operation takes time $O(\log N)$. Total time complexity of this algorithm is $O(n) + \log(n)\sqrt{n} = O(n)$ because $\log(n)\sqrt{n} = O(n)$, because $\log(n) = O\left(\sqrt{n}\right)$