Section 4.6

In Problems 1–18 solve each differential equation by variation of parameters.

3.
$$y'' + y = \sin(x)$$

4.
$$y'' + y = \sec(\theta)\tan(\theta)$$

11.
$$y'' + 3y' + 2y = \frac{1}{1+e^x}$$

15.
$$y'' + 2y' + y = e^{-t}\ln(t)$$

In Problems 23–26 proceed as in Example 3 and solve each differential equation by variation of parameters.

24.
$$y'' - 4y = \frac{e^{2x}}{x}$$

In Problems 29–32 solve the given third-order differential equation by variation of parameters.

31.
$$y''' - 2y'' - y' + 2y = e^{4x}$$

Section 4.1

In Problems 23–30 verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Form the general solution.

29. (Use the Wronskian to show linear independence)

$$x^3y''' + 6x^2y'' + 4xy' - 4y = 0; \quad x, \quad x^{-2}, \quad x^2\ln(x) \quad (0,\infty)$$

39. (a) Verify that $y_1 = x^3$ and $y_2 = |x|^3$ are linearly independent solutions of the differential equation $x^2y'' - 4xy' + 6y = 0$ on the interval $(-\infty,\infty)$.

    (b) For the functions y_1 and y_2 in part(a), show that $W(y_1, y_2) = 0$ for every real number x. Does this result violate Theorem 4.1.3? Explain.

(c) Verify that $Y_1 = x^3$ and $Y_2 = x^2$ are linearly independent solutions of the differential equation in part (a) on the interval $(-\infty, \infty)$.

(d) Besides the functions y_1, y_2, Y_1, and Y_2 in parts (a) and (c), find a solution of the differential equation that satisfies $y(0) = 0$, $y'(0) = 0$.

(e) By the superposition principle, Theorem 4.1.2, both linear combinations $y = c_1 y_1 + c_2 y_2$ and $Y = c_1 Y_1 + c_2 Y_2$ are solutions of the differential equation. Discuss whether one, both, or neither of the linear combinations is a general solution of the differential equation on the interval $(-\infty, \infty)$.