Chapter 2

First-Order Differential Equations

Solution Curves Without a Solution

Given a 1st order D.E. $y' = f(x, y)$, y' is the slope of the tangent line at any point (x_0, y_0) on a solution curve

$$y' = f(x, y) = x + y$$

- $f(0, 0)$=0
- $f(1, 0)$=1

2.1.1 – Slope/Direction Fields

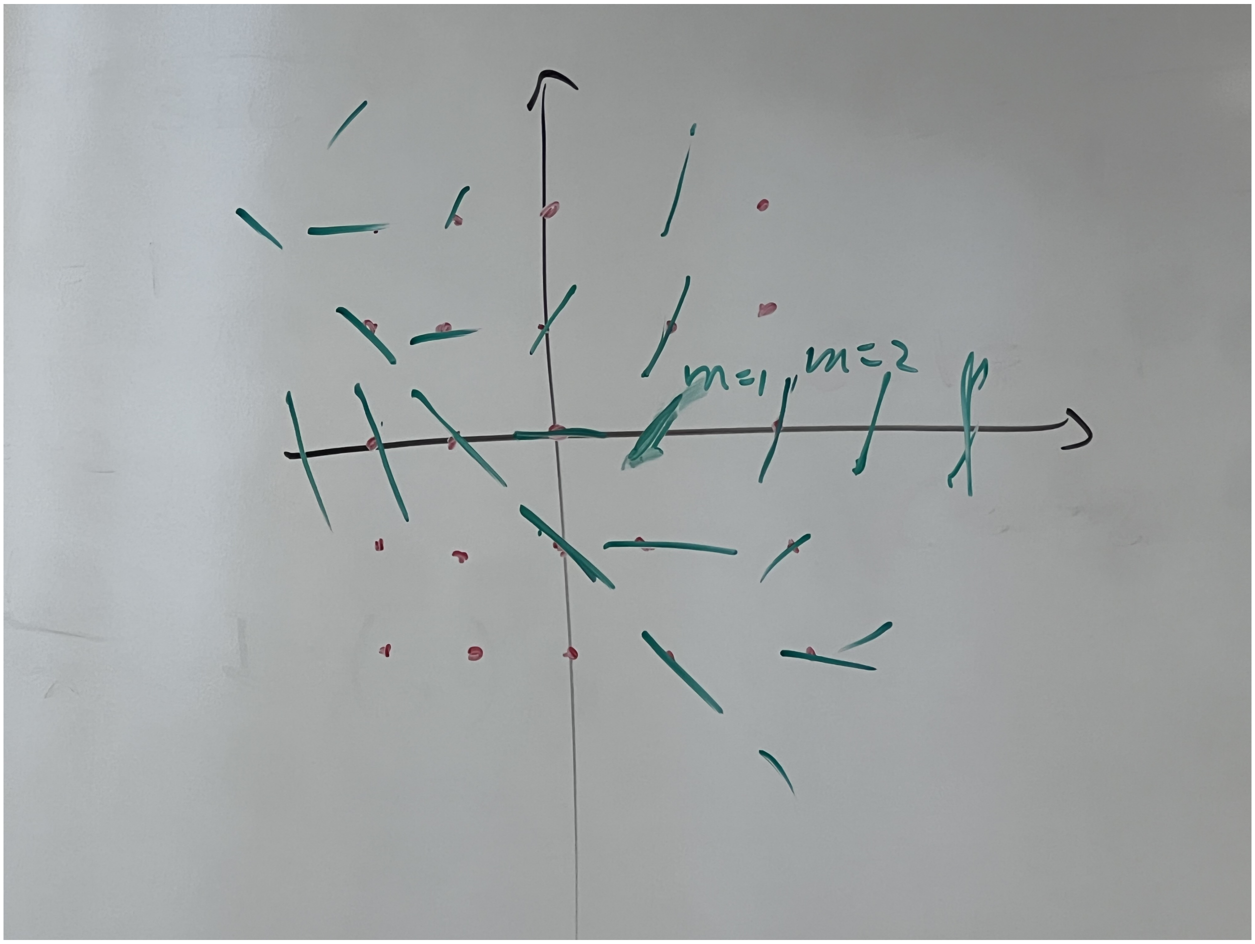

Figure 2.1: The direction field for the previous example

If the function $f(x, y)$ in the D.E. $y' = f(x, y)$ is reasonably simple so that we can solve $f(x, y) = 0$, we can make a "phase portrait diagram". We will also assume $f(x, y)$ only involves the y-variable.

$$y' = (y + 2)(y - 3)(y - 5)$$
$$f(x, y) = (y + 2)(y - 3)(y - 5)$$

An "equilibrium solution" is a solution where y is a constant. In this example: $y = 3$, $y = 5$, $y = -2$ are each constant functions.

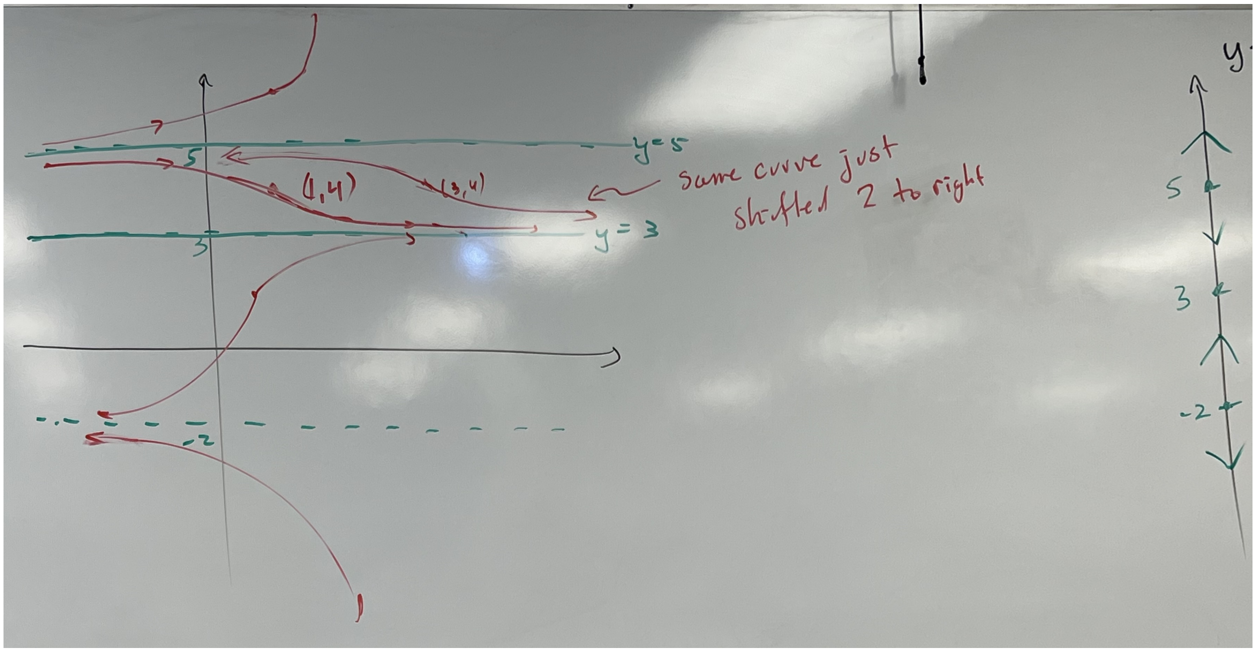

Figure 2.2: The equilibrium solution for the previous example.

The area around $y = 5$ is an unstable equilibrium since the solutions diverge and go in separate directions away from $y = 5$. The area around $y = 3$ is a stable equilibrium because the slopes above and below it converge to $y = 3$. The area around $y = -2$ is semi-stable, since all the slopes around it will converge in one direction, but the point isn't always $y = -2$.