Section 4.3

In Problems 1–14 find the general solution of the given second-order differential equation.

3.
$$y'' - y' - 6y = 0$$

5.
$$y'' + 8y' + 16y = 0$$

6.
$$y'' - 10y' + 25y = 0$$

9.
$$y'' + 9y = 0$$

11.
$$y'' - 4y' + 5y = 0$$

12.
$$2y'' + 2y' + y = 0$$

In Problems 15–28 find the general solution of the given higher-order differential equation.

15.
$$y''' - 4y'' - 5y' = 0$$

17.
$$y''' - 5y'' + 3y' + 9y = 0$$

25.
$$16\frac{d^4y}{dx^4} + 24\frac{d^2y}{dx^2} + 9y = 0$$

26.
$$\frac{d^4y}{dx^4} - 7\frac{d^2y}{dx^2} - 18y = 0$$

In Problems 29–36 solve the given initial-value problem.

31.
$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 5y = 0, \quad y(1) = 0, \quad y'(1) = 2$$

32.
$$4y'' - 4y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 5$$

In Problems 49–58 find a homogenous linear differential equation with constant coefficients whose general solution is given.

49.
$$y = c_1e^x + c_2e^{5x}$$

50.
$$y = c_1e^{-4x} + c_2e^{-3x}$$

53.
$$y = c_1\cos(3x) + c_2\sin(3x)$$

56.
$$y = c_1 + c_2e^{2x}\cos(5x) + c_3e^{2x}\sin(5x)$$

Section 4.1

In Problems 23–30 verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Form the general solution.

23.
$$y'' - y' - 12y = 0; \quad e^{-3x}, \quad e^{4x}, \quad (-\infty, \infty)$$

25.
$$y'' - 2y' + 5y = 0; \quad e^x\cos(2x), \quad e^x\sin(2x), \quad (-\infty, \infty)$$

40. Is the set of functions $f_1(x) = e^{x+2}, f_2(x) = e^{x-3}$ linearly dependent or linearly independent on $(-\infty, \infty)$? Discuss.

41. Suppose $y_1,\ y_2,\ \ldots,\ y_k$ are k linearly independent solutions on $(-\infty, \infty)$ of a homogeneous linear nth-order differential equation with constant coefficients. By Theorem 4.1.2 it follows that $y_{k+1} = 0$ is also a solution of the differential equation. Is the set of functions $y_1, y_2, \ldots, y_k, y_{k+1}$ linearly dependent or linearly independent on $(-\infty, \infty)$? Discuss.

42. Suppose that $y_1, y_2, \ldots, y_k$ are k nontrivial solutions of a homogeneous linear nth-order differential equation with constant coefficients and that $k = n+1$. Is the set of solutions $y_1, y_2, \ldots, y_k$ linearly dependent or linearly independent on $(-\infty, \infty)$? Discuss.