Section 4.2

In Problems 1–16 the indicated function $y_1(x)$ is a solution of the given differential equation. Use reduction of order or formula (5), as instructed, to find a second solution y_2(x).

1.

$$y'' - 4y' + 4y = 0; y_1 = e^{2x}$$

3.

$$y'' + 16y = 0; y_1 = \cos(4x)$$

9.

$$x^2y'' - 7xy' + 16y = 0; y_1 = x^4$$

11.

$$xy'' + y' = 0; y_1 = \ln(x)$$

12.

$$4x^2y'' + y = 0; y_1 = \sqrt{x}\ln(x) = x^{\frac{1}{2}}\ln(x)$$

In Problems 21 and 22 the indicated function $y_1(x)$ is a solution of the given differential equation. Use formula (5) to find a second solution $y_2(x)$ expressed in terms of an integral-defined function. See (*iii*) in the *Remarks.*

21.

$$x^2y'' + (x^2 - x)y' + (1 - x)y = 0; y_1 = x$$

Section 4.1

In Problems 15–22 determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$.

15.

$$f_1(x) = x, f_2(x) = x^2, f_3(x) = 4x - 3x^2$$

17.

$$f_1(x) = 5, f_2(x) = \cos^2(x), f_3 = \sin^2(x)$$

19.

$$f_1(x) = x, f_2(x) = x - 1, f_3(x) = x + 3$$

20.

$$f_1(x) = 2 + x, f_2(x) = 2 + |x|$$