

In Problems 1–20 find either $F(t)$ or $f(t)$, as indicated.

3.

$$\mathcal{L} \{t^3 e^{-2t}\}$$

7.

$$\mathcal{L} \{e^t \sin(3t)\}$$

8.

$$\mathcal{L} \{e^{-2t} \cos(4t)\}$$

11.

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s+2)^3} \right\}$$

15.

$$\mathcal{L} \left\{ \frac{s}{s^2 + 4s + 5} \right\}$$

17.

$$\mathcal{L} \left\{ \frac{s}{(s+1)^2} \right\}$$

18.

$$\mathcal{L} \left\{ \frac{5s}{(s-2)^2} \right\}$$

In Problems 21–30 use the Laplace transform to solve the given initial-value problem.

23.

$$y'' + 2y' + y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

27.

$$y'' - 6y' + 13y = 0, \quad y(0) = 0, \quad y'(0) = -3$$

In Problems 37–48 find either $F(s)$ or $f(t)$, as indicated.

37.

$$\mathcal{L} \{(t-1)\mathcal{U}(t-1)\}$$

38.

$$\mathcal{L}\{e^{2-t}\mathcal{U}(t-2)\}$$

43.

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\}$$

47.

$$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$$

In Problems 55–62 write each function in terms of unit step functions. Find the Laplace transform of the given function.

55.

$$f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ -2, & t \geq 3 \end{cases}$$

56.

$$f(t) = \begin{cases} 1, & 0 \leq t < 4 \\ 0, & 4 \leq t < 5 \\ 1, & t \geq 5 \end{cases}$$

59.

$$f(t) = \begin{cases} t, & 0 \leq t < 2 \\ 0, & t \geq 2 \end{cases}$$

In Problems 63–70 use the Laplace transform to solve the given initial-value problem.

65.

$$y' + 2y = f(t), \quad y(0) = 0, \quad \text{where}$$

$$f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

66.

$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = -1, \quad \text{where}$$

$$f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$