In Problems 1–22 solve the given differential equation by separation of variables.

1. $$\frac{dy}{dx} = \sin 5x$$

3. $$dx + e^{3x}dy = 0$$

5. $$x\frac{dy}{dx} = 4y$$

6. $$\frac{dy}{dx} + 2xy^2 = 0$$

7. $$\frac{dy}{dx} = e^{3x+2y}$$

9. $$y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2$$

11. $$\csc y \; dx + \sec^2 x \; dy = 0$$

13. $$(e^y + 1)^2 e^{-y}dx + (e^x + 1)^3 e^{-x}dy = 0$$

16. $$\frac{dQ}{dt} = k(Q - 70)$$

In Problems 23–28 find an explicit solution of the given initial-value problem.

24. $$\frac{dy}{dx} = \frac{y^2 - 1}{x^2 - 1}, \; y(2) = 2$$

28. $$\left(1 + x^4\right) dy + x \left(1 + 4y^2\right) dx = 0, \; y(1) = 0$$

In Problems 29 and 30 proceed as in Example 5 and find an explicit solution of the given initial-value problem.

29. $$\frac{dy}{dx} = ye^{-x^2}, \; y(4) = 1$$

Often a radical change in the form of the solution of a differential equation corresponds to a very small change in either the initial condition or the equation itself. In Problems 39–42 find an explicit solution of the given initial-value problem. Use a graphing utility to plot the graph of each solution. Compare each solution curve in a neighborhood of $(0, 1)$.

39.
$$\frac{dy}{dx} = (y-1)^2,\ y(0) = 1$$

In Problems 45–50 use a technique of integration or a substitution to find an explicit solution of the given differential equation or initial-value problem.

45.
$$\frac{dy}{dx} = \frac{1}{1+\sin x}$$

47.
$$\left(\sqrt{x}+x\right)\frac{dy}{dx} = \sqrt{y}+y$$

49.
$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{y},\ y(1) = 4$$