Chapter 7

Method of Laplace Transforms for Solving DE's

7.3 Operational Rules Part 1

Shifting Theorem Shifting on t-axis

$$\mathscr{L}\left\{f(t-a)\mathscr{U}(t-a)\right\} = e^{-as}\mathscr{L}\left\{f(t)\right\}$$

Proof:

$$\begin{aligned} \mathscr{L}\left\{f(t-a)\mathscr{U}(t-a)\right\} &= \int_0^\infty e^{-st} f(t-a)\mathscr{U}(t-a) \\ &= \int_0^a 0dt + \int_a^\infty e^{-st} f(t-a)dt \\ \text{Let } \tau = t-a,\ d\tau = dt \qquad &= 0 + \int_a^\infty e^{-s(\tau+a)} f(\tau)d\tau \\ &= e^{-sa}\int_a^\infty e^{-s\tau} f(\tau)d\tau \\ &= e^{-sa}\mathscr{L}\left\{f(\tau)\right\} \end{aligned}$$

$$\mathscr{L}\left\{f(\tau)\mathscr{U}(\tau)\right\} = e^{-as}\mathscr{L}\left\{f(\tau)\right\}$$

$$\mathscr{L}\left\{f(t-a)\mathscr{U}(t-a)\right\} = e^{-as}\mathscr{L}\left\{f(t-a)\right\} \tag{7.1}$$

$$f(t-a)\mathscr{U}(t-a) = \mathscr{L}^{-1}\left\{e^{-as}F(s)\right\}$$

$$f(t-a)\mathscr{U}(t-a) = \mathscr{L}^{-1}\left\{e^{-as}\mathscr{L}\left\{f(t)\right\}\right\} \tag{7.2}$$