In Problems 3–6, $y = \frac{1}{x^2+c}$ is a one-parameter family of solutions of the first-order DE $y' + 2xy^2 = 0$. Find a solution of the first-order IVP consisting of this differential equation and the given initial condition. Give the largest interval I over which the solution is defined.

3.
$$y(2) = \frac{1}{3}$$

6.
$$y\left(\frac{1}{2}\right) = -4$$

In Problems 7–10, $x = c_1 \cos(t) + c_2 \sin(t)$ is a two-parameter family of solutions of the second-order DE $x'' + x = 0$. Find a solution of the second-order IVP consisting of this differential equation and the given initial conditions.

7.
$$x(0) = -1, x'(0) = 8$$

10.
$$x\left(\frac{\pi}{4}\right) = \sqrt{2}, x'\left(\frac{\pi}{4}\right) = 2\sqrt{2}$$

In Problems 11–14, $y = c_1 e^x + c_2 e^{-x}$ is a two-parameter family of solutions of the second-order DE $y'' - y = 0$. Find a solution of the second-order IVP consisting of this differential equation and the given initial conditions.

11.
$$y(0) = 1, y'(0) = 2$$

12.
$$y(1) = 0, y'(1) = e$$

In Problems 15 and 16 determine by inspection at least two solutions of the given first-order IVP.

15.
$$y' = 3y^{\frac{2}{3}}, y(0) = 0$$

In Problems 17–24 determine a region of the xy-plane for which the given differential equation would have a unique solution whose graph passes through a point (x_0, y_0) in the region.

17.
$$\frac{dy}{dx} = y^{\frac{2}{3}}$$

21.
$$\left(4 - y^2\right) y' = x^2$$

22.
$$\left(1 + y^3\right) y' = x^2$$

In Problems 25–28 determine whether Theorem 1.2.1 guarantees that the differential equation $y' = \sqrt{y^2 - 9}$ possesses a unique solution through the given point.

25.
$$(1, 4)$$

28.
$$(-1, 1)$$

31. (a) Verify that $y = -\frac{1}{x+c}$ is a one-parameter family of solutions of the differential equation $y' = y^2$.

(b) Since $f(x, y) = y^2$ and $\frac{\partial f}{\partial y} = 2y$ are continuous everywhere, the region R in Theorem 1.2.1 can be taken to be the entire xy-plane. Find a solution from the family in part (a) that satisfies $y(0) = 1$. Then find a solution from the family in part (a) $y(0) = -1$. Determine the largest interval I of definition for the solution of each initial-value problem.

(c) Determine the largest interval I of definition for the solution of the first-order value problem $y' = y^2, y(0) = 0$. [*Hint*: The solution is not a member of the family of solutions in part (a).]

32. (a) Show that a solution from the family in part (a) of Problem 31 that satisfies $y' = y^2$, $y(1) = 1$, is $y = \frac{1}{2-x}$

(b) Then show that a solution from the family in part (a) of Problem 31 that satisfies $y' = y^2, y(3) = -1$, is $y = \frac{1}{2-x}$.

(c) Are the solutions in parts (a) and (b) the same?