

In Problems 1–8 state the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear by matching it with (6).

1.

$$(1 - x)y'' - 4xy' + 5y = \cos x$$

4.

$$\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r + u)$$

5.

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

6.

$$\frac{d^2R}{dt^2} = -\frac{k}{R^2}$$

9. Determine whether the given first-order differential equation is linear in the indicated dependent variable by matching it with the first differential equation given in (7).

$$(y^2 - 1)dx + x dy = 0; \text{ in } y; \text{ in } x$$

In Problems 11–14 verify that the indicated function is an explicit solution of the given differential equation. Assume an appropriate interval  $I$  of definition for each solution.

11.

$$2y' + y = 0; y = e^{-x/2}$$

12.

$$\frac{dy}{dt} + 20y = 24; y = \frac{6}{5} - \frac{6}{5}e^{-20t}$$

In Problems 15–18 verify that the indicated function  $y = \phi(x)$  is an explicit solution of the given first-order differential equation. Proceed as in Example 6, by considering  $\phi$  simply as a *function* and give its domain. Then by considering  $\phi$  as a *solution* of the differential equation, give at least one interval  $I$  of definition.

15.

$$(y - x)y' = y - x + 8; y = x + 4\sqrt{x + 2}$$

17.

$$y' = 2xy^2; y = \frac{1}{4 - x^2}$$

18.

$$2y' = y^3 \cos x; y = (1 - \sin x)^{-\frac{1}{2}}$$

In Problems 21–24 verify that the indicated family of functions is a solution of the differential equation. Assume an appropriate interval  $I$  of definition for each solution.

21.

$$\frac{dP}{dt} = P(1 - P); P = \frac{c_1 e^t}{1 + c_1 e^t}$$

22.

$$\frac{dy}{dx} + 4xy = 8x^3; y = 2x^2 - 1 + c_1 e^{-2x^2}$$

In Problems 25–28 use (12) to verify that the indicated function is a solution of the given differential equation. Assume an appropriate interval  $I$  of definition of each solution.

27.

$$x^2 \frac{dy}{dx} + xy = 10 \sin x; y = \frac{5}{x} + \frac{10}{x} \int_1^x \frac{\sin t}{t} dt$$

28.

30. In Example 7 we saw that  $y = \phi_1(x) = \sqrt{25 - x^2}$  and  $y = \phi_2(x) = -\sqrt{25 - x^2}$  are solutions of  $\frac{dy}{dx} = -\frac{x}{y}$  on the interval  $(-5, 5)$ . Explain why the piecewise-defined function

$$y = \begin{cases} \sqrt{25 - x^2}, & -5 < x < 0 \\ -\sqrt{25 - x^2}, & 0 \leq x < 5 \end{cases}$$

is *not* a solution of the differential equation on the interval  $(-5, 5)$ .

In Problems 35 and 36 find values of  $m$  so that the function  $y = x^m$  is a solution of the given differential equation.

35.

$$xy'' + 2y' = 0$$