3. If $\mathbf{A} = \begin{bmatrix} 2 & -3 \\ -5 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 6 \\ 3 & 2 \end{bmatrix}$, find

(a) $\mathbf{AB}$

(b) $\mathbf{BA}$

(c) $\mathbf{A}^2 = \mathbf{AA}$

(d) $\mathbf{B}^2 = \mathbf{BB}$

4. If $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 5 & 10 \\ 8 & 12 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -4 & 6 & -3 \\ 1 & -3 & 2 \end{bmatrix}$, find

(a) $\mathbf{AB}$

(b) $\mathbf{BA}$

9. If $\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 8 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 & 10 \\ -2 & -5 \end{bmatrix}$, find

(a) $(\mathbf{AB})^T$

(b) $\mathbf{B}^T\mathbf{A}^T$

In Problems 11–14 write the given sum as a single column matrix.

11.

$$4\begin{bmatrix} -1 \\ 2 \end{bmatrix} - 2\begin{bmatrix} 2 \\ 8 \end{bmatrix} + 3\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

14.

$$\begin{bmatrix} 1 & -3 & 4 \\ 2 & 5 & -1 \\ 0 & -4 & -2 \end{bmatrix}\begin{bmatrix} t \\ 2t-1 \\ -t \end{bmatrix} + \begin{bmatrix} -t \\ 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 2 \\ 8 \\ -6 \end{bmatrix}$$

In Problems 15–22 determine whether the given matrix is singular or non-singular. If it is non-singular, find $\mathbf{A}^{-1}$ using Theorem B.2.

15.

$$\mathbf{A} = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}$$

16.

$$\mathbf{A} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$$

19.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

In Problems 23 and 24 show that the given matrix is non-singular for every real value of t. Find $\mathbf{A}^{-1}(t)$ using Theorem B.2.

23.

$$\mathbf{A}(t) = \begin{bmatrix} 2e^{-t} & e^{4t} \\ 4e^{-t} & 3e^{4t} \end{bmatrix}$$

29. Let $\mathbf{A}(t) = \begin{bmatrix} e^{4t} & \cos(\pi t) \\ 2t & 3t^2 - 1 \end{bmatrix}$. Find

(a)

$$\frac{d\mathbf{A}}{dt}$$

(b)

$$\int_0^2 \mathbf{A}(t)dt$$

(c)

$$\int_0^t \mathbf{A}(s)ds$$

In Problems 31–38 solve the given system of equations by either Gaussian elimination or Gauss-Jordan elimination.

31.

$$\begin{aligned} x + y \quad & -2z = 14 \\ 2x - y \quad & +z = 0 \\ 6x + 3y \quad & +4z = 1 \end{aligned}$$

32.

$$\begin{aligned} 5x - 2y \quad & +4z = 10 \\ x + y \quad & +z = 9 \\ 4x - 3y \quad & +3z = 1 \end{aligned}$$

In Problems 39 and 40 use Gauss-Jordan elimination to demonstrate that the given system of equations has not solution.

39.

$$\begin{aligned} x + 2y \quad & +4z = 2 \\ 2x + 4y \quad & +3z = 1 \\ x + 2y \quad & -z = 7 \end{aligned}$$

In Problems 41–46 use Theorem B.3 to find $\mathbf{A}^{-1}$ for the given matrix or show that no inverse exists. If it is non-singular, find $\mathbf{A}^{-1}$ using Theorem B.2.

41.

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & -2 & 0 \end{bmatrix}$$

42.

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 2 & -2 \\ 8 & 10 & -6 \end{bmatrix}$$

In Problems 47–54 find the eigenvalues and eigenvectors of the given matrix.

47.

$$\begin{bmatrix} -1 & 2 \\ -7 & 8 \end{bmatrix}$$

48.

$$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

51.

$$\begin{bmatrix} 5 & -1 & 0 \\ 0 & -5 & 9 \\ 5 & -1 & 0 \end{bmatrix}$$

52.

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

In Problems 55 and 56 show that the given matrix has complex eigenvalues. Find the eigenvectors of the matrix.

55.

$$\begin{bmatrix} -1 & 2 \\ -5 & 1 \end{bmatrix}$$