In Problems 1–6 write the given linear system in matrix form.

3.

$$\frac{dx}{dt} = -3x + 4y - 9z$$
$$\frac{dy}{dt} = 6x - y$$
$$\frac{dz}{dt} = 10x + 4y + 3z$$

4.

$$\frac{dx}{dt} = x - y$$
$$\frac{dy}{dt} = x + 2z$$
$$\frac{dz}{dt} = -x + z$$

In Problems 7–10 write the given linear system without the use of matrices.

7.

$$\mathbf{X}' = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} \mathbf{X} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t}$$

In Problems 11–16 verify that the vector $\mathbf{X}$ is a solution of the given homogenous linear system.

13.

$$\mathbf{X}' = \begin{bmatrix} -1 & \frac{1}{4} \\ 1 & -1 \end{bmatrix} \mathbf{X}; \quad \mathbf{X} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-\frac{3t}{2}}$$

In Problems 17–20 the given vectors are solutions of a system $\mathbf{X}' = \mathbf{A}\mathbf{X}$. Determine whether the vectors form a fundamental set on the interval $(-\infty, \infty)$.

17.

$$\mathbf{X}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}, \quad \mathbf{X}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-6t}$$

20.

$$\mathbf{X}_1 = \begin{bmatrix} 1 \\ 6 \\ -13 \end{bmatrix}, \quad \mathbf{X}_2 = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} e^{-4t}, \quad \mathbf{X}_3 = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} e^{3t}$$

In Problems 21–24 verify that the vector $\mathbf{X}_p$ is a particular solution of the given nonhomogenous linear system.

21.

$$\frac{dx}{dt} = x + 4y + 2t - 7$$
$$\frac{dy}{dt} = 3x + 2y - 4t - 18$$
$$\mathbf{X}_p = \begin{bmatrix} 2 \\ -1 \end{bmatrix} t + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

24.

$$\mathbf{X}' = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 2 & 0 \\ -6 & 1 & 0 \end{bmatrix} \mathbf{X} + \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} \sin(3t); \quad \mathbf{X}_p = \begin{bmatrix} \sin(3t) \\ 0 \\ \cos(3t) \end{bmatrix}$$

25. Prove that the general solution of the homogenous linear system

$$\mathbf{X}' = \begin{bmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \mathbf{X}$$

on the interval $(-\infty, \infty)$ is

$$\mathbf{X} = c_1 \begin{bmatrix} 6 \\ -1 \\ -5 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} e^{-2t} + c_3 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} e^{-2t}.$$