In Problems 1–4 reproduce the given computer-generated direction field. Then sketch, by hand, an approximate solution curve that passes through each of the indicated points. Use different colored pencils for each solution curve.

1.
$$\frac{dy}{dx} = x^2 - y^2$$

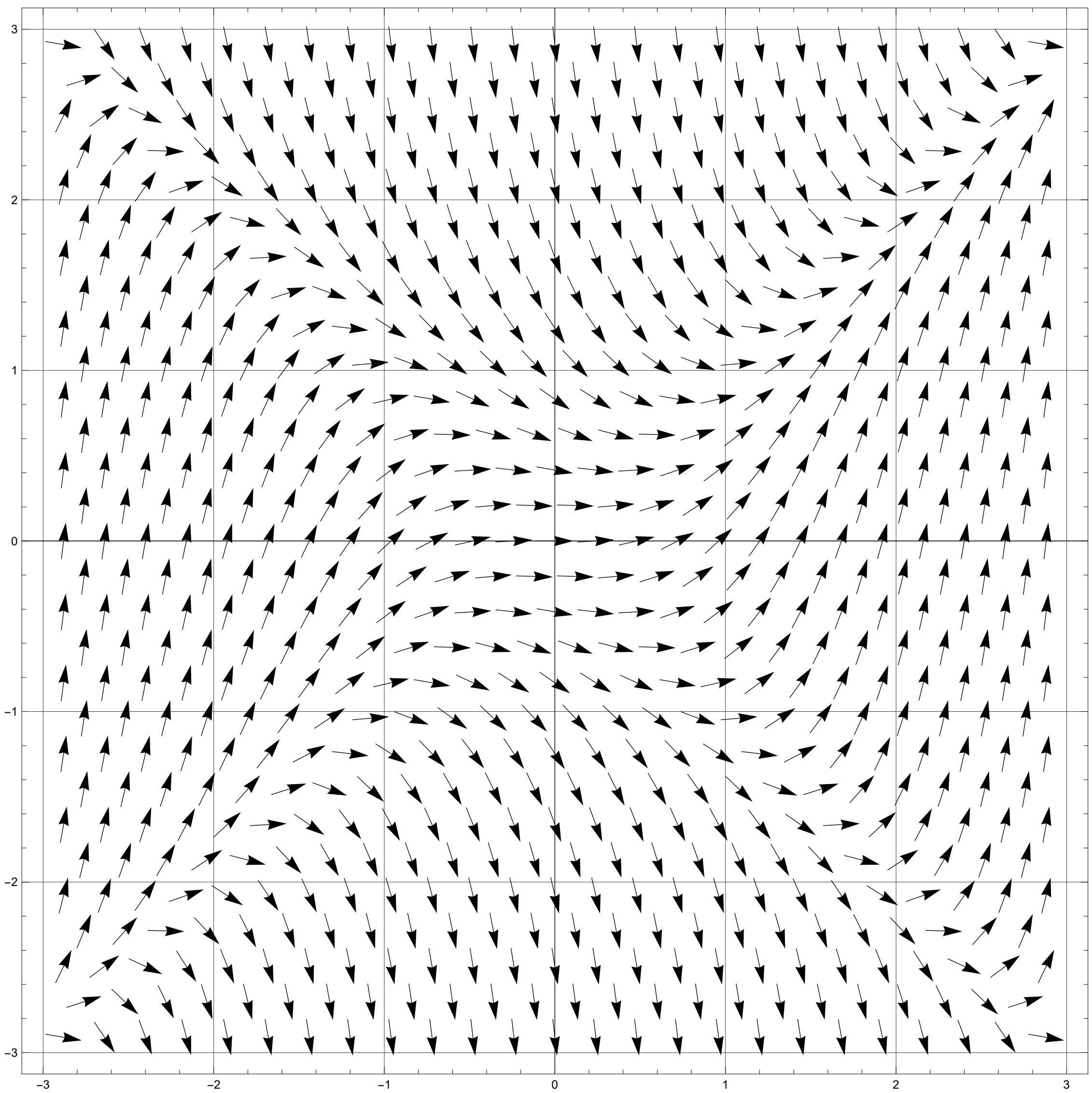

Figure 2.1: The direction field for Problem 1.

(a) $y(-2) = 1$

(b) $y(3) = 0$

(c) $y(0) = 2$

(d) $y(0) = 0$

3.

$$\frac{dy}{dx} = 1 - xy$$

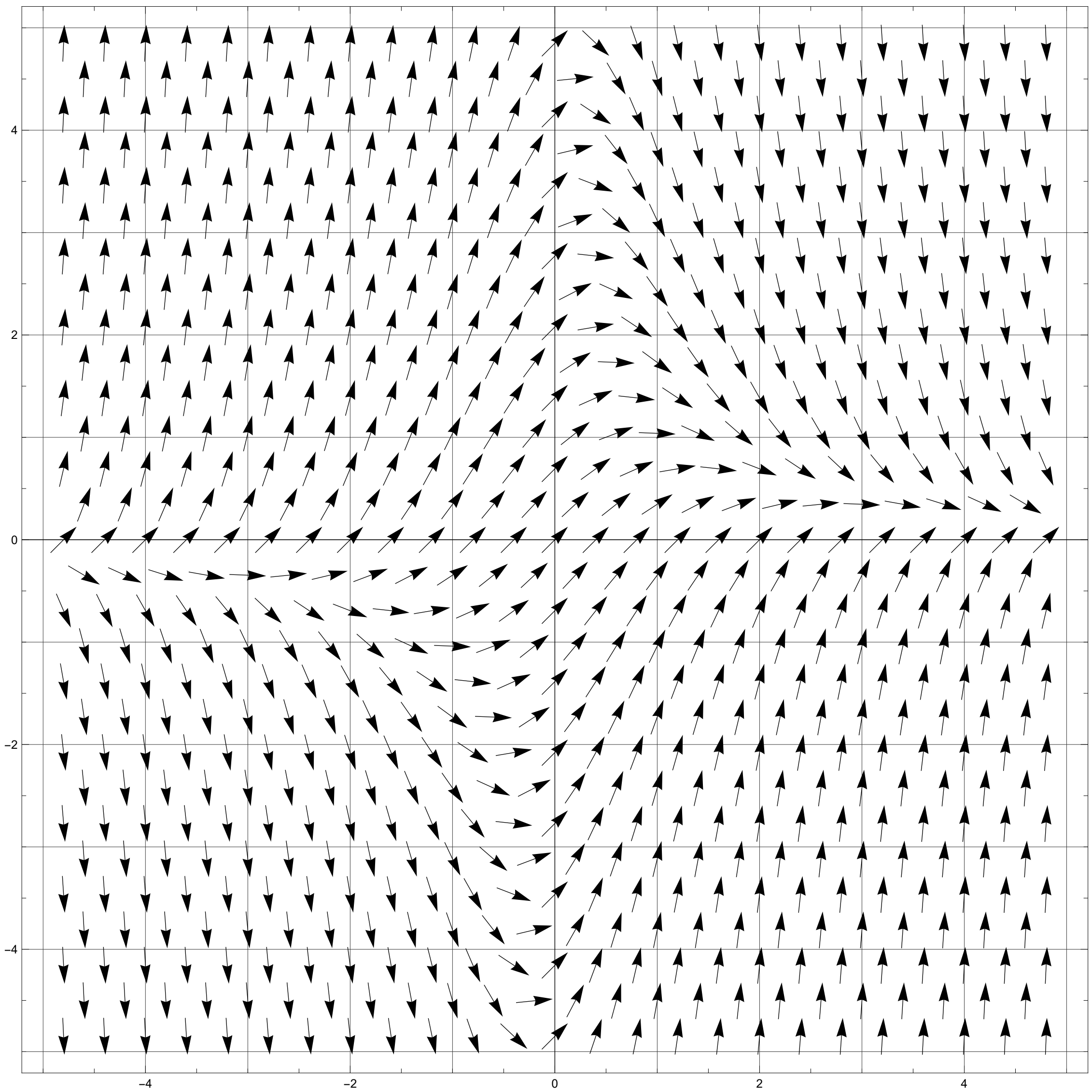

Figure 2.2: The direction field for Problem 3.

(a) $y(0) = 0$

(b) $y(-1) = 0$

(c) $y(2) = 2$

(d) $y(0) = -4$

4.

$$\frac{dy}{dx} = (\sin x) \cos y$$

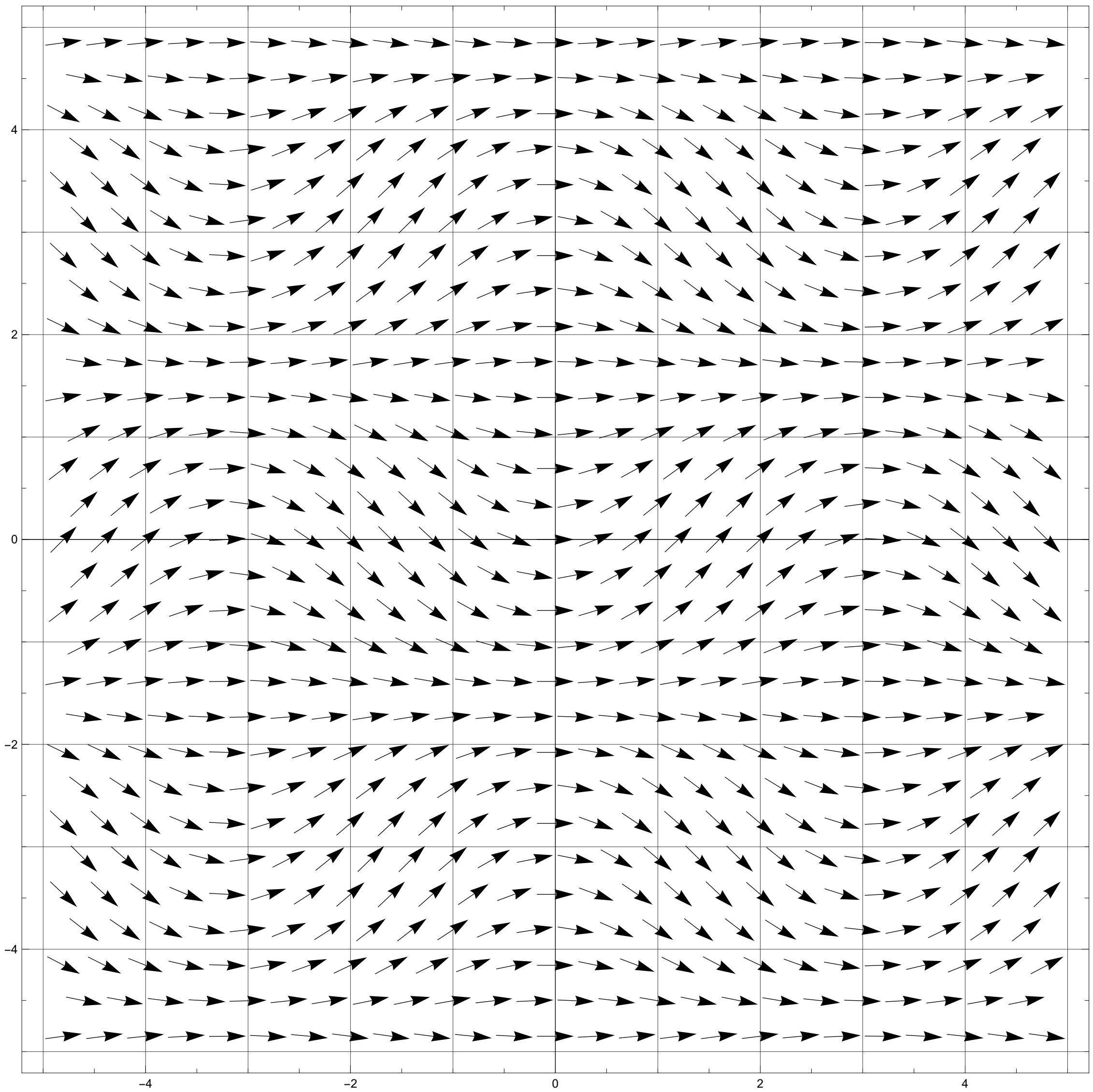

Figure 2.3: The direction field for Problem 4.

(a) $y(0) = 1$

(b) $y(1) = 0$

(c) $y(3) = 3$

(d) $y(0) = -\frac{5}{2}$

In Problems 5–12 use computer software to obtain a direction field for the given differential equation. By hand, sketch an approximate solution curve passing through each of the given points.

8.

$$\frac{dy}{dx} = \frac{1}{y}$$

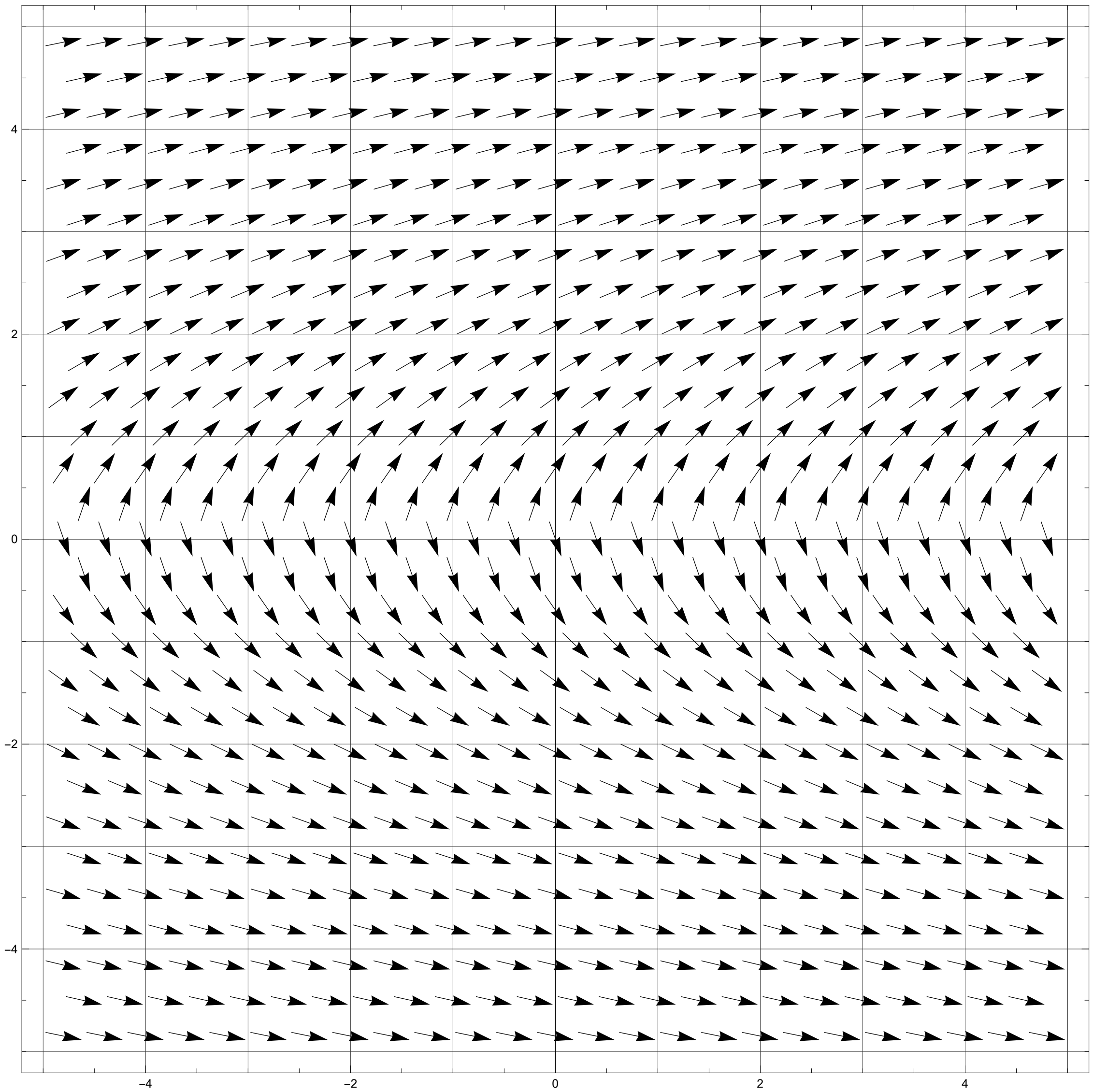

Figure 2.4: The direction field for Problem 8.

(a) $y(0) = 1$

(b) $y(-2) = -1$

In Problems 21–28 find the critical points and phase portrait of the given autonomous first-order differential equation. Classify each critical point as asymptotically stable, unstable, or semi-stable. By hand, sketch typical solution curves in the regions in the xy-plane determined by the graphs of the equilibrium solutions.

21.

$$\frac{dy}{dx} = y^2 - 3y$$

25.
$$\frac{dy}{dx} = y^2\left(4 - y^2\right)$$

26.
$$\frac{dy}{dx} = y(2 - y)(4 - y)$$

28.
$$\frac{dy}{dx} = \frac{ye^y - 9y}{e^y}$$

33. Suppose that $y(x)$ is a nonconstant solution of the autonomous equation $\frac{dy}{dx} = f(y)$ and that c is a critical point of the DE. Discuss: Why can't the graph of $y(x)$ cross the graph of the equilibrium solution $y = c$? Why can't $f(y)$ change signs in one of the subregions discussed on page 40? Why can't $y(x)$ be oscillatory or have a relative extremum (maximum or minimum)?