In Problems 1–8 use Theorem 7.4.1 to evaluate the given Laplace transform.

1.

$$\mathscr{L}\left\{te^{-10t}\right\}$$

3.

$$\mathscr{L}\left\{t\cos(2t)\right\}$$

6.

$$\mathscr{L}\left\{t\sinh(3t)\right\}$$

In Problems 9–14 use the Laplace transform to solve the given initial-value problem. Use the table of Laplace transforms in Appendix C as needed.

9.

$$y' + y = t\sin t, \quad y(0) = 0$$

12.

$$y'' + y = \sin t, \quad y(0) = 1, \quad y'(0) = -1$$

13.

$$y'' + 16y = f(t), \quad y(0) = 0, \quad y'(0) = 1, \text{ where } f(t) = \begin{cases} \cos 4t, & 0 \le t < \pi, \\ 0, & t \ge \pi \end{cases}$$

In Problems 19–22 proceed as in Example 3 and find the convolution $f \times g$ of the given functions. After integrating, find the Laplace transform of $f \times g$.

19.

$$f(t) = 4t, \quad g(t) = 3t^2$$

20.

$$f(t) = t, \quad g(t) = e^{-t}$$

In Problems 23–34 proceed as in Example 4 and find the Laplace transform of $f \times g$ using Theorem 7.4.2. Do not evaluate the convolution integral before transforming.

25.

$$\mathscr{L}\left\{e^{-t} \times e^{t}\cos(t)\right\}$$

26.

$$\mathscr{L}\left\{e^{2t} \times \sin t\right\}$$

27.

$$\mathscr{L}\left\{\int_0^t e^{\tau} d\tau\right\}$$

28.

$$\mathscr{L}\left\{\int_0^t \cos(\tau) d\tau\right\}$$

31.

$$\mathscr{L}\left\{\int_0^t e\tau^{t-\tau} d\tau\right\}$$

33.

$$\mathscr{L}\left\{\int_0^t \sin(\tau) d\tau\right\}$$

In Problems 41–50 use the Laplace transform to solve the given integral equation or integrodifferential equation.

41.

$$f(t) + \int_0^t (t-\tau) f(\tau) d\tau = t$$

43.

$$f(t) = te^t + \int_0^t \tau f(t-\tau) d\tau$$

44.

$$f(t) + 2\int_0^t f(\tau) \cos(t-\tau) d\tau = 4e^{-t} + \sin t$$

45.

$$f(t) + \int_0^t f(\tau) d\tau = 1$$

In Problems 53–58 use Theorem 7.4.3 to find the Laplace transform of the given periodic function.

53.

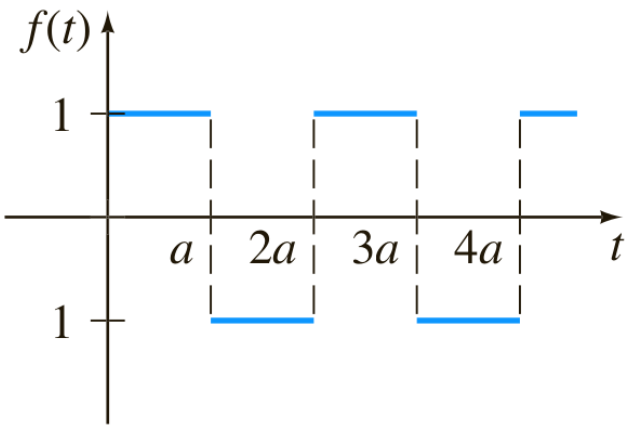

FIGURE 7.4.53 Graph for Problem 53

55.

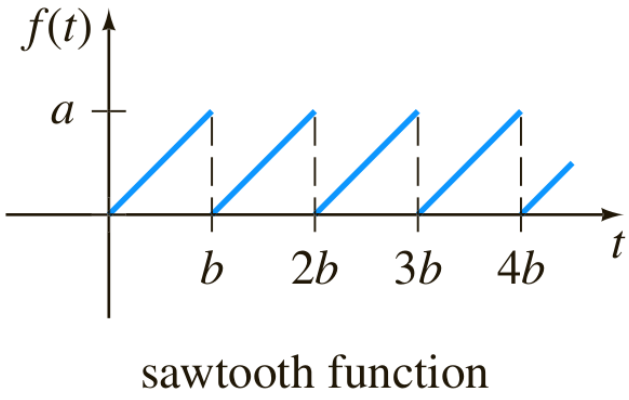

FIGURE 7.4.55 Graph for Problem 55

56.

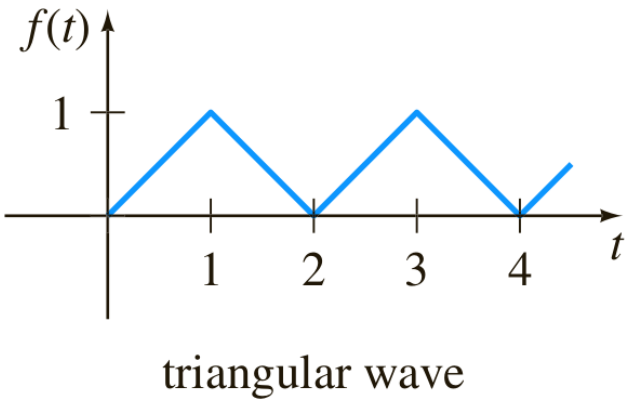

FIGURE 7.4.56 Graph for Problem 56