In Problems 1–20 find either $F(t)$ or $f(t)$, as indicated.

3.

$$\mathscr{L}\left\{t^3 e^{-2t}\right\}$$

7.

$$\mathscr{L}\left\{e^t \sin(3t)\right\}$$

8.

$$\mathscr{L}\left\{e^{-2t} \cos(4t)\right\}$$

11.

$$\mathscr{L}^{-1}\left\{\frac{1}{(s+2)^3}\right\}$$

15.

$$\mathscr{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\}$$

17.

$$\mathscr{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\}$$

18.

$$\mathscr{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\}$$

In Problems 21–30 use the Laplace transform to solve the given initial-value problem.

23.

$$y'' + 2y' + y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

27.

$$y'' - 6y' + 13y = 0, \quad y(0) = 0, y'(0) = -3$$

In Problems 37–48 find either $F(s)$ or $f(t)$, as indicated.

37.

$$\mathscr{L}\left\{(t-1)\mathscr{U}(t-1)\right\}$$

38.

$$\mathscr{L}\left\{e^{2-t}\mathscr{U}(t-2)\right\}$$

43.

$$\mathscr{L}^{-1}\left\{\frac{e^{-2s}}{s^3}\right\}$$

47.

$$\mathscr{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$$

In Problems 55–62 write each function in terms of unit step functions. Find the Laplace transform of the given function.

55.

$$f(t)=\begin{cases}2, & 0\le t<3\\ -2, & t\ge 3\end{cases}$$

56.

$$f(t)=\begin{cases}1, & 0\le t<4\\ 0, & 4\le t<5\\ 1, & t\ge 5\end{cases}$$

59.

$$f(t)=\begin{cases}t, & 0\le t<2\\ 0, & t\ge 2\end{cases}$$

In Problems 63–70 use the Laplace transform to solve the given initial-value problem.

65.

$$y'+2y=f(t),\quad y(0)=0,\ \text{where}$$
$$f(t)=\begin{cases}t, & 0\le t<1\\ 0, & t\ge 1\end{cases}$$

66.

$$y''+4y=f(t),\quad y(0)=0,\quad y'(0)=-1,\ \text{where}$$
$$f(t)=\begin{cases}1, & 0\le t<1\\ 0, & t\ge 1\end{cases}$$