In Problems 1–8 state the order of the given ordinary differential equation. Determine whether the equation is linear or nonlinear by matching it with (6).

1.
$$(1-x)y'' - 4xy' + 5y = \cos x$$

4.
$$\frac{d^2u}{dr^2} + \frac{du}{dr} + u = \cos(r+u)$$

5.
$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

6.
$$\frac{d^2R}{dt^2} = -\frac{k}{R^2}$$

9. Determine whether the given first-order differential equation is linear in the indicated dependent variable by matching it with the first differential equation given in (7).

$$(y^2-1)dx + x\ dy = 0;\ \text{in } y;\ \text{in } x$$

In Problems 11–14 verify that the indicated function is an explicit solution of the given differential equation. Assume an appropriate interval I of definition for each solution.

11.
$$2y' + y = 0;\ y = e^{-x/2}$$

12.
$$\frac{dy}{dt} + 20y = 24;\ y = \frac{6}{5} - \frac{6}{5}e^{-20t}$$

In Problems 15–18 verify that the indicated function $y = \phi(x)$ is an explicit solution of the given first-order differential equation. Proceed as in Example 6, by considering ϕ simply as a *function* and give its domain. Then by considering ϕ as a *solution* of the differential equation, give at least one interval I of definition.

15.
$$(y-x)y' = y - x + 8;\ y = x + 4\sqrt{x+2}$$

17.
$$y' = 2xy^2;\ y = \frac{1}{4-x^2}$$

18.
$$2y' = y^3 \cos x;\ y = (1-\sin x)^{-\frac{1}{2}}$$

In Problems 21–24 verify that the indicated family of functions is a solution of the differential equation. Assume an appropriate interval I of definition for each solution.

21.
$$\frac{dP}{dt} = P(1-P);\ P = \frac{c_1 e^t}{1 + c_1 e^t}$$

22.
$$\frac{dy}{dx} + 4xy = 8x^3;\ y = 2x^2 - 1 + c_1 e^{-2x^2}$$

In Problems 25–28 use (12) to verify that the indicated function is a solution of the given differential equation. Assume an appropriate interval I of definition of each solution.

27.
$$x^2 \frac{dy}{dx} + xy = 10 \sin x; y = \frac{5}{x} + \frac{10}{x} \int_1^x \frac{\sin t}{t} dt$$

28.

30. In Example 7 we saw that $y = \phi_1(x) = \sqrt{25 - x^2}$ and $y = \phi_2(x) = -\sqrt{25 - x^2}$ are solutions of $\frac{dy}{dx} = -\frac{x}{y}$ on the interval $(-5, 5)$. Explain why the piecewise-defined function

$$y = \begin{cases} \sqrt{25 - x^2}, & -5 < x < 0 \\ -\sqrt{25 - x^2}, & 0 \leq x < 5 \end{cases}$$

is *not* a solution of the differential equation on the interval $(-5, 5)$.

In Problems 35 and 36 find values of m so that the function $y = x^m$ is a solution of the given differential equation.

35.
$$xy'' + 2y' = 0$$