

In Problems 1–10 find the interval and radius of convergence for the given power series.

3.

$$\sum_{n=1}^{\infty} \frac{2^n}{n} x^n$$

6.

$$\sum_{k=0}^{\infty} k!(x-1)^k$$

7.

$$\sum_{k=1}^{\infty} \frac{1}{k^2 + k} (3x-1)^k$$

In Problems 11–16 use an appropriate series in (2) to find the Maclaurin series of the given function. Write your answer in summation notation.

11.

$$e^{-\frac{x}{2}}$$

12.

$$xe^{3x}$$

13.

$$\frac{1}{2+x}$$

In Problems 23 and 24 use substitution to shift the summation index so that the general term of given power series involves x^k .

23.

$$\sum_{n=1}^{\infty} nc_n x^{n+2}$$

24.

$$\sum_{n=3}^{\infty} (2n-1)c_n x^{n-3}$$

In Problems 25–30 proceed as in Example 3 to rewrite the given expression using a single power series whose general term involves x^k

27.

$$\sum_{n=1}^{\infty} 2nc_n x^{n-1} + \sum_{n=0}^{\infty} 6c_n x^{n+1}$$

30.

$$\sum_{n=2}^{\infty} n(n-1)c_n x^n + 2 \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + 3 \sum_{n=1}^{\infty} nc_n x^n$$

In Problems 35–38 proceed as in Example 4 and find a power series solution $y = \sum_{n=0}^{\infty} c_n x^n$ of the given linear first-order differential equation.

35.

$$y' - 5y = 0$$

37.

$$y' = xy$$

38.

$$(1+x)y' + y = 0$$