In Problems 1–4 the given family of functions is the general solution of the differential equation on the indicated interval. Find a member of the family that is a solution of the initial-value problem.

1.

$$y = c_1e^x + c_2e^{-x}, (-\infty, \infty); y'' - y = 0, y(0) = 0, y'(0) = 1$$

2.

$$y = c_1e^{4x} + c_2e^{-x}, (-\infty, \infty); y'' - 3y' - 4y = 0, y(0) = 1, y'(0) = 2$$

5. Given that $y = c_1 + c_2x^2$ is a two-parameter family of solutions of $xy'' - y' = 0$ on the interval $(-\infty, \infty)$, show that constants c_1 and c_2 cannot be found so that a member of the family satisfies the initial conditions $y(0) = 0$, $y'(0) = 1$. Explain why this does not violate Theorem 4.1.1.

6. Find two members of the family of solutions in Problem 5 that satisfy the initial conditions $y(0) = 0$, $y'(0) = 0$.

In Problems 9 and 10 find an interval centered about $x = 0$ for which the given initial-value problem has a unique solution.

9.

$$(x - 2)y'' + 3y = x, y(0) = 0, y'(0) = 1$$