

Section 4.6

In Problems 1–18 solve each differential equation by variation of parameters.

3.

$$y'' + y = \sin(x)$$

4.

$$y'' + y = \sec(\theta) \tan(\theta)$$

11.

$$y'' + 3y' + 2y = \frac{1}{1 + e^x}$$

15.

$$y'' + 2y' + y = e^{-t} \ln(t)$$

In Problems 23–26 proceed as in Example 3 and solve each differential equation by variation of parameters.

24.

$$y'' - 4y = \frac{e^{2x}}{x}$$

In Problems 29–32 solve the given third-order differential equation by variation of parameters.

31.

$$y''' - 2y'' - y' + 2y = e^{4x}$$

Section 4.1

In Problems 23–30 verify that the given functions form a fundamental set of solutions of the differential equation on the indicated interval. Form the general solution.

29. (Use the Wronskian to show linear independence)

$$x^3 y''' + 6x^2 y'' + 4xy' - 4y = 0; \quad x, \quad x^{-2}, \quad x^2 \ln(x) \quad (0, \infty)$$

39. (a) Verify that $y_1 = x^3$ and $y_2 = |x|^3$ are linearly independent solutions of the differential equation $x^2 y'' - 4xy' + 6y = 0$ on the interval $(-\infty, \infty)$.

(b) For the functions y_1 and y_2 in part(a), show that $W(y_1, y_2) = 0$ for every real number x . Does this result violate Theorem 4.1.3? Explain.

- (c) Verify that $Y_1 = x^3$ and $Y_2 = x^2$ are linearly independent solutions of the differential equation in part (a) on the interval $(-\infty, \infty)$.
- (d) Besides the functions y_1, y_2, Y_1 , and Y_2 in parts (a) and (c), find a solution of the differential equation that satisfies $y(0) = 0, y'(0) = 0$.
- (e) By the superposition principle, Theorem 4.1.2, both linear combinations $y = c_1y_1 + c_2y_2$ and $Y = c_1Y_1 + c_2Y_2$ are solutions of the differential equation. Discuss whether one, both, or neither of the linear combinations is a general solution of the differential equation on the interval $(-\infty, \infty)$.