Math 486/522 - Homework 7

Fall 2024 **Len Washington III**

1. Consider the irreversible chemical reaction: $A \overset{k_1}{\to} B \overset{k_2}{\to} C$.

(a) Derive the system of differential equations for the concentrations $A = A(t)$, $B = B(t)$, and $C = C(t)$ with initial conditions $A(0) = A_0$, $B(0) = 0$, and $C(0) = 0$. Problem 1a answer here.

(b) Find a conservation law that replaces the need for the dC/dt equation. Problem 1b answer here.

(c) Solve the equations in **(a)** for A and B then use the conservation law to find C. Problem 1c answer here.

(d) If $k_1 = k_2 = k$, solve for A, B, and C. Problem 1d answer here.

(e) If $k_1 = k_2 = k$, find the maximum amount of B that is produced. Problem 1e answer here.

2. Consider the dimerization to two monomers example from class

$$A + A \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C$$

(a) Derive the system of differential equations for the concentrations $A = A(t)$ and $C = C(t)$ with initial conditions $A(0) = A_0$ and $C(0) = 0$. Problem 2a answer here.

(b) Find a conservation law that replaces the need for the dC/dt equation and derive the equation for $A(t)$. Problem 2b answer here.

(c) If $k_1 = k_{-1} = k$ and $A_0 = 1$, find the steady-state values for $A(t)$ and $C(t)$, i.e. limits as $t \to \infty$. Problem 2c answer here.

3. Consider the ODE:

$$\frac{dx}{dt} = f(x) = \frac{2x^2}{1 + x^4} - x, \quad x(0) \geq 0.$$

(a) Locate all the critical points ($x \geq 0$). Problem 3a answer here.

(b) Classify the stability of all the critical points. Problem 3b answer here.

(c) Draw the phase line labeling the critical points and indicate the direction of flow of the solution. Problem 3c answer here.

4. Consider the system

$$\frac{dx}{dt} = 1 - y,$$
$$\frac{dy}{dt} = x^2 = y^2.$$

(a) Determine all critical points and nullclines. Problem 4a answer here.

(b) Find the corresponding linear system near each critical point. Problem 4b answer here.

(c) Find the eigenvalues and eigenvectors of each linear system. What conclusions can you draw about the nonlinear system? Problem 4c answer here.

(d) Generate a phase plane portrait for the nonlinear system. Use Mathematica command *StreamPlot* with $-2 < x < 2$ and $-1 < y < 2$. Problem 4d answer here.

If your result in **(c)** consistent with the figure?

5. Consider the Michaelis-Menton reaction:

$$S + E \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} C \overset{k_2}{\to} P + E$$

where all the rate constants are positive and the concentrations S = substrate, E = enzyme, C = complex, and P = product.

(a) Using the system of equations for S and C derived in class, (i.e equations (3.51) and (3.52) on page 101 of Holmes' book), find the equations of the nullclines and locate all the critical points. Problem 5a answer here.

(b) Test the stability of the critical points. Find the linearized system, compute the solution, and explain your conclusion. Problem 5b answer here.