Math 486/522 - Homework 1 - Review of Matrices

Fall 2024   **Len Washington III**

Review solving linear equations using matrices in any linear algebra text or check out Paul Dawkins' linear algebra notes that are linked on the homework page.

For problems 1 and 2, convert the system of equations into a matrix problem $A\vec{x} = \vec{b}$ by defining A and $\vec{b}$. Define an augmented matrix $(A|b)$. Solve using Gaussian elimination to convert A into upper triangular or row-echelon form. Show all steps.

1. Consider the linear system:

$$x_1 - x_2 - x_3 = -3$$
$$2x_1 + 3x_2 + 5x_3 = 7$$
$$x_1 - 2x_2 + 3x_3 = -11$$

Problem 1 answer here.

2. Consider the linear system:

$$x_1 - x_2 - x_3 = 8$$
$$x_1 - x_2 + x_3 = 3$$
$$-x_1 + x_2 + x_3 = 4$$

Problem 2 answer here.

3. Consider the linear system:

$$kx_1 + 6x_3 = 51$$
$$12x_2 - 6x_3 = -6$$
$$x_1 - x_2 - x_3 = 0$$

(a) Write the system in matrix form $A\vec{x} = \vec{b}$ by defining A and $\vec{b}$.   Problem 3a answer here.

(b) For what values of k does the system have a unique solution and when does it not have an inverse?   Problem 3b answer here.

4. Consider $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & -1 & 4 & -1 \\ -2 & -4 & 7 & -1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}.$$

(a) Use Gaussian elimination with an augmented matrix $(A|b)$ to solve for $\vec{x}$.   Problem 4a answer here.

(b) Solve the associated homogeneous equation $A\vec{x} = 0$.   Problem 4b answer here.

(c) What is the definition of the kernel $K(A)$ or nullspace $N(A)$?   Problem 4c answer here.

5. Consider $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & -2 & 1 & 2 \\ 1 & -5 & 0 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 3 \\ 8 \\ 5 \end{bmatrix}.$$

    (a) Use Gaussian elimination with an augmented matrix $(A|b)$ to solve for $\vec{x}$.   Problem 5a answer here.

    (b) Solve the associated homogeneous equation $A\vec{x} = 0$.   Problem 5b answer here.

    (c) What is the definition of the kernel $K(A)$ or nullspace $N(A)$?   Problem 5c answer here.

6. Find the kernel or nullspace of:

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

Problem 6 answer here.

7. The velocity v at which flow in a pipe will switch from laminar to turbulent depends on the diameter d of the pipe as well as the density ρ and the dynamic viscosity μ of the fluid.

    (a) Find a dimensionally reduced form for v.   Problem 7a answer here.

    (b) Suppose the pipe has diameter $d = 100$ and for water (where $\rho = 1$ and $\mu = 10^{-2}$) it is found that $v = 0.25$. What is the v for the olive oil (where $\rho = 1$ and $\mu = 1$). The units here are in cgs.   Problem 7b answer here.