Math 486/522 - Homework 5

Fall 2024 **Len Washington III**

1. Strep throat, sinus infections, etc. usually require an antibiotic to help bring the infection under control. Zithromax (azithromycin) is often prescribed for these infections as a Z Pak containing 6 pills. Suppose we design a two compartment model for Zithromax with the first compartment being the GI tract $x(t)$ and the second compartment being the blood stream, $y(t)$ with the following system of ODE's:

$$x' = -k_1 x + I(t)$$
$$y' = k_1 x - k_2 y$$

where $I(t)$ is the input of the pills. The initial amount in each compartment equal to 0. The dosing regimen for a Z Pak is 2 pills the first day and then 1 pill for the following 4 days (5 day regimen). The time between the doses is 1 day and each pill delivers D units of the drug.

(a) Find the amount of the drug in each compartment from days 1 to 8. Model each pill dose by a Dirac delta function spiked at the appropriate time. Problem 1a answer here.

(b) If each pill is 400mg, $k_1 = 0.9$, and half-life of the drug in the blood is 2.3 days, graph $x(t)$ and $y(t)$ on the same axes from day 1 to day 8. Problem 1b answer here.

2. Consider a system of ODE's with initial conditions.

$$\frac{d\vec{x}}{dt} = A\vec{x} + \vec{b} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \vec{x} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \vec{x}(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \tag{1}$$

(a) Problem 2a answer here.

(b) Problem 2b answer here.

(c) Problem 2c answer here.

(d) Problem 2d answer here.

(e)
- **(i)** Problem 2e(i) answer here.
- **(ii)** Problem 2e(ii) answer here.
- **(iii)** Problem 2e(iii) answer here.

3. **(a)** Problem 3a answer here.

(b)
- **(i)** Problem 3b(i) answer here.
- **(ii)** Problem 3b(ii) answer here.
- **(iii)** Problem 3b(iii) answer here.

(c) Problem 3c answer here.