

Math 486/522 - Homework 4 - Pharmacokinetics**Fall 2024****Len Washington III**

Please review Dawkin's ODE notes, particularly Laplace Transforms, if needed. You **must** include your computer code for any computations and graphs.

1. Consider the system of ODEs

$$\begin{aligned}x' &= -2y + \delta(t - 1), \quad x(0) = 0 \\y' &= -3x = y, \quad y(0) = 0\end{aligned}$$

- (a) Solve the system using **Laplace Transforms**. Clearly show **all** steps. [Problem 1a answer here.](#)
 - (b) Graph the solutions $x(t)$ and $y(t)$ from $t = 0$ to $t = 7$ on the same axes and label each curve. [Problem 1b answer here.](#)
2. A pharmacokinetics model with one compartment (plasma) is used to model a drug that is administered with dose D at $t = 0$. A booster of dose $D/3$ is given at $t = 5$. The clearance rate from the compartment is $k = 1/4$. Define $x(t)$ be the amount of drug in the plasma at time t .
- (a) Derive a differential equation models for $x(t)$ with the proper initial condition. You must use Dirac delta functions in your model. [Problem 2a answer here.](#)
 - (b) Use Laplace transforms to solve the ODE. [Problem 2b answer here.](#)
 - (c) Plot $x(t)$ from $t = 0$ to $t = 10$ with $D = 4$. [Problem 2c answer here.](#)
3. Consider a one compartment model for the amount $x(t)$, a drug in the plasma with $x(0) = 0$. Assume the elimination rate is $k > 0$. The drug is administered using intravenous infusion (IV) for 2 hours but the amount of drug in the IV (drip rate) is reduced in time by a factor of e^{-t} . The drug input $I(t)$ is given by

$$I(t) = \begin{cases} Ae^{-t}, & 0 \leq t < 2, \\ 0, & t \geq 2. \end{cases}$$

Here time is in hours.

- (a) Set up a differential equation for $x(t)$ by converting $I(t)$ to a single line function using the Heaviside step function. [Problem 3a answer here.](#)
- (b) Use Laplace transform to solve the ODE assuming $x(0) = 0$. Show all details. [Problem 3b answer here.](#)
- (c) The total amount of drug that is removed or eliminated is equal to $k \int_0^\infty x(t) dt$. Verify that all the drug is removed. [Problem 3c answer here.](#)
- (d) Plot $x(t)$ for $0 < t < 8$ is the half-life of the drug in the blood is 5 hours, and $A = 3$. [Problem 3d answer here.](#)

4. A maintenance drug such as Lipitor is taken daily for a long period of time. It reduces cholesterol levels and lowers the risk of a heart attack. A single compartment pharmacokinetics model is used to track the concentration of the drug in the plasma. Suppose each dose has concentration C and it taken at fixed time intervals τ . The elimination rate for the compartment is k .
- (a) Derive difference equations for the sequence of local maxima, u_n , and local minima, v_n , assuming the concentration in the blood is 0 just before the first dose. [Problem 4a answer here.](#)
- (b) Solve the equations in (a). Compute the limits of the sequences as $n \rightarrow \infty$. These need to be inside the therapeutic window for the maintenance drug. [Problem 4b answer here.](#)
- (c) The data for the drug is:
- half-life is plasma in 2 hours.
 - $C = 150\text{mg}/15.8\text{L}$. Here the volume of distribution is 15.8L, which is not realistic.

Find the smallest does interval τ such that the drug remains in the therapeutic window $0.5\text{mg/L} < \text{conc.} < 10.3\text{mg/L}$. The value of τ must be an integer to be useful for the patient. [Problem 4c answer here.](#)

Listing 1 Source Code for the Graphs

[Source Code Here](#)
