

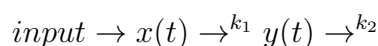
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Example 1 Multiple Dose Model

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Two Compartment Model



System of ODE's

$$\begin{aligned}\frac{dx}{dt} &= -k_1x + I(t), \quad x(0) = 0 \\ \frac{dy}{dt} &= k_1x - k_2y, \quad y(0) = 0\end{aligned}$$

0.2.1 Methods of Solution for Systems of Equations

- (1) Elimination
- (2) Laplace Transforms
- (3) Matrix methods
 - eigenvalue problem
- (4) Numerical methods
 - Mathematica, Matlab and Python

Example 2

$$\begin{aligned}\frac{dx}{dt} &= x_1 + 3x_2, \quad x_1(0) = 5 \\ \frac{dy}{dt} &= 5x_1 + 3x_2, \quad x_2(0) = 1\end{aligned}$$

$$\begin{aligned}sX_1(s) - x_1(0) &= \mathcal{L}\{x_1(t)\} & sX_2(s) - x_2(0) &= \mathcal{L}\{x_2(t)\} \\ sX_1(s) - x_1(0) &= \mathcal{L}\{x_1\} + \mathcal{L}\{3x_2\} & sX_2(s) - x_2(0) &= \mathcal{L}\{5x_1\} + \mathcal{L}\{3x_2\} \\ sX_1(s) - 5 &= X_1(s) + 3X_2(s) & sX_2(s) - 1 &= 5X_1(s) + 3X_2(s) \\ sX_1(s) - 5 - X_1(s) &= 3X_2(s) & sX_2(s) - 1 - 5X_1(s) &= 3X_2(s) \\ sX_1(s) - 5 - X_1(s) &= sX_2(s) - 1 - 5X_1(s) \\ sX_1(s) + 4X_1(s) - sX_2(s) &= 4 \\ (s + 4)X_1(s) - sX_2(s) &= 4\end{aligned}$$

$$\begin{aligned}\frac{dx}{dt} &= -k_1x + D\delta(t), \quad x(0) = 0 \\ \frac{dy}{dt} &= k_1x - k_2y, \quad y(0) = 0\end{aligned} \tag{1}$$

Use Laplace Transforms

$$X(s) = \mathcal{L}\{x(t)\} \quad Y(s) = \mathcal{L}\{y(t)\}$$

$\mathcal{L}\{(1)\}$:

$$\begin{aligned}sX(s) - x(0) &= \mathcal{L}\{-k_1x\} + \mathcal{L}\{D\delta(t)\} & sY(s) - y(0) &= \mathcal{L}\{k_1x\} - \mathcal{L}\{k_2y\} \\ sX(s) - 0 &= -k_1X(s) + D(1) & sY(s) - 0 &= k_1X(s) - k_2Y(s) \\ sX(s) + k_1X(s) &= D & sY(s) + k_2Y(s) &= k_1X(s) \\ (s + k_1)X(s) &= D & (s + k_2)Y(s) &= k_1X(s) \\ X(s) &= \frac{D}{s + k_1} & Y(s) &= \frac{k_1X(s)}{s + k_2} \\ & & Y(s) &= \frac{Dk_1}{(s + k_2)(s + k_1)} \\ X(s) &= \frac{D}{s + k_1} & Y(s) &= \frac{Dk_1}{(s + k_2)(s + k_1)}\end{aligned}$$

$$\begin{aligned}\frac{Dk_1}{(s + k_1)(s + k_2)} &= \frac{A}{s + k_2} + \frac{B}{s + k_1} \\ \frac{Dk_1}{(s + k_1)(s + k_2)} &= \frac{A(s + k_1)}{(s + k_2)(s + k_1)} + \frac{B(s + k_2)}{(s + k_1)(s + k_2)} \\ Dk_1 &= A(s + k_1) + B(s + k_2)\end{aligned}$$

$$\begin{array}{ll}
s = -k_1 & s = -k_2 \\
Dk_1 = A(-k_1 + k_1) + B(-k_1 + k_2) & Dk_1 = A(-k_2 + k_1) + B(-k_2 + k_2) \\
Dk_1 = A(0) + B(k_2 - k_1) & Dk_1 = A(k_1 - k_2) + B(0) \\
Dk_1 = B(k_2 - k_1) & Dk_1 = A(k_1 - k_2) \\
B = \frac{Dk_1}{k_2 - k_1} & A = \frac{Dk_1}{k_1 - k_2} \\
X(s) = \frac{D}{s + k_1} & Y(s) = \frac{Dk_1}{k_2 - k_1} + \frac{Dk_1}{k_1 - k_2} \\
x(t) = \mathcal{L}^{-1} \left\{ \frac{D}{s + k_1} \right\} & y(t) = \mathcal{L}^{-1} \left\{ \frac{Dk_1}{k_2 - k_1} \right\} + \mathcal{L}^{-1} \left\{ \frac{Dk_1}{k_1 - k_2} \right\}
\end{array}$$

Lead Uptake Model

- Excess of lead in body leads to lead (Pb) poisoning.
- Symptoms of Pb poisoning

Adults GI tract – vomiting pain

Children Serious – central nervous system disorders, anemia

- Pb in the environment:

Natural small

Human-made

- gasoline
- paint
- waterpipes
- make-up
- plastics

- How does Pb enter the body?
 - Inhalation
 - Eat or drinking
- How is Pb removed from the body?
 - urine
 - hair, nails, sweat