

Function

- 1.
2. Appropriate analytic calculations
- 3.
4. Data alone!
 - Machine Learning (ML)
 - Deep Learning (DL)

This class will concentrate on 3 methods to develop models developing.

Example 1

Let v be the velocity when the flow in a pipe switches from laminar flow (smooth) to turbulent. Let ρ = density of fluid. Let μ = dynamic viscosity

Find $v = f(d, \rho, \mu)$ function when no other info is given!! (some data?)

$$ma = m \frac{dv}{dt} = \sum \text{forces} \quad (1)$$

$$m \frac{dv}{dt} = -mg + av^2$$

$$\frac{dv}{dt} = -g + \frac{av^2}{m}$$

$$dv = -g \, dt + \frac{av^2}{m} \, dt$$

$$dv - \frac{av^2}{m} \, dt = -g \, dt$$

Method 1 - No law, no rule, “no data”

Tool: **Dimensional Analysis** – Understand the parameters and variables, simplify problems by reducing the number of variables and parameters and, maybe obtain a function!

Warm Up

$$2x_1 - x_2 = 6$$

Example 2

$$\begin{aligned}
 A &= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\
 \vec{x} &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\
 \vec{b} &= \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \\
 \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \\
 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right] &= (r_2 \leftarrow r_2 - r_3) \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right] \\
 &= (r_1 \leftarrow r_1 - r_3) \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right] \\
 &= (r_1 \leftarrow r_1 - r_2) \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]
 \end{aligned}$$

Example 3

$$\begin{aligned}
 \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 3 & -1 & -3 & -1 \\ 2 & 3 & 1 & 6 \end{array} \right] &= (r_2 \leftarrow r_2 - r_3) \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 1 & -4 & -4 & -7 \\ 2 & 3 & 1 & 6 \end{array} \right] \\
 &= (r_2 \leftarrow r_2 - r_1) \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -6 & -5 & -11 \\ 2 & 3 & 1 & 6 \end{array} \right] \\
 &= (r_3 \leftarrow r_3 - 2r_1) \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & -6 & -5 & -11 \\ 0 & -1 & -1 & -2 \end{array} \right] \\
 &= (r_2 \leftarrow r_2 - 7r_1) \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & -3 \\ 0 & -1 & -1 & -2 \end{array} \right] \\
 &= (r_3 \leftarrow r_3 + r_2) \left[\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & -5 \end{array} \right]
 \end{aligned}$$

