$$AUC = \int_{t_1}^{t_2} c(t)dt, \quad [AUC] = \frac{MT}{Volume}$$

$$\bar{c(t)} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2}$$

Half-life

The time to reduce by $\frac{1}{2}$,

Example 1

Find $\tau_{\frac{1}{2}}$

$$\frac{D}{2} = De^{-k\tau_{\frac{1}{2}}}$$
$$\frac{1}{2} = e^{-k\tau_{\frac{1}{2}}}$$
$$\ln\left(\frac{1}{2}\right) = -k\tau_{\frac{1}{2}}$$
$$k = \frac{\ln\left(\frac{1}{2}\right)}{\tau_{\frac{1}{2}}}$$

Example 2

One compartment (plasma) with inhection

$$\frac{dx}{dt} = -kx, \quad k > 0, \quad x(0) = D$$

Solution:

$$x(t) = De^{-kt}$$

This is exponential decay

$$c(t) = \frac{x(t)}{V_D} = \frac{D}{V_D}e^{-kt} \tag{1}$$

$$AUC = \int_0^\infty x(t)dt = \frac{D}{k}$$

Total amount eliminated $= kAUC = D$

Example 3

Patient A patient gets a *loading* does of 400mg of a drug.

$$\begin{aligned} V_D &= 30L \\ k &= 0.115hr^{-1}(\text{removal rate}) \\ \tau_{\frac{1}{2}} &= \frac{\ln 2}{} \end{aligned}$$

Assume a simple one compartment model as above with injection $D = 400$ mg at $t = 0$. What is the concentration after 4 hours.

$$\begin{aligned} c(t) &= \frac{400}{30}e^{-0.115(4)} \\ &= 8.4\text{mg/L} \end{aligned}$$

Alternate formulation

One compartment model injection at $t = 0$.

$$\frac{dx}{dt} = -kx + D\delta(t), \quad x(0) = 0 \tag{2}$$

0.5.1 Properties

$$\int_{-\infty}^{\infty} \delta(t)dt = 1 \tag{3}$$

$$\int_{-\infty}^{\infty} \delta(t)f(t)dt = f(0) \tag{4}$$

0.5.2 Dimensional Analysis

$$\begin{aligned} \frac{dx}{dt} &= -kx + D\delta(t) \\ \left[\frac{dx}{dt}\right] &= \frac{M}{T} \\ &= [kx] \\ [k] &= \frac{1}{T} \\ [D\delta(t)] &= \frac{M}{T} \\ [\delta(t)] &= \frac{1}{T} \end{aligned}$$

Check: Since $\int_{-\infty}^{\infty} \delta(t)dt = 1$

Method of Solution to Integral Transforms

$\mathscr{L}\left\{\ \right\}$ = Laplace Transform Operator
Solve (2) using Laplace Transforms:

$$\begin{aligned} X(s) &= \mathscr{L}\left\{x(t)\right\} \\ &= \int_0^\infty e^{-st}x(t)dt \end{aligned}$$

Rules: ...

$$\begin{aligned} \mathscr{L}\left\{\frac{dx}{dt}\right\} &= \mathscr{L}\left\{-kx + D\delta(t)\right\} \\ &= \mathscr{L}\left\{-kx\right\} + \mathscr{L}\left\{D\delta(t)\right\} \\ &= -k\mathscr{L}\left\{x(t)\right\} + D\mathscr{L}\left\{\delta(t)\right\} \\ sX(s) - x(0) &= -kX(s) + D \times 1 \\ sX(s) - 0 &= -kX(s) + D \\ sX(s) + kX(s) &= D \\ (s+k)X(s) &= D \\ X(s) &= \frac{D}{s+k} \\ \mathscr{L}^{-1}\left\{X(s)\right\} &= D\mathscr{L}^{-1}\left\{\frac{1}{s+k}\right\} \\ x(t) &= De^{-k} \end{aligned}$$

Multiple Dosing

Two injections Injection at $t = 0$ and a 2nd injection at $t = t_1$.

$$D\delta(t) + D\delta(t - t_1)$$

$$\frac{dx}{dt} = -kx + D\delta(t) + D\delta(t - t_1), \quad x(0) = 0 \tag{5}$$

Apply Laplace Transforms to (5).

$$\begin{aligned} X(s) &= \mathscr{L}\left\{x(t)\right\} \\ \mathscr{L}\left\{\delta(t)\right\} &= 1 \\ \mathscr{L}\left\{\delta(t - t_1)\right\} &= \int_0^\infty \dots \\ &= e^{-st_1} \end{aligned}$$

$$
\begin{aligned}
\mathscr{L}\left\{\frac{dx}{dt}\right\} &= \mathscr{L}-kx + D\mathscr{L}\left\{\delta(t)\right\} + D\mathscr{L}\left\{\delta(t-t_1)\right\} \\
sX(s) - x(0) &= -kX(s) + D(1) + De^{-st_1} \\
sX(s) - 0 + kX(s) &= D + De^{-st_1} \\
(s+k)X(s) &= D + De^{-st_1} \\
X(s) &= \frac{D}{s+k} + \frac{De^{-st_1}}{s+k}
\end{aligned}
$$