Contents

Parameter fitting or Calibration.
Crucial step in modeling (machine learning). Theory → practice.
Compartment models (linear) ___ to functions that are sums of exponentials

$$\begin{aligned} x(t) &= c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} \\ &= f(t, c_1, c_2, \lambda_1, \lambda_2) \end{aligned} \tag{1}$$

Given training data $\{t_i, x_i\}_{i=1}^N$
If there was only 1 exponential

$$x = c_1 e^{-\lambda_1 t} \tag{2}$$

Linear: $\ln x = -\lambda_1 t + \ln c_1$

1. Closest solution: $A^T A\vec{y} = A^T\vec{b}$
2. Minimize the sum of squares of the errors (SSE)

$$SSE = \sum_{i=1}^{N} \left[-\lambda t_i + b - \ln x_i\right]^2$$

Could try

$$\begin{aligned} \min SSE &= \min \sum_{i=1}^{N} \left(x(t_i) - x_i\right)^2 \\ &= F(c_1, \lambda_1, c_2, \lambda_2) \end{aligned}$$

Alternate method:

Exponential peeling

$$x(t) = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} \qquad \lambda_x > 0 \tag{3}$$

Assume $|\lambda_2| > |\lambda_1|$.
If $t \gg 1$: $x(t) \approx c_1 e^{-\lambda_1 t}$ i.e. $e^{-\lambda_2 t} \ldots$
$x(t)$ is asymptotic to $c_1 e^{\lambda_1 t}$

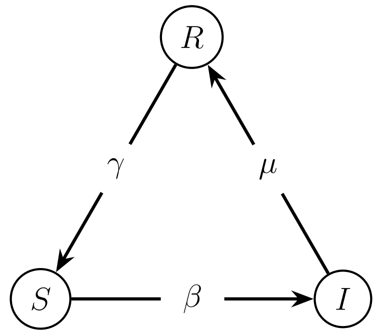

Figure 1: SIRS Model

Let S, I, R be fractions (already sealed) (i.e. $S + I + R = 1$)

$$\begin{aligned}\frac{dS}{dt} &= -\beta SI + \gamma R \\ \frac{dI}{dt} &= \beta SI - \mu I \\ \frac{dR}{dt} &= \mu I - \gamma R\end{aligned}$$

$$\begin{aligned}\frac{dS}{d\tau} &= -R_0 SI + c(1 - I - S) \\ &= F(S, 1) \\ \frac{dI}{d\tau} &= R_0 SI - I \\ &= G(S, 1)\end{aligned}$$

where $\tau = \mu t$, $R_0 = \frac{\beta}{\mu}$, $c = \frac{\gamma}{\mu}$

$$\begin{aligned}\frac{dS}{dt} &= dN - \beta \frac{I}{N} S - dS \\ \frac{dI}{dt} &= \beta \frac{I}{N} S - \mu I - dI \\ \frac{dR}{dt} &= \mu I - \gamma R - dR\end{aligned}$$

Next COVID Model – SEIR

E = Exposed – latent infection period

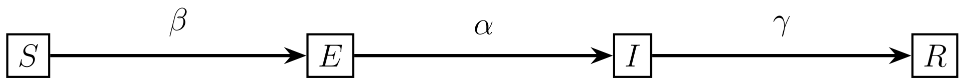

Figure 2: SEIR Model

$$\begin{aligned}
\frac{dS}{dt} &= -\beta IS, & S(0) &= S_0 \\
\frac{dE}{dt} &= \beta IS - \alpha E, & E(0) &= E_0 \\
\frac{dI}{dt} &= \alpha E - \gamma I, & I(0) &= I_0 \\
\frac{dR}{dt} &= \gamma I
\end{aligned}$$

Conservation Law: $S + E + I + R = 1$