

Math 486/522 - Homework 7

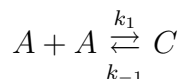
Fall 2024

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1. Consider the irreversible chemical reaction: $A \xrightarrow{k_1} B \xrightarrow{k_2} C$.

- (a) Derive the system of differential equations for the concentrations $A = A(t)$, $B = B(t)$, and $C = C(t)$ with initial conditions $A(0) = A_0$, $B(0) = 0$, and $C(0) = 0$.
[Problem 1a answer here.](#)
- (b) Find a conservation law that replaces the need for the dC/dt equation. [Problem 1b answer here.](#)
- (c) Solve the equations in (a) for A and B then use the conservation law to find C .
[Problem 1c answer here.](#)
- (d) If $k_1 = k_2 = k$, solve for A , B , and C . [Problem 1d answer here.](#)
- (e) If $k_1 = k_2 = k$, find the maximum amount of B that is produced. [Problem 1e answer here.](#)

2. Consider the dimerization to two monomers example from class



- (a) Derive the system of differential equations for the concentrations $A = A(t)$ and $C = C(t)$ with initial conditions $A(0) = A_0$ and $C(0) = 0$. [Problem 2a answer here.](#)
- (b) Find a conservation law that replaces the need for the dC/dt equation and derive the equation for $A(t)$. [Problem 2b answer here.](#)
- (c) If $k_1 = k_{-1} = k$ and $A_0 = 1$, find the steady-state values for $A(t)$ and $C(t)$, i.e. limits as $t \rightarrow \infty$. [Problem 2c answer here.](#)

3. Consider the ODE:

$$\frac{dx}{dt} = f(x) = \frac{2x^2}{1+x^4} - x, \quad x(0) \geq 0.$$

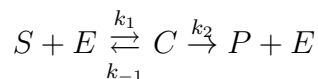
- (a) Locate all the critical points ($x \geq 0$). [Problem 3a answer here.](#)
- (b) Classify the stability of all the critical points. [Problem 3b answer here.](#)
- (c) Draw the phase line labeling the critical points and indicate the direction of flow of the solution. [Problem 3c answer here.](#)

4. Consider the system

$$\begin{aligned} \frac{dx}{dt} &= 1 - y, \\ \frac{dy}{dt} &= x^2 - y^2. \end{aligned}$$

- (a) Determine all critical points and nullclines. [Problem 4a answer here.](#)
- (b) Find the corresponding linear system near each critical point. [Problem 4b answer here.](#)
- (c) Find the eigenvalues and eigenvectors of each linear system. What conclusions can you draw about the nonlinear system? [Problem 4c answer here.](#)
- (d) Generate a phase plane portrait for the nonlinear system. Use Mathematica command *StreamPlot* with $-2 < x < 2$ and $-1 < y < 2$. Is your result in (c) consistent with the figure? [Problem 4d answer here.](#)

5. Consider the Michaelis-Menton reaction:



where all the rate constants are positive and the concentrations S = substrate, E = enzyme, C = complex, and P = product.

- (a) Using the system of equations for S and C derived in class, (i.e equations (3.51) and (3.52) on page 101 of Holmes' book), find the equations of the nullclines and locate all the critical points. [Problem 5a answer here.](#)
- (b) Test the stability of the critical points. Find the linearized system, compute the solution, and explain your conclusion. [Problem 5b answer here.](#)