Contents

Example 1 Multiple Dose Model

...

Two Compartment Model

$$input \to x(t) \to^{k_1} y(t) \to^{k_2}$$

System of ODE's

$$\frac{dx}{dt} = -k_1 x + I(t), \quad x(0) = 0$$
$$\frac{dy}{dt} = k_1 x - k_2 y, \quad y(0) = 0$$

0.2.1 Methods of Solution for Systems of Equations

(1) Elimination

(2) Laplace Transforms

(3) Matrix methods

- eigenvalue problem

(4) Numerical methods

- Mathematica, Matlab and Python

Example 2

$$\frac{dx}{dt} = x_1 + 3x_2, \quad x_1(0) = 5$$
$$\frac{dy}{dt} = 5x_1 + 3x_2, \quad x_2(0) = 1$$

$$\begin{aligned} sX_1(s) - x_1(0) &= \mathscr{L}\left\{x_1(t)\right\} \\ sX_1(s) - x_1(0) &= \mathscr{L}\left\{x_1\right\} + \mathscr{L}\left\{3x_2\right\} \\ sX_1(s) - 5 &= X_1(s) + 3X_2(s) \\ sX_1(s) - 5 - X_1(s) &= 3X_2(s) \end{aligned}$$

$$\begin{aligned} sX_2(s) - x_2(0) &= \mathscr{L}\left\{x_2(t)\right\} \\ sX_2(s) - x_2(0) &= \mathscr{L}\left\{5x_1\right\} + \mathscr{L}\left\{3x_2\right\} \\ sX_2(s) - 1 &= 5X_1(s) + 3X_2(s) \\ sX_2(s) - 1 - 5X_1(s) &= 3X_2(s) \end{aligned}$$

$$\begin{aligned} sX_1(s) - 5 - X_1(s) &= sX_2(s) - 1 - 5X_1(s) \\ sX_1(s) + 4X_1(s) - sX_2(s) &= 4 \\ (s+4)X_1(s) - sX_2(s) &= 4 \end{aligned}$$

$$\begin{aligned} \frac{dx}{dt} &= -k_1 x + D\delta(t), \quad x(0) = 0 \\ \frac{dy}{dt} &= k_1 x - k_2 y, \quad y(0) = 0 \end{aligned} \tag{1}$$

Use Laplace Transforms

$$X(s) = \mathscr{L}\left\{x(t)\right\} \qquad Y(s) = \mathscr{L}\left\{y(t)\right\}$$

$\mathscr{L}\left\{(1)\right\}$:

$$\begin{aligned} sX(s) - x(0) &= \mathscr{L}\left\{-k_1 x\right\} + \mathscr{L}\left\{D\delta(t)\right\} \\ sX(s) - 0 &= -k_1 X(s) + D(1) \\ sX(s) + k_1 X(s) &= D \\ (s+k_1)X(s) &= D \\ X(s) &= \frac{D}{s+k_1} \end{aligned}$$

$$\begin{aligned} sY(s) - y(0) &= \mathscr{L}\left\{k_1 x\right\} - \mathscr{L}\left\{k_2 y\right\} \\ sY(s) - 0 &= k_1 X(s) - k_2 Y(s) \\ sY(s) + k_2 Y(s) &= k_1 X(s) \\ (s+k_2)Y(s) &= k_1 X(s) \\ Y(s) &= \frac{k_1 X(s)}{s+k_2} \\ Y(s) &= \frac{Dk_1}{(s+k_2)(s+k_1)} \end{aligned}$$

$$X(s) = \frac{D}{s+k_1} \qquad Y(s) = \frac{Dk_1}{(s+k_2)(s+k_1)}$$

$$\begin{aligned} \frac{Dk_1}{(s+k_1)(s+k_2)} &= \frac{A}{s+k_2} + \frac{B}{s+k_1} \\ \frac{Dk_1}{(s+k_1)(s+k_2)} &= \frac{A(s+k_1)}{(s+k_2)(s+k_1)} + \frac{B(s+k_2)}{(s+k_1(s+k_2))} \\ Dk_1 &= A(s+k_1) + B(s+k_2) \end{aligned}$$

$$
\begin{aligned}
s &= -k_1 & s &= -k_2 \\
Dk_1 &= A(-k_1 + k_1) + B(-k_1 + k_2) & Dk_1 &= A(-k_2 + k_1) + B(-k_2 + k_2) \\
Dk_1 &= A(0) + B(k_2 - k_1) & Dk_1 &= A(k_1 - k_2) + B(0) \\
Dk_1 &= B(k_2 - k_1) & Dk_1 &= A(k_1 - k_2) \\
B &= \frac{Dk_1}{k_2 - k_1} & A &= \frac{Dk_1}{k_1 - k_2}
\end{aligned}
$$

$$
X(s) = \frac{D}{s + k_1} \qquad Y(s) = \frac{Dk_1}{k_2 - k_1} + \frac{Dk_1}{k_1 - k_2}
$$

$$
x(t) = \mathscr{L}^{-1}\left\{\frac{D}{s + k_1}\right\} \qquad y(t) = \mathscr{L}^{-1}\left\{\frac{Dk_1}{k_2 - k_1}\right\} + \mathscr{L}^{-1}\left\{\frac{Dk_1}{k_1 - k_2}\right\}
$$

Lead Uptake Model

- Excess of lead in body leads to lead (Pb) poisoning.
- Symptoms of Pb poisoning

  Adults GI tract – vomiting pain

  Children Serious – central nervous system disorders, anemia

- Pb in the environment:

  Natural small

  Human-made

    - gasoline
    - paint
    - waterpipes
    - make-up
    - plastics

- How does Pb enter the body?
  - Inhalation
  - Eat or drinking
- How is Pb removed from the body?
  - urine
  - hair, nails, sweat