Contents

SEIR Model with vital statistics

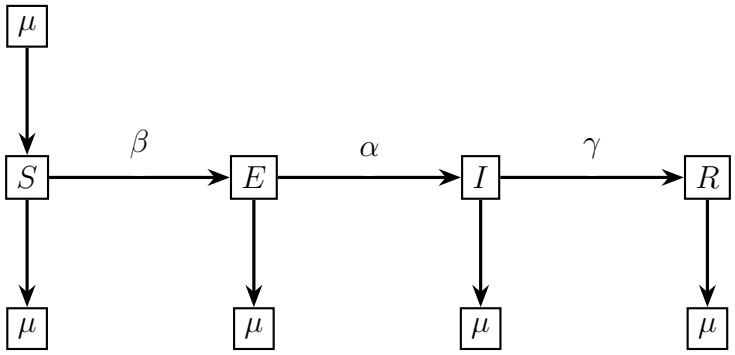

Figure 1: SEIR Model with Vital Statistics

$$
\begin{aligned}
S' &= \mu - \mu S - \beta IS \\
&= 0 \\
E' &= \beta IS - (\mu + \alpha)E \\
&= 0 \\
I &= \alpha E - (\mu + \gamma)I \\
&= 0
\end{aligned}
$$

Guess:

$$
\begin{aligned}
I^* &= 0 \\
&\Rightarrow S^* = 1 \\
&\Rightarrow E^* = 0
\end{aligned}
$$

Null clines:

$$
\begin{aligned}
S' = 0 \quad I &= \frac{\mu}{\beta}\left(\frac{1}{S} - 1\right) \\
E' = 0 \quad E &= \frac{\beta}{\mu + \alpha} SI \\
I' = 0 \quad E &= \frac{\mu + \gamma}{\alpha} I
\end{aligned}
$$

$$
\begin{aligned}
E' &= I' \\
\frac{\beta}{\mu+\alpha} SI &= \frac{\mu+\gamma}{\alpha} I \\
\frac{\beta}{\mu+\alpha} S &= \frac{\mu+\gamma}{\alpha} \\
S_2^* &= \frac{\mu+\alpha}{\beta}\frac{\mu+\gamma}{\alpha} \\
&= \frac{(\mu+\alpha)(\mu+\gamma)}{\beta\alpha} \\
I_2^* &= \frac{\mu\alpha}{(\mu+\gamma)(\mu+\alpha)} - \frac{\mu}{\beta} \\
E_2^* &= \frac{\mu+\gamma}{\alpha} I_2^* \\
&= \frac{\mu+\gamma}{\alpha}\left(\frac{\mu\alpha}{(\mu+\gamma)(\mu+\alpha)} - \frac{\mu}{\beta}\right) \\
&= \frac{\mu}{\mu+\alpha} - \frac{\mu(\mu+\gamma)}{\beta\alpha}
\end{aligned}
$$

When is Q_2^* in the domain?

$$E_2^* = \frac{\mu}{\mu+\gamma}\left[1 - \frac{1}{R_0}\right]$$

Obtain R_0 without math above Claim:

$$
\begin{aligned}
R_0 &= (\text{number of contacts per unit time})\,(\text{probability of transmission per coontact}) \\
&\quad \times (\text{duration of infection})\,(\text{probability of surviving}) \\
&= (\beta)\,(1)\left(\frac{1}{\gamma+\mu}\right)\left(\frac{\alpha}{\alpha+\mu}\right)
\end{aligned}
$$

How do vaccinations affect the models?

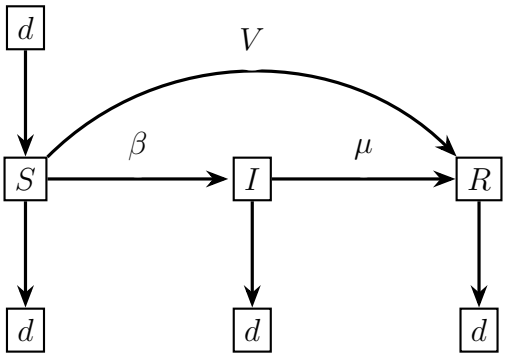

Figure 2: SEIR Model with Vital Statistics