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0.1 Next COVID model (SIRS) 3

$$x = F(\tau, R_0)$$

where $x = \frac{I}{N}$, $\tau = \mu t$, and $R_0 = N\frac{\beta}{\mu}$

$$\begin{aligned}
\frac{dx}{d\tau} &= R_0(1-x)x - x \\
&= R_0 x - x^2 - x \\
&= x(R_0 - 1 - x)
\end{aligned}$$

$$\begin{aligned}
\frac{ds}{dt} &= -\beta i s \\
\frac{di}{dt} &= \beta i s - \mu i
\end{aligned}$$

$$\begin{aligned}
\frac{di}{dt} &= \beta i s - \mu i \\
\frac{di}{dt} &= i(\beta s - \mu) \\
\frac{di}{i} &= (\beta s - \mu) dt \\
\int \frac{di}{i} &= \int (\beta s - \mu) dt \\
\ln |i| &= \beta s t - \mu t + C \\
i &= e^{\beta s t - \mu t + C} \\
&= e^{\beta s t - \mu t} e^{C} \\
&= A e^{(\beta s - \mu) t}
\end{aligned}$$

$$\begin{aligned}
\frac{ds}{d\tau} &= -R_0 i s \\
\frac{di}{d\tau} &= R_0 i s - i
\end{aligned}$$

Stability Analysis: Locate nullclines:

$$\begin{aligned} 0 &= \frac{ds}{d\tau} \\ &= -R_0 is \\ &= is \end{aligned}$$

$$s = 0 \qquad i = 0$$

$$\begin{aligned} 0 &= \frac{di}{d\tau} \\ &= R_0 is - i \\ &= i(R_0 s - 1) \end{aligned}$$

$$i = 0 \qquad \begin{aligned} R_0 s - 1 &= 0 \\ R_0 s &= 1 \\ s &= \frac{1}{R_0} \end{aligned}$$

Domain: $0 \leq s + i \leq 1$
If $R_0 > 1$, the line will be inside the domain, else ($R_0 < 1$), it is outside the domain.
If $R_0 > 1$, the infection increases when $s > \frac{1}{R_0}$ and dies out when $s < \frac{1}{R_0}$.
If $R_0 < 1$, then the rate of change of the infection dying out will be constant.
$s(t)$, $i(t)$ - set of parametric equations. We want the equation of the parametric curve.

$$\begin{aligned} \frac{ds}{d\tau} &= -R_0 is \\ \frac{di}{d\tau} &= R_0 is - i \\ \frac{di}{ds} &= \frac{di}{d\tau} \times \frac{d\tau}{ds} \\ &= \frac{R_0 is - i}{-R_0 is} \\ &= -\frac{R_0 s - 1}{R_0 s} \\ &= -1 + \frac{1}{R_0 s} \\ &= -1 + \frac{1}{R_0}\frac{1}{s} \\ \int \frac{di}{ds} &= \int \left(-1 + \frac{1}{R_0}\frac{1}{s}\right) \\ \int di &= \int \left(-1 + \frac{1}{R_0}\frac{1}{s}\right) ds \\ i &= -s + \frac{1}{R_0}\ln(s) + C \end{aligned}$$

$$\begin{aligned}\frac{ds}{di} &= \frac{ds}{d\tau} \times \frac{d\tau}{di} \\ &= \frac{-R_0 is}{R_0 is - i} \\ &= -\frac{R_0 s}{R_0 s - 1} \\ &= \frac{R_0 s}{1 - R_0 s}\end{aligned}$$

Next COVID model (SIRS)

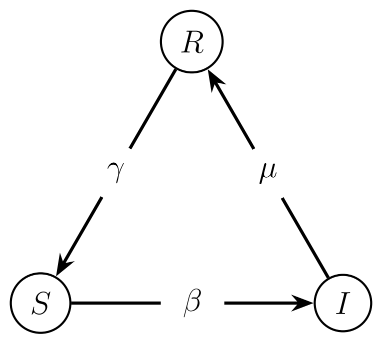

Figure 1: SIRS Model

Assume fraction: $s + i + r = 1$

$$\begin{aligned}\frac{ds}{dt} &= -\beta si + \gamma r, & s(0) &= s_0 < 1 \\ \frac{di}{dt} &= \beta si - \mu i, & i(0) &= i_0 < 1 \\ \frac{dR}{dt} &= \mu i - \gamma r, & r(0) &= 0\end{aligned}$$