Multiple Dosing. Two injections. Injection at $t = 0$ and $t = t_1$.

$$\begin{aligned}
D\delta(t) + D\delta(t - t_1)& \\
\frac{dx}{dt} &= -kx + D\delta(t) + D\delta(t - t_1), \quad x(0) = 0 \\
\mathscr{L}\left\{\frac{dx}{dt}\right\} &= \mathscr{L}\left\{-kx\right\} + \mathscr{L}\left\{D\delta(t)\right\} + \mathscr{L}\left\{D\delta(t - t_1)\right\} \\
sX(s) &= -k\mathscr{L}\left\{x\right\} + D\mathscr{L}\left\{\delta(t)\right\} + D\mathscr{L}\left\{\delta(t - t_1)\right\} \\
sX(s) &= -kX(s) + D(1) + De^{st_1} \\
sX(s) + kX(s) &= D + De^{st_1} \\
(s + k)X(s) &= D + De^{st_1} \\
X(s) &= \frac{D}{s + k} + D\frac{e^{st_1}}{s + k} \\
\mathscr{L}^{-1}\left\{X(s)\right\} &= \mathscr{L}^{-1}\left\{\frac{D}{s + k}\right\} + \mathscr{L}^{-1}\left\{D\frac{e^{st_1}}{s + k}\right\} \\
x(t) &= D\mathscr{L}^{-1}\left\{\frac{1}{s + k}\right\} + D\mathscr{L}^{-1}\left\{\frac{e^{st_1}}{s + k}\right\} \\
x(t) &= De^{-kt} + De^{-kt}\mathscr{U}(t + t_1) \\
&= De^{-kt}\left(1 + \mathscr{U}(t + t_1)\right) \\
&= \begin{cases} De^{-kt} & 0 \le t < t_1 \\ De^{-kt} + De^{-k(t - t_1)} & t \ge t_1 \end{cases}
\end{aligned}$$

Intermittent Infusion (IV)

Over finite time intended

$$I(t) = \begin{cases} D & 0 \le t < 1 \\ 0 & t > 1 \end{cases}$$

$$\frac{dx}{dt} = -kx + I(t), \quad x(0) = 0, \quad c = \frac{x}{volume}$$

How to model $I(t)$? Want a one line formula to use Laplace Transform Table

$$I(t) = D[\mathscr{U}(t) - \mathscr{U}(t - 1)] \tag{1}$$

$$\begin{aligned}
\mathscr{L}\left\{\frac{dx}{dt}\right\} &= -kX(s) + D\left[\mathscr{L}\left\{\mathscr{U}(t)\right\} - \mathscr{L}\left\{\mathscr{U}(t-1)\right\}\right] \\
sX(s) - x(0) &= -kX(s) + D\left[\frac{1}{s} - \frac{e^{-s}}{s}\right] \\
sX(s) - 0 + kX(s) &= \frac{D}{s} - D\frac{e^{-s}}{s} \\
(s+k)X(s) &= \frac{D}{s} - D\frac{e^{-s}}{s} \\
X(s) &= \frac{D}{s(s+k)} - D\frac{e^{-s}}{s(s+k)} \\
\frac{D}{s(s+k)} &= \frac{A}{s} + \frac{B}{s+k} \\
\frac{D}{s(s+k)} &= \frac{A(s+k)}{s(s+k)} + \frac{Bs}{s(s+k)} \\
D &= A(s+k) + Bs
\end{aligned}$$

Must be true for any s

$$\begin{aligned}
s &= 0 \\
D &= A(0+k) + B(0) \\
D &= Ak \\
A &= \frac{D}{k}
\end{aligned}$$

$$\begin{aligned}
s &= -k \\
D &= A(-k+k) + B(-k) \\
D &= A(0) - Bk \\
D &= -Bk \\
B &= -\frac{D}{k}
\end{aligned}$$

$$\begin{aligned}
\frac{D}{s(s+k)} &= \frac{A}{s} + \frac{B}{s+k} \\
\frac{D}{s(s+k)} &= \frac{D}{ks} - \frac{Dk^{-1}}{s+k}
\end{aligned}$$

$$\begin{aligned}
X(s) &= \frac{D}{s(s+k)} - D\frac{e^{-s}}{s(s+k)} \\
X(s) &= \frac{Dk^{-1}}{s} - \frac{Dk^{-1}}{s+k} - De^{-s}\left[\frac{Dk^{-1}}{s} - \frac{Dk^{-1}}{s+k}\right] \\
\mathscr{L}^{-1}\left\{X(s)\right\} &= \mathscr{L}^{-1}\left\{\frac{Dk^{-1}}{s}\right\} - \mathscr{L}^{-1}\left\{\frac{Dk^{-1}}{s+k}\right\} - \mathscr{L}^{-1}\left\{De^{-s}\frac{Dk^{-1}}{s}\right\} - \mathscr{L}^{-1}\left\{De^{-s}\frac{Dk^{-1}}{s+k}\right\} \\
x(t) &= \frac{D}{k}\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{D}{k}\mathscr{L}^{-1}\left\{\frac{1}{s+k}\right\} - \frac{D^2}{k}\mathscr{L}^{-1}\left\{\frac{e^{-s}}{s}\right\} - \frac{D^2}{k}\mathscr{L}^{-1}\left\{\frac{e^{-s}}{s+k}\right\} \\
x(t) &= \frac{D}{k}(1) - \frac{D}{k}(e^{-kt}) - \frac{D^2}{k}(1\mathscr{U}(t-1)) - \frac{D^2}{k}\left(e^{-kt}\mathscr{U}(t-1)\right) \\
x(t) &= \frac{D}{k}\left(1 - e^{-kt} - D\mathscr{U}(t-1) - De^{-kt}\mathscr{U}(t-1)\right)
\end{aligned}$$