

Contents

0.1	Example 1	1
0.2	M-M reaction	2
0.3	Briggs-Haldane (1928)	3
0.4	Quasi-Steady State Assumption	3

Example 1

$$\frac{dx}{dt} = x^3 - x$$

$$\frac{dy}{dt} = -2y$$

$$\frac{dx}{dt} = x^3 - x$$

$$\frac{dx}{x^3 - x} = dt$$

$$\frac{dy}{dt} = -2y$$

$$\frac{dy}{-2y} = dt$$

$$\int \frac{dy}{-2y} = \int dt$$

$$-\frac{1}{2} \ln |y| = t + C_1$$

$$\ln |y| = -2t + C_2$$

$$y = e^{-2t} + e^{C_2}$$

$$= e^{-2t} + C$$

$$\frac{dx}{dt} = 0$$

$$x^3 - x = 0$$

$$x(x^2 - 1) = 0$$

$$x^2 - 1 = 0 \quad x = 0$$

$$x^2 = 1$$

$$x = \pm 1 \quad x = 0$$

$$\frac{dy}{dt} = 0$$

$$-2y = 0$$

$$y = 0$$

Critical points: $(-1, 0)$, $(0, 0)$, $(1, 0)$. The behavior around each crit point on the x -axis is

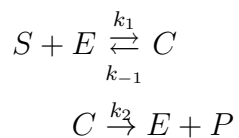
positive, negative, positive, (checking the derivative of $\frac{dx}{dt}$.)

$$\begin{aligned}\frac{d\vec{u}}{dt} &= J_{ss}\vec{u} \\ J_{ss} &= \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \\ &= \begin{bmatrix} 3x^2 - 1 & 0 \\ 0 & -2 \end{bmatrix} \\ \frac{d\vec{u}}{dt} &= \begin{bmatrix} 3x^2 - 1 & 0 \\ 0 & -2 \end{bmatrix} \vec{u} \\ J(0,0) &= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \\ \lambda_1 &= -2 & \lambda_2 &= -1 \\ \lambda_1 &< 0 & \lambda_2 &< 0 \\ &\Rightarrow \textit{stable} & &\Rightarrow \textit{stable} \\ J(-1,0) &= \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\end{aligned}$$

M-M reaction

Overall reaction: $S \rightarrow P$.

Reaction steps:



Kinetic equations:

$$\begin{aligned}\frac{ds}{dt} &= -k_1SE + k_{-1}C, & S(0) &= S_0 \\ \frac{dE}{dt} &= -k_1SE + k_{-1}C + k_2C, & E(0) &= E_0 \\ \frac{dC}{dt} &= k_1SE - k_{-1}C - k_2C, & C(0) &= 0 \\ \frac{dP}{dt} &= k_2C, & P(0) &= 0\end{aligned}$$

Two conservation laws:

$$\begin{aligned}\frac{d}{dt}(S + C + P) &= 0 \\ S + C + P &= S_0\end{aligned}$$

$$\frac{ds}{dt} = -k_1 E_0 S + (k_1)$$

Stability Analysis – HW (0,0) is the only critical point. Conservation Law $\Rightarrow P = S_0 \Rightarrow$ completion ($S + C + P = S_0$).

New idea – approximation:

Briggs-Haldane (1928)

Assumption: substrate binds to “all” enzyme *very quickly* and stays that way!

Key: $E_0 \ll S_0$

$$\frac{dC}{dt} \approx 0 \text{ (slow for most time)}$$

i.e. set $\frac{dC}{dt} = 0$:

$$\begin{aligned} \frac{dS}{dt} &= -k_1 E_0 S + (k_{-1} + k_2)C \\ 0 &= k_1 E_0 S - (k_2 + k_{-1} + k_1)C \end{aligned}$$

Solve for C :

$$C = \frac{k_1 E_0 S}{k_2 + k_{-1} + k_1}$$

$$\frac{ds}{dt} = -k_1 E_0 S + (k_1 + k_1)S \left[\frac{k_1 E_0 S}{k_2 + k_{-1} + k_1} \right]$$

Quasi-Steady State Assumption

Velocity of reaction:

$$\begin{aligned} \frac{dS}{dt} &= -\frac{v_{max}S}{k_m + S} \\ &= -Velocity \end{aligned} \tag{1}$$

Conservation Law: $\frac{d}{dt}(S + C + P) = 0$

if $\frac{dc}{dt} \approx 0 \Rightarrow \frac{dP}{dt} \approx -\frac{dS}{dt} = \frac{V_{max}s}{k_m+s}$

Velocity of MM reaction: $Vel = \frac{dP}{dt}$

Can we solve:

$$\frac{dS}{dt} = -\frac{V_{max}S}{k_m + S} \quad S(0) = S_0$$

First order, nonlinear, separable?

$$\begin{aligned} \frac{k_m + S}{S} ds &= -V_{max} dt \\ \int \frac{k_m + S}{S} ds &= -\int V_{max} dt \\ \int \left(k_m \frac{1}{S} + 1 \right) ds &= -\int V_{max} dt \\ k_m \ln |S| + S &= -V_{max} t + C \end{aligned}$$

Use $S(0) = S_0$

$$k_m \ln |S(0)| + S(0) = -V_{max}(0) + C$$

$$k_m \ln |S_0| + S_0 = C$$

$$k_m \ln |S| + S = -V_{max}t + k_m \ln |S_0| + S_0$$

$$k_m \ln |S| - k_m \ln |S_0| + S - S_0 = -V_{max}t$$

$$k_m \ln \left| \frac{S}{S_0} \right| + S - S_0 = -V_{max}t$$