

Math 486/522 - Homework 1 - Review of Matrices**Fall 2024****Len Washington III**

Review solving linear equations using matrices in any linear algebra text or check out Paul Dawkins' linear algebra notes that are linked on the homework page.

For problems 1 and 2, convert the system of equations into a matrix problem $A\vec{x} = \vec{b}$ by defining A and \vec{b} . Define an augmented matrix $(A|\vec{b})$. Solve using Gaussian elimination to convert A into upper triangular or row-echelon form. Show all steps.

1. Consider the linear system:

$$\begin{aligned}x_1 - x_2 - x_3 &= -3 \\2x_1 + 3x_2 + 5x_3 &= 7 \\x_1 - 2x_2 + 3x_3 &= -11\end{aligned}$$

[Problem 1 answer here.](#)

2. Consider the linear system:

$$\begin{aligned}x_1 - x_2 - x_3 &= 8 \\x_1 - x_2 + x_3 &= 3 \\-x_1 + x_2 + x_3 &= 4\end{aligned}$$

[Problem 2 answer here.](#)

3. Consider the linear system:

$$\begin{aligned}kx_1 + 6x_3 &= 51 \\12x_2 - 6x_3 &= -6 \\x_1 - x_2 - x_3 &= 0\end{aligned}$$

- (a) Write the system in matrix form $A\vec{x} = \vec{b}$ by defining A and \vec{b} . [Problem 3a answer here.](#)
- (b) For what values of k does the system have a unique solution and when does it not have an inverse? [Problem 3b answer here.](#)

4. Consider $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & -1 & 4 & -1 \\ -2 & -4 & 7 & -1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}.$$

- (a) Use Gaussian elimination with an augmented matrix $(A|\vec{b})$ to solve for \vec{x} . [Problem 4a answer here.](#)
- (b) Solve the associated homogeneous equation $A\vec{x} = 0$. [Problem 4b answer here.](#)
- (c) What is the dimension of the kernel $K(A)$ or nullspace $N(A)$? [Problem 4c answer here.](#)

5. Consider $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & -2 & 1 & 2 \\ 1 & -5 & 0 & 1 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 3 \\ 8 \\ 5 \end{bmatrix}.$$

- (a) Use Gaussian elimination with an augmented matrix $(A|b)$ to solve for \vec{x} . [Problem 5a answer here.](#)
- (b) Solve the associated homogeneous equation $A\vec{x} = 0$. [Problem 5b answer here.](#)
- (c) What is the dimension of the kernel $K(A)$ or nullspace $N(A)$? [Problem 5c answer here.](#)

6. Find the kernel or nullspace of:

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

[Problem 6 answer here.](#)

7. The velocity v at which flow in a pipe will switch from laminar to turbulent depends on the diameter d of the pipe as well as the density ρ and the dynamic viscosity μ of the fluid.

- (a) Find a dimensionally reduced form for v . [Problem 7a answer here.](#)
- (b) Suppose the pipe has diameter $d = 100$ and for water (where $\rho = 1$ and $\mu = 10^{-2}$) it is found that $v = 0.25$. What is the v for the olive oil (where $\rho = 1$ and $\mu = 1$). The units here are in cgs. [Problem 7b answer here.](#)