Math 486/522 - Homework 2 - Dimensional Analysis

Fall 2024 **Len Washington III**

1. The drag D_F on an object with characteristic length L_p (such as the radius in the class example), moving with velocity V in air with density ρ and dynamic viscosity μ is:

$$\frac{D_F}{\rho V^2 L_p^2} = f(Re), \quad Re = \frac{\rho V L_p}{\mu}$$

where Re is the Reynold's number. See the class notes.
An auto company is designing a new sports car with length L_p and wants to know the aerodynamic drag at speed 50 mph and air temperature 25°C. The engineers will build of 1/5 scale model for a wind tunnel test. The wind tunnel temperature is 5°C. How fast must the wind tunnel speed be to achieve similarity between the scale model and the prototype? Here is some data:

At 25°C	$\rho = 1.184$ Kg/m^3	$\mu = 1.849 \times 10^{-5}$
At 5°C	$\rho = 1.269$ Kg/m^3	$\mu = 1.754 \times 10^{-5}$

Problem 1 answer here.

2. Consider the flow of water in a circular pipe of length l and radius r. The pressure difference p between the ends of the pipe influence the velocity v of the flow. In addition, the flow depends on the dynamic viscosity μ of water and the density of water ρ. Find the functional dependence of v on the above quantities using dimensional analysis. Clearly identify all the dimensionless quantities as Π_j. Problem 2 answer here.

3. An object of mass m is launched upward with initial velocity V. The air resistance force is proportional to either the velocity ($b = 1$) or the square of the velocity ($b = 2$) in the equation below. Let $h(t)$ be the height of the object which satisfies the initial value problem

$$mh''(t) = -mg - a[h'(t)]^b, \quad h(0) = 0, \quad h'(0) = V.$$

   (a) Find the dimensions of the constant a for both cases $b = 1$ and $b = 2$. Problem 3a answer here.

   (b) For case $b = 2$, use dimensional analysis to find a formula for the height h of the ball in terms of t and all the parameters in the problem. Identify all the dimensionless qualities. Write your result with one dimensionless time $\tau = tg/V$ and a dimensionless height - the class notes might be useful. Problem 3b answer here.

4. A rocket is launched from the surface of the Earth with an initial velocity V and its engine shuts off immediately. Let $x(t)$ be its distance from the surface and R is the radius

of the Earth. Holmes' equation (1.1) is a simplified form of Newton's Universal Law of Gravitation

$$\frac{d^2x}{dt^2} = -\frac{gR^2}{(R+x)^2} \tag{1}$$

where the mass m of the rocket has been cancelled.

(a) Show (1) is dimensionally correct. Problem 4a answer here.

(b) Assuming the $x = f(t, R, g, V)$, use dimensional reduction to find a formula for x. Problem 4b answer here.

(c) Do you see any issues with your formula? Problem 4c answer here.