

## Math 486/522 - Homework 7

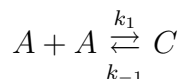
Fall 2024

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1. Consider the irreversible chemical reaction:  $A \xrightarrow{k_1} B \xrightarrow{k_2} C$ .

- (a) Derive the system of differential equations for the concentrations  $A = A(t)$ ,  $B = B(t)$ , and  $C = C(t)$  with initial conditions  $A(0) = A_0$ ,  $B(0) = 0$ , and  $C(0) = 0$ . [Problem 1a answer here.](#)
- (b) Find a conservation law that replaces the need for the  $dC/dt$  equation. [Problem 1b answer here.](#)
- (c) Solve the equations in (a) for  $A$  and  $B$  then use the conservation law to find  $C$ . [Problem 1c answer here.](#)
- (d) If  $k_1 = k_2 = k$ , solve for  $A$ ,  $B$ , and  $C$ . [Problem 1d answer here.](#)
- (e) If  $k_1 = k_2 = k$ , find the maximum amount of  $B$  that is produced. [Problem 1e answer here.](#)

2. Consider the dimerization to two monomers example from class



- (a) Derive the system of differential equations for the concentrations  $A = A(t)$  and  $C = C(t)$  with initial conditions  $A(0) = A_0$  and  $C(0) = 0$ . [Problem 2a answer here.](#)
- (b) Find a conservation law that replaces the need for the  $dC/dt$  equation and derive the equation for  $A(t)$ . [Problem 2b answer here.](#)
- (c) If  $k_1 = k_{-1} = k$  and  $A_0 = 1$ , find the steady-state values for  $A(t)$  and  $C(t)$ , i.e. limits as  $t \rightarrow \infty$ . [Problem 2c answer here.](#)

3. Consider the ODE:

$$\frac{dx}{dt} = f(x) = \frac{2x^2}{1+x^4} - x, \quad x(0) \geq 0.$$

- (a) Locate all the critical points ( $x \geq 0$ ). [Problem 3a answer here.](#)
- (b) Classify the stability of all the critical points. [Problem 3b answer here.](#)
- (c) Draw the phase line labeling the critical points and indicate the direction of flow of the solution. [Problem 3c answer here.](#)

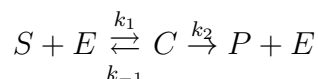
4. Consider the system

$$\begin{aligned} \frac{dx}{dt} &= 1 - y, \\ \frac{dy}{dt} &= x^2 - y^2. \end{aligned}$$

- (a) Determine all critical points and nullclines. [Problem 4a answer here.](#)
- (b) Find the corresponding linear system near each critical point. [Problem 4b answer here.](#)
- (c) Find the eigenvalues and eigenvectors of each linear system. What conclusions can you draw about the nonlinear system? [Problem 4c answer here.](#)
- (d) Generate a phase plane portrait for the nonlinear system. Use Mathematica command *StreamPlot* with  $-2 < x < 2$  and  $-1 < y < 2$ . [Problem 4d answer here.](#)

If your result in (c) consistent with the figure?

5. Consider the Michaelis-Menton reaction:



where all the rate constants are positive and the concentrations  $S$  = substrate,  $E$  = enzyme,  $C$  = complex, and  $P$  = product.

- (a) Using the system of equations for  $S$  and  $C$  derived in class, (i.e equations (3.51) and (3.52) on page 101 of Holmes' book), find the equations of the nullclines and locate all the critical points. [Problem 5a answer here.](#)
- (b) Test the stability of the critical points. Find the linearized system, compute the solution, and explain your conclusion. [Problem 5b answer here.](#)