Math 486/522 - Homework 8

Fall 2024 **Len Washington III**

The $LambertW(t)$ function arises in the pharmacokinetics problem as well as other applications in engineering and life sciences. $LambertW(t)$ is defined in Mathematica as ProductLog(t). MatLab also has the $LambertW(t)$ built-in. Be sure to use the principal branch (branch 1 in class) of $xe^x = t$ where $x = LambertW(t)$ for problems 1 and 2 below.

1. **(a)** Plot the function xe^x and explain why it is not 1 to 1. The principal branch is defined on $x > -1$. Problem 1a answer here.

(b) Plot $LambertW(t)$ for $-\frac{1}{e} < t < 1$ and compare with the graph in **(a)**. Problem 1b answer here.

2. Nonlinear Pharmacokinetics: In class we discussed a single compartment, nonlinear pharmacokinetics model where the clearance is based on the Michaelis-Menton model:

$$\frac{dc}{dt} = -\frac{V_m c}{K_m + c}, \quad c(0^+) = c_0 \tag{1}$$

Here $c(t)$ and c_0 are concentrations and V_m and K_m are the Michaelis-Menton constants. Following what was done in class:

(a) Graph the explicit solution of (1) that was derived in class (Day 19 page 5) for t from 0 to 10 hours. Use the parameter values: $c_0 = 4$, $K_m = 1.2$, and $V_m = 1$. Problem 2a answer here.

(b) Derive the $c(t)$ using a linear model with first order clearance and clearance constant k. Problem 2b answer here.

(c) Graph the solution of the linear model with $c_0 = 4$, $k = \frac{1}{1.2}$ on the same axes with the graph in **(a)**. How do they compare? Problem 2c answer here.

3. Review the notes for the nonlinear model with multiple doses of size c_0, see Day 19 page 5a.

(a) Using the notes, derive a difference equation relating c_{n+1} to c_n. Remember c_n is the value just after the nth dose. Each dose is size c_0. Problem 3a answer here.

(b) Find an explicit formula for the rest or critical value, i.e. $\lim_{n\to\infty} c_n$ using the parameters given in class. This is the maximum concentration, which must be below the minimum toxic level. Problem 3b answer here.

(c) Graph the max and min sequences for $c_0 = 4$, $K_m = 1.2$, $V_m = 1$ and $T = 6$. Does c_n approach the critical value in **(b)**. Problem 3c answer here.