Chapter 9

Hypothesis Testing

LO 9.1: Define the null hypothesis and the alternative hypothesis.

LO 9.2: Distinguish between Type I and Type II errors.

LO 9.3: Explain the steps of a hypothesis test using the p-value approach.

LO 9.4: Explain the steps of a hypothesis test using the critical value approach.

LO 9.5: Differentiate between the test statistics for the population mean.

LO 9.6: Specify the test statistic for the population proportion.

9.1 Point Estimators and Their Properties

LO 9.1 **Define the null hypothesis and the alternative hypothesis.**

- Hypothesis tests resolve conflicts between two competing opinions (hypotheses).
- In a hypothesis test, define

  H_0 the null hypothesis, the presumed default state of nature or status quo.

  H_A the alternative hypothesis, a contradiction of the default state of nature or status quo.
- In statistics, we use sample information to make inferences regarding the unknown population parameters of interest.
- We conduct hypothesis tests to determine if sample evidence contradicts H_0.
- On the basis of sample information, we either
  - "Reject the null hypothesis"
    * Sample evidence <u>is</u> inconsistent with H_0.

  - "Do not reject the null hypothesis"
    * Sample evidence is not inconsistent with H_0.
    * We do not have enough evidence to "accept" H_0.

9.1.1 Defining the Null Hypothesis and Alternative Hypothesis

General guidelines:

- Null hypothesis, H_0, states the status quo.
- Alternative hypothesis, H_A, states whatever we wish to establish (i.e., contests the status quo)
- Note that H_0 always contains the "equality".

9.1.2 One-Tailed vs Two-Tailed Hypothesis Tests

Two-Tailed Test

- Reject H_0 on either side of the hypothesized value of the population parameter.
- For example:
  - H_0: $\mu = \mu_0$ versus H_A: $\mu \neq \mu_0$
  - H_0: $p = p_0$ versus H_A: $p \neq p_0$
- The $\neq$ symbol in H_A indicates that both tail areas of the distribution will be used to make the decision regarding the rejection of H_0.

One-Tailed Test

- Reject H_0 only on one side of the hypothesized value of the population parameter.
- For example:
  - H_0: $\mu \leq \mu_0$ versus H_A: $\mu > \mu_0$ (right-tail test)
  - H_0: $\mu \geq \mu_0$ versus H_A: $\mu < \mu_0$ (left-tail test)
- Note that the inequality in H_A determines which tail area will be used to make the decision regarding the rejection of H_0.

9.1.3 Three Steps to Formulate Hypotheses

1. Identify the relevant population parameter of interest (e.g., μ or p).
2. Determine whether it is a one- or a two-tailed test.
3. Include some form of the equality sign in H_0 and use H_A to establish a claim.

H_0	H_A	Test Type
$=$	$\neq$	Two-tail
$\geq$	$<$	One-tail, Left-tail
$\leq$	$>$	One-tail, Right-tail

9.1.4 Type I and Type II Errors

LO 9.2 **Distinguish between Type I and Type II errors.**

- Type I Error – Committed when we reject H_0 when H_0 is actually true.
  - Occurs with probability α. α is chosen a priori.
- Type II Error – Committed when we do not reject H_0 when H_0 is actually false.
  - Occurs with probability β. Power of the test $= 1 - \beta$
- For a given sample size n, a decrease in α will increase β and vice versa.
- Both α and β decreases as n increases.

Decision	**Null hypothesis is true**	**Null hypothesis is false**
Reject the null hypothesis	Type I error	Correct decision
Do not reject the null hypothesis	Correct decision	Type I error

9.2 Hypothesis Test of the Population Mean When σ Is Known

LO 9.3 **Explain the steps of a hypothesis test using the p-value approach.**

- Hypothesis testing enables us to determine whether the sample evidence is inconsistent with what is hypothesized under the null hypothesis (H_0).
- Basic principle: First assume that H_0 is true and then determine if sample evidence contradicts this assumption.
- Two approaches to hypothesis testing:
  - The p-value approach.
  - The critical value approach.

9.2.1 The p-value Approach

- The value of the test statistic for the hypothesis test of the population mean μ when the population standard deviation σ is known is computed as

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \tag{9.1}$$

  where μ_0 is the hypothesized mean value.

- p-value: the likelihood of obtaining a sample mean that is at least as extreme as the one derived from the given sample, under the assumption that the null hypothesis is true.

- Under the assumption that $\mu = \mu_0$, the p-value is the likelihood of observing a sample mean that is at least as extreme as the one derived from the given sample.

- The calculation of the p-value depends on the ______.

Alternative hypothesis	***p*-value**
$H_A : \mu > \mu_0$	Right-tail probability: $P(Z \geq z)$
$H_A : \mu < \mu_0$	Left-tail probability: $P(Z \leq z)$
$H_A : \mu \neq \mu_0$	Two-tail probability: $2P(Z \geq z)$ if $z > 0$ or $2P(Z \leq z)$ if $z < 0$

- Decision rule: Reject H_0 if p-value $< \alpha$.

9.2.2 Four Step Procedure Using the p-value Approach

Step 1. Specify the null and the alternative hypotheses.

Step 2. Specify the test statistic and compute its value.

Step 3. Calculate the p-value.

Step 4. State the conclusion and interpret the results.

LO 9.4 **Explain the steps of a hypothesis test using the critical value approach.**

9.2.3 The Critical Value Approach

- Rejection region – a region of values such that if the test statistic falls into this region, then we reject H_0.
  - The location of this region is determined by H_A.
- Critical value – a point that separates the rejection region from the nonrejection region.
- The critical value approach specifies a region such that if the value of the test statistic falls into the region, the null hypothesis is rejected.
- The critical value depends on the alternative.

Alternative hypothesis	**Critical Value**
$H_A : \mu > \mu_0$	Right-tail critical value is z_α, where $P(Z \geq z_\alpha) = \alpha$
$H_A : \mu < \mu_0$	Left-tail critical value is $-z_\alpha$, where $P(Z \leq -z_\alpha) = \alpha$
$H_A : \mu \neq \mu_0$	Two-tail critical value $-z_{\alpha/2}$ and $z_{\alpha/2}$, where $P(Z \geq z_{\alpha/2}) = \frac{\alpha}{2}$

- Decision Rule: Reject H_0 if:
  - $z > z_\alpha$ for a right-tailed test
  - $z < -z_\alpha$ for a left-tailed test
  - $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ for a two-tailed test

9.2.4 Four Step Procedure Using the Critical Value Approach

Step 1. Specify the null and the alternative hypotheses.

Step 2. Specify the test statistic and compute its value.

Step 3. Find the critical value or values.

Step 4. State the conclusion and interpret the results.

9.2.5 Confidence Intervals and Two-Tailed Hypothesis Tests

- Given the significance level α, we can use the sample data to construct a $100(1-\alpha)\%$ confidence interval for the population mean μ.
- Decision Rule
  - Reject H_0 if the confidence interval does not contain the value of the hypothesized mean μ_0.
  - Do not reject H_0 if the confidence interval does contain the value of the hypothesized mean μ_0.

9.2.6 Implementing a Two-Tailed Test Using a Confidence Interval

- The general specification for a $100(1-\alpha)\%$ confidence interval of the population mean μ when the population standard deviation σ is known as

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \left[\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] \tag{9.2}$$

- Decision Rule: Reject H_0 if $\mu_0 < \bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ or if $\mu_0 > \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

9.3 Hypothesis Test of the Population Mean When σ Is Unknown

9.3.1 Test Statistic for μ When σ is Unknown

LO 9.5 **Differentiate between the test statistics for the population mean.**

- When the population standard deviation σ is unknown, the test statistic for testing the population mean μ is assumed to follow the t_{df} distribution with $(n-1)$ degrees of freedom (df).
- The value of t_{df} is computed as

$$t_{df} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \tag{9.3}$$

9.4 Hypothesis Test of the Population Proportion

LO 9.6 **Specify the test statistic for the population proportion.**

- $\bar{P}$ can be approximated by a normal distribution if $np \geq 5$ and $n(1-p) \geq 5$.
- Test statistic for the hypothesis test of the population proportion p is assumed to follow the z distribution:

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{9.4}$$

where $\bar{p} = \frac{x}{n}$ and p_0 is the hypothesized value of the population proportion.