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7.1 Sampling

LO 7.1 **Differentiate between a population parameter and a sample statistic.**

- **Population** – consists of all items of interest in a statistical problem.
  - **Population Parameter** is unknown.
- **Sample** – a subset of the population.
  - **Sample statistic** is calculated from sample and used to make inferences about the population.
- **Bias** – the tendency of a sample statistic to systematically over- or under-estimate a population parameter.

LO 7.2 **Explain common sample biases.**

- Classic Case of a "Bad" Sample: The *Literary Digest* Debacle of 1936
  - During the 1936 presidential election, the *Literary Digest* predicted a landslide victory of Alf Landon over Franklin D. Roosevelt (FDR) with only a 1% margin or error.
  - They were wrong! FDR won in a landslide election.
  - The *Literary Digest* had committed **selection bias** by randomly sampling from their own subscriber/membership lists, etc.
  - In addition, with only a 24% response rate, the *Literary Digest* had a great deal of non-response bias.
- **Selection bias** – a systematic exclusion of certain groups from consideration for the sample.
  - The *Literary Digest* committed selection bias by excluding a large portion of the population (e.g., lower income voters).
- **Nonresponse bias** – a systematic difference in preferences between respondents and non-respondents to a survey or a poll.
  - The *Literary Digest* had only a 24% response rate. This indicates that only those who cared a great deal about the election took the time to respond to the survey. These respondents may be atypical of the population as a whole.

LO 7.3 **Describe simple random sampling.**

7.1.1 Sampling Methods

- Simple random sample is a sample of n observations which have the sample probability of being selected from the population as any other sample of n observations.
  - Most statistical methods presume simple random samples.
  - However, in some situations, other sampling methods have an advantage over simple random samples.

LO 7.4 **Distinguish between stratified random sampling and cluster sampling.**

7.1.2 Stratified Random Sampling

- Divide the population into mutually exclusive and collectively exhaustive groups, called **strata**.
- Randomly select observations from each stratum, which are proportional to the stratum's size.
- Advantages:
  - Guarantees that each population's subdivision is represented in the sample.
  - Parameter estimates have greater precision than those estimated from simple random sampling.

7.1.3 Cluster Sampling

- Divide population into mutually exclusive and collectively exhaustive groups, called clusters.
- Random select clusters.
- Sample every observation in those randomly selected clusters.
- Advantages and disadvantages:
  - Less expensive than other sampling methods.
  - Less precision than simple random sampling or stratified sapling.
  - Useful when clusters occur naturally in the population.

Table 7.1: Stratified vs. Cluster Sampling

Stratified Sampling	**Cluster Sampling**
Sample consists of elements from each group.	Sample consists of elements from the selected groups.
Preferred when the objective is to increase precision.	Preferred when the objective is to reduce costs.

7.2 The Sampling Distribution of the Means

LO 7.5 **Describe the properties of the sampling distribution of the same mean.**

- Population is described by parameters.
  - A *parameter* is a constant, whose value may be unknown.
  - Only one population.
- Sample is described by statistics.
  - A **statistic** is a random variable whose value depends on the chosen random sample.
  - Statistics are used to make **inferences** about the population parameters.
  - Can draw multiple random samples of size n.

7.2.1 Estimator

- A statistic that is used to estimate a population parameter.
- For example, $\bar{X}$, the mean of the sample, is an estimate of μ, the mean of the population.

7.2.2 Estimate

- A particular value of the estimator.
- For example, the mean of the sample $\bar{x}$ is an estimate of μ, the mean of the population.

7.2.3 Sampling Distribution of the Mean $\bar{x}$

- Each random sample size n drawn from the population provides an estimate of μ–the sample mean $\bar{x}$.
- Drawing many samples of size n results in many different sample means, one for each sample.
- The sampling distribution of the mean is the frequency or probability distribution of these sample means.

7.2.4 The Expected Value and Standard Deviation of the Sample Mean

- The expected value of X,
$$E(X) = \mu \tag{7.1}$$

- The expected value of the mean,
$$E(\bar{X}) = E(X) = \mu \tag{7.2}$$

- Variance of X
$$\text{Var}(X) = \sigma^2 = \sum \frac{(X_i - \bar{X})^2}{n-1} \tag{7.3}$$

- Standard Deviation
  - of X
$$SD(X) = \sqrt{\sigma^2} = \sigma \tag{7.4}$$
  - of $\bar{X}$
$$SD(\bar{X}) = \frac{\sigma}{\sqrt{n}} \tag{7.5}$$

    where n is the sample size. Also known as the **standard error of the mean**.

7.2.5 Sampling from a Normal Distribution

- For any sample size n, the sampling distribution of $\bar{X}$ is **normal** if the population X from which the sample is drawn is normally distributed.

- If X is normal, then we can transform it into the **standard normal random variable** as:

  - For a sampling distribution:
$$\begin{aligned} Z &= \frac{\bar{X} - E(\bar{X})}{\text{SD}(\bar{X})} \\ &= \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \end{aligned} \tag{7.6}$$

  - For a distribution of the values of X.
$$\begin{aligned} Z &= \frac{X - E(X)}{\text{SD}(X)} \\ &= \frac{X - \mu}{\sigma} \end{aligned} \tag{7.7}$$

7.2.6 The Central Limit Theorem

LO 7.6 **Explain the importance of the central limit theorem.**

- For any population X with expected value μ and standard deviation σ, the sampling distribution of $\bar{X}$ will be approximately normal if the sample size n is sufficiently large.
- As a general guideline, the normal distribution approximation is justified when $n \geq 30$.
- As before, if $\bar{X}$ is approximately normal, then we can transform it using (7.6).

7.3 The Sampling Distribution of the Sample Proportion

LO 7.7 **Describe the properties of the sample distribution of the sample proportion.**

- **Estimator** – Sample proportion $\bar{P}$ is used to estimate the population parameter p.
- **Estimate** – a particular value of the estimator $\bar{p}$.

7.3.1 The Expected Value and Standard Deviation of the Sample Proportion

- The expected value of $\bar{P}$ is

$$E(\bar{P}) = p \tag{7.8}$$

- The standard deviation of $\bar{P}$ is

$$\mathrm{SD}(\bar{P}) = \sqrt{\frac{p(1-p)}{n}} \tag{7.9}$$

7.3.2 The Central Limit Theorem for the Sample Proportion

- For any population proportion p, the sampling distribution of $\bar{P}$ is approximately normal if the sample size n is sufficiently large.
- As a general guideline, the normal distribution approximation is justified when $np \geq 5$ and $n(1-p) \geq 5$.
- If $\bar{P}$ is normal, we can transform it into the standard normal random variable as

$$\begin{aligned} Z &= \frac{\bar{P} - E(\bar{P})}{SD(\bar{P})} \\ &= \frac{\bar{P} - p}{\sqrt{\frac{p(1-p)}{n}}} \end{aligned} \tag{7.10}$$

- Therefore, any value $\bar{p}$ on $\bar{P}$ has a corresponding value z on Z given by

$$z = \frac{\bar{p} - p}{\sqrt{\frac{p(1-p)}{n}}} \tag{7.11}$$

7.4 The Finite Population Correction Factor

LO 7.8 **Use a finite population correction factor.**

- Used to reduce the sampling variation of $\bar{X}$.
- The resulting standard deviation is

$$SD(\bar{X}) = \frac{\sigma}{\sqrt{n}} \left(\sqrt{\frac{N-n}{N-1}} \right) \tag{7.12}$$

- The transformation of $\bar{x}$ to Z is made accordingly.
- Apparently, only used when $\frac{n}{N} > 5\%$.

7.4.1 The Finite Population Correction Factor for the Sample Proportion

- Used to reduce the sampling variation of the sample proportion $\bar{P}$.
- The resulting standard deviation is:

$$SD(\bar{P}) = \sqrt{\frac{p(1-p)}{n}} \left(\sqrt{\frac{N-n}{N-1}} \right) \tag{7.13}$$

- The transformation of $\bar{P}$ to Z is made accordingly.

7.5 Statistical Quality Control

LO 7.9 **Construct and interpret control charts from quantitative and qualitative data.**

- Involves statistical techniques used to develop and maintain a firm's ability to produce high-quality goods and services.
- Two Approaches for Statistical Quality Control
  - Acceptance Sampling
  - Detection Approach

7.5.1 Acceptance Sampling

- Used at the completion of a production process or service.
- If a particular product does not conform to certain specifications, then it is either discarded or repaired.
- Disadvantages
  - It is costly to discard or repair a product.
  - The detection of all defective products is not guaranteed.

7.5.2 Detection Approach

- Inspection occurs during the production process in order to detect any nonconformance to specifications.
- Goal is to determine whether the production process should be continued or adjusted before producing a large number of defects.
- Types of variation.
  - Chance variation.
  - Assignable variation.

Chance Variation (Common Variation)

- Caused by a number of randomly occurring events that are part of the production process.
- Not controllable by the individual worker or machine.
- Expected, so not a source of alarm as long as its magnitude is tolerable and the end product meets specifications.

Assignable variation

- Caused by specific events or factors that can usually be identified and eliminated.
- Identified and corrected or removed.

7.5.3 Control Charts

- Developed by Walter A. Shewhart.
- A plot of calculated statistics of the production process over time.
- Production process is "in control" if the calculated statistics fall in an expected range.

- Production process is "out of control" if calculated statistics reveal an undesirable trend.
  - For quantitative data–$\bar{x}$ chart.
  - For qualitative data–$\bar{p}$ chart.

Control Charts for Quantitative Data

- Centerline–the mean when the process is under control.
- Upper control limit (UCL)–set at $+3\sigma$ from the mean.

$$\mu + 3\frac{\sigma}{\sqrt{n}} \tag{7.14}$$

  - Points falling above the upper control limit are considered to be **out of control**.
- Lower control limit (LCL)–set at -3σ from the mean.

$$\mu - 3\frac{\sigma}{\sqrt{n}} \tag{7.15}$$

  - Points falling below the lower control limit are considered to be **out of control**.
- Process is in control–all points fall within the control limits.

Control Charts for Qualitative Data

- $\bar{p}$ chart (fraction defective or percent defective chart).
- Tracks proportion of defects in a production process.
- Relies on central limit theorem for normal approximation for the sampling distribution of the sample proportion.
- Centerline–the mean when the process is under control.
- Upper control limit (UCL)–set at $+3\sigma$ from the mean.

$$p + 3\sqrt{\frac{p(1-p)}{n}} \tag{7.16}$$

  - Points falling above the upper control limit are considered to be **out of control**.
- Lower control limit (LCL)–set at -3σ from the mean.

$$p - 3\sqrt{\frac{p(1-p)}{n}} \tag{7.17}$$

  - Points falling below the lower control limit are considered to be **out of control**.

- Process is out of control–some points fall above the UCL.

Chapter 8

Estimation

LO 8.1: Discuss point estimators and their desirable properties.

LO 8.2: Explain an interval estimator.

LO 8.3: Calculate a confidence interval for the population mean when the population standard deviation is known.

LO 8.4: Describe the factors that influence the width of a confidence interval.

LO 8.5: **Discuss features of the t distribution.**

LO 8.6: **Calculate a confidence interval for the population mean when the population standard deviation is not known.**

LO 8.7: **Calculate a confidence interval for the population proportion.**

LO 8.8: **Select a sample size to estimate the population mean and the population proportion.**

8.1 Point Estimators and Their Properties

LO 8.1 **Discuss point estimators and their desirable properties.**

8.1.1 Point Estimator

- A function of the random sample used to make inferences about the value of an unknown population parameter.
- For example, $\bar{X}$ is a point estimator for μ and $\bar{P}$ is a point estimator for p.

8.1.2 Point Estimate

- The value of the point estimator derived from a given sample.
- For example, $\bar{x} = 96.5$ is a point estimate of the mpg for all ultra-green cars.

8.1.3 Properties of Point Estimators

- **Unbiased** – an estimator is unbiased if its expected value equals the unknown population parameter being estimated.
- **Efficient** – an unbiased estimator is efficient if its standard error is lower than that of other unbiased estimators.
- **Consistent** – an estimator is consistent if it approaches the unknown population parameter being estimated as the sample size grows larger.

8.2 Confidence Interval of the Population Mean When σ Is Known

LO 8.2 **Explain an interval estimator.**

- **Confidence Interval** – provides a range of values that, with a certain level of confidence, contains the population parameter of interest.
  - Also referred to as an **interval estimate**.
- Construct a confidence interval as: Point estimate ± Margin of error.
  - **Margin of error** accounts for the variability of the estimator and the desired confidence level of the interval.

8.2.1 Constructing a Confidence Interval for μ When σ is Known

LO 8.3 **Calculate a confidence interval for the population mean when the population standard deviation is known.**

- Consider a standard normal random variable:

$$P(-1.96 \leq Z \leq 1.96) = 0.95$$

- Because of (7.6), we get:

$$P\left(-1.96 \leq \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq 1.96\right) = 0.95$$

- Which, after algebraically manipulating, is equal to:

$$P\left(\mu - 1.96\frac{\sigma}{\sqrt{n}} \leq \bar{X} \leq \mu + 1.96\frac{\sigma}{\sqrt{n}}\right) = 0.95 \tag{8.1}$$

- Note that (8.1) implies there is a 95% probability that the sample mean $\bar{X}$ will fall within the interval $\mu \pm 1.96\frac{\sigma}{\sqrt{n}}$.
  - Thus, if samples of size n are drawn repeatedly from a given population, 95% of the computed sample means, ___, will fall within the interval and the remaining 5% will fall outside the interval.
- Since we do not know μ, we cannot determine if a particular $\bar{x}$ falls within the interval of not.
  - However, we do know that $\bar{X}$ will fall within the interval $\mu \pm 1.96\frac{\sigma}{\sqrt{n}}$ iff μ falls within the interval $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$.
- This will happen 95% of the time given the interval construction. Thus, this is a 95% confidence interval for the population mean.
- Level of significance (i.e., probability of error) $= \alpha$.
- Confidence coefficient $= 1 - \alpha \Rightarrow \alpha = 1-$ confidence coefficient.
- A $100(1-\alpha)\%$ confidence interval of the population mean μ when the standard deviation σ is known is computed as

$$\bar{x} \pm z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \tag{8.2}$$

  or equivalently

$$\left[\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right] \tag{8.3}$$

- $z_{\frac{\alpha}{2}}$ is the z-value associated with the probability of $\frac{\alpha}{2}$ being in the upper-tail.
- Confidence Intervals:
  - 90%, $\alpha = 0.10$, $\frac{\alpha}{2} = 0.05$, $z_{0.05} = 1.645$.
  - 95%, $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$, $z_{0.025} = 1.96$.
  - 99%, $\alpha = 0.01$, $\frac{\alpha}{2} = 0.005$, $z_{0.005} = 2.575$.

8.2.2 Interpreting a Confidence Interval

- Interpreting a confidence interval requires care.
- Incorrect: the probability that μ falls in the interval is 0.95.

- Correct: If numerous samples of size n are drawn from a given population, then 95% of the intervals formed by the ____ $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{n}$ will contain μ.
  - Since there are many possible samples, we will be right 95% of the time, thus giving us 95% confidence.

8.2.3 The Width of a Confidence Interval

LO 8.4 **Describe the factors that influence the width of a confidence interval.**

- Margin of Error: $z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
- Confidence Interval Width: $2\left(z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)$
- The width of the confidence interval is influenced by the:
  - Sample size n,
  - Standard deviation σ, and
  - Confidence level $100(1-\alpha)\%$.

8.2.4 Summary of the t_{df} Distribution

- Bell-shaped and symmetric around 0 with asymptotic tails (the tails get closer and closer to the horizontal axis, but never touch it).
- Has slightly broader tails than the z distribution.
- Consists of a family of distributions where the actual shape of each one depends on the df. As df increases, the t_{df} distribution becomes similar to the z distribution; it is identical to the z distribution when $df \to \infty$.

8.3 Confidence Interval of the Population Mean When σ Is Unknown

8.3.1 The t-Distribution

LO 8.5 **Discuss features of the t distribution.**

- If repeated samples of size n are taken from a normal population with a finite variance, then the statistic T follows the t-distribution __ with $n-1$ degrees of freedom, __.
- **Degrees of freedom** – determines the extent of the broadness of the tails of the distribution; the fewer the degrees of freedom, the broader the tails.

$$T = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \tag{8.4}$$

8.3.2 Constructing a Confidence Interval for μ When σ Is Unknown

LO 8.6 **Calculate a confidence interval for the population mean when the population standard deviation is not known.**

- A $100(1-\alpha)\%$ confidence interval of the population mean μ when the population standard deviation σ is not known is ____

$$\bar{x} \pm t_{\alpha/2,df}\frac{s}{\sqrt{n}} \tag{8.5}$$

or equivalently

$$\left[\bar{x} - t_{\alpha/2,df}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,df}\frac{s}{\sqrt{n}}\right] \tag{8.6}$$

where s is the sample standard deviation.

8.4 Confidence Interval of the Population Proportion

LO 8.7 **Calculate a confidence interval for the population proportion.**

- Let the parameter p represent the proportion of successes in the population, where success is defined by a particular output.
  - p is the point estimator of the population proportion p.
- By the central limit theorem, $\bar{P}$ can be approximated by a normal distribution for large samples (i.e., $np \geq 5$ and $n(1-p) \geq 5$).
- Thus, a $100(1-\alpha)\%$ confidence interval of the population proportion is

$$\bar{p} \pm z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \text{ or } \left[\bar{p} - z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \bar{p} + z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right] \tag{8.7}$$

where $\bar{p}$ is used to estimate the population parameter p.

8.5 Selecting a Useful Sample Size

LO 8.8 **Select a sample size to estimate the population mean and the population proportion.**

- **Precision** in interval estimates is implied by a low margin of error.
- The larger n reduces the margin of error for the interval estimates.
- How large should the sample size by for a given margin of error?

8.5.1 Selecting n to Estimate μ

- Consider a confidence interval for μ with a known σ and let D denote the desired margin or error.
- Since

$$D = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \tag{8.8}$$

  we may rearrange to get

$$n = \left(\frac{z_{\frac{\alpha}{2}} \sigma}{D} \right)^2. \tag{8.9}$$

- If σ is unknown, estimate it with $\hat{\sigma}$.
- For a desired margin of error D, the minimum sample size n required to estimate a $100(1-\alpha)\%$ confidence interval of the population mean μ is

$$n = \left(\frac{z_{\frac{\alpha}{2}} \hat{\sigma}}{D} \right)^2. \tag{8.10}$$

  where $\hat{\sigma}$ is a reasonable estimate of σ in the planning stage.

8.5.2 Selecting n to Estimate p

- Consider a confidence interval for p and let D denote the desired margin of error.
- Since

$$D = z_{\frac{\alpha}{2}} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \tag{8.11}$$

  (where $\bar{p}$ is the sample proportion), we may rearrange to get

$$n = \left(\frac{z_{\alpha/2}}{D} \right)^2 \bar{p}(1-\bar{p}) \tag{8.12}$$

- Since $\bar{p}$ comes from a sample, we must use a reasonable estimate of p, that is, $\hat{p}$.
- For a desired margin of error D, the minimum sample size n required to estimate a $100(1-\alpha)\%$ confidence interval of the population proportion p is

$$n = \left(\frac{z_{\alpha/2}}{D} \right)^2 p(1-p) \tag{8.13}$$

  where $\hat{p}$ is a reasonable estimate of p in the planning stage.

Chapter 9

Hypothesis Testing

LO 9.1: Define the null hypothesis and the alternative hypothesis.

LO 9.2: Distinguish between Type I and Type II errors.

LO 9.3: Explain the steps of a hypothesis test using the p-value approach.

LO 9.4: Explain the steps of a hypothesis test using the critical value approach.

LO 9.5: Differentiate between the test statistics for the population mean.

LO 9.6: Specify the test statistic for the population proportion.

9.1 Point Estimators and Their Properties

LO 9.1 **Define the null hypothesis and the alternative hypothesis.**

- Hypothesis tests resolve conflicts between two competing opinions (hypotheses).
- In a hypothesis test, define

  H_0 the null hypothesis, the presumed default state of nature or status quo.

  H_A the alternative hypothesis, a contradiction of the default state of nature or status quo.
- In statistics, we use sample information to make inferences regarding the unknown population parameters of interest.
- We conduct hypothesis tests to determine if sample evidence contradicts H_0.
- On the basis of sample information, we either
  - "Reject the null hypothesis"
    - Sample evidence **is** inconsistent with H_0.

- "Do not reject the null hypothesis"
  - Sample evidence **is not** inconsistent with H_0.
  - We do not have enough evidence to "accept" H_0.

9.1.1 Defining the Null Hypothesis and Alternative Hypothesis

General guidelines:

- Null hypothesis, H_0, states the status quo.
- Alternative hypothesis, H_A, states whatever we wish to establish (i.e., contests the status quo)
- Note that H_0 always contains the "equality".

9.1.2 One-Tailed vs Two-Tailed Hypothesis Tests

Two-Tailed Test

- Reject H_0 on either side of the hypothesized value of the population parameter.
- For example:
  - H_0: $\mu = \mu_0$ versus H_A: $\mu \neq \mu_0$
  - H_0: $p = p_0$ versus H_A: $p \neq p_0$
- The $\neq$ symbol in H_A indicates that both tail areas of the distribution will be used to make the decision regarding the rejection of H_0.

One-Tailed Test

- Reject H_0 only on one side of the hypothesized value of the population parameter.
- For example:
  - H_0: $\mu \leq \mu_0$ versus H_A: $\mu > \mu_0$ (right-tail test)
  - H_0: $\mu \geq \mu_0$ versus H_A: $\mu < \mu_0$ (left-tail test)
- Note that the inequality in H_A determines which tail area will be used to make the decision regarding the rejection of H_0.

9.1.3 Three Steps to Formulate Hypotheses

1. Identify the relevant population parameter of interest (e.g., μ or p).
2. Determine whether it is a one- or a two-tailed test.
3. Include some form of the equality sign in H_0 and use H_A to establish a claim.

H_0	H_A	Test Type
$=$	$\neq$	Two-tail
$\geq$	$<$	One-tail, Left-tail
$\leq$	$>$	One-tail, Right-tail

9.1.4 Type I and Type II Errors

LO 9.2 **Distinguish between Type I and Type II errors.**

- **Type I Error** – Committed when we reject H_0 when H_0 is actually true.
  - Occurs with probability α. α is chosen **a priori**.
- **Type II Error** – Committed when we do not reject H_0 when H_0 is actually false.
  - Occurs with probability β. Power of the test $= 1 - \beta$
- For a given sample size n, a decrease in α will increase β and vice versa.
- Both α and β decreases as n increases.

Decision	**Null hypothesis is true**	**Null hypothesis is false**
Reject the null hypothesis	Type I error	Correct decision
Do not reject the null hypothesis	Correct decision	Type I error

9.2 Hypothesis Test of the Population Mean When σ Is Known

LO 9.3 **Explain the steps of a hypothesis test using the p-value approach.**

- Hypothesis testing enables us to determine whether the sample evidence is inconsistent with what is hypothesized under the null hypothesis (H_0).
- Basic principle: First assume that H_0 is true and then determine if sample evidence contradicts this assumption.
- Two approaches to hypothesis testing:
  - The p-value approach.
  - The critical value approach.

9.2.1 The p-value Approach

- The value of the test statistic for the hypothesis test of the population mean μ when the population standard deviation σ is known is computed as

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \tag{9.1}$$

  where μ_0 is the hypothesized mean value.

- p-value: the likelihood of obtaining a sample mean that is at least as extreme as the one derived from the given sample, under the assumption that the null hypothesis is true.

- Under the assumption that $\mu = \mu_0$, the p-value is the likelihood of observing a sample mean that is at least as extreme as the one derived from the given sample.

- The calculation of the p-value depends on the ______.

Alternative hypothesis	**p-value**
$H_A : \mu > \mu_0$	Right-tail probability: $P(Z \geq z)$
$H_A : \mu < \mu_0$	Left-tail probability: $P(Z \leq z)$
$H_A : \mu \neq \mu_0$	Two-tail probability: $2P(Z \geq z)$ if $z > 0$ or $2P(Z \leq z)$ if $z < 0$

- Decision rule: Reject H_0 if p-value $< \alpha$.

9.2.2 Four Step Procedure Using the p-value Approach

Step 1. Specify the null and the alternative hypotheses.

Step 2. Specify the test statistic and compute its value.

Step 3. Calculate the p-value.

Step 4. State the conclusion and interpret the results.

LO 9.4 **Explain the steps of a hypothesis test using the critical value approach.**

9.2.3 The Critical Value Approach

- **Rejection region** – a region of values such that if the test statistic falls into this region, then we reject H_0.
  - The location of this region is determined by H_A.

- **Critical value** – a point that separates the rejection region from the nonrejection region.
- The critical value approach specifies a region such that if the value of the test statistic falls into the region, the null hypothesis is rejected.
- The critical value depends on the alternative.

Alternative hypothesis	**Critical Value**
$H_A : \mu > \mu_0$	Right-tail critical value is z_α, where $P(Z \geq z_\alpha) = \alpha$
$H_A : \mu < \mu_0$	Left-tail critical value is $-z_\alpha$, where $P(Z \leq -z_\alpha) = \alpha$
$H_A : \mu \neq \mu_0$	Two-tail critical value $-z_{\alpha/2}$ and $z_{\alpha/2}$, where $P(Z \geq z_{\alpha/2}) = \frac{\alpha}{2}$

- Decision Rule: Reject H_0 if:
  - $z > z_\alpha$ for a right-tailed test
  - $z < -z_\alpha$ for a left-tailed test
  - $z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ for a two-tailed test

9.2.4 Four Step Procedure Using the Critical Value Approach

Step 1. Specify the null and the alternative hypotheses.

Step 2. Specify the test statistic and compute its value.

Step 3. Find the critical value **or** values.

Step 4. State the conclusion and interpret the results.

9.2.5 Confidence Intervals and Two-Tailed Hypothesis Tests

- Given the significance level α, we can use the sample data to construct a $100(1-\alpha)\%$ confidence interval for the population mean μ.
- Decision Rule
  - Reject H_0 if the confidence interval **does not** contain the value of the hypothesized mean μ_0.
  - Do not reject H_0 if the confidence interval **does** contain the value of the hypothesized mean μ_0.

9.2.6 Implementing a Two-Tailed Test Using a Confidence Interval

- The general specification for a $100(1-\alpha)\%$ confidence interval of the population mean μ when the population standard deviation σ is known as

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \left[\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] \tag{9.2}$$

- Decision Rule: Reject H_0 if $\mu_0 < \bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ or if $\mu_0 > \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

9.3 Hypothesis Test of the Population Mean When σ Is Unknown

9.3.1 Test Statistic for μ When σ is Unknown

LO 9.5 **Differentiate between the test statistics for the population mean.**

- When the population standard deviation σ is unknown, the test statistic for testing the population mean μ is assumed to follow the t_{df} distribution with $(n-1)$ degrees of freedom (df).
- The value of t_{df} is computed as

$$t_{df} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \tag{9.3}$$

9.4 Hypothesis Test of the Population Proportion

LO 9.6 **Specify the test statistic for the population proportion.**

- $\bar{P}$ can be approximated by a normal distribution if $np \geq 5$ and $n(1-p) \geq 5$.
- Test statistic for the hypothesis test of the population proportion p is assumed to follow the z distribution:

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \tag{9.4}$$

where $\bar{p} = \frac{x}{n}$ and p_0 is the hypothesized value of the population proportion.