1 Introduction to Universal Turing Machines and Diagonalization Language

1.1 Review and Introduction

- Continuation from Friday's discussion on universal Turing machines.
- Introduction to the **Diagonalization Language** (LD), a language no Turing machine can accept.
- Connection to the **halting problem**, a similar concept to the Hello World problem.
- Goal is to understand how a machine can be fed its own encoding.

1.2 Encoding Turing Machines

- Turing machines are encoded as strings of 0s and 1s.
- Encoding of transition functions:
  - **States** are represented by a number of 0s (e.g., state 1 = 1 zero, state 2 = 2 zeros, etc.).
  - **Tape head symbols** are encoded as 0s, separated from the state encoding with a single 1.
  - **Resulting states** are also encoded as 0s.
  - **Tape writes** (what we'll write on the tape), also as a number of 0's.
  - **Move direction** is encoded as 1 zero for right, and 2 zeros for left.
- Individual transitions are encoded, and **transitions are separated by two 1s**.
- The **input (W)** is separated from the transitions with three 1s.
- Example: encoding with two transitions and input *0, 0, 1.*
- Different possible encodings of the same Turing machine due to transition function ordering

1.3 Universal Turing Machine and Self-Feeding

- The encoding of a Turing machine with input can also be seen as just an input to another Turing machine.
- A universal Turing machine can accept any language and can be fed its own encoding.
- This self-feeding property is a key step towards defining the diagonalization language.

2 Diagonalization Language (LD)

2.1 Definition of LD

- **LD** is the set of strings such that one of those encodings, WI, is not in the language of the Turing Machine, MII.
- LD consists of strings W such that the Turing machine M whose code is W does not accept W as input.
- LD is defined as the opposite of what a machine would accept.

2.2 Construction via Enumeration

- Enumeration of encodings of Turing machines (M_{ii}).
- Concept of an infinite table T, where cells T_{ij} represent whether the string w_i is in the language of Turing machine M_j
- If w_i is in the language of M_i, then the value of the cell is 1, otherwise it is 0.
- Each row represents a characteristic vector for a specific Turing machine.

3 Key Takeaways

- Turing Machines can be encoded as strings of 0s and 1s allowing for self-referential operations

- The Diagonalization Language (LD) is defined based on Turing machine encodings and their acceptance behavior
- The concept of an enumerated table is crucial for understand the construct of LD.