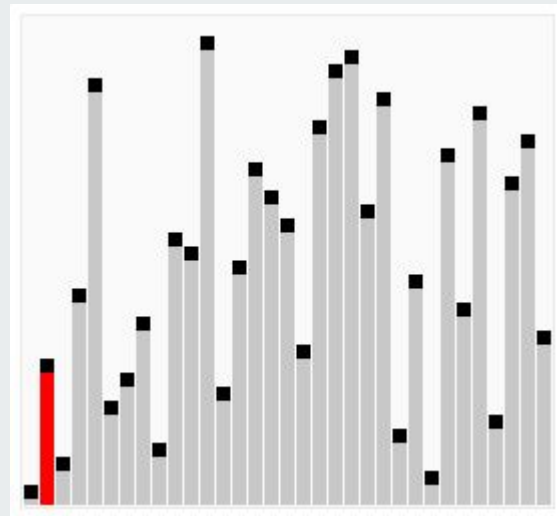




Sort an Array

Wenbo





Data types

Double: 1.35

Integer: 1



Array

Array: `my_arr = [1, 3, 5, 7, 9]`

Index starts from 1 in Julia!

`My_arr[3] = 5`

`My_arr[1.3] = Error!`



For loop

```
My_arr = [0, 0, 0, 0, 0]
```

```
My_arr[1] = 1
```

```
My_arr[1] = 2
```

```
My_arr[1] = 3
```

```
My_arr[1] = 4
```

```
My_arr[1] = 5
```

```
My_arr = [1, 2, 3, 4, 5]
```



For loop

```
My_arr = [0, 0, 0, 0, 0]
```

```
My_arr[1] = 1
```

```
My_arr[1] = 2
```

```
My_arr[1] = 3
```

```
My_arr[1] = 4
```

```
My_arr[1] = 5
```

```
My_arr = [1, 2, 3, 4, 5]
```

```
My_arr = [0, 0, 0, 0, 0]
```

```
for i = 1:6
```

```
    My_arr[i] = i
```

```
end
```



Big O notation

For input size = n ,

$O(n)$ means the amount of work = $n, 2n, 3n, 4n \dots$

$O(1)$ means the amount of work is constant.

$O(n^2)$ means work = $n^2, 2n^2, 3n^2 \dots$

Can be used to present **time** and **space** complexity.



In-Place functions

Functions with $O(1)$ space complexity, with an exclamation mark at the end, for example:

Swap!(arr, i, j) swaps the 'i'-th element and 'j'-th element of array 'arr'

```
My_arr = [1, 2, 3, 4, 5]
```

```
Swap!(My_arr, 2, 4)
```

```
→ My_arr = [1, 4, 3, 2, 5]
```

Given an array of integers `nums`, sort the array in ascending order and return it.

You must solve the problem **without using any built-in** functions in $O(n \log(n))$ time complexity and with the smallest space complexity possible.

Example 1:

Input: `nums = [5,2,3,1]`

Output: `[1,2,3,5]`

Explanation: After sorting the array, the positions of some numbers are not changed (for example, 2 and 3), while the positions of other numbers are changed (for example, 1 and 5).

Example 2:

Input: `nums = [5,1,1,2,0,0]`

Output: `[0,0,1,1,2,5]`

Explanation: Note that the values of `nums` are not necessarily unique.

Constraints:

- $1 \leq \text{nums.length} \leq 5 * 10^4$
- $-5 * 10^4 \leq \text{nums}[i] \leq 5 * 10^4$

- Selection Sort
- Bubble Sort
- Insertion Sort
- Merge Sort
- Quick Sort
- Heap Sort
- Counting Sort
- Radix Sort
- Bucket Sort
- Bingo Sort Algorithm
- ShellSort
- TimSort
- Comb Sort
- Pigeonhole Sort
- Cycle Sort
- Cocktail Sort
- Strand Sort
- Bitonic Sort
- Pancake sorting
- BogoSort or Permutation Sort
- Gnome Sort
- Sleep Sort – The King of Laziness
- Structure Sorting in C++
- Stooge Sort
- Tag Sort (To get both sorted and original)
- Tree Sort
- Odd-Even Sort / Brick Sort
- 3-way Merge Sort

Double Loop Sort (Bubble Sort)

Pair-wise comparison

$$\text{Num comparisons: } C_n^2 = \frac{n \times (n - 1)}{2 \times 1} = O(n^2)$$

Time = $O(n^2)$

Space = $O(1)$ for in-place implementation

i = 0	j	0	1	2	3	4	5	6	7
	0	5	3	1	9	8	2	4	7
	1	3	5	1	9	8	2	4	7
	2	3	1	5	9	8	2	4	7
	3	3	1	5	9	8	2	4	7
	4	3	1	5	8	9	2	4	7
	5	3	1	5	8	2	9	4	7
	6	3	1	5	8	2	4	9	7
i = 1	j	0	1	2	3	4	5	6	7
	0	3	1	5	8	2	4	7	9
	1	1	3	5	8	2	4	7	
	2	1	3	5	8	2	4	7	
	3	1	3	5	8	2	4	7	
	4	1	3	5	2	8	4	7	
	5	1	3	5	2	4	8	7	
i = 2	j	0	1	2	3	4	5	6	7
	0	1	3	5	2	4	7	8	
	1	1	3	5	2	4	7		
	2	1	3	5	2	4	7		
	3	1	3	2	5	4	7		
	4	1	3	2	4	5	7		
i = 3	j	0	1	2	3	4	5	6	7
	0	1	3	2	4	5	7		
	1	1	3	2	4	5			
	2	1	2	3	4	5			
	3	1	2	3	4	5			
i = 4	j	0	1	2	3	4	5	6	7
	0	1	2	3	4	5			
	1	1	2	3	4				
	2	1	2	3	4				
i = 5	j	0	1	2	3	4	5	6	7
	0	1	2	3	4				
	1	1	2	3					
i = 6	j	0	1	2	3	4	5	6	7
	0	1	2	3					
	1	1	2						

Merge Sort

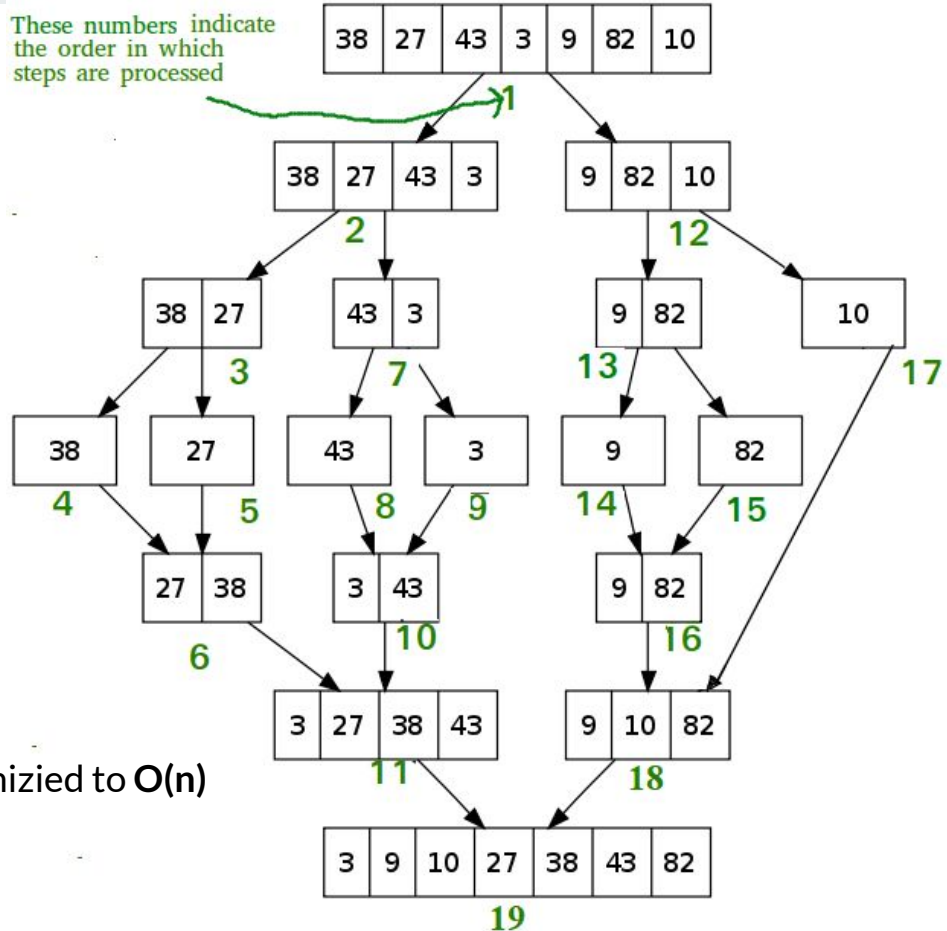
Two phases: Divide and Conquer

Depth for each phase = $\log(n)$

Work for each level = n

So Time = $O(n \log(n))$

Space = $O(n \log(n))$ but can be optimized to $O(n)$



Quick Sort (Still Divide and Conquer)

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i j
j = j+1

aaaaaaaaa|xxxxxxxxxxxxxxxxx|bbbbbbbbbb

i j

(≤ pivot) (undiscovered) (> pivot)

aaaaaaaaaaaaaaaaaaaaa|bbbbbbbbbbbbbbbbbb

j i

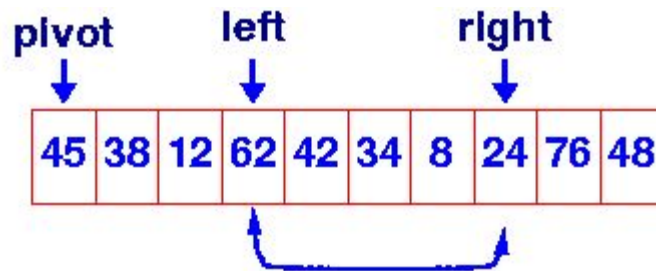
(≤ pivot) (> pivot)

Quick Sort (Still Divide and Conquer)

Default: choose the left element as the pivot

Average time complexity: $O(n \log n)$

Space: $O(1)$ by in-place implementation



Quick Sort (Still Divide and Conquer)

Average time complexity: $O(n \log n)$

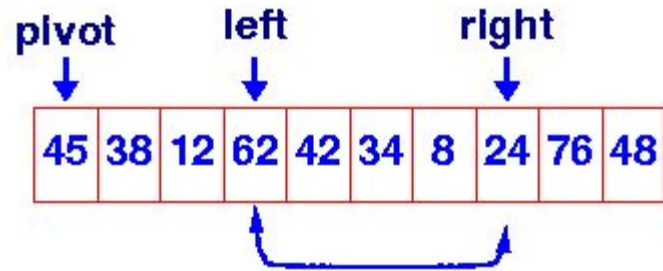
Worst case time complexity: $O(n^2)$

Space: $O(1)$ by in-place implementation

Worst case, Sort $[1, 2, 3, 4, 5, \dots, n]$ in ascending order.

1. Pivot = 1, work = n
2. Pivot = 2, work = $n-1$
3. ...
4. Pivot = $n-2$, work = 1

Total work = $\sum(1:n) = O(n^2)$



Quick Sort (Still Divide and Conquer)

Average time complexity: $O(n \log n)$

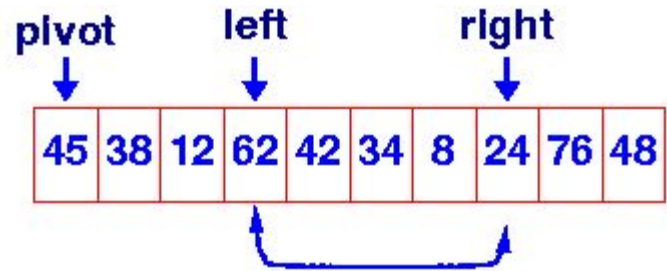
Worst case time complexity: $O(n^2)$

Space: $O(1)$ by in-place implementation

Worst case, Sort $[1, 2, 3, 4, 5, \dots, n]$ in ascending order.

1. Pivot = 1, work = n
2. Pivot = 2, work = $n-1$
3. ...
4. Pivot = $n-2$, work = 1

Total work = $\sum(1:n) = O(n^2)$



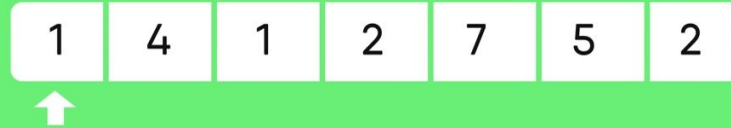
Choose random pivot



Extra: Counting Sort

1st pass: Counting

For simplicity, consider data in range of 0 to 9



Index :	0	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0	0

Count each element in the given array and place the count at the appropriate index.

1st pass: Counting

For simplicity, consider data in range of 0 to 9

1	4	1	2	7	5	2
---	---	---	---	---	---	---



Index : 0 1 2 3 4 5 6 7 8 9

0	2	0	0	1	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---

1st pass: Counting

For simplicity, consider data in range of 0 to 9

		1	4	1	2	7	5	2		
Index :	0	1	2	3	4	5	6	7	8	9
	0	2	2	0	1	1	0	1	0	0

Modify the count array by adding the previous counts.

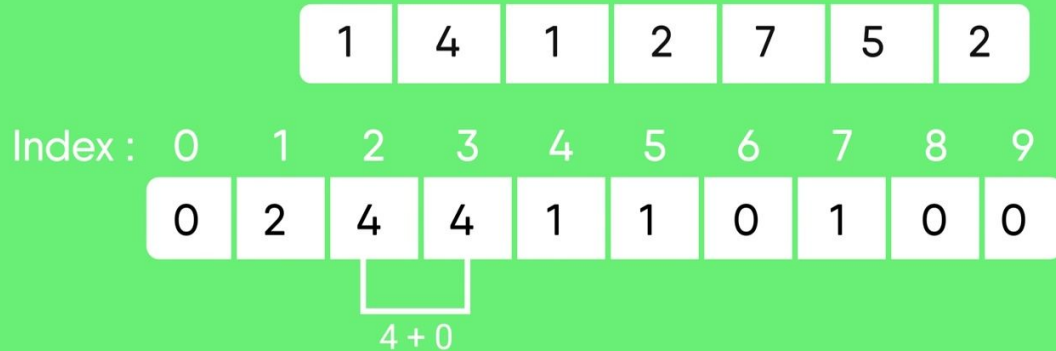
2nd pass: Ranking

For simplicity, consider data in range of 0 to 9



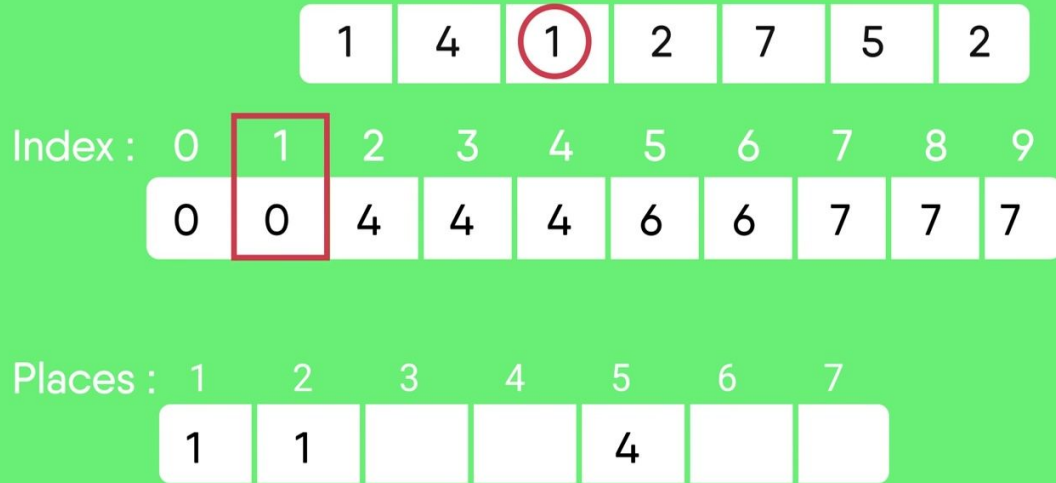
2nd pass: Ranking

For simplicity, consider data in range of 0 to 9



3rd pass: Build the output

For simplicity, consider data in range of 0 to 9



3rd pass: Build the output

For simplicity, consider data in range of 0 to 9





Extra: Counting Sort

Fastest but not space efficient(Heavy load to be deployed on server)

Time: $O(n+K)$

Space: $O(n+K)$

Trade off between time and space



Follow Up: A better Quick Sort?

Follow Up: A better Quick Sort?

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aaaaaaaaa|pppppppppp|xxxxxxxxxxxxxxx|bbbbbbbbbbb

i

j

k

(<pivot) (==pivot) (undiscovered) (>pivot)

aaaaaaaaa|pppppppppp|bbbbbbbbbbb

jk

i

(<pivot) (==pivot) (>pivot)

Implement `pow(x, n)`, which calculates `x` raised to the power `n` (i.e., `xn`).

Next Week: Pow(x, n)

Constraints:

- $-100.0 < x < 100.0$
- $-2^{31} \leq n \leq 2^{31}-1$
- `n` is an integer.
- $-10^4 \leq x^n \leq 10^4$

Example 1:

Input: `x = 2.00000`, `n = 10`

Output: `1024.00000`

Example 2:

Input: `x = 2.10000`, `n = 3`

Output: `9.26100`

Example 3:

Input: `x = 2.00000`, `n = -2`

Output: `0.25000`

Explanation: $2^{-2} = 1/2^2 = 1/4 = 0.25$