残差雅可比推导

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1 线特征残差雅可比

根据ppt:

$$J_\varepsilon = \frac{\partial d_\varepsilon}{\partial T} = \frac{\partial d_\varepsilon}{\partial \widetilde{p_i}} \frac{\partial \widetilde{p_i}}{\partial T}$$

根据视觉slam14讲，可知:

$$\frac{\partial \widetilde{p_i}}{\partial T} = -(Rp_i + t)^\wedge I$$

令 $X = d_\varepsilon = \frac{|(\widetilde{p_i}-p_b)\times(\widetilde{p_i}-p_a))|}{|p_a-p_b|}$，则:

$$\frac{\partial d_\varepsilon}{\partial \widetilde{p_i}} = \frac{\partial |X|}{\partial \widetilde{p_i}} = \frac{X}{|X|}\frac{\partial X}{\partial \widetilde{p_i}} = \frac{X}{|X|}\frac{(p_a-p_b)^\wedge}{|p_a-p_b|} \quad (according \quad to \quad ppt)$$

最终，线特征的残差雅可比为:

$$J_\varepsilon = \frac{X}{|X|}\frac{(p_a-p_b)^\wedge}{|p_a-p_b|}[-(Rp_i+t)^\wedge I]$$

2 面特征残差雅可比

由上述推导可知，取模长于否在于是否往最终的结果里添加 $\frac{X}{|X|}$ 项，因此，面特征残差雅可比可以简单的推导出来:

$$J_H = \frac{\partial d_H}{\partial T} = \frac{\partial d_H}{\partial \widetilde{p_i}} \frac{\partial \widetilde{p_i}}{\partial T}$$

其中:

$$\frac{\partial d_H}{\partial \widetilde{p_i}} = \frac{(p_l-p_j)\times(p_m-p_j)}{|(p_l-p_j)\times(p_m-p_j)|} \quad (according \quad to \quad ppt)$$

最终，面特征的残差雅可比为:

$$J_H = \frac{(p_l-p_j)\times(p_m-p_j)}{|(p_l-p_j)\times(p_m-p_j)|}[-(Rp_i+t)^\wedge I]$$