Contents

Color code: highlight, key concepts and definitions, to be added.

Reference: PTE, Folland and Tao, Cinlar.

Chapter 1

From Reading Group on Durrett

1.1 Measure and Integration

1.1.1 Measure and Probability Spaces

First of all, why should we discuss measure theory. It's for a better integration theory than the usual Riemann integral. Because we need strong conditions for arguments like:

$$\lim_n \int f_n = \int \lim_n f_n$$

to hold for Riemann integral. TBA: Relationship with Riemann Integral We'll talk about

1. Measure space, construction of measure, Lebesgue measures;
2. Integration theory
3. Convergence concepts
4. Differentiation

Let's start with an abstract measure space. Let Ω be a space. A σ-algebra $\mathcal{F}$ is a collection of subsets called *measurable sets* of Ω such that it's closed under complements and countable unions. Also the $\emptyset \in \mathcal{F}$. We can assign *measure* μ to members in $\mathcal{F}$. The reason we need to restrict attention from the power set to $\mathcal{F}$ is that there are examples that violate some properties we want a measure to have. μ satisfies:

$$\mu(\varnothing) = 0 \quad \text{and} \quad \mu\big(\sqcup_i^\infty A_i\big) = \sum_i \mu(A_i) \quad \text{for} \quad (A_i) \subset \mathcal{F}$$

The triplet $(\Omega, \mathcal{F}, \mu)$ is called a *measure space*.

A *measurable function* is $X : (\Omega, \mathcal{F}, \mu) \to (Y, \mathcal{H})$ such that for all $H \in \mathcal{H}$, $X^{-1}(H) \in \mathcal{F}$. Meaning that we can assign a measure to the subsets of $\mathcal{H}$ based on the measure μ and X. Given two measurable spaces, we can check whether a function X is measurable.

1.1.1 Lemma. *Suppose $\mathcal{H} = \sigma(\mathcal{G})$, if for all $B \in \mathcal{G}$, $X^{-1}(B) \in \mathcal{F}$, then X is measurable.*

Proof. Let $\mathcal{H}' = \big\{B \subset S \mid X^{-1}(B) \in \mathcal{F}\big\}$, then we can show $\mathcal{H}'$ is a σ-algebra because X^{-1} preserves complements and union. Then we have $\mathcal{G} \subset \sigma(\mathcal{G}) = \mathcal{H} \subset \mathcal{H}'$, meaning that all $H \in \mathcal{H}$ satisfies the condition. □

There are useful lemmas for checking a collection of subsets is indeed a σ-algebra.

1.1.2 Lemma (Dykin's $\pi - \lambda$). *If a p-system $\mathcal{P}$ is contained in a λ-system $\mathcal{D}$, then $\sigma(\mathcal{P}) \subset \mathcal{D}$.*

Proof. Let $E \in$ □

1.1.3 Lemma (Monotone Class Theorem). *A collection of subsets $\mathcal{M}$ is a monotone class if $(A_n) \subset \mathcal{A}$*

1. *$A_n \uparrow A$ then $A \in \mathcal{A}$.*

2. *$A_n \downarrow A$ implies $A \in \mathcal{A}$.*

Themonotone class generated by an algebra $\mathcal{A}$ $\mathcal{M}(\mathcal{A}) = \sigma(\mathcal{A})$.

1.1.4 Open Sets on $\mathbb{R}$. *Every open set $U \subset \mathbb{R}$ is an at most countable union of disjoint open intervals.*

1.1.5 Measurability. *For countable collections of random variables X_n, Y_n, $X + Y, XY$ and $\sup X_n$, $\inf X_n$, $\limsup X_n$, $\liminf X_n$ are measurable. If $\lim_n X_n$ exists, it's also measurable.*

Proof. For $\sup_n X_n$, let $t \in \mathbb{R}$, $\sup_n X_n \in (-\infty, t) \iff X_n < t$, for all t, hence $(\sup_n X_n)^{-1} = \cap_n X_n^{-1}(-\infty, t)$. □

Probability Space and Random Variables

A measure space is a *probability space* if $\mu = 1$, we write it as $(\Omega, \mathcal{F}, P)$. A random variable/vector is a measurable function on probability space $X : \Omega \to \mathbb{R}/\mathbb{R}^d/\mathbb{C}$ with the Borel σ-algebra, the σ-algebra generated by the open sets.

1.1.2 Construction of Measures

Motivation

Now we want to show that we can actually find measure on $\mathbb{R}$ and $\mathbb{R}^d$ that are the most useful spaces. We start from abstract construction method inspired by the following observation of measure on $\mathbb{R}$.

In order to have a natural measure λ on $\mathbb{R}$, we want the following properties:

1. $\lambda_0(a, b] = b - a$;
2. Measure of union of disjoint intervals should be the sum;(we can't have uncountable sum, hence restricted to countable union)
3. Apply to all intervals(we can't apply to the power set, hence restrict to the measurable sets).

Now consider the set $\mathcal{S}_{\mathbb{R}} = \{(a, b] : a \le b \in \mathbb{R}\}$, it's too small, we want to extend the natural measure funtion λ_0 to a larger set(we can extend to $\mathcal{B}_{\mathbb{R}}$, even to $\mathcal{L}_{\mathbb{R}}$).

Semi-algebra $\mathcal{S}$ is collection of subsets of a space Ω that satisfies the following conditions: closed under intersection and the complements are union of finite disjoint sets in $\mathcal{S}$. Let μ_0 be a what I call *semi-pre-measure* to (*algebra*, *premeasure*) to (outer-measurable sets, outer measure) to (sigma-algebra, measure) where the *semi-pre-measure* μ_0 satisfies if both $S_N = \sqcup^N S_j, S_\infty = \sqcup^\infty S_j \in \mathcal{S}$

$$\mu_0(\varnothing) = 0; \quad \mu_0(S_N) = \sum^N \mu_0 S_j; \quad \mu_0(S_\infty) \le \sum^\infty \mu_0 S_j$$

We follow the steps of extension

$$(\mathcal{S}, \mu_0) \to (\mathcal{A}, \mu_1) \to (\mathcal{M}, \mu^*) \to (\sigma(\mathcal{S}), \mu)$$

that is semi-algebra and *pre-measure* μ_1 satisfies $\mu_1 \sqcup_i^\infty A_i = \sum_i^\infty \mu_1 A_i$ as long as the union is also in $\mathcal{A}$. And we will show that $\mathcal{B}_\mathbb{R} \subset \sigma(\mathcal{S})$.

Construction of Abstract Measure

1.1.6 $(\mathcal{S}, \mu_0) \to (\mathcal{A}, \mu_1)$. *We have that* $\mathcal{A} = \left\{ \bigsqcup_j^N S_j : S_j \in \mathcal{S} \right\}$ *is an algebra. And* $\mu_1(A) = \sum_j^N \mu_0 S_j$ *is a premeasure.*

Proof. $\mu_1(\varnothing) = 0$. Suppose $A = \bigsqcup^\infty A_i \in \mathcal{A}$, then there exists $T_l \in \mathcal{S} : l = 1, \dots N_T$, such that $A = \sqcup_l T_l$, also for each $A_j = \sqcup_k^{N_j} S_{j,k}$. □

Let an *outer measure* be a function over the power set such that:

$$\mu^*(\varnothing) = 0 \quad \text{and} \quad \mu^*\big(\sqcup^\infty A_j\big) \leq \sum_j \mu^* A_j \quad \text{and} \quad \mu^*(A) \leq \mu^*(B) \quad \text{for} \quad A \subset B$$

Also let $\mathcal{M}$ be the *outer-measurable sets*, that is the collections of A such that

$$\mu^*(E) = \mu^*(E \cap A) + \mu^*(E \cap A^c) \quad \text{for all} \quad E \subset X$$

1.1.7 $(\mathcal{A}, \mu_1) \to (\mathcal{M}, \mu^*)$ **Caratheodory's Theorem applied to premeasures.** *We can find the outer measure induced by* μ_1*:*

$$\mu_1^*(B) = \inf \left\{ \sum_j^\infty \mu_1 A_j : B \subset \cup_j A_j \right\}$$

we show μ_1^* *is indeed an outer measure, Caratheodory states that for outer measures,* $\mathcal{M}$ *of* μ_1^* *is a* σ*-algebra and* $\mu^*|_\mathcal{M}$ *is complete.*

Proof.

1. Monotone property is easy, for countable subadditivity, we use ε-room method, find a cover C_{jk} of A_j such that :

$$\mu_1^*(A_j) > \sum_k \mu_1^* C_{jk} - \varepsilon$$

2. To show that $\mathcal{M}$ is a σ-algebra we only need to show that for $E_j \in \mathcal{M}$ and any $B \subset \Omega$, we have:
$$\mu^*(B) \geq \mu^*(B \cap E) + \mu^*(B \cap E)$$
which can be seen by considering finite sum, taking out one E_j at a time and then take limit.

3. Show that the restriction is complete.

□

1.1.8 $(\mathcal{M}, \mu^*) \to (\sigma(\mathcal{S}), \mu)$. *We can restrict μ^* to define $\mu = \mu^*|_{\sigma(\mathcal{S})}$, and the resulting measure is complete.*

Proof. We show that $\sigma(\mathcal{S}) \subset \mathcal{M}$ and the restriction is complete. □

1.1.3 Borel and Lebesgue-Stietjes Measure on $\mathbb{R}$

We verify that for any nondecreasing right-continuous function $F : \mathbb{R} \to \mathbb{R}$, the semi-algebra $\mathcal{S}_{\mathbb{R}}$ together with the semi-pre-measure defined by

$$\lambda_0(a, b] = F(b) - F(a)$$

satisfies the conditions set in the previous section. $\mathcal{A}_{\mathbb{R}}$ be the algebra generated by $\mathcal{S}_{\mathbb{R}}$ in turn will generate $\mathcal{B}_{\mathbb{R}}$. We can extend it to a something larger, a complete measure whose domain contains $\mathcal{B}_{\mathbb{R}}$. It will be called the *Lebesgue-Stietjes measure* and *Lebesgue measurable sets* $\mathcal{M}_\lambda$. The restriction on $\mathcal{B}$ is called *Borel measure*. It's just the completion.

1.1.9 **Regularity of $\mathcal{M}_\lambda$.** *Lebesgue measurable sets are of simple form if you allow for a small error.*

1. $\lambda(A) = \inf\{\lambda U : A \subset U, U \text{ is open}\} = \sup\{\lambda K : K \subset A, K \text{ is compact}\}$
2. $A = G_\delta \setminus N_1 = F_\sigma \cup N_2$ *where N are null sets.*
3. *Littlewood's First Principle: Suppose $\lambda(A) < \infty$ then for all $\varepsilon > 0$, there exists B that is a finite union of open intervals such that $A \triangle B < \varepsilon$.*

Proof. □

1.1.4 Integration

Integration and measure are the two sides of same coin: integration is the extension of measure from the space of *indicator functions* $\mathbf{1}_A$ of measurable sets to a larger set of functions.

For *simple functions* $f_s = \sum_j^N \mathbf{1}_{A_j}$, we can define $\mu f_s = \sum_j^N \mu(A_j)$

Then for *nonnegative measurable* functions f, we can define

$$\mu f = \sup\{\mu f_s : f_s \leq f \quad \text{pointwise}\}$$

If $f = f^+ - f^-$ and the integrals of two nonnegative parts are not ∞, then we say f is *integrable* or in L^1 and $\mu f = \mu f^+ - \mu f^-$.

1.1.10 Littlewood's Second Principle. *Measurable and integrable functions can be approximated by simple well-behaved functions.*

1.1.11 Lusin's Theorem.

1.1.12 Properties of Integral.

1. *Monotone $\mu f \leq \mu g$ if $f \leq g$ pointwise.*
2. *Linearity for $a > 0$, $\mu(af) = a\mu f$, $\mu(f+g) = \mu f + \mu g$.*
3. *Monoton Convergence: if $0 \leq f_n \uparrow f$ a.e., then $\mu f_n \uparrow \mu f$.*

Proof. Folland proves by choosing simple function and a scaling factor α. Tao's proof is essentially the same. PTE proves via Fatou's lemma.

Suppose $f_n \uparrow f$ a.e., if μf exists, we know μf_n must converge because it's a nondecreasing bounded sequence of numbers. Also $\mu f_n \leq \mu f$ by monotonicity.

To show that $\mu f \leq \lim_n \mu f_n$, where $\mu f = \sup\{\mu g : g \text{ is simple}, g \leq f\}$, we only need to show that for all simple $g \leq f$, we have $\mu g \leq \mu f_n$. We can use ε-room method, let $E_n = \{\omega : f_n \geq (1-\varepsilon)g\}$, $E_n \uparrow \Omega$, and we have $\int f_n \geq \int_{E_n}(1-\varepsilon)g$, taking lim (we have yet to show the monotone convergence for simple functions), we have $\lim \int f_n \geq (1-\varepsilon)\int g$ for all $\varepsilon > 0$. □

Remark. These three properties fully characterise the integral. That is given a functional that satisfies the three properties we can find a measure for which the function is the integral.

1.1.13 Inequalities and Controls. *Useful inequalities and controls, suppose that* $f \in L^1$;

1. *Jensen's inequality: suppose we have a convex function* $\Phi(x) : (a, b) \to \mathbb{R}$, *suppose* $\mu(\Omega) = 1$, *and* $f \in L^1 : \Omega \to (a, b)$ *then* $\mu(\Phi(f)) \geq \Phi\mu f$
2. *Holder's inequality. Suppose* $1 \leq p \leq \infty$ *For* $q : \frac{1}{p} + \frac{1}{q} = 1$ *and* f, g *being measurable, we have* $\|fg\|_1 \leq \|f\|_p \|g\|_q$ *with equality iff* $\alpha, \beta \neq 0$, $\alpha|f|^p = \beta|g|^q$ *a.e.*
3. *Minkowski's inequality:* $\|f + g\|_p \leq \|f\|_p + \|g\|_p$.
4. *Chebyshev's Inequality:*
5. *For nonnegative measurable functions,* $\mu f < \infty \implies f < \infty$ *a.e.,* $\mu f = 0$ *implies* $f = 0$ *a.e.*
6. *Moment and tail behaviour, if* μ *is finite,*

$$\mu f^r < \infty \implies \sum x^{r-1} \mu(f > x) < \infty$$

Proof. We prove the above inequalities.

1. First we show that if Φ is convex and $t_0 \in (a, b)$ then there exists β such that $F(t) - F(t_0) \geq \beta(t - t_0)$ for all $t \in (a, b)$. Second Φ is measurable, because for any t, $\Phi^{-1}(x : x < t)$ is an interval.
2. Suppose $\|f\|_p = 0$, then $f = 0, a.e.$, then LHS is also 0. If $\|f\|_p = \infty$ then it obviously is true. Then one proof is based on the lemma:

1.1.14 Lemma. *If* $a, b \geq 0$ *and* $0 < \lambda < 1$ *then*

$$a^\lambda b^{1-\lambda} \leq \lambda a + (1 - \lambda) b$$

with equality iff $a = b$.

□

1.1.5 Product Space and Lebesgue Measure on $\mathbb{R}^d$

Product Measurable Space

Let $(Y_\alpha, \mathcal{H}_\alpha) : \alpha \in \mathcal{A}$ be a collection of measurable spaces, then we can define a *product σ-algebra* $\otimes_\alpha \mathcal{H}_\alpha = \sigma\{\pi_\alpha^{-1}(E_\alpha) : E_\alpha \in \mathcal{H}_\alpha, \alpha \in \mathcal{A}\}$ on the product space $\prod_\alpha \Omega_\alpha$,

1.1.15 **Proposition.** *Let $\Omega, \mathcal{F}$ and $(Y_\alpha, \mathcal{H}_\alpha)$ be measurable spaces, let $(Y, \mathcal{H}) = (\prod Y_\alpha, \otimes_\alpha \mathcal{H}_\alpha)$. $f : \Omega \to Y$ is $(\mathcal{F}, \mathcal{H})$-measurable if and only if $f_\alpha = \pi_\alpha f$ is $\mathcal{F}, \mathcal{H}_\alpha$-measurable for all α.*

1.1.16 **Proposition (TBA).** *Product σ-algebra is generally smaller than the σ-algebra on the product space itself. On separable metric space, they coincide.*

A measure space is a *probability space* if $\mu = 1$, we write it as $(\Omega, \mathcal{F}, P)$. A random variable/vector is a measurable function on probability space $X : \Omega \to \mathbb{R}/\mathbb{R}^d/\mathbb{C}$ with the Borel σ-algebra, the σ-algebra generated by the open sets.

Now consider finite vector of random variables $X^d := (X_1, \dots, X_d) : \Omega \to \mathbb{R}^d$. Then X^d is measurable iff all X_i are measurable. *Product measurable* and *product measure*.

Finite Product Measure

Let $(\Omega_1, \mathcal{F}_1, \mu_1)$ and $(\Omega_2, \mathcal{F}_2, \mu_2)$ be two σ-finite measure spaces. We can definte a product measure on the *rectangles*: $E_1 \times E_2 \in \mathcal{F}_1 \times \mathcal{F}_2$:

$$\tilde{\mu}(E_1 \times E_2) = \mu_1(E_1)\mu_2(E_2)$$

we can show that rectangles form a semi-algebra $\mathcal{S}$ on $\Omega_1 \times \Omega_2$: they are closed under intersection and $\Omega_1 \times \Omega_2 \in \mathcal{S}$, and $(E_1 \times E_2)^c$ is a finite disjoint union of members of $\mathcal{S}$.

The product σ-algebra $\mathcal{F}_1 \otimes \mathcal{F}_2 = \sigma(\mathcal{S})$, we can show that $\tilde{\mu}$ is σ-additive, then we can uniquely extend it to a measure on $\mathcal{F}_1 \otimes \mathcal{F}_2$ by Caratheodory's theorem.

Proof. We show that for any $A \in \mathcal{F}_1 \otimes \mathcal{F}_2$, the *section* of A at x: $A_x := \{y \in \Omega_2 : (x, y) \in A\}$ is in $\mathcal{F}_2$. □

Countable Product Measure

Fubini-Tonelli

1.1.17 **Fubini-Tonelli**. *Suppose we have a product space,*

1.1.6 Convergence Concepts

1.1.18 **Fatou's Lemma.** *Suppose f_n is measurable on Ω, then we have the following inequality:*

$$\int \liminf f_n \leq \liminf \int f_n$$

Proof. Based on Monotone Convergence. Because for each fixed n and $m \geq n$, $\inf_{m\geq n} f_m \leq f_m$, we have $\int \inf_{m\geq n} f_m \leq \int f_m$, then $\int \inf_{m\geq n} f_m \leq \inf_{m\geq n} \int f_m$. The integrand on the left is an increasing sequence of functions, we apply MCT and get the result. ☐

1.1.19 **Dominated Convergence.** *Suppose we have measurable $f_n \to f$ a.e., and a function $g \in L^1$ such that $|f| < g$ a.e., then $\lim \int f_n = \int \lim f_n$.*

Proof. Because $|f_n| < g$,we know that $|f| < g$ as well and hence integrable. Apply Fatou's Lemma twice. $|f_n - g + g - f| \leq |f_n - g| + |g - f|$. We know that $\left|\int f_n - f\right| \leq \int |f_n - f|$, which is integrable because ☐

1.1.20 **Proposition (Defect version of Fatou's Lemma).** *From Tao Exercise 1.4.48, which I find interesting. $f_n \to f$ where f_n, f are all integrable. Show that*

$$\int f_n - \int f - \|f_n - f\|_{L^1} \to 0$$

Proof. Apply dominated convergence to $\min(f_n, f)$. ☐

1.1.21 **Proposition (Uniqueness of Limits).** *For different convergence modes, they agree on the limit, if exists, a.e.*

Uniform integrability is a substitute for *dominance condition*. A sequence of $f_n \in L^1$ is uniformly integrable if

1. $\sup_n \|f_n\|_{L^1} < \infty$;

2. $\sup_n \|f_n \mathbf{1}(|f_n| > M)\| \to 0$ as $M \to \infty$;

3. $\sup_n \|f_n \mathbf{1}(|f_n| \le \delta)\| \to 0$ as $\delta \to 0$.

For functions $f_n : E \to \mathbb{R}$ there are a lot of ways to define $f_n \to f$. $f_n \to f$ *pointwise* if for each $x \in E$, $f_n(x)$ as a sequence of number converges to $f(x)$. $f_n \rightrightarrows f$ if for all x, the convergence rate is bounded from below.

With E equipped with a measure, we can have a relaxation of *uniform convergence*. $f_n \to f$ *almost uniformly* if for any $\varepsilon > 0$, there exists a $N_\varepsilon \subset \mathcal{E}$ with measure $< \varepsilon$ such that on $E \setminus N_\varepsilon$, $f_n \rightrightarrows f$. We have the following

1.1.22 Egoroff's Theorem. *Suppose $f_n \to f$ a.e. on a finite measure space, then $f_n \to f$ almost uniformly.*

Proof. Suppose $E_{m,n} := \{x : |f_n(x) - f(x)| > \frac{1}{m}\}$. We know for fixed m, $E_{m,n} \to \varnothing$ a.e. The set $E_m(k) := \{x : \sup_{n>k} |f_n(x) - f(x)| > \frac{1}{m}\}$ □

We introduce *convergence in measure* and *convergence in distribution*. $f_n \to f$ in measure if $\mu(\|f_n - f(x)\| > \varepsilon) \to 0$ for any $\varepsilon > 0$.

Convergence in distribution has a *Skorokhod's representation* as independent random variables with corresponding distributions.

We relate the convergence of sets to the sum of measures.

1.1.23 Borel-Cantelli Lemma. *Let $E_1, E_2, \ldots, E_3$ be a sequence of $\mathcal{B}$-measurable sets such that $\sum_n \mu(E_n) < \infty$, show that $\mu(\limsup E_n) = 0$.*

Proof. $\sum \mu(E_n) < \infty$, meaning that $\sum_{n>m} \mu E_n \to 0$ as $m \to \infty$. Then for any $\varepsilon > 0$ there exists m such that $\mu(\limsup E_n) < \mu\big(\sup_{n>m} E_n\big) < \mu\big(\sum_{n>m} E_n\big) < \varepsilon$. □

Remark. Borel-Cantelli is useful for stong law of large numbers which turns the statement of $\mu\{x : \lim |f_n - f|(x) > \varepsilon\} = 0$ to $\sum_n \mu(x : |f_n - f| > \varepsilon) < \infty$.

1.1.7 Differentiation

1.2 Independence

1.2.1 Distribution and Density

Distribution is the probability induced by a random variable X, such that $\mu(A) = P(X \in A)$. *Density* is defined via some methods(To be added). It's used to change measure, for example,

$$\int g(X)\,\mathrm{d}P = \int g(x)f(x)\,\mathrm{d}x$$

the latter being the Lebesgue integral over $\mathbb{R}$ to be defined later.

1.2.2 Independence

Independence is the first concept that has nontrivial meaning in probability other than inheriting measure theoretical names.

The independence is defined via the σ-algebras. Let $(\Omega, \mathcal{F}, P)$ be a probability space. A collection of sub-σ-algebras $(\mathcal{F}_\alpha)$ are independent if for all finite subcollection, for all $F_1, F_2, \ldots, F_n \in \mathcal{F}_1, \mathcal{F}_2, \ldots \mathcal{F}_n$

$$P(F_1F_2) = P(F_1)P(F_2)\ldots P(F_n)$$

Can we extend this concept? For countable collection of sub-σ-algebras, $(\mathcal{F}_n)$, it's *independency* if for any finite collection $F_j \in \mathcal{F}_j$ we have

$$P\left(\cap_j^J F_j\right) = \prod_j P\left(F_j\right)$$

What about uncountable, let's assume it can be defined in the same way.

There are several questions: can we find independency, how can we show that two σ-algebras are independent.

The use of independence is that if $\sigma(X_n)$ are independent, then we have for any measurable functions of X_n, then we have

$$\int f_1(X_1) f_2(X_2) \ldots f_n(X_n) \,\mathrm{d}P = \int f_1(X_1) \,\mathrm{d}P \cdots \int f_n(X_n) \,\mathrm{d}P$$

This is related to the Fubini-Tonelli theorem in that we know the joint distribution will be the product measure of each distribution.

The Kolmogorov's Extension Theorem asserts that we can find infinite sequence of independent random variables that are consistent on **nice** spaces.

1.2.3 Law of Large Numbers

1.2.4 Convergence of Random Series

1.2.5 Characteristic Functions

1.2.6 Central Limit Theorem

1.3 Conditional Expectation and Martingale

Conditional expectation is very important, it's a way to capture the dependence among random variables.

We begin with the definition of *filtration*. A filtration is a collection of sub-σ-algebra of $(\Omega, \mathcal{F}, P)$, such that

$$\mathcal{F}_n \uparrow \mathcal{F}$$

It represents the accumulation of information over time.

1.4 Further Stochastic Processes

Will be based on Dexter's Notes and Durrett.