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/* Heng Low Wee U096901R
Problem Set 2, Problem 1
```

Write a Prolog predicate definition that counts the number of occurrences of an operator inside an expression. For instance, the query:

```
?- count(a+b*c-(2+3*4)/(5*(2+a)+(b+c)^f((d-e)*(x-y))), *, C).
```

would count the number of occurrences of operator `*` in the expression given as the first argument. The return value should be bound to the variable passed as the third argument. For the query given above, the answer should be `C=4`.

```
*/
```

```
count(Expr, Op, C) :-
    (Expr =.. [Operator, LeftOperand, RightOperand]
    -> (count(LeftOperand, Op, C1),
        count(RightOperand, Op, C2),
        (Operator = Op
        -> C is C1+C2+1
        ; C is C1+C2
        ),
        !)
    ; (Expr =.. [Operator, LeftOperand],
        count(LeftOperand, Op, C1),
        (Operator = Op
        -> C is C1+1
        ; C is C1
        ),
        !)
    ).
```

```
count(Expr, _, 0) :- atomic(Expr) ; number(Expr).
```

```
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Problem Set 2, Problem 2
```

Consider the language of arithmetic expressions with the operators `+`, `-`, `*`, and `/`, where the operands are either numeric constants, or Prolog atoms denoting mathematical variables. Thus, the Prolog term:

$(x+2)*(a-x)$

stands for the mathematical function  $f(x)=(x+2)(a-x)$ , where  $a$  would be symbolic constant. Write a Prolog program that computes the derivative of such an expression. For instance, the query:

```
?- derive((x+2)*(a-x),x,D).
```

should derive the expression given as the first argument with respect to the variable given as the second argument. The result should be placed in the variable given as the third argument. For the query above, the answer should be

$$C = (1+0)*(a-x)+(x+2)*(0-1)$$

Do not perform any arithmetic simplification in your solution to this problem. Simplification is the topic of the next problem.

\*/

```
derive(Expr, Wrt, Result) :-
    Expr =.. [Operator, Left, Right],
    derive(Left, Wrt, R1),
    derive(Right, Wrt, R2),
    d(Operator, Left, Right, R1, R2, Result), !.

% Base cases
derive(Expr, _, 0) :- number(Expr), !.
derive(Expr*Wrt, Wrt, Expr) :- number(Expr), !.
derive(Wrt*Expr, Wrt, Expr) :- number(Expr), !.
derive(Expr, Wrt, 0) :- Expr \= Wrt, !.
derive(Expr, Wrt, 1) :- Expr = Wrt, !.

d(+, _, _, DL, DR, Result1) :- Result1 = DL + DR, !.
d(-, _, _, DL, DR, Result1) :- Result1 = DL - DR, !.
d(*, L, R, DL, DR, Result1) :- Result1 = DL * R + L * DR, !.
d(/, L, R, DL, DR, Result1) :- Result1 = (DL * R - L * DR)/(R*R), !.
```

Write a Prolog program that performs arithmetic expression simplification. Your program should at least eliminate multiplications by 0 and 1, and additions with 0. Ideally, it should also convert any subexpression containing only constants into the value of that expression, and convert multiplications with -1 into the negative of the multiplicand. For instance, the expression

$$(1+0)*(a-x)+(x+2)*(0-1)$$

could be simplified into:

$$a-x-(x+2)$$

Further simplifications may include distribution of high-precedence operators, and grouping of terms that contain the same variable. The expression above could be further simplified into:

$$a-2*x-2$$

READ HERE:

Sample Run...

```
simplify((1+0)*(a-x)+(x+2)*(0-1), C).
```

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```
simplify(X, Result) :- number(X), Result is X, !.
```

```
simplify(X, Result) :- atom(X), Result = X.
```

```
simplify(Expr, Result) :-
```

```
    (Expr =.. [Operator, Left, Right]
```

```
    -> (simplify(Left, R1),
        simplify(Right, R2),
        d1(Operator, R1, R2, Result))
```

```
    ; (Expr =.. [Operator, Right],
        simplify(Right, R3),
        d2(Operator, R3, Result))
```

```
),
```

```
!.
```

```
% Additions with numbers
```

```
d1(+, DL, DR, Result1) :- number(DL), number(DR), Result1 is DL+DR, !.
```

```
% Subtraction with numbers
```

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```
d1(-, DL, DR, Result1) :- number(DL), number(DR), Result1 is DL-DR, !.  
% Multiplication with numbers  
d1(*, DL, DR, Result1) :- number(DL), number(DR), Result1 is DL*DR, !.  
% Division with numbers  
d1(/, DL, DR, Result1) :- number(DL), number(DR), Result1 is DL/DR, !.  
  
% Multiplication/Division by 0  
d1(*, 0, _, Result1) :- Result1 is 0, !.  
d1(*, _, 0, Result1) :- Result1 is 0, !.  
d1(/, 0, _, Result1) :- Result1 is 0, !.  
d1(/, _, 0, Result1) :- Result1 = infinity, !.  
  
% Multiplication/Division by 1  
d1(*, 1, DR, Result1) :- Result1 = DR, !.  
d1(*, DL, 1, Result1) :- Result1 = DL, !.  
d1(/, 1, DR, Result1) :- (number(DR) -> Result1 is 1/DR ; Result1 = 1/DR), !.  
d1(/, DL, 1, Result1) :- Result1 = DL, !.  
  
% Multiplication/Division by -1  
d1(*, -1, DR, Result1) :- Result1 = -DR, !.  
d1(*, DL, -1, Result1) :- Result1 = -DL, !.  
d1(/, -1, DR, Result1) :- (number(DR) -> Result1 is -1/DR; Result1 = -1/DR),  
!.  
d1(/, DL, -1, Result1) :- Result1 = -DL, !.  
  
% Additions with 0  
d1(+, 0, DR, Result1) :- Result1 = DR, !.  
d1(+, DL, 0, Result1) :- Result1 = DL, !.  
% Subtraction with 0  
d1(-, 0, DR, Result1) :- Result1 = -DR, !.  
d1(-, DL, 0, Result1) :- Result1 = DL, !.  
  
% Additions with variables  
d1(+, DL, DR, Result1) :- DR =.. [-, Tail], Result1 = DL-Tail, !.  
d1(+, DL, DR, Result1) :- DR =.. [_, _, _], Result1 = DL+DR, !.  
d1(+, DL, DR, Result1) :- DL=DR, Result1 = 2*DL, !.  
d1(+, DL, DR, Result1) :- Result1 = DL+DR, !.  
  
% Subtraction with variables  
d1(-, DL, DR, Result1) :- DR =.. [-, Tail], Result1 = DL+Tail, !.  
d1(-, DL, DR, Result1) :- DR =.. [_, _, _], Result1 = DL-DR, !.  
d1(-, DL, DR, Result1) :- DL=DR, Result1 is 0, !.  
d1(-, DL, DR, Result1) :- Result1 = DL-DR, !.  
  
% Multiplication with variables
```

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```
d1(*, DL, DR, Result1) :- ((number(DL),atom(DR)) -> Result1 = DL*DR ; Result1
= DR*DL), !.
```

```
% Division with variables
```

```
d1(/, DL, DR, Result2) :- atom(DL), number(DR), Result2 = Result1 + DL,
Result1 is 1/DR, !.
```

```
d1(/, DL, DR, Result1) :- (DL=DR -> Result1 is 1 ; Result1 = DL/DR), !.
```

```
d2(-, Term, Result1) :- number(Term), Result1 is -Term, !.
```

```
d2(-, Term, Result1) :- Result1 = -Term, !.
```