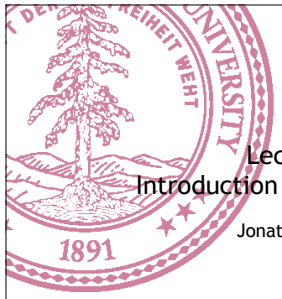


Lecture 2: Introduction to Segmentation

Jonathan Krause

Goal

- Goal: Identify groups of pixels that go together

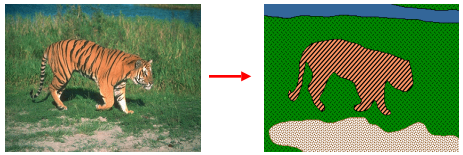


image credit: Steve Seitz, Kristen Grauman

Types of Segmentation

- Semantic Segmentation: Assign labels

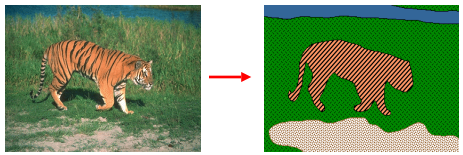


image credit: Steve Seitz, Kristen Grauman

We're going to learn a number of techniques that are useful for semantic segmentation, but will focus on techniques that are more generally applicable to several types of segmentation problems. For example, most semantic segmentation methods will learn appearance models for each class, which is not something we're going to talk about much.

Types of Segmentation

- Figure-ground segmentation: Foreground/background

image credit: Carsten Rother

This is the type of segmentation done by GrabCut, the focus of project 1.

Types of Segmentation

- Co-segmentation: Segment common object

image credit: Armand Joulin

Co-segmentation is a relatively recent line of work in segmentation. Historically, it started with segmenting the same object instance in multiple images, but eventually grew to segmenting out instances of the same category.

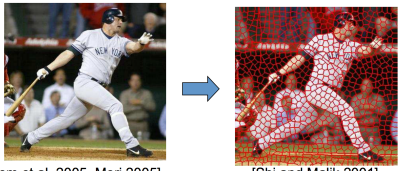
Application: As a result

Rother et al. 2004

Useful in graphics applications, e.g. image matting

Application: Speed up Recognition

[Felzenszwalb and Huttenlocher 2004]



[Hoiem et al. 2005, Mori 2005]

[Shi and Malik 2001]

Slide: Derek Hoiem

Superpixels (also see last slide) make computation faster. Region proposals speed up detection.

Application: Better Classification

Angelova and Zhu, 2013

By getting rid of the background we can remove irrelevant information

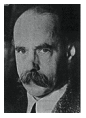
History: Before Computer Vision

Gestalt Theory

- Gestalt: whole or group
  - Whole is greater than sum of its parts
  - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

Max Wertheimer
(1880-1943)



Gestalt Factors

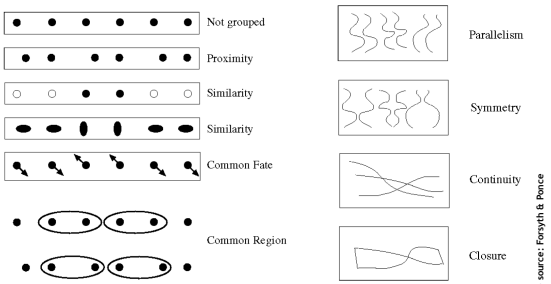


Image source: Forsyth & Ponce

- These factors make intuitive sense, but are very difficult to translate into algorithms.

Outline

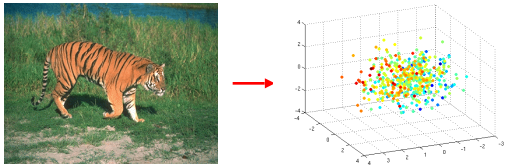
1. Segmentation as clustering
2. Graph-based segmentation — CS 131 review
3. Segmentation as energy minimization — new stuff

Outline

1. Segmentation as clustering
   1. K-Means
   2. GMMs and EM
   3. Mean Shift
2. Graph-based segmentation
3. Segmentation as energy minimization

Segmentation as Clustering

- Pixels are points in a high-dimensional space
  - color: 3d
  - color + location: 5d
- Cluster pixels into segments



Clustering isn't used as a segmentation approach too much anymore, but highlights many of the key ideas still used in modern algorithms in terms of modeling appearance.

Clustering: K-Means

Algorithm:

1. Randomly initialize the cluster centers, $c_1, \ldots, c_K$
2. Given cluster centers, determine points in each cluster
   - For each point p, find the closest c_i. Put p into cluster i
3. Given points in each cluster, solve for c_i
   - Set c_i to be the mean of points in cluster i
4. If c_i have changed, repeat Step 2

- Properties
  - Will always converge to some solution
  - Can be a "local minimum"
    - Does not always find the global minimum of objective function:

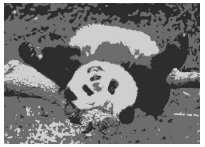
$$\sum_{i=1}^{k} \sum_{x \in C_i} \|x - c_i\|^2$$

slide credit: Steve Seitz

Clustering: K-Means

k=2

k=3



slide credit: Kristen Grauman

Clustering: K-Means

Note: Visualize segment with average color

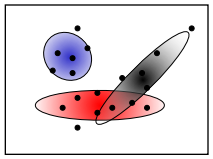
K-Means

Pro:

- Extremely simple
- Efficient

Con:

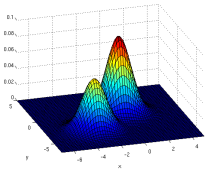
- Hard quantization in clusters
- Can't handle non-spherical clusters



Gaussian Mixture Model

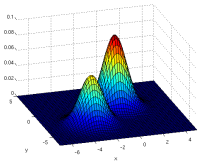
- Represent data distribution as mixture of multivariate Gaussians.

$$P(x) = \sum_{i=1}^{K} \pi_i \cdot pdf(x; \mu_i, \Sigma_i)$$



How do we actually fit this distribution?

Expectation Maximization (EM)



- Goal
  - Find parameters θ (for GMMs: π_i, μ_i, Σ_i) that maximize the likelihood function:

$$P(data; \theta) = \prod_{i=1}^{N} P(x_i; \theta)$$

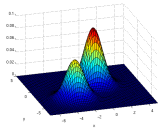
- Approach:
  1. E-step: given current parameters, compute ownership of each point
  2. M-step: given ownership probabilities, update parameters to maximize likelihood function
  3. Repeat until convergence

See CS229 material if this is unfamiliar!

CS229's treatment of EM is quite nice. Look it up if you don't know this!

Clustering: Expectation Maximization (EM)

GMMs



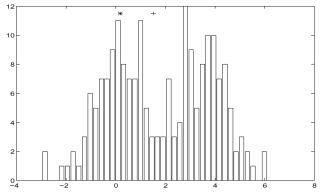
Pro:

- Still fairly simple and efficient
- Model more complex distributions

Con:

- Need to know number of components in advance — hard to know unless you're looking at the data yourself!

Clustering: Mean-shift



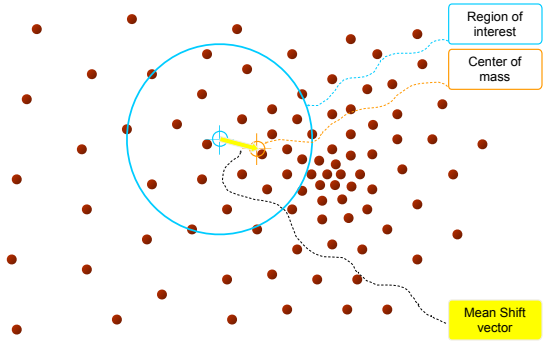
1. Initialize random seed, and window W
2. Calculate center of gravity (the "mean") of W: $\frac{1}{|W|}\sum_{x \in W} x$
   - Can generalize to arbitrary windows/kernels
3. Shift the search window to the mean
4. Repeat Step 2 until convergence

Only parameter: window size

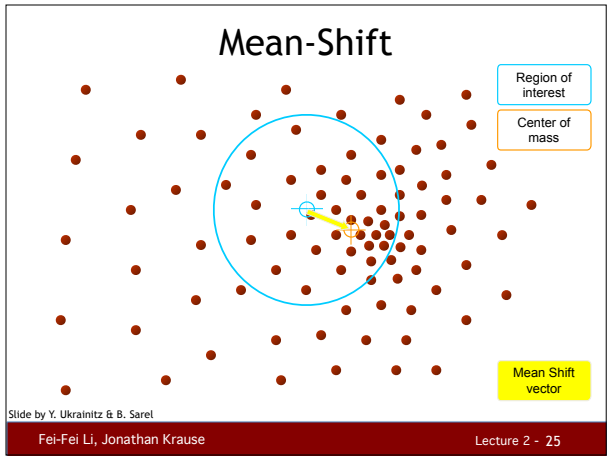
slide credit: Steve Seitz

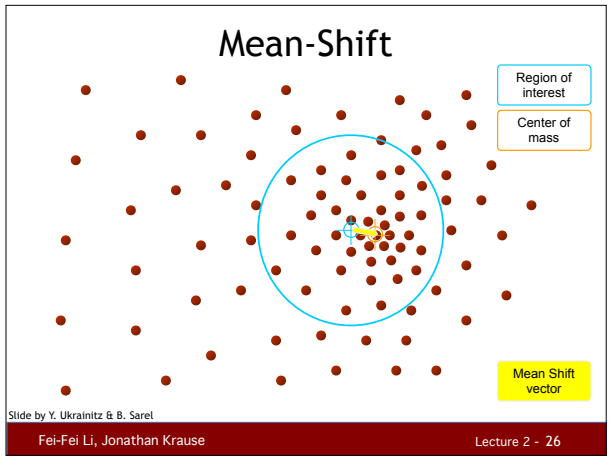
The original mean shift paper generalizes all of this.

Mean-Shift

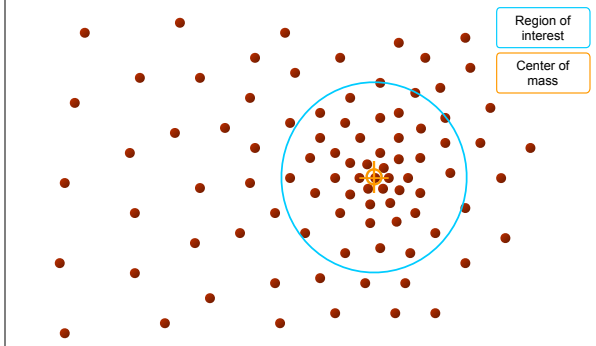


Slide by Y. Ukrainitz & B. Sarel


Mean-Shift
Region of interest
Center of mass
Mean Shift vector
Slide by Y. Ukrainitz & B. Sarel
Fei-Fei Li, Jonathan Krause
Lecture 2 - 25


Mean-Shift
Region of interest
Center of mass
Mean Shift vector
Slide by Y. Ukrainitz & B. Sarel
Fei-Fei Li, Jonathan Krause
Lecture 2 - 26

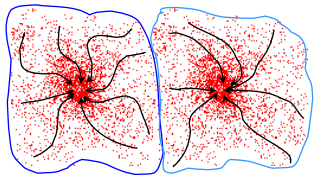
Mean-Shift



Slide by Y. Ukrainitz & B. Sarel

Clustering: Mean-shift

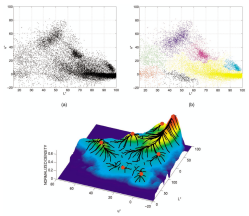
- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode



slide credit: Y. Ukrainitz & B. Sarel

Mean-shift for segmentation

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same “peak” or mode

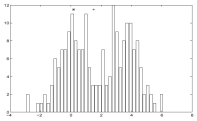


Mean-shift for segmentation

Mean Shift

Pro:

- No number of clusters assumption
- Handle unusual distributions
- Simple



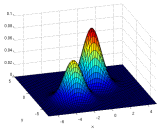
Con:

- Choice of window size
- Can be somewhat expensive

Clustering

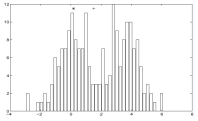
Pro:

- Generally simple
- Can handle most data distributions with sufficient effort.



Con:

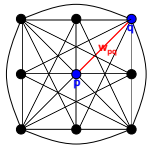
- Hard to capture global structure
- Performance is limited by simplicity



Outline

1. Segmentation as clustering
2. Graph-based segmentation
   1. General Properties
   2. Spectral Clustering
   3. Min Cuts
   4. Normalized Cuts
3. Segmentation as energy minimization

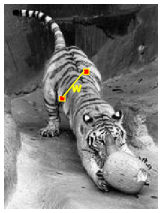
Images as Graphs



- Node (vertex) for every pixel
- Edge between pairs of pixels, (p,q)
- Affinity weight w_{pq} for each edge
  - w_{pq} measures similarity
  - Similarity is inversely proportional to difference (in color and position…)

slide credit: Steve Seitz

Images as Graphs



Which edges to include?

Fully connected:
- Captures all pairwise similarities
- Infeasible for most images

Neighboring pixels:
- Very fast to compute
- Only captures very local interactions

Local neighborhood:
- Reasonably fast, graph still very sparse
- Good tradeoff

Measuring Affinity

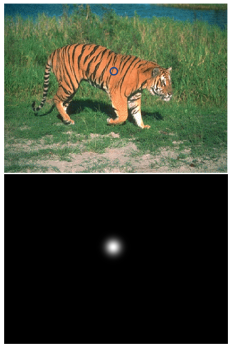
- In general: $aff(x, y) = \exp\left(-\frac{1}{2\sigma_d^2}\|f(x) - f(y)\|^2\right)$
- Examples:
  - Distance: $f(x) = location(x)$
  - Intensity: $f(x) = intensity(x)$
  - Color: $f(x) = color(x)$
  - Texture: $f(x) = filterbank(x)$
- Note: Can also modify distance metric

slide credit: Forsyth & Ponce

Measuring Affinity

Distance:

$f(x) = location(x)$

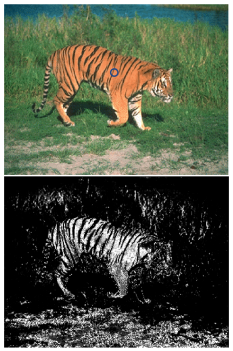


slide credit: Forsyth & Ponce

Measuring Affinity

Intensity:

$f(x) = intensity(x)$

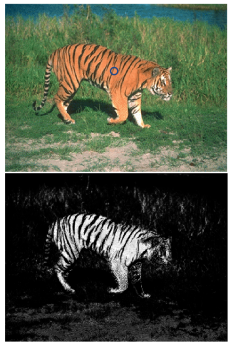


slide credit: Forsyth & Ponce

Measuring Affinity

Color:

$f(x) = color(x)$

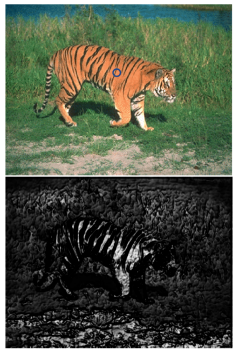


slide credit: Forsyth & Ponce

Measuring Affinity

Texture:

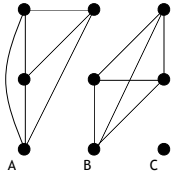
$f(x) = filterbank(x)$



slide credit: Forsyth & Ponce

In practice you'd combine several of these affinity measures in one.

Segmentation as Graph Cuts



- Break Graph into Segments
  - Delete links that cross between segments
  - Easiest to break links that have low similarity (low weight)
    - Similar pixels should be in the same segments
    - Dissimilar pixels should be in different segments

slide credit: Steve Seitz

Graph Cut with Eigenvalues

- Given: Affinity matrix *W*
- Goal: Extract a single good cluster *v*
  - *v(i)*: score for point *i* for cluster *v*

$$\max_{v} \; v^T W v$$
$$\text{s.t.} \; v^T v = 1$$

Optimizing

$$\max_v \ v^T W v \quad \text{s.t.} \ v^T v = 1 \quad \longleftrightarrow \quad \min_v \ -\tfrac{1}{2} v^T W v \quad \text{s.t} \quad v^T v = 1$$

Lagrangian: $-\frac{1}{2} v^T W v + \lambda(v^T v - 1)$

$$-Wv + \lambda v = 0$$

$$Wv = \lambda v$$

v is an eigenvector of *W*

See EE364 (Convex Optimization) to learn more about formulating and solving optimization problems.

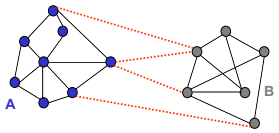
Clustering via Eigenvalues

1. Construct affinity matrix *W*
2. Compute eigenvalues and vectors of *W*
3. Until done
   1. Take eigenvector of largest unprocessed eigenvalue
   2. Zero all components of elements that have already been clustered
   3. Threshold remaining components to determine cluster membership

Note: This is an example of a *spectral clustering* algorithm

The spectrum of a matrix is its set of eigenvalues.

Graph Cuts - Another Look



- Set of edges whose removal makes a graph disconnected
- Cost of a cut
  - Sum of weights of cut edges: $cut(A, B) = \sum_{p \in A, q \in B} w_{pq}$
- A graph cut gives us a segmentation
  - What is a "good" graph cut and how do we find one?

slide credit: Steve Seitz

The key here is that we want to be more precise about what we mean by a "good" graph cut. This lets us investigate new formulations.

Formulation: Min Cut

- We can do segmentation by finding the *minimum cut*
  - either smallest number of elements (unweighted) or smallest sum of weights (weighted)
  - efficient algorithms exist
- Drawback
  - Weight of cut proportional to number of edges
  - Biased towards cutting small, isolated components

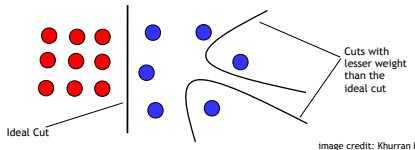


image credit: Khurran Hassan-Shafique

Formulation: Normalized Cuts

- Key idea: normalize segment size
  - Fixes min cut's bias
- Formulation:

$$Ncut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

$$= cut(A,B)\left[\frac{1}{\sum_{p\in A} w_{p,q}} + \frac{1}{\sum_{q\in B} w_{p,q}}\right]$$

$assoc(A,V)$ = sum of weights of edges in *V* that touch *A*

- NP-hard, but can approximate

J. Shi and J. Malik. Normalized cuts and image segmentation. PAMI 2000

This is one of the biggest contributions of computer vision to the rest of computer science — normalized cuts is a bona fide contribution to theoretical CS.

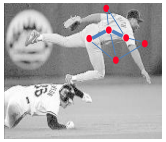
NCuts as Generalized Eigenvector Problem

Definitions:

W : **affinity** matrix

D : **diagonal** matrix $D(i,i) = \sum_j w_{i,j}$

z : **vector** in $\{-1,1\}^N, z_i = 1 \Leftrightarrow i \in A$



In matrix form:

$$NCut(A,B) = \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(A,B)}{assoc(B,V)}$$

$$= \frac{(1+z)^T(D-W)(1+z)}{k1^T D1} + \frac{(1-z)(D-W)(1-z)}{(1-k)1^T D1}; \quad k = \frac{\sum_{z_i>0} D(i,i)}{\sum_i D(i,i)}$$

$$= \ldots$$

Slide credit: Jitendra Malik

After a lot of math…

- After simplification, we get

$$NCut(A,B) = \frac{y^T(D-W)y}{y^TDy},\ y_i \in \{1,-b\},\ y^TD1 = 0$$

This is hard, y is discrete!

- This is a Rayleigh Quotient
  - Solution given by the "generalized" eigenvalue problem

$$(D-W)y = \lambda Dy$$

Relaxation: continuous y

- Subtleties
  - Optimal solution is second smallest eigenvector
  - Gives continuous result—must convert into discrete values of y

Slide credit: Alyosha Efros

Another spectral clustering algorithm! But we typically simply call this one Normalized Cuts.

NCuts example

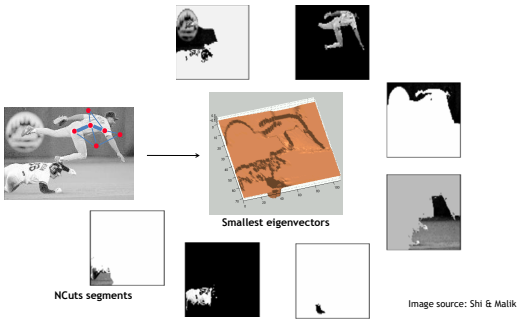


Image source: Shi & Malik

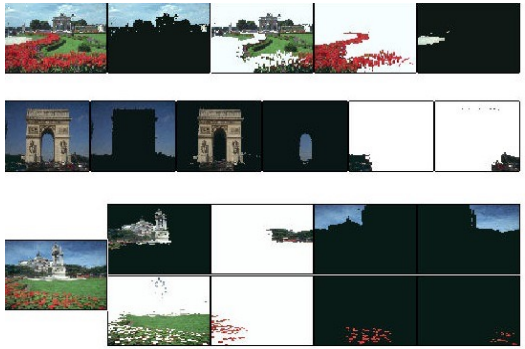
NCuts: Algorithm Summary

1. Construct weighted graph $G = (V, E)$
2. Construct affinity matrix W
3. Solve $(D - W)y = \lambda Dy$ for smallest few eigenvectors.
   - This is a continuous solution
4. Threshold eigenvectors to get a discrete cut
   - This is the approximation
   - As before, several heuristics for doing this
5. Recursively subdivide as desired.

If you want k clusters, one common approach is to take the k smallest eigenvectors. Another common approach is to always use the smallest eigenvector (i.e. do a single cut), and then recursively subdivide until you have k clusters.

Normalized cuts has other uses besides image segmentation, too — can use for arbitrary clustering!

NCuts examples



NCuts examples

NCuts Pro and Con

- Pro
  - Flexible to choice of affinity matrix
  - Generally works better than other methods we've seen so far
- Con
  - Can be expensive, especially with many cuts.
  - Bias toward balanced partitions
  - Constrained by affinity matrix model

Slide source: Kristen Grauman

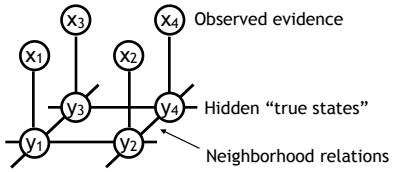
Normalized cuts is still in use today — a number of region proposal methods (which we'll see when we get to object detection) use normalized cuts.

Outline

1. Segmentation as clustering
2. Graph-based segmentation
3. Segmentation as energy minimization
   1. MRFs + CRFs
   2. Segmentation with CRFs
   3. GrabCut

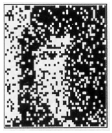
Conditional Random Fields (CRFs)

- Rich probabilistic model for images
- Built in local, modular way
  - Get global effects from only learning/modeling local ones
- After conditioning, get a Markov Random Field (MRF)



Pixels as CRF Nodes

Original image



Degraded image

Reconstruction from MRF modeling pixel neighborhood statistics

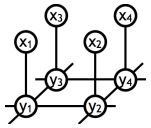


Image source: Bastian Liebe

In this case, the degraded image would correspond to the observed variables, and the reconstruction corresponds to getting the MAP (maximum a posteriori) assignment of the CRF with respect to the x variables.

CRF Probability

$$P(x,y) = \frac{1}{Z}\prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(y_i, y_j)$$

Image

Scene

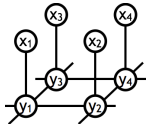
Partition Function

Image-scene compatibility function

Local observations

Scene-scene compatibility function

Neighboring scene nodes



For MAP problems we don't care about the partition function. Fortunately for us, we typically are doing MAP problems.

Energy Formulation

$$P(x, y) = \frac{1}{Z} \prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(y_i, y_j)$$

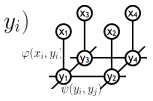
take logs, drop Z

$$E(x, y) = \sum_i \varphi(x_i, y_i) + \sum_{i,j} \psi(y_i, y_i)$$

- We call E an *energy function*
  - named from free-energy problems in statistical mechanics
- Individual terms are *potentials*
- *Note: Derived this way, it's energy maximization. Be careful and check each formulation individually.*

Energy Formulation

$$E(x, y) = \sum_i \varphi(x_i, y_i) + \sum_{i,j} \psi(y_i, y_i)$$



- Unary potentials φ
  - Local information about each pixel
  - e.g. how likely a pixel/patch belongs to a certain class
- Pairwise potentials ψ
  - Neighborhood information, enforces consistency
  - e.g. how different a pixel is from its neighbor in appearance

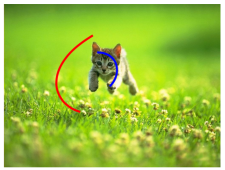
slide credit: Bastian Liebe

CRF segmentation example

- Boykov and Jolly (2001)

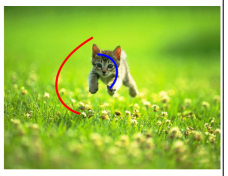
$$E(x,y) = \sum_i \varphi(x_i, y_i) + \sum_{i,j} \psi(y_i, y_i)$$

- Variables
  - x_i : Annotation (Input)
    - Foreground/background/empty
  - y_i : Binary variable
    - Foreground/background
- Unary term
  - $\varphi(x_i, y_i) = K[x_i \neq y_i]$
  - Penalty for disregarding annotation
- Pairwise term
  - $\psi(y_i, y_j) = [y_i \neq y_j] w_{ij}$
  - Encourage smooth annotations
  - w_{ij} is affinity between pixels *i* and *j*



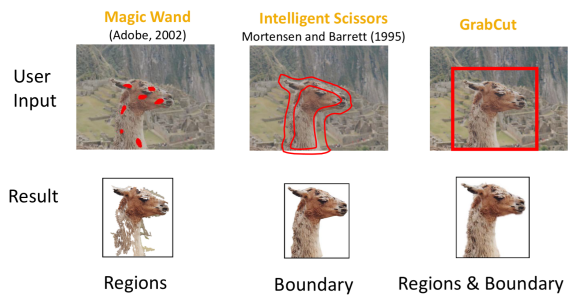
Solving Efficiently

- Grid structured random fields
  - Maxflow/mincut
  - Optimal for binary labeling
    - submodular energy functions
  - Boykov & Kolmogorov, "An Experimental Comparison of Min-Cut/Max-Flow Algorithms for Energy Minimization in Vision", PAMI 2004
- Fully connected models
  - Efficient solution with convolution mean-field
  - Krähenbühl and Koltun, "Efficient Inference in Fully-Connected CRFs with Gaussian Edge Potentials", NIPS 2011



submodular energy functions are pairwise energy functions e(x1,x2) such that e(0,0) + e(1,1) <= e(0,1) + e(1,0)

GrabCut: Interactive Foreground Extraction



The key idea here is that with it's easy to draw a bounding box.
Outside the box, pixels are marked as "definitely background".

GrabCut formulation

$$E(x, y, \theta, k) = \sum_i \varphi(x_i, y_i, \theta, k_i) + \sum_{ij} \psi(y_i, y_j, x_i, x_j)$$

- Variables
  - x_i: pixel
  - $y_i \in \{0,1\}$: foreground/background label {0,1}
  - $k_i \in \{0, \ldots, K\text{-}1\}$: GMM mixture component
  - θ: GMM model parameters
  - $I = \{z_1, \ldots, z_m\}$: RGB image
- Unary Term $\varphi(x_i, y_i, \theta, k_i)$
  - -log of GMM probability
- Pairwise Term

$$\psi(y_i, y_j, x_i, x_j) = \gamma[y_i \neq y_j] \exp\left(-\beta \|x_i - x_j\|^2\right)$$

GMMs are typically in RGB colorspace.
A common number of components is K=5

GrabCut - Iterative Optimization

1. Initialize Mixture Models based on user annotation
2. Assign GMM components

$$k_i = \arg\min \varphi(x_i, y_i, \theta, k_i)$$

3. Learn GMM parameters

$$\theta = \arg\min \sum_i \varphi(x_i, y_i, \theta, k_i)$$

4. Estimate segmentations (mincut)

$$y = \arg\min E(x, y, \theta, k)$$

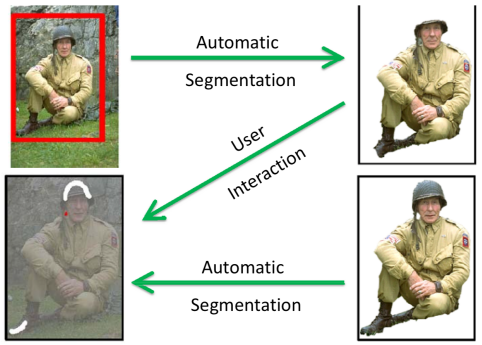
5. Repeat 2-4 until convergence

5 iterations is normally sufficient, and there's typically no reason to do more than 10

GrabCut results

Middle case shows that it doesn't have to be a bounding box if that's not convenient

Further editing with GrabCut



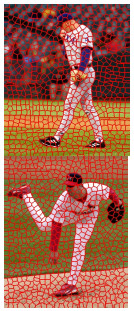
Another easy generalization.

Summary: Graph Cuts with CRFs

- Pros
  - Very powerful, get global results by defining local interactions
  - Very general
  - Rather efficient
  - Becoming more or less standard for many segmentation problems (GrabCut was 2004!)
- Cons
  - Only works for sub modular energy functions (binary)
  - Only approximate algorithms work for multi-label case

Extra: Improving Segmentation Efficiency

- Images contain a lot of pixels
  - Even efficient methods can be slow
- Efficiency trick: Superpixels
  - Group together similar pixels
  - Cheap and local over segmentation
  - Must be high precision!
  - Many methods exist, try several.
- Another trick: Resize the image
  - Do segmentation in lower-res version, then scale back up to high-res.



Felzenszwalb and Huttenlocher (2004) is a common super pixel method, as is SLIC.

Takes a little bit of work to get the energy functions right when you have multiple pixels and coding the optimization to only be over superpixels instead of pixels, but the speedups can be rather dramatic.

Additional Reading

- CS 131/CS231a slides (all)
- CS 229 (clustering, EM)
- CS 228/228T (MRFs, CRFs, energy minimization)
- EE 364 (optimization)