Analysis of Positional Competition in the Boundary Contour System*

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Abstract

This work builds upon the proposal of the Boundary Contour System proposed in [GrossbergMingolla85] and [GrossbergMingolla87] as a subsystem of the FACADE architecture introduced in [Grossberg90]. We analyse the positional sharpening capabilities of the competitive layers in the competitive cooperative loop. Positional sharpening as set up by Grossberg and Mingolla may be seen as a two-stage process, where a threshold-linear signal function transforms the input and drives the subsequent positional competition stage. We draw the conclusion that positional sharpening is enforced by thresholding rather than by positional competition.

1 Introduction and Motivation

A large part of human and mammal brains is dedicated to visual perception and the combination of locally ambiguous visual information into a globally consistent and unambiguous representation of the visual environment. So the question arises, how multiple sources of visual information like texture, stereo, and motion cooperate to generate a visual percept.

One of the key ideas for the development of the FACADE by Stephen Grossberg and coworkers is the conviction that modular approaches with special purpose procedures are not very useful for an understanding of real-world vision. The working hypothesis of the FACADE architecture is, that every stage of visual processing multiplexes together several key properties of the scenic representation.

The FACADE architecture aims at an explanation of how the visual system is able to detect relatively invariant surface colours under variable illumination conditions, to detect relatively invariant object boundaries under occlusion conditions, and to recognize familar objects or events in the environment. More elaborated versions of the FACADE architecture include processing of stereo and motion. We deal here with the simplified variant defined in [Grossberg90], where the Feature Contour System (FCS), Boundary Contour System (BCS), and the Object Recognition System (ORS) are designed in correspondence to the above processing goals of the overal architecture.

Processing of the visual input in retina and LGN relies on local measurements which introduce some amount of uncertainty into the representation at the corresponding processing stages. The FACADE architecture sets up parallel and hierarchical interaction schemes that can resolve these uncertainties by means of several processing stages.

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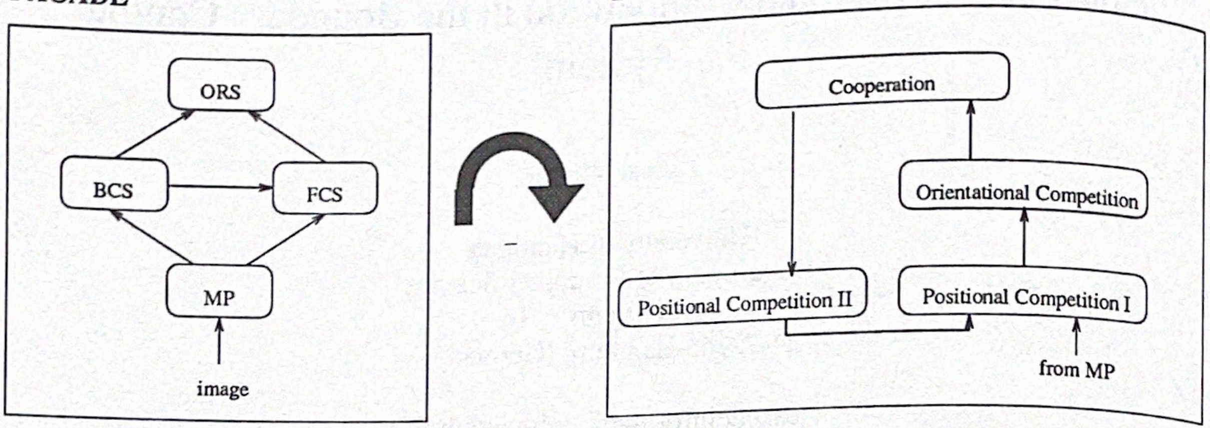
The following diagramm visualizes the dataflow bewteen the subsystems of the FACADE architecture and within the Boundary Contour System and its competitive and cooperative layers.

FACADE **Boundary Contour System**



The MP stage defines oriented receptive fields for each perceptual location which are sensitive to local contrast in the intensity function in accordance to the hypercolumn model of the primary visual cortex by Hubel and Wiesel. The local contrast detectors feed into the competitive-cooperative loop of the Boundary Contour System which generates an emergent segmentation of boundaries in the scene by means of spatially short-range competitive interactions and spatially long-range cooperative interactions. Two successive stages of spatially short-range competitive interactions feed into a cooperative stage with in turn feeds back inbtwo the first competitive stage via an intermediate competitive stage. Preattentive processing in the Boundary Contour System does not rely on memorized templates or expectancies but is purely data-driven. Emergent segmentation provides a way to generate boundaries which have no direct physical correlate in the intensity function but are perceived by human subjects in psychophysical experiments. The Kanizsa square and the Ehrenstein illusion are prominent examples used in those experiments. The proposal of the Boundary Contour System aims at an explanation of how those *illusory contours* are generated by the visual system.

In the first competitive layer a cell of prescribed orientation excites like oriented cells at the same location and inhibits like-oriented cells at nearby positions. This on-center off-surround organization of like-oriented cells exists around every perceptual location. In the second competitive layer cells compete that represent different orientations — notably perpendicular ones — at the same perceptual location. An additional net effect of the first and second competitive layer is to generate end-cuts. The competitive layers feed into the cooperative stage which defines spatially long-range interactions for boundary completion. The output of the cooperative stage is enhanced by a further competitive layer and fed back into the first competitive layer. This feedback allows for the discontinuous completion of continuous boundaries. (For details see [GrossbergMingolla85],[GrossbergMingolla87].)

2 Positional Sharpening and Positional Competition

In [GrossbergMingolla85] and [GrossbergMingolla87] the authors define the following process for the second positional competition layer to realize the postulate of positional sharpening

$$v_{ijk} = \frac{h(z_{ijk})}{1 + \sum_{(p,q)} h(z_{pqk}) \cdot W_{pqij}}$$

with a circular inhibitory weighting kernel and a threshold-linear signal function.

$$W_{pqij} = \begin{cases} W & \text{if} \quad (p-i)^2 + (q-j)^2 \leq W_0^2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad h(z) = L \cdot [z - M]^+$$

The variables z and v denote the output of the cooperative stage and the subsequent competitive stage. The incdices i,j, and k have the following meaning. The pair $< i, j >$ indexes a perceptual location whereas k is an index to an orientation band. Since there are no interactions between different orientation bands, we will leave out the index k in our further considerations. The definition of the first competitive stage is quite similar and uses the same mechanism, but is not so well suited for an analysis of positional sharpening since it mixes two inputs.

> "Functionall, the $z_{ijk} \rightarrow v_{ijk}$ transformation enables the most favored cooperations to enhance their preferred positions and orientations as they suppress nearby positions with the same orientation."

3 Analysis of Positional Competition

For an analysis of the positional competition stage and its relation to the postulate of positional sharpening we first state that the transformation defined in the preceeding section may by seen as a two stage process, i.e. the transformation given by

$$\{\mathcal{T}(I)\}(x,y) = \frac{\{h(I)\}(x,y)}{1 + \{h(I) \star W\}(x,y)}$$

may be rewritten as the concatenation of two transformations where a point operation (thresholding) is followed by a positional competition transformation:

$$\mathcal{T}(I) = \mathcal{T}_2(\mathcal{T}_1(I)) \qquad \text{with} \qquad \{\mathcal{T}_1(I)\}(x,y) = h(I(x,y)) \qquad \text{and} \qquad \{\mathcal{T}_2(I)\}(x,y) = \frac{I(x,y)}{1 + \{I \star W\}(x,y)}$$

Before we can analyze positional sharpening we have to clarify our understanding of this notion. We want to relate two one-dimensional stimuli I_1 and I_2 where I_2 is a transformed version of stimulus I_1. We assume that I_1 and I_2 are positive unimodal functions having their maximum in $t = 0$. Defining the meaning of I_2 is *sharper* than I_1 is a debatable issue. We start with the definition of the opposite.

Definition 1: Given two positive unimodal functions I_1 und I_2 having their maximum in $t = 0$, we say that I_2 is **broader** than I_1, if

$$\frac{I_2(t)}{I_2(0)} = \overline{I_2(t)} > \overline{I_1(t)} = \frac{I_1(t)}{I_1(0)} \qquad \text{for} \qquad t \neq 0$$

•

The intuition behind this definition is that the function I_2 is something like a cheese cover of the same height for the function I_1, if I_2 is *broader* than I_1. (See the illustration after proposition 1.) One alternative for the definition of "*sharper*" is the inversion of the above inequality. Other possible definition include the reduction of the full-width-at-half-height or the reduction of the area under the normalized function. There is no need for a final decision since our further investigations will show that the transformation $\mathcal{T}_2$ as defined above will essentially broaden the input. So all of the broadening-effect of the overall transformation $\mathcal{T}$ is due to the threshold operation $\mathcal{T}_1$ for a large class of inputs and inhibitory kernels.

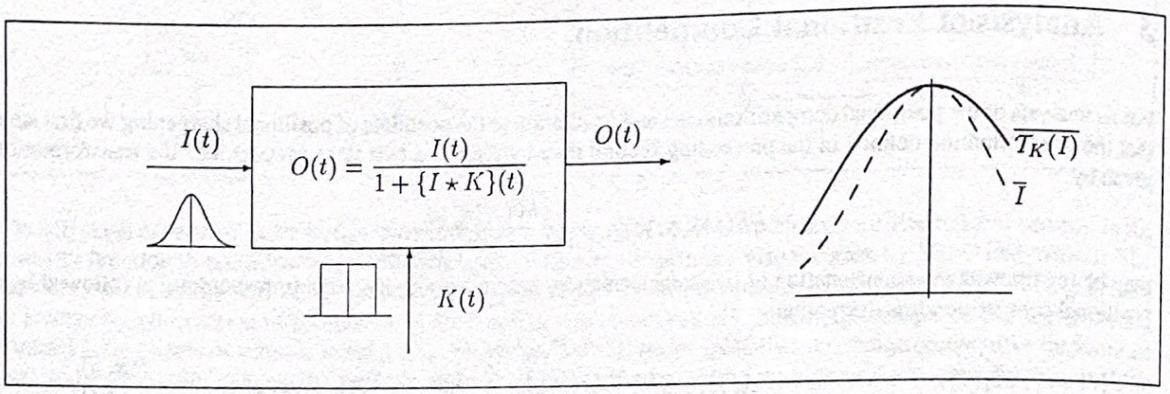
We establish our results as follows. In proposition 1 we investigate the behaviour of the transformation $\mathcal{T}_2$ for the one-dimensional case with a Gaussian input and a boxed weighting kernel. We extend this result to the two-dimensional case and study the transformation of a Gaussian activity distribution under positional competition with the inhibitory weighting kernel proposed by Grossberg and Mingolla. This extension leads us to a useful lemma which allows us to draw more general conclusions on the *broadening* behaviour for the one-dimensional and two-dimensional positional competition transformation in proposition 3 and proposition 4.

Proposition 1: Given a Gaussian input I ($\sigma > 0$) and a boxed weighting kernel K ($h > 0$ and $w > 0$)

$$I(t) = G_\sigma(t) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{t^2}{2\sigma^2}) \qquad K(t) = \begin{cases} h & |t| \le w \\ 0 & \text{otherwise} \end{cases}$$

the positional competition transformation $\mathcal{T}_K$ broadens I, i.e. $\mathcal{T}_K(I)$ is *broader* than I.

$$\{\mathcal{T}_K(f)\}(t) = \frac{f(t)}{1 + \{f \star K\}(t)}$$



Proof of Proposition 1: We have to show, that

$$\frac{\{\mathcal{T}_K(I)\}(t)}{I(t)} = \frac{1 + \{I \star K\}(0)}{1 + \{I \star K\}(t)} > 1 \quad \text{if } |t| > 0.$$

and we see that $\{I \star K\}(0) > \{I \star K\}(t)$ has to be shown for $|t| > 0$. With

$$\{I \star K\}(t) = \frac{1}{2} \cdot \left(\operatorname{erf}(\frac{t+w}{2\sqrt{2}}) - \operatorname{erf}(\frac{t-w}{2\sqrt{2}})\right)$$

we find that this function has a global maximum at $t = 0$ by differentiating

$$\frac{\partial}{\partial t}\left(\operatorname{erf}(\frac{t+w}{2\sqrt{2}}) - \operatorname{erf}(\frac{t-w}{2\sqrt{2}})\right) = G_\sigma(t+w) - G_\sigma(t-w)$$

The first derivative vanishes and the second derivative is negative for at $t = 0$, but the first derivative does not vanish for $t \neq 0$, which completes the proof. ✓

Proposition 2: For a Gaussian-shaped input I with $\sigma > 0$ and a circular weighting kernel K with $h > 0$ and $w > 0$

$$I(x, y) = I(x) = G_\sigma(x) \qquad K(x, y) = \begin{cases} h & x^2 + y^2 \le w^2 \\ 0 & \text{otherwise} \end{cases}$$

the positional competition transformation $\mathcal{T}_K$ broadens I, i.e. $\{\mathcal{T}_K(I)\}(x, y) = \{\mathcal{T}_K(I)\}(x)$ is *broader* than $I(x, y) = I(x)$.

$$\{\mathcal{T}_K(f)\}(x, y) = \frac{f(x, y)}{1 + \{f \star K\}(x, y)}$$

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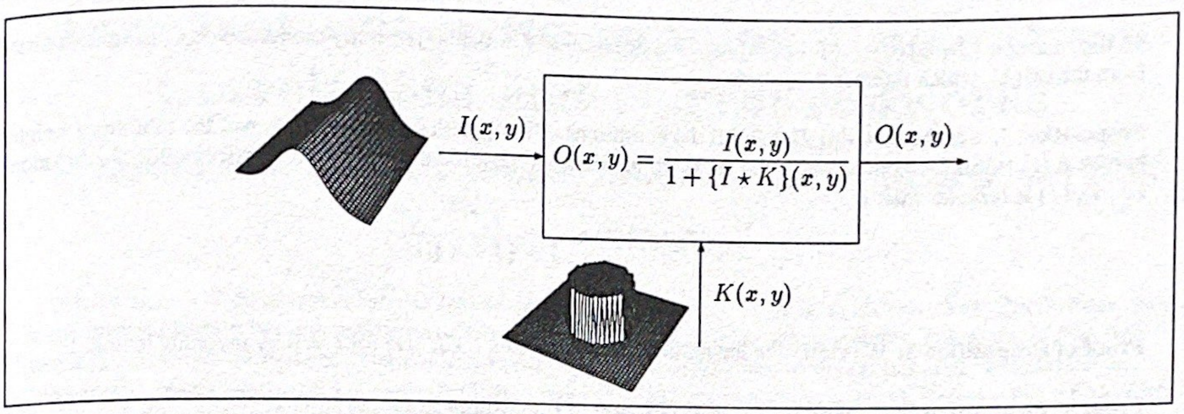

We postpone the proof of proposition 2 and motivate a useful lemma by simplifying the convolution integral

$$\{I\star K\}(x,y) = \int_{-w}^{w} I(x-u)\cdot\left[\int_{-\sqrt{w^2-u^2}}^{\sqrt{w^2-u^2}} h\,dv\right]du = \int_{-w}^{w} I(x-u)\cdot\left[2\cdot h\cdot\sqrt{w^2-u^2}\right]du$$

The last integral indicates that the two dimensional transformation with the proposed circular weighting kernel may be seen as the one-dimansional transformation with a monotone decreasing kernel given by

$$K(t) = \begin{cases} 2\cdot h\cdot\sqrt{w^2-t^2} & |t|\leq w \\ 0 & \text{otherwise} \end{cases}$$

The extension to the 2D case motivates an extension of the 1D case studied in proposition 1. We may ask whether more general inhibitory weighting kernels also yield broadening of the input by the corresponding positional competition.. We prepare these generalization by the following lemma.

Lemma 1: Let f and g be positive, even functions which are montone decreasing with distance from the origin. we require f to be strictly decreasing such that

$$\begin{array}{rclcrcll} f & \in & C^\infty & \qquad & g & \in & W^{1,1} & \\ f(t) & > & 0 & & g(t) & \geq & 0 & \\ f(t) & = & f(-t) & & g(t) & = & g(-t) & \\ f(t_1) & > & f(t_2) & & g(t_1) & \geq & g(t_2) & 0\leq t_1 < t_2 \\ & & & & \int_0^\infty \Phi(t)\cdot g'(t)\,dt & < & 0 & \forall\Phi\in C^\infty, \Phi(t)>0 \end{array}$$

Then their convolution product

$$\{f\star g\}(t) = \int_{-\infty}^{\infty} f(u)\cdot g(t-u)du$$

takes its maximum in $t=0$. •

Proof of Lemma 1: We show that $\{f\star g\}(0)-\{f\star g\}(t)$ takes a global minimum in $t=0$. So

$$\{f\star g\}(0)-\{f\star g\}(t) = \int_{-\infty}^{\infty} f(u)\cdot(g(u)-g(t-u))\,du$$

Now

$$\frac{\partial}{\partial t}\left(\{f\star g\}(0)-\{f\star g\}(t)\right) = \int_0^\infty (f(t-u)-f(t+u))\cdot g'(u)\,du$$

The sign of this derivative directly depends on the sign of $f(t-u)-f(t+u)$. For $t>0$ we have $f(t-u)-f(t+u)>0$ since f is even and strict decreasing with distance from the origin. If $t=0$ the difference is 0. For negative t we conclude that the difference is negative because of the identity $f(t-u)-f(t+u) = -(f(|t|-u)-f(|t|+u))$.

So the function $\{f \star g\}(0) - \{f \star g\}(t)$ has an extremum in $t = 0$, and is strictly monotone decreasing with distance from the origin, which finishes the proof. ✓

Proposition 3: Let the stimulus $I(t)$ fulfill the conditions of the function f in lemma 1, and the inhibitory weighting kernel $K(t)$ fulfill the conditions of the function g. Then the positional competition transformation $\mathcal{T}_K$ broadens I, i.e. $\mathcal{T}_K(I)$ is *broader* than I.

$$\{\mathcal{T}_K(f)\}(t) = \frac{f(t)}{1 + \{f \star K\}(t)}$$

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Proof of Proposition 3: We verify the inequality $\{I \star K\}(0) > \{I \star K\}(t)$ for $t \neq 0$ by invoking lemma 1. ✓

Proof of Proposition 2: Proposition 2 is a special case of Proposition 3. ✓

Proposition 4: Let the stimulus $I(x, y) = I(x)$ fulfill the conditions of the function f in lemma 1, and the inhibitory weighting kernel $K(x, y) = K(\sqrt{x^2 + y^2})$ fulfill the conditions of the function g. Then the positional competition transformation $\mathcal{T}_K$ broadens I, i.e. $\{\mathcal{T}_K(I)\}(x, y) = \{\mathcal{T}_K(I)\}(x)$ is *broader* than $I(x, y) = I(x)$.

$$\{\mathcal{T}_K(f)\}(x, y) = \frac{f(x, y)}{1 + \{f \star K\}(x, y)}$$

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Proof of Proposition 4: Transforming th convolution integral

$$\{I \star K\}(x, y) = \int_{\infty}^{\infty} I(x - u) \cdot \left[\int_{\infty}^{\infty} K(u, v)\right] du$$

we see that the bracketed integral is a function of u which fulfills the properties of the one-dimensional kernel $K(t)$ in proposition 3 and may use the result reported there. ✓

4 Conclusion

We have analyzed the positional sharpening capabilities of the Boundary Contour System as a subsystem of the FACADE architecture. Positional sharpening as set up by Grossberg and Mingolla may be seen as a two-stage process, where a threshold-linear signal functions transforms the input and drives the subsequent positional competition stage. We come to the conclusion that the sharpening effect of the positional sharpening process is due to the transformation by the threshold-linear signal function. A similar analysis has been given in [ElliasGrossberg75] several for other types of shunting equations.

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