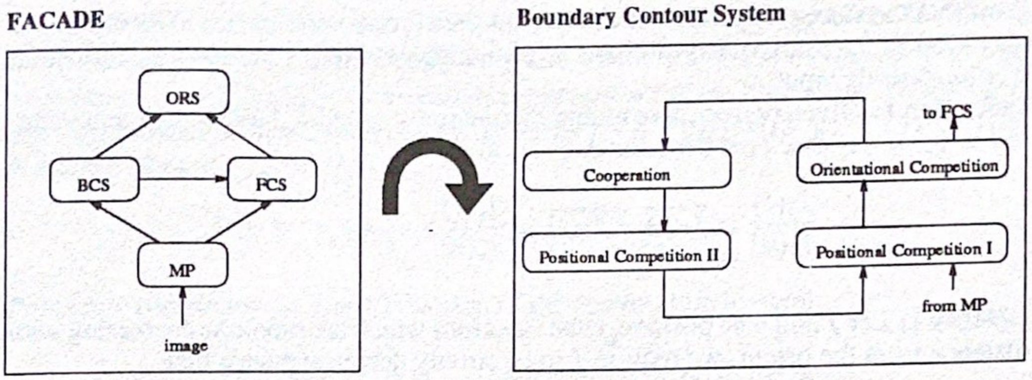
Positional Competition in the BCS*

Lothar Wieske
Department of Computer Science, University of Hamburg
Hamburg, Germany

1 Introduction and Motivation

A large part of human and mammal brains is dedicated to visual perception and the combination of locally ambiguous visual information into a globally consistent and unambiguous representation of the visual environment. So, one of the key ideas for the development of the FACADE architecture by Stephen Grossberg and coworkers is the conviction that modular approaches with special purpose procedures for texture, stereo, and motion are not very useful for an understanding of real-world vision.
The FACADE architecture aims at an explanation of how the visual system is able to detect relatively invariant surface colours under variable illumination conditions, to detect relatively invariant object boundaries under occlusion conditions, and to recognize familar objects or events in the environment. The Feature Contour System (FCS), Boundary Contour System (BCS), and the Object Recognition System (ORS) are designed in correspondence to the above processing goals of the overall architecture ([Grossberg90]). Processing of the visual input in retina and LGN relies on local measurements which introduce some amount of uncertainty into the representation of the visual environment. The FACADE architecture sets up parallel and hierarchical interaction schemes that can resolve these uncertainties.
The MP (monocular preprocessing) stage defines oriented receptive fields for each perceptual location which are sensitive to local contrast in the intensity function in accordance to the hypercolumn model of the primary visual cortex by Hubel and Wiesel. The local contrast detectors feed into the competitive-cooperative loop of the Boundary Contour System which generates an emergent segmentation of boundaries in the scene by means of spatially short-range competitive interactions and spatially long-range cooperative interactions. Preattentive processing in the Boundary Contour System is purely data-driven and does not rely on memorized templates or expectancies. Emergent segmentation provides a way to generate boundaries (*illusory contours*) which have no direct physical correlate in the intensity function but are perceived by human subjects in psychophysical experiments.
In the first competitive layer a cell of prescribed orientation excites like oriented cells at the same location and inhibits like-oriented cells at nearby positions. This on-center off-surround organization of like-oriented cells exists around every perceptual location. In the second competitive layer cells compete that represent different orientations at the same perceptual location. The competitive layers feed into the cooperative stage which defines spatially long-range interactions for boundary completion. The output of the cooperative stage is enhanced by a further competitive layer and fed back into the first competitive layer. This feedback allows for the discontinuous completion of continuous boundaries. (For details see [GrossbergMingolla85],[GrossbergMingolla87].)

*This work was funded by the German Federal Ministry of Science and Technology (NAMOS/413-4001-01 IN 101 C 1)



2 Positional Sharpening and Positional Competition

In [GrossbergMingolla85] and [GrossbergMingolla87] the authors define the following process for the second positional competition layer to realize the postulate of positional sharpening

$$v_{ijk} = \frac{h(z_{ijk})}{1 + \sum_{(p,q)} h(z_{pqk}) \cdot W_{pqij}}$$

with a circular inhibitory weighting kernel and a threshold-linear signal function.

$$W_{pqij} = \begin{cases} W & \text{if} \quad (p-i)^2 + (q-j)^2 \leq W_0^2 \\ 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad h(z) = L \cdot [z - M]^+$$

The variables z and v denote the output of the cooperative stage and the subsequent competitive stage. The pair $< i, j >$ indexes a perceptual location whereas k is an index to an orientation band. Since there are no interactions between different orientation bands, we will leave out the index k in our further considerations. The definition of the first positional competition layer is quite similar and uses the same competitive mechanism, but is not so well suited for an analysis of positional sharpening since it superposes two inputs.

3 Analysis of Positional Competition

The above transformation may be rewritten as the concatenation of two transformations where a point operation (thresholding) is followed by a positional competition transformation, i.e. $\mathcal{T}(I) = \mathcal{T}_2(\mathcal{T}_1(I))$.

$$\{\mathcal{T}_1(I)\}(x,y) = h(I(x,y)) \qquad \text{and} \qquad \{\mathcal{T}_2(I)\}(x,y) = \frac{I(x,y)}{1 + \{I \star W\}(x,y)}$$

We want to relate two one-dimensional stimuli I_1 and I_2 where I_2 is a transformed version of stimulus I_1. We assume that I_1 and I_2 are positive unimodal functions having their maximum in $t = 0$. Defining the meaning of I_2 is *sharper* than I_1 is a debatable issue. We start with the definition of the opposite. The intuition of this definition is that the function I_2 is something like a cheese cover of the same height for the function I_1, if I_2 is *broader* than I_1. (See the illustration at the end.) We will show that the sharpening-effect of the overall transformation $\mathcal{T}$ is due to the threshold operation $\mathcal{T}_1$ for a large class of inputs and inhibitory kernels, since the transformation $\mathcal{T}_2$ *broadens* its input.

Definition 1: Given two positive unimodal functions I_1 und I_2 having their maximum in $t = 0$, we say that I_2 is **broader** than I_1, if

$$\frac{I_2(t)}{I_2(0)} = \overline{I_2(t)} > \overline{I_1(t)} = \frac{I_1(t)}{I_1(0)} \qquad \text{for} \qquad t \neq 0$$

•

Lemma 1: Let f and g be positive, even functions which are montone decreasing with distance from the origin. we require f to be strictly decreasing such that

$$\begin{array}{rcl} f & \in & C^\infty \\ f(t) & > & 0 \\ f(t) & = & f(-t) \\ f(t_1) & > & f(t_2) \end{array} \qquad \begin{array}{rcll} g & \in & W^{1,1} & \\ g(t) & \geq & 0 & \\ g(t) & = & g(-t) & \\ g(t_1) & \geq & g(t_2) & 0 \leq t_1 < t_2 \\ \int_0^\infty \Phi(t) \cdot g'(t)\, dt & < & 0 & \forall \Phi \in C^\infty, \Phi(t) > 0 \end{array}$$

Then their convolution product

$$\{f \star g\}(t) = \int_{-\infty}^{\infty} f(u) \cdot g(t-u) du$$

takes its maximum in $t = 0$.

•

Proof of Lemma 1: We show that $\{f \star g\}(0) - \{f \star g\}(t)$ takes a global minimum in $t = 0$. Now

$$\frac{\partial}{\partial t} (\{f \star g\}(0) - \{f \star g\}(t)) = \int_0^\infty (f(t-u) - f(t+u)) \cdot g'(u)\, du$$

The sign of this derivative directly depends on the sign of $f(t-u) - f(t+u)$. For $t > 0$ we have $f(t-u) - f(t+u) > 0$ since f is even and strict decreasing with distance from the origin. If $t = 0$ the difference is 0. For negative t we conclude that the difference is negative because of the identity $f(t-u) - f(t+u) = -(f(|t| - u) - f(|t| + u))$. So the function $\{f \star g\}(0) - \{f \star g\}(t)$ has an extremum in $t = 0$, and is strictly monotone decreasing with distance from the origin, which finishes the proof. ✓

Proposition 1: Let the stimulus $I(t)$ fulfill the conditions of the function f in lemma 1, and the inhibitory weighting kernel $K(t)$ fulfill the conditions of the function g. Then the positional competition transformation $\mathcal{T}_K$ broadens I, i.e. $\mathcal{T}_K(I)$ is *broader* than I.

$$\{\mathcal{T}_K(f)\}(t) = \frac{f(t)}{1 + \{f \star K\}(t)}$$

Proof of Proposition 1: We have to show, that

$$\frac{\overline{\{\mathcal{T}_K(I)\}(t)}}{\overline{I(t)}} = \frac{1 + \{I \star K\}(0)}{1 + \{I \star K\}(t)} > 1 \quad \text{if } |t| > 0.$$

and we see that $\{I \star K\}(0) > \{I \star K\}(t)$ has to be shown for $|t| > 0$. We verify the inequality $\{I \star K\}(0) > \{I \star K\}(t)$ for $t \neq 0$ by invoking lemma 1. ✓

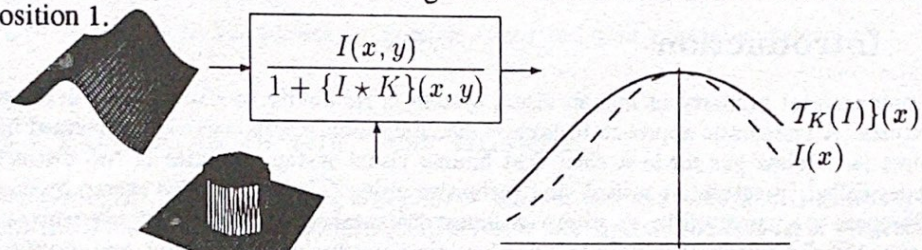
Proposition 2: Let the stimulus $I(x, y) = I(x)$ fulfill the conditions of the function f in lemma 1, and the circular inhibitory weighting kernel ($K(x, y) = K(\sqrt{x^2 + y^2})$) be such that $g(u) = \int_{-\infty}^{\infty} K(u, v)dv$ fullfills the properties of the function g. Then the positional competition transformation $\mathcal{T}_K$ broadens I, i.e. $\{\mathcal{T}_K(I)\}(x, y) = \{\mathcal{T}_K(I)\}(x)$ is *broader* than $I(x, y) = I(x)$.

$$\{\mathcal{T}_K(f)\}(x, y) = \frac{f(x, y)}{1 + \{f \star K\}(x, y)}$$

Proof of Proposition 2: Transformation of the convolution integral

$$\{I \star K\}(x, y) = \int_{\infty}^{\infty} I(x - u) \cdot \left[\int_{\infty}^{\infty} K(u, v)\right] du$$

shows that we may see the bracketed integral as an one-dimensional kernel and invoke proposition 1. ✓



4 Conclusion

We have analyzed the positional sharpening capabilities of the Boundary Contour System. Positional sharpening as set up by Grossberg and Mingolla may be seen as a two-stage process, where a threshold-linear signal functions transforms the input and drives the subsequent positional competition stage. We come to the conclusion that the sharpening effect of the positional sharpening process is due to the transformation by the threshold-linear signal function. A similar analysis has been given in [EliasGrossberg75] for other types of feedforward shunting equations.

Acknowledgement. I would like to thank O. Ludwig and R. Sprengel for fruitful and stimulating discussions.

References

[EliasGrossberg75] S.A. Ellias and S. Grossberg. Pattern formation, contrast control, and oscillations in the short term memory of shunting on-center off-surround networks. *Biological Cybernetics*, 20:69–98, 1975.

[Grossberg90] S. Grossberg. Neural facades: Visual representations of static and moving form-and-color-and-depth. *Mind & Language*, 5:411–456, 1990.

[GrossbergMingolla85] S. Grossberg and E. Mingolla. Neural dynamics of perceptual grouping: Textures, boundaries, and emergent segmentation. *Perception and Psychophysics*, 38:141–171, 1985.

[GrossbergMingolla87] S. Grossberg and E. Mingolla. Neural dynamics of surface perception: Boundary webs, illuminants, and shape-from-shading. *Computer Vision, Graphics, and Image Processing*, 37:116–165, 1987.