BS 730 HW2

1.) $p(\mu | y) \propto p(\mu)\, p(y|\mu)$

Know: $\mu \sim N(m_0, S_{\mu_0}^2)$

$\theta_i = \mu + r_i$ $\quad$ r_i = known/fixed reporting error

$y_i | \theta_i, \sigma^2 \sim N(\theta_i, \sigma^2)$ independent for $i = 1, 2, \dots, n$

$$p(\mu) = \exp\left(\frac{-1}{2S_{\mu_0}^2}(\mu - m_0)^2\right)$$

$$p(y_i | \theta_i, \sigma^2) = \exp\left(\frac{-1}{2\sigma_2} \sum_{i=1}^{n} (y_i - (\mu + r_i))^2\right)$$

$$\therefore\ p(\mu | \vec{y}) = \exp\left(-\frac{1}{2S_{\mu_0}^2}(\mu - m_0)^2\right) \cdot \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - (\mu + r_i))^2\right)$$

→ Because r_i is fixed, and doesn't depend on μ, probably can get it out → not completely b/c y_i depends on it s[o] it has factor posterior somehow

$$= \exp\left(-\frac{1}{2S_{\mu_0}^2}(\mu - m_0)^2\right) \cdot \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (y_i - r_i - \mu)^2\right)$$

$$\propto \exp\left(-\frac{1}{2S_{\mu_0}^2}(\mu - m_0)^2\right) \cdot \exp\left(-\frac{n}{2\sigma^2}\left(\overline{(y - r)} - \mu\right)^2\right)$$

Can you do this?

↳ yes, mean diff = diff of means

↳ $\overline{y - r} = \bar{y} - \bar{r}$

$$\propto \exp\left(-\frac{1}{2} f(\mu)\right)$$

where $f(\mu) = \frac{1}{S_{\mu_0}^2}(\mu^2 - 2m_0\mu) + \frac{n}{\sigma^2}(\mu^2 - 2(\bar{y} - \bar{r}))$

$$\propto \mu^2\left(\frac{1}{S_{\mu_0}^2} + \frac{n}{\sigma^2}\right) - 2\mu\left(\frac{m_0}{S_{\mu_0}^2} + \frac{n(\bar{y} - \bar{r})}{\sigma^2}\right)$$

want form: $f(\mu) \propto \frac{1}{\text{var}}(\mu^2 - 2(\text{mean})\mu)$

Bayes